Grade

10

Mathematics

Teachers' Guide

(Implemented from year 2015)

Department of Mathematics
Faculty of Science & Technology
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Message of the Director General

The first phase of the new competency based curriculum, with 8 years curriculum cycle was introduced to secondary education in Sri Lanka in 2007 replacing the existed content based education system with basic objective of developing the national level competencies recommended by the National Education Commission.

The second phase of the curriculum cycle to be introduced to grades 6 and 10 starts from 2015. For this purpose, National Institute of Education has introduced a rationalization process and developed rationalized syllabi for these grades using research based outcomes and various suggestions made by different stakeholders.

In the rationalization process, vertical integration has been used to systematically develop the competency levels in all subjects from fundamentals to advanced levels using the bottom up approach. Horizontal integration is used to minimize the overlapping in the subject content and to reduce the content over loading in the subjects to produce more students friendly and implementable curricular.

A new format has been introduced to the teachers' guide with the aim of providing the teachers with the required guidance in the areas of lesson planning, teaching, carrying out activities and measurement and evaluation. These guidelines will help the teachers to be more productive and effective in the classroom.

The new teachers' guides provide freedom to the teachers in selecting quality inputs and additional activities to develop the competencies of the students. The new teachers' guides are not loaded with subject content that is covered in the recommended textbooks. Therefore, it is essential for the teacher to use the new teachers' guides simultaneously with the relevant textbooks prepared by Education Publication Department as reference guides to be more aware of the syllabi.

The basic objectives of the rationalized syllabi and the new format of teachers' guide and newly developed textbooks are to bring a shift from the teacher centered education system into a student centered and more activity based education system in order to develop the competencies and skills of the school leavers and to enable the system to produce suitable human resource to the world of work.

I would like to take this opportunity to thank the members of Academic Affairs Board and Council of National Institute of Education and all the resource persons who have immensely contributed in developing these new teacher guides.

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National Institute of Education

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Instructions on the use of the Teacher's Manual

The Department of Mathematics of the National Institute of Education has been preparing for the new education reforms to be implemented in 2015 for the first time since 2007, in accordance with the education reforms policy which is implemented once every eight years. The Grade 10 Mathematics Teacher's Manual which has been prepared accordingly has many special features.

The **Grade 10 syllabus** is included in the first chapter. The syllabus has been organized under the titles Competencies, Competency Levels, Content, Learning Outcomes and Number of Periods. The proposed **lesson sequence** is given in the second chapter. The **Learning-Teaching-Evaluation methodology** has been introduced in the third chapter. A special feature of this is that the best method to develop each of the subject concepts in students has been identified from various methods such as the discovery method, the guided discovery method, the lecture-discussion method etc and the lesson plan has been developed based on it.

Following the proposed lesson sequence, the relevant competency and competency levels as well as the number of periods required for each lesson have been included at the beginning under each topic. Specimen lesson plans have been prepared with the aim of achieving one or two of the learning outcomes related to a selected competency level under each competency. These lesson plans have been carefully prepared to be implemented during a period or a maximum of two periods.

To create awareness amongst the students regarding the practical applications of the subject content that is learnt, a section titled '**Practical Use'** which contains various such applications has been introduced in some of the lessons.

You have been provided with the opportunity to prepare suitable lesson plans and appropriate assessment criteria for the competency levels and related learning outcomes for which specimen lesson plans have not been included in this manual. Guidance on this is provided under the title '**For your attention** ….'.

Another special feature of this Teacher's Manual is that under each lesson, websites which can be used by the teacher or the students, in the classroom or outside which contain resources that include videos and games to enhance students' knowledge is given under the title '**For further use'** and the symbol  **.** Although it is not essential to make use of these, the learning-teaching-evaluation process can be made more successful and students' subject knowledge can be enhanced by their use, if the facilities are available.

Further, in selected lessons, under the title "**For the teacher only**" and the symbol  , facts which are especially for the teacher are included. This information is only to enhance the teacher's knowledge and is not given to be discussed with the students directly.

The teacher has the freedom to make necessary amendments to the specimen lesson plan given in the new teacher's manual which includes many new features, depending on the classroom and the abilities of the students.

We would be grateful if you would send any amendments you make or any new lessons you prepare to the Director, Department of Mathematics, National Institute of Education. The mathematics department is prepared to incorporate any new suggestions that would advance mathematics educations in the secondary school system.

Project Leader

Syllabus

1.0 Syllabus

Competency	Competency Level	Content	Learning Outcomes	Periods

Competency – 1
Manipulates the mathematical operations in the set of real numbers to fulfill the needs of day to day life.
	1.1
	Finds the square roots of numbers using various methods.
		• Square root
		  • Approximation (First approximation only)
		  • Division Method
			• Recognizes that the square root of a number which is not a perfect square is a decimal number.
			• Finds an approximate value for the square root of a number that lies between two consecutive perfect squares.
			• Finds the square root of a number which is not a perfect square to the first approximation.
			• Finds the square root of a perfect square number using the division method.
			• Using the division method, finds the square root of a number which is not a perfect square, to the second decimal place.
			• Using the division method, finds the square root of a decimal number to the second decimal place.
				04

Competency – 2 Makes decisions for future requirements by investigating the various relationships between numbers.	**2.1** Identifies arithmetic progressions and solves related problems.	• Arithmetic Progressions • Introduction • n^{th} term	• Identifies a number sequence in which the difference between consecutive terms is a constant, as an **arithmetic progression.** • Identifies the technical terms relevant to arithmetic progressions. • Develops the formula $T_n = a + (n - 1)d$ for the n^{th} term of an arithmetic progression. • Finds the n^{th} term of an arithmetic progression using the formula $T_n = a + (n - 1)d$. • Finds the value of n using the formula when the n^{th} term (T_n) of an arithmetic progression is given. • Solves problems using the formula $T_n = a + (n - 1)d$.	**03**
	2.2 Investigates the various behavioral patterns of arithmetic progressions.	• Arithmetic Progressions • Sum of the first n terms	• Develops the formulae $S_n = \frac{n}{2}\{2a + (n - 1)d\}$ and $S_n = \frac{n}{2}\{a + l\}$ for the sum of the first n terms of an arithmetic progression. • Finds the sum of the first n terms of an arithmetic progression using the formulae.	**04**

			• Using the formulae, finds the number of terms in a progression, when the sum of the first n terms is given. • Solves problems related to arithmetic progressions with solving of simultaneous equations.	
Competency – 3 Manipulates units and parts of units under the mathematical operations to easily fulfill the requirements of day to day life.	**3.1** Solves problems involving fractions.	• Solving problems involving fractions	• Analyses instances where fractions are used in day to day life. • Solves problems related to day to day life using fractions with BODMAS rule.	**04**
Competency – 4 Uses ratios to facilitate day to day activities.	**4.1** Investigates the relationships between quantities using ratios.	• Introducing inverse proportions • Problems related to inverse proportions • Work and time • Representing inverse proportions algebraically • $x \propto \frac{1}{y} \rightarrow xy = k$, k is a constant • Solving problems using x $y = k$	• Analyses the relationship between two quantities and identifies inverse proportions. • Solves problems related to work and time using the knowledge on inverse proportions. • Identifies that when x and y are two inversely proportional quantities, the proportion is denoted by $x \propto \frac{1}{y}$. • Solves problems related to inverse proportions by using $xy = k$, where k is a constant.	**05**

Competency - 5 Uses percentages to make successful transactions in the modern world.	**5.1** Solves problems related to taxes using percentages.	• Types of taxes (Duty, income tax, rates, VAT) • Introduction • Installments	• Identifies the taxes - rates, duty, income tax and VAT. • Identifies how taxes are used to develop the country. • Engages in calculations related to rates. • Engages in calculations related to duty. • Engages in calculations related to income tax. • Engages in calculations related to VAT. • Solves problems related to taxes.	**04**
	5.2 Makes decisions by calculating the interest.	• Simple interest • Interest Rate • Annual/monthly • Calculating the interest	• Identifies **simple interest** as the interest calculated by taking into consideration the initial amount, the time and the interest rate. • Recognizes that the interest received during equal time periods and same interest rate is the same for a given amount of money. • Calculates the interest for a given amount, for a given period and given interest rate. • Solves problems involving finding the interest or the interest rate or the time or the amount, when the necessary	**03**

			information is given. • Makes effective decisions on transactions in day to day life by taking into consideration the interest.	
Competency – 6 Uses logarithms and calculators to easily solve problems in day to day life.	**6.1** Analyses the relationship between indices and logarithms.	• Relationship between indices and logarithms • Logarithm ⇔ Power conversion	• Describes the logarithm of a number in terms of the base, when the number is expressed in index form. • Converts an expression in index form to logarithm form and an expression in logarithm form to index form.	**02**

	6.2
	Uses the laws of logarithms for multiplication and division.
		• Laws of Logarithms
		  • Multiplication
		  • Division
			• Identifies the laws of logarithms for multiplication and division.
			• Simplifies expressions involving logarithms using the laws of logarithms.
				03

	6.3
	Simplifies numerical expressions using the logarithm tables.
		• Use of the logarithm tables
		  • Logarithms of numbers greater than 1
		  • Expressions involving numbers greater than 1
		    • Multiplication
		    • Division
			• Using the logarithm tables, finds the logarithm of numbers greater than 1.
			• Using the logarithm tables, multiplies and divides numbers greater than 1.
			• Using the logarithm tables, simplifies expressions involving the multiplication and division of numbers greater than 1.
				04

	6.4 Uses the calculator to solve mathematical problems.	• Use of the keys of the calculator • The keys $\boxed{+}$ $\boxed{-}$, $\boxed{\times}$ $\boxed{\div}$ $\boxed{=}$ • • The keys $\boxed{)}$ and $\boxed{(}$ Simplifying expressions with decimals $\boxed{(}$	• Identifies the keys $\boxed{+}$ $\boxed{-}$ $\boxed{\times}$, $\boxed{\div}$ and $\boxed{=}$. • Obtains the value of a numerical expression involving +, - , ×, ÷ and = using the calculator. • Identifies the keys $\boxed{(}$ and . $\boxed{)}$ • Simplifies numerical expressions by using the 'bracket' keys appropriately. • Simplifies expressions with decimals using the calculator. • Verifies using the calculator, the accuracy of the solutions of products and quotients of numbers obtained using the logarithm tables.	**01**
Competency – 7 Investigates the various methods of finding the perimeter to carry out daily tasks effectively.	**7.1** Extends measurements related to length to find the perimeter of plane figures containing sectors.	• Perimeter • Sectors • Compound figures containing sectors	• Develops the relationship $\frac{\theta}{360} \times 2\pi r$ for the length of an arc of a sector when the angle at the centre is θ and the radius of the circle is *r*. • Calculates the perimeter of sectors. • Solves problems related to the perimeter of compound figures which contain sectors of circles.	**04**

Competency – 8
Makes use of a limited space in an optimal manner by investigating the area.
	8.1
	Solves problems related to the area of plane figures containing sectors.
		• Area
		  • Sectors
		  • Compound plane figures involving sectors
			• Develops the relationship $A = \frac{\theta}{360} \times \pi r^2$ for the area A of a sector when the angle at the centre is θ and the radius of the circle is r.
			• Finds the area of a sector using the formula $A = \frac{\theta}{360} \times \pi r^2$.
			• Constructs an algebraic expression for the area of a sector when the measurements of the sector are given as algebraic terms.
			• Solves problems related to the area of compound plane figures which contain sectors of circles.
				04

	8.2 Investigates the surface area of cylinders.	• Surface Area • Cylinder	• Develops the formula $A = 2\pi r^2 + 2\pi rh$ for the surface area A of a closed right circular cylinder of radius r and height h. • Calculates the surface area of a closed right circular cylinder using the formula $A = 2\pi r^2 + 2\pi rh$. • Solves problems related to the surface area of a right circular cylinder.	**02**
	8.3 Investigates the surface area of prisms.	• Surface Area • Right prism with a triangular cross-section	• Identifies the shapes of the faces of a right prism with a triangular cross-section.	**03**

			• Calculates the surface area of a right prism with a triangular cross-section. • Solves problems related to the surface area of a right prism with a triangular cross-section.	

Competency – 10
Gets the maximum out of space by working critically with respect to volume.
	10.1
	Has an awareness about the volumes of cylinders.
		• Cylinder
		  • Formula for the volume
		  • Application of the formula
			• Develops the formula $V = \pi r^2 h$ for the volume V of a right circular cylinder of radius r and height h.
			• Finds the volume of a right circular cylinder using the formula $V = \pi r^2 h$.
			• Solves problems related to the volumes of cylinders.
				02

	10.2
	Has an awareness about the volumes of prisms.
		• A right prism with a triangular cross-section
		  • Formula for the volume
		  • Application of the formula
			• Develops the formula $V = Ah$ for the volume V of a right prism with a triangular cross-section of area A and height/length h.
			• Calculates the volume of a right prism with a triangular cross-section.
			• Solves problems related to the volume of a right prism with a triangular cross section.
				02

Competency – 12 Manages time to fulfill the needs of the world of work.	**12.1** Manages time to perform daily duties efficiently.	• Distance and time • Representation by a distance time graph (when the speed is	• Recognizes that the rate of change of distance with respect to time is **speed**.	**05**

		uniform) • Gradient of the graph = $\frac{distance}{time}$ = speed • Volume and time (Including liquids flowing through pipes)	• Writes down the relationship between distance, time and speed. • Represents the information on distance and time by a graph. • Recognizes that the **speed** is given by the **gradient** of a distance-time graph. • Solves problems related to distance, time and speed. • Solves problems related to volume and time. (Including liquids flowing through pipes) • Performs daily tasks efficiently using speed and rate.	
Competency – 13 Uses scale diagrams in practical situations by exploring various methods.	**13.1** Investigates the various locations in the environment using scale diagrams.	• Drawing scale diagrams of a vertical plane • Location of an object in terms of the angle of elevation and the angle of depression • Drawing scale diagrams using the location • Describing the location using a scale diagram • Drawing 2-dimensional vertical scale diagrams	• Identifies the angle of depression. • Identifies the angle of elevation. • Describes the location of an object in terms of the angle of depression and the angle of elevation. • Draws scale diagrams to represent the information on measurements in a vertical plane. • Describes locations in the environment using scale diagrams.	**05**

			• Solves problems related to scale diagrams of vertical planes.	
Competency – 14 Simplifies algebraic expressions by systematically exploring various methods.	**14.1** Squares a binomial expression.	• Expansion of binomial expressions • Of the form $(a\,x + by)\,(cx + dy)$; $a, b, c, d \in \mathbb{Q}$ • Expansion of $(a\,x + by)^2$; $a, b \in \mathbb{Z}$	• Simplifies the product of two binomial expressions of the form $(a\,x + by)\,(cx + dy)$. • Verifies the product of two binomial expressions by means of the areas of squares/rectangles. • Expands $(ax + by)^2$ by considering the product of two binomial expressions. • Expands $(ax + by)^2$ by considering the relationship between the terms of the expansion of $(a + b)^2$. • Verifies the product and square of two binomial expressions by substituting values.	**04**
. **Competency – 15** Factorizes algebraic expressions by systematically exploring various methods.	**15.1** Factorizes a trinomial quadratic expression.	• Finding factors • Difference of two squares • Of the form $ax^2 + bx + c$; $a \neq 0$ $b^2 - 4ac$ is a perfect square	• Finds the factors of the difference of two squares which include algebraic expressions. • Finds the factors of expressions of the form $ax^2 + bx + c$. • Verifies the accuracy of the factors of an expression of the form $ax^2 + bx + c$ using various methods.	**04**

Competency - 16 Explores the various methods of simplifying algebraic fractions to solve problems encountered in day to day life.	**16.1** Finds the least common multiple of several algebraic expressions.	• Least common multiple of algebraic expressions (No more than 3 expressions, no more than 2 variables, index not more than 2)	• Recognizes that the smallest algebraic expression that can be divided by several algebraic expressions is the **least common multiple** of these expressions. • Finds the least common multiple of several given algebraic terms. • Using factors, finds the least common multiple of several algebraic expressions. • Logically determines the least common multiple of several algebraic expressions.	**04**
	16.2 Manipulates algebraic fractions under addition and multiplication.	• Algebraic fractions (With unequal denominators) • Addition • Subtraction	• Explains the necessity of equivalent fractions for the addition and subtraction of algebraic fractions. • Adds and simplifies algebraic fractions with unequal denominators. • Subtracts and simplifies algebraic fractions with unequal denominators. • Simplifies algebraic fractions with unequal denominators.	**04**

Competency – 17 Manipulates the methods of solving equations to fulfill the needs of day to day life.	**17.1** Uses linear equations to solve problems.	• Linear equations with algebraic fractions • Solving • Constructing	• Recognizes that the methods of simplifying algebraic fractions can be used to solve simple equations involving algebraic fractions. • Solves simple equations involving algebraic fractions. • Expresses the relationship between the data of a given problem by a simple equation involving algebraic fractions and solves it.	**03**
	17.2 Uses simultaneous equations to solve problems.	• Simultaneous equations (With 2 variables of unequal integral coefficients) • Constructing • Solving	• Solves pairs of simultaneous equations with distinct coefficients. • Expresses the relationships in the given information by a pair of simultaneous equations and solves it. • Verifies with reasons the accuracy of the solution of a pair of simultaneous equations by substituting the obtained values back into the equations. • Solves problems using simultaneous equations.	**03**

	17.3 Uses quadratic equations to solve problems.	• Solving quadratic equations • Using factors	• Factorizes the algebraic expression of an algebraic equation. • Recognizes that for a product of two expressions to be zero, at least one of the expressions should be zero. • Solves quadratic equations using factors. • Solves problems involving quadratic equations.	**02**

Competency – 18
Analyzes the relationships between various quantities related to real-life problems.
	18.1
	Solves problems in daily life related to the inequality of two quantities.
		• Solving inequalities and representing solutions on a number line
		  • Of the form
		    $ax + b \gtreqless c$
		    ($a, b, c \in \mathbb{Z}, a \neq 0$)
		  • Integral solutions
		  • Intervals of solutions
		• Representing inequalities on a coordinate plane
		  • Of the form $x \gtreqless a$
		  • Of the form $y \gtreqless b$
		  • Of the form $x \gtreqless y$
			• Writes down the integral solution set of the inequalities $ax + b < c$; $ax + b > c$; $ax + b \leq c$; $ax + b \geq c$.
			• Represents the solutions of the inequalities $ax + b < c$; $ax + b > c$; $ax + b \leq c$; $ax + b \geq c$ on a number line.
			• Represents inequalities of the form $x < a$, $x > a$, $x \leq a$, $x \geq a$ on a coordinate plane.
			• Represents inequalities of the form $y < b$, $y > b$, $y \leq b$, $y \geq b$ on a coordinate plane.
			• Represents inequalities of the form $y < x$, $y > x$, $y \leq x$, $y \geq x$ on a coordinate plane.
			• Recognizes that inequalities
				06

			can be used to represent information in day to day life. • Uses inequalities to solve problems in day to day life.	

Competency – 19
Explores the methods by which formulae can be applied to solve problems encountered in day to day life.
	19.1
	Investigates the methods by which formulae can be used to solve problems.
		• Formulae
		  • Changing the subject of formulae (With squares and square roots)
		  • Substitution
			• Changes the subject of a formula which contains squares and square roots.
			• Finds the value of a given term by substituting the given values into a formula which contains squares and square roots.
			• Uses formulae to solve problems.
				03

Competency – 20 Easily communicates the mutual relationships that exist between two variables by exploring various methods.	**20.1** Determines the nature of the linear relationship between two variables.	• Calculating the gradient of a straight line of the form $y = mx + c$ (Using coordinates)	• Calculates the gradient of an equation of the form $y = mx + c$ when the coordinates of two points on the line are given. • Calculates the gradient of an equation of the form $y = mx + c$ when the graph of the equation is given. • Determines the relationship between the two variables using the gradient of the straight line.	**02**

	20.2 Graphically analyzes the mutual quadratic relationships between two variables.	• Quadratic functions of the form $y = ax^2$ and $y = ax^2 + b$ ($a, b \in \mathbb{Q}$ and $a \neq 0$) • Drawing the graph • Maximum/minimum value • Coordinates of the turning point • Equation of the axis of symmetry • Behaviour of the graph	• Calculates the y values corresponding to several given x values of a function of the form $y = ax^2$ or $y = ax^2 + b$. • Draws the graph of a function of the form $y = ax^2$ or $y = ax^2 + b$ for a given domain. • Finds the maximum/minimum value of the function, equation of the axis of symmetry and the coordinates of the turning point of the graph of the function of the form $y = ax^2$ or $y = ax^2 + b$, using its graph. • Using the graph, finds the interval of values of x corresponding to a given interval of values of the function, for a function of the form $y = ax^2$ or $y = ax^2 + b$. • Finds the roots of the equation $y = 0$ by considering the graph of the function y which is of the form $y = ax^2$ or $y = ax^2 + b$. • Determines a function similar to a function of the form $y = ax^2$ or $y = ax^2 + b$ by considering its graph.	**04**

	20.3 Analyzes the properties of a quadratic function by observing the function.	• Properties of quadratic functions of the form $y = ax^2$ and $y = ax^2 + b$ ($a, b \in \mathbb{Q}$ and $a \neq 0$) (Without drawing the graph) • Maximum/minimum value • Coordinates of the turning point • Equation of the axis of symmetry	• Finds the inter-relationships between a function of the form $y = ax^2$ or $y = ax^2 + b$ and its graph, to determine its maximum/minimum value, the coordinates of the turning point and the equation of the axis of symmetry, by observing the function. • Determines the maximum/minimum value, the coordinates of the turning point and the equation of the axis of symmetry of a function of the form $y = ax^2$ or $y = ax^2 + b$, by observing the function.	**03**
Competency – 23 Makes decisions regarding day to day activities based on geometrical concepts related to rectilinear plane figures.	**23.1** Formally investigates the sum of the three interior angles of a triangle.	• Proof of the theorem "The sum of the three interior angles of a triangle is 180^{o}" and related problems	• Performs calculations using the theorem, "The sum of the three interior angles of a triangle is 180^{o}". • Proves riders using the theorem, "The sum of the three interior angles of a triangle is 180^{o}". • Formally proves the theorem, "The sum of the three interior angles of a triangle is 180^{o}".	**02**

	23.2 Investigates the relationship between the exterior angle formed by producing a side of a triangle and the interior opposite angles.	• Proof of the theorem "If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles" and related problems	• Performs calculations using the theorem, "If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles". • Proves riders using the theorem, "If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles". • Formally proves the theorem, "If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles".	**03**
	23.3 Investigates the requirements for two triangles to be congruent.	• Identifying the conditions under which two triangles are congruent and applications • S.A.S. • A.A.S • S.S.S • Hyp. S	• Identifies two plane figures which coincide with each other as **congruent figures**. • Identifies the properties of congruent plane figures. • Identifies the conditions that are necessary and sufficient for two triangles to be congruent as S.A.S., A.A.S., S.S.S. and Hyp.S. • Proves riders using the congruency of triangles.	**05**

	23.4 Formally proves the relationship between the sides and the angles of an isosceles triangle.	• Isosceles Triangles • Application and proof of the theorem "If two sides of a triangle are equal, the angles opposite those sides are equal"	• Identifies the theorem, "If two sides of a triangle are equal, the angles opposite those sides are equal". • Verifies the theorem, "If two sides of a triangle are equal, the angles opposite those sides are equal". • Performs calculations using the theorem, "If two sides of a triangle are equal, the angles opposite those sides are equal". • Proves riders using the theorem, "If two sides of a triangle are equal, the angles opposite those sides are equal". • Formally proves the theorem, "If two sides of a triangle are equal, the angles opposite those sides are equal".	**03**
	23.5 Uses the converse of the theorem on the relationship between the sides and the angles of an isosceles triangle.	• Application of the converse of the theorem "If two sides of a triangle are equal, the angles opposite those sides are equal" (Proof is not expected)	• Performs calculations using the converse of the theorem, "If two sides of a triangle are equal, the angles opposite those sides are equal". • Proves riders using the converse of the theorem, "If two sides of a triangle are equal, the angles opposite those sides are equal".	**02**

	23.6 Formally proves the relationships between the sides and the relationships between the angles of parallelograms.	• Properties • Application and proof of the theorem "In a parallelogram, opposite sides are equal; opposite angles are equal; and each diagonal bisects the area of the parallelogram"	• Identifies the theorem, "In a parallelogram, opposite sides are equal; opposite angles are equal; and each diagonal bisects the area of the parallelogram". • Verifies by various methods the theorem "In a parallelogram, opposite sides are equal; opposite angles are equal; and each diagonal bisects the area of the parallelogram". • Performs simple calculations using the theorem, "In a parallelogram, opposite sides are equal; opposite angles are equal; and each diagonal bisects the area of the parallelogram". • Proves riders using the theorem, "In a parallelogram, opposite sides are equal; opposite angles are equal; and each diagonal bisects the area of the parallelogram". • Formally proves the theorem, "In a parallelogram, opposite sides are equal; opposite angles are equal; and each diagonal bisects the area of the parallelogram".	**04**

	23.7 Identifies and uses the relationship between the diagonals of a parallelogram.	• Application of the theorem "The diagonals of a parallelogram bisect each other" (Proof is not expected)	• Identifies the theorem, "The diagonals of a parallelogram bisect each other". • Verifies the theorem, "The diagonals of a parallelogram bisect each other". • Proves riders using the theorem, "The diagonals of a parallelogram bisect each other".	**03**
	23.8 Identifies and uses the necessary conditions on the sides of a quadrilateral for it to be a parallelogram.	• Necessity • Application of the theorem "If the opposite sides of a quadrilateral are equal, it is a parallelogram" (Proof is not expected)	• Identifies the theorem, "If the opposite sides of a quadrilateral are equal, it is a parallelogram". • Verifies the theorem, "If the opposite sides of a quadrilateral are equal, it is a parallelogram". • Proves riders using the theorem, "If the opposite sides of a quadrilateral are equal, it is a parallelogram".	**03**
	23.9 Identifies and uses the necessary conditions on the angles of a quadrilateral for it to	• Application of the theorem "If the opposite angles of a quadrilateral are equal, it is a parallelogram"	• Identifies the theorem, "If the opposite angles of a quadrilateral are equal, it is a parallelogram".	**03**

	be a parallelogram.	(Proof is not expected)	• Verifies the theorem, "If the opposite angles of a quadrilateral are equal, it is a parallelogram". • Proves riders using the theorem, "If the opposite angles of a quadrilateral are equal, it is a parallelogram".	
	23.10 Identifies and uses the fact that a quadrilateral with certain special features is a parallelogram.	• Application of the theorem "If the diagonals of a quadrilateral bisect each other then it is a parallelogram" (Proof is not expected) • Application of the theorem "If a pair of opposite sides of a quadrilateral are equal and parallel, the quadrilateral is a parallelogram" (Proof is not expected)	• Identifies the theorem, "If the diagonals of a quadrilateral bisect each other then it is a parallelogram". • Verifies the theorem, "If the diagonals of a quadrilateral bisect each other then it is a parallelogram". • Proves riders using the theorem, "If the diagonals of a quadrilateral bisect each other then it is a parallelogram". • Identifies the theorem, "If a pair of opposite sides of a quadrilateral are equal and parallel, the quadrilateral is a parallelogram". • Proves riders using the theorem, "If a pair of opposite sides of a quadrilateral are equal and parallel, the quadrilateral is a parallelogram".	**03**

Competency – 24 Thinks logically to make decisions based on geometrical concepts related to circles.	**24.1** Identifies and applies the theorem on the relationship between a chord and the centre of a circle.	• Chord • Application and proof of the theorem "The straight line joining the mid-point of a chord of a circle to the centre is perpendicular to the chord"	• Identifies the theorem, "The straight line joining the mid-point of a chord of a circle to the centre is perpendicular to the chord". • Verifies the theorem "The straight line joining the mid-point of a chord of a circle to the centre is perpendicular to the chord". • Performs calculations using the theorem, "The straight line joining the mid-point of a chord of a circle to the centre is perpendicular to the chord". • Proves riders using the theorem, "The straight line joining the mid-point of a chord of a circle to the centre is perpendicular to the chord". • Formally proves the theorem, "The straight line joining the mid-point of a chord of a circle to the centre is perpendicular to the chord".	**03**
	24.2 Applies the converse of the theorem on the relationship between a chord and the centre of a circle.	• Application of the theorem "The perpendicular drawn from the centre of a circle to a chord bisects the chord" (Proof is not expected)	• Identifies the theorem, "The perpendicular drawn from the centre of a circle to a chord bisects the chord".	**03**

			• Verifies the theorem "The perpendicular drawn from the centre of a circle to a chord bisects the chord". • Performs calculations using the theorem, "The perpendicular drawn from the centre of a circle to a chord bisects the chord". • Proves riders using the theorem, "The perpendicular drawn from the centre of a circle to a chord bisects the chord".	
	24.3 Formally proves and applies the relationships between the angles that are subtended by an arc of a circle.	• Angles • Application and proof of the theorem "The angle subtended at the centre of a circle by an arc is equal to twice the angle subtended on the circumference by the same arc"	• Identifies the theorem, "The angle subtended at the centre of a circle by an arc is equal to twice the angle subtended on the circumference by the same arc". • Verifies the theorem, "The angle subtended at the centre of a circle by an arc is equal to twice the angle subtended on the circumference by the same arc". • Performs calculations using the theorem, "The angle subtended at the centre of a circle by an arc is equal to twice the angle subtended on the circumference by the same	**04**

			arc”. • Proves riders using the theorem, “The angle subtended at the centre of a circle by an arc is equal to twice the angle subtended on the circumference by the same arc”. • Formally proves the theorem, “The angle subtended at the centre of a circle by an arc is equal to twice the angle subtended on the circumference by the same arc”.	
	24.4 Solves problems using the relationships between the angles in a circle.	• Application of the theorem “Angles in the same segment of a circle are equal” (Proof is not expected) • Application of the theorem “The angle in a semi-circle is a right-angle” (Proof is not expected)	• Identifies the theorem, “Angles in the same segment of a circle are equal”. • Verifies the theorem, “Angles in the same segment of a circle are equal”. • Performs calculations using the theorem, “Angles in the same segment of a circle are equal”. • Proves riders using the theorem, “Angles in the same segment of a circle are equal”. • Identifies the theorem, “The angle in a semi-circle is a right-angle”.	**02**

			• Verifies the theorem "The angle in a semi-circle is a right-angle". • Performs calculations using the theorem, "The angle in a semi-circle is a right-angle". • Proves riders using the theorem, "The angle in a semi-circle is a right-angle".	
Competency – 27 Analyzes according to geometric laws, the nature of the locations in the surroundings.	**27.1** Uses the knowledge on the basic loci to determine locations.	• Using a straight edge and a pair of compasses • Constructs the four basic loci	• Demonstrates using various methods, the locus of a point moving at a constant distance from a fixed point. • Using a straight edge and a compass, constructs the locus of a point moving at a constant distance from a fixed point. • Demonstrates using various methods, the locus of a point moving at an equal distance from two fixed points. • Using a straight edge and a compass, constructs the locus of a point moving at an equal distance from two fixed points. • Demonstrates using various methods, the locus of a point moving at a constant distance from a straight line.	**04**

			• Using a straight edge and a compass, constructs the locus of a point moving at a constant distance from a straight line. • Demonstrates using various methods, the locus of a point moving at an equal distance from two intersecting straight lines. • Using a straight edge and a compass, constructs the locus of a point moving at an equal distance from two intersecting straight lines. • Locates various geometric positions using the knowledge of the four basic loci.	

	27.2
	Constructs triangles using the given data.
		• Construction of triangles
		  • When the three sides are given
		  • When two sides and the included angle are given
		  • When two angles and a side are given
			• Constructs a triangle with a straight edge and a pair of compasses when the lengths of the three sides are given.
			• Constructs a triangle with a straight edge and a pair of compasses when the lengths of two sides and the magnitude of the included angle are given.
			• Constructs a triangle with a straight edge and a pair of compasses when the magnitudes of two angles and the length of a side are given.
				03

			• Constructs various other plane figures using the constructions of triangles.	
	27.3 Constructs rectilinear plane figures involving parallel lines by considering the relationships between the angles related to parallel lines.	• Constructs quadrilaterals connected with parallel lines.	• Constructs a parallelogram when the lengths of a pair of adjacent sides and the magnitude of the included angle are given. • Constructs a parallelogram when the lengths of a pair of adjacent sides and the perpendicular distance between a pair of parallel sides are given. • Constructs a trapezium when the measurements are given. • Constructs plane figures using the given measurements and thereby obtains the other measurements.	**03**

Competency – 28
Facilitates daily work by investigating the various methods of representing data.
	28.1
	Extends frequency tables to easily communicate data.
		• Types of date
		  • Continuous
		  • Discrete
		• Grouped frequency distribution
		    • Mid-values
			• Identifies continuous and discrete data.
			• Provides reasons as to why a given data is either discrete or continuous.
			• Identifies the mid-values of a frequency distribution.
			• Finds the mid-values of a frequency distribution.
				03

	28.2 Represents data graphically to facilitate comparison and to solves problems.	• Data representation • Pie charts	• Represents a given set of data by a pie chart. • Uses pie charts to communicate information efficiently and effectively. • Solve problems related to pie charts.	**03**

Competency – 29
Makes predictions after analyzing data by various methods to facilitate daily activities.
	29.1
	Uses representative values to interpret data.
		• Interpretation of data
		  • Mean of a grouped frequency distribution
		    • Using the mid-value
		    • Using the assumed mean
			• Calculates the mean of a grouped set of data using the mid-value.
			• Calculates the mean of a grouped set of data using the assumed mean.
			• Identifies the easiest method of finding the mean of a grouped set of data.
			• Expresses the advantages/disadvantages of calculating the mean as the central tendency measurement to interpret data.
			• Recognizes that the mean can be used to numerically estimate daily requirements.
			• Makes predictions for daily requirements by using the mean.
				07

Competency – 30
Manipulates the principles related to sets to facilitate daily activities.
	30.1
	Uses methods of denoting sets to facilitate problem solving.
		• Set notation
		  • Descriptive form
		  • As a collection of elements
		  • By Venn diagrams
		  • Generating form
			• Identifies methods of denoting sets.
			• Writes a set in descriptive form, as a collection of elements, in Venn diagram form and in generating form.
			• Solves problems by using the various methods of denoting sets.
				03

	30.2 Solves problems using sets.	• Solving problems related to sets (For 2 sets) • Application of the formula for two finite sets in Venn diagrams $n(A\cup B) = n(A) + n(B) - n(A\cap B)$	• Expresses $n(A\cup B)$ in terms of $n(A)$, $n(B)$ and $n(A\cap B)$, when A and B are two finite sets. • Represents two finite sets in a Venn diagram. • Illustrates in a Venn diagram, a region relevant to a given set operation. • Describes in words, regions in a Venn diagram that contain information relevant to set operations. • Solves problems related to two sets using Venn diagrams. • Solves problems related to two finite sets using the formula $n(A\cup B) = n(A) + n(B) - n(A\cap B)$.	**05**

Competency – 31 Analyzes the likelihood of an event occurring to predict future events.	**31.1** Analyses the mutual relationships between events.	• Events • Simple • Compound • Complement • Not mutually exclusive Application of the formula $P(A\cup B) = P(A) + P(B) - P(A\cap B)$ • Mutually exclusive Application of the formula $P(A\cup B) = P(A) + P(B)$	• Separates out and identifies simple events and compound events. • Expresses that if A is an event in a sample space S, the probability of A occurring is $P(A) = \frac{n(A)}{n(S)}$. • Identifies the complement of an event. • Expresses the probability of a compound event. • Expresses that if the complement of the event A is A', then $P(A') = 1 - P(A)$. • Explains mutually exclusive events using examples. • Explains events that are not mutually exclusive using examples. • Finds the probability of a compound event consisting of events which are not mutually exclusive by using the formula $P(A\cup B) = P(A) + P(B) - P(A\cap B)$. • Finds the probability of a compound event consisting of mutually exclusive events by using the formula $P(A\cup B) = P(A) + P(B)$.	**03**

	31.2 Illustrates the occurrences of a compound event pictorially.	• Sample space of a random experiment (With independent events) • Representation on a grid • Representation in a tree diagram • Solving problems involving independent events using a grid or a tree diagram (Not more than two stages)	• Provides examples of independent events. • Solves problems using $P(A \cap B) = P(A) \times P(B)$ for independent events. • Represents the sample space of a random experiment on a grid. • Represents all the equally likely outcomes of a process involving two stages in a tree diagram. • When solving problems related to probabilities, explains with reasons whether a grid or a tree diagram is the more suitable method of representing the sample space. • Solves problems using a grid or a tree diagram.	**05**
			Total	**190**

2.0 Lesson Sequence

Content	Number of Periods	Competency Levels
Term 1		
1. Perimeter	04	7.1
2. Square Root	04	1.1
3. Fractions	04	3.1
4. Binomial Expressions	04	14.1
5. Congruency	05	23.3
6. Area	04	8.1
7. Factors of Quadratic Expressions	04	15.1
8. Triangles	10	23.1, 23.2, 23.4, 23.5
9. Inverse Proportion	05	4.1
10. Data Representation	03	28.2
	47	
Term 2		
11. Least Common Multiple of Algebraic	04	16.1
Expressions	04	16.2
12. Algebraic Fractions	07	5.1, 5.2
13. Percentages	08	17.1, 17.2, 17.3
14. Equations	07	23.6, 23.7
15. Parallelograms I	09	23.8, 23.9, 23.10
16. Parallelograms II	08	30.1, 30.2
17. Sets	05	6.1, 6.2
18. Logarithms I	05	6.3, 6.4
19. Logarithms II	09	20.1, 20.2, 20.3
20. Graphs	05	12.1
21. Rate		
	71	
Term 3		
22. Formulae	03	19.1
23. Arithmetic Progressions	07	2.1, 2,2
24. Algebraic Inequalities	06	18.1
25. Frequency Distributions	10	28.1, 29.1
26. Chords of a Circle	06	24.1, 24.2
27. Constructions	10	27.1, 27.2, 27.3
28. Surface Area and Volume	09	8.2 8.3, 10.1, 10.2
29. Probability	08	31.1, 31.2
30. Angles in a Circle	08	24.3, 24.4
31. Scale Diagrams	05	13.1
	72	
TOTAL	190	

Instructions for the Learning-Teaching Evaluation Process

1. Perimeter

Competency 7: Investigates the various methods of finding the perimeter to carry out daily tasks effectively.

Competency Level 7.1: Extends measurements related to length to find the perimeter of plane figures containing sectors.

Number of periods: 04

Introduction:
The locus of a point on a plane, a constant distance from a point on the same plane is a circle. The length of the boundary of a circle is known by the special name circumference, and the circumference of a circle of radius r is given by $2\pi r$. Any part of a circle is known as an arc. A sector of a circle is formed by two radii of the circle and the arc between the two radii. The length of the respective arc is known as the **arc length** of the sector. The angle formed at the centre by the two radii is known as the angle at the centre. The length of an arc varies when the angle at the centre varies. The arc length of a sector of a circle of radius r and angle at the centre θ is $2\pi r \times \frac{\theta}{360}$.

The perimeter of the sector is obtained when the lengths of the two radii are added to the arc length. Thus the perimeter of the above sector is $2r + 2\pi r \times \frac{\theta}{360}$.

The perimeter of a closed figure including sectors of circles is obtained by adding together the relevant side lengths and arc lengths of the sides and arcs by which the figure is bounded. In this section, it is expected to discuss the subject content mentioned above which come under competency level 7.1.

Learning outcomes relevant to competency level 7.1:

1. **Develops the relationship $\frac{\theta}{360} \times 2\pi r$ for the arc length of a sector when the angle at the centre is θ and the radius of the circle is r.**
2. Calculates the perimeter of sectors.
3. Solves problems related to the perimeter of compound figures which contain sectors of circles.

Glossary of Terms:

Perimeter	- පරිමිතිය	- சுற்றளவு
Sector	- කේන්ද්‍රික ඛණ්ඩය	- ஆரைச்சிறை
Angle at the centre	- කේන්ද්‍ර කෝණය	- ஆரைச்சிறைக் கோணம்
Arc	- චාපය	- வில்
Arc length	- චාප දිග	- வில்லின் நீளம்

Instructions to plan the lesson:

It is expected to build up the concepts relevant to learning outcome 1 of competency level 7.1 in this lesson. An exemplar lesson plan to be used to assist students to develop a formula for the arc length of a sector using the guided discovery method is given below.

Time: 40 minutes

Quality Inputs:

- Set of figures given below - a set per group
  - Circle drawn on a Bristol board with the centre and one diameter marked.
  - Sectors of circles cut out from Bristol board of radius equal to the radius of the above circle and with angle at the centre 180^0, 90^0, 60^0 and 45^0 (The magnitude of the angle at the centre should be marked)
- Copies of the student's work sheet

Instructions for the teacher:

Approach:

- Exhibit on the chalk board a circle of radius *r* and centre O, and a sector of a circle of radius *r,* centre O and angle at the centre 50^0. Elicit from the students the special name with which we identify the length around the circle and how this length is obtained.
- Discuss with the students about the radius, arc, arc length and the angle at the centre by drawing their attention to the sector.
- Mention that the circumference of a circle with radius *r* can be obtained using the formula $2\pi r$, that the sector with angle at the centre 50^0 is a plane figure bounded by two radii and an arc, that the arc is a part of the circumference of the circle, and that the aim is to find the length of an arc through a group activity.

Development of the lesson:

- Group the students and give each group a copy of the student's worksheet and a set of figures which consists of a circle and sectors.
- Once the students have completed the group activity, give them the opportunity to present the formula they developed to find the arc length of a sector and how they developed it.
- Lead a discussion to clarify that the ratio between the angle at the centre of a sector, and the angle around the centre of the circle (which is 360^0) is equal to the ratio of the arc length and the whole circumference of the circle, that when the angle at the centre is θ degrees, then the arc length is $\frac{\theta}{360}$ of the circumference and that when the radius is *r* and the angle at the centre is θ degrees, the arc length is given by the formula $2\pi r \times \frac{\theta}{360}$.

Student's worksheet:

- Identify the centre of the circle your group received as well as the centre, radius, angle at the centre and the arc length of the sector with angle at the centre equal to 180^0 which you received.
- Colour the length of the arc for the purpose of identification.
- Place the sector on the circle such that the two centres coincide.
- Check what fraction of the angle around the centre the angle at the centre of the sector is.
- Considering the angle at the centre of the sector, express the fraction you have obtained as a fraction with denominator 360^0.
- Placing the sector on the circle once again, investigate what fraction of the circumference the length of the arc is.
- Copy the table given below in your exercise book and compare the information you have collected about the sector with angle at the centre 180^0 with the information that is recorded.

Angle at the centre of the sector	The angle at the centre as a fraction of the angle around the centre	Arc length as a fraction of the circumference	Arc length
180^0	$\frac{1}{2} = \frac{180}{360}$	$\frac{1}{2}$	$\frac{1}{2} \times$ circumference
90^0			
60^0			
45^0			
50^0			
θ^0			

- In the same manner, examine the sectors with angles at the centre 90^0, 60^0 and 45^0 and note down the relevant information in the table.
- For each sector, compare the fraction you obtained when you considered the angle at the centre of the sector as a fraction of 360^0 with the fraction you obtained when you considered the arc length as a fraction of the circumference.
- Accordingly, note down in the table the relevant information for a sector with angle at the centre 50^0.
- Repeat this for an angle at the centre equal to θ too.
- Develop a formula in terms of r, θ and π for the **arc length** of a sector of radius r and angle at the centre θ, by considering the circumference of the circle.
- Prepare to present to the class how you developed the formula for the arc length of a sector.

Assessment and Evaluation:

- Assessment criteria
  - Describes the arc length as a part of the circumference of the circle from which the sector was obtained.
  - Expresses that the arc length varies depending on the angle at the centre.
  - Satisfies himself that for different sectors with different angles at the centre, the ratio of the angle at the centre to the angle around the centre is equal to the ratio of the arc length to the circumference.
  - Presents his ideas using the experience he has gained.
  - Develops new relationships by engaging in activities.
- Direct the students to the relevant exercise in lesson 1of the text book.

Practical situations:

- Discuss with the students the following practical situation where finding the perimeters of sectors is used.
  - Vesak pandols include sectors of circles and knowledge on the lengths of arcs is necessary to assemble bulbs.
  - In building constructions where circular sections are used, arc lengths are required (e.g.: balconies)
  - In constructing circular patterns, arc lengths are taken into consideration.

For your attention…

Development of the lesson:

- Plan and implement a suitable activity which highlights the fact that the perimeter of a sector of a circle is found by adding the lengths of the two radii of the boundary of the sector to the arc length, which is the subject content relevant to learning outcome 2.
- Plan and implement suitable activities in relation to learning outcome 3, to create various compound plane figures which include squares, rectangles and sectors of circles and to find their perimeters by having an awareness of the sides of the plane figures included in the compound figure, which do not contribute to the perimeter.

Assessment and Evaluation:

- Direct the students to create various compound plane figures by using rectilinear plane figures and sectors of circles and then to find their perimeters. (Instruct the students to use numbers which can easily be divided by 7 as the radii, to facilitate calculations)
- Direct the students to the relevant exercise in lesson 1 of the textbook.

For further reference: • http://www.youtube.com/watch?v=tVcasOt55Lc

• http://www.youtube.com/watch?v=1BH2TNzAAik

2. Square root

Competency 1: Manipulates the mathematical operations in the set of real numbers to fulfill the needs of day to day life.

Competency Level 1.1: Finds the square roots of numbers using various methods.

Number of periods: 04

Introduction:
The square root of a perfect square can be found by expressing the perfect square as a product of its prime factors. The square root of a number which is not a perfect square lies between the square roots of the two perfect squares which are closest to it. By considering all the numbers up to one decimal place that lie between the square roots of these two perfect squares and examining their squares to find the value which is closest to the given number, an approximate value for the square root of the number can be found.
This approximate value is called the **first approximation**.
Finding the square root of a number by pairing digits from right to left and considering the square root of the perfect square number which is less than and closest to the number at the left most end (which is either a one digit or two digit number), is known as the **division method** of finding the square root.
Finding the square root of a number which is not a perfect square to one decimal place by the method of approximation and finding the square root of a number which is not a perfect square or is a decimal number using the division method is discussed in this section.

Learning outcomes relevant to competency level 1.1:

1. **Recognizes that the square root of a number which is not a perfect square is a decimal number.**
2. **Finds an approximate value for the square root of a number that lies between two consecutive perfect squares.**
3. **Finds the square root of a number which is not a perfect square to the first approximation.**
4. Using the division method, finds the square root of a number which is not a perfect square, to two decimal places.
5. Using the division method, finds the square root of a decimal number to two decimal places.

Glossary of Terms:

Square root	-	වර්ගමූලය	-	வர்க்கமூலம்
Approximation	-	සන්නිකර්ෂණය	-	அண்ணளவாக்கம்
Square numbers	-	වර්ග සංඛ්‍යා	-	வர்க்க எண்கள்
Decimal numbers	-	දශම සංඛ්‍යා	-	தசம எண்கள்
Whole numbers	-	පූර්ණ සංඛ්‍යා	-	முழு எண்கள்
First approximation	-	පළමුවන සන්නිකර්ෂණය	-	முதலாம் அண்ணளவாக்கம்
Perfect square	-	පූර්ණ වර්ගය	-	நிறை வர்க்கம்

Instructions to plan the lesson:

It is expected to inculcate the concepts relevant to learning outcomes 1, 2 and 3 under competency level 1.1. An exemplar lesson plan based on a group assignment to assist students to find the square root by considering the first approximation is given below.

Time: 40 minutes

Quality inputs:

- A calculator
- Demy paper, platignum pens
- Copies of the pupils work sheet
- Copies of the assignment sheet sufficient for all the students

Instructions for the teacher:

Approach:

- Lead a discussion and show that if a number can be expressed as a product of two identical factors, then one of those factors is the square root of the said number.
- Present the students with a number, the square root of which cannot be found by expressing it as a product of its prime factors and explain that to find an approximate value for the square root of the relevant number, the square roots of the perfect square number greater than and the perfect square number less than this number, which are closest to it should be obtained.
- Describe how the two perfect square numbers greater than and less than a given number and closest to it are found using several examples.

Development of the lesson:

- Group the students and give each group a copy of the student's work sheet.
- Distribute the quality inputs required for the activity.
- After the students have engaged in the group activity, allow them to present how the square root of a number is found. Lead a discussion clarifying that to find the

square root of a number which is not a perfect square, we have to first find the square roots of the perfect square greater than and the perfect square less than the relevant number which are closest to it. Then the numbers between the square roots of these two perfect squares have to be expressed to one decimal place. In the first approximation method, the suitable value is found by first considering which of the square roots of the two perfect squares is closest to the square root of the given number. The number written to one decimal place between the square roots of these two perfect squares which is closest to the required square root is taken as the first approximation of the relevant square root value.

Assignment

Let us find an approximate value for $\sqrt{5}$.

• Write the two square numbers on either side of 5 which are closest to it in the two cages.

$$\square < 5 < \square$$

• Fill in the blanks by considering the square roots of these two numbers.

$$\sqrt{\ldots} < \sqrt{5} < \sqrt{9}$$
$$2 < \sqrt{5} < \ldots$$

• Write down the numbers between these two square roots to one decimal place in ascending order.

2.1, 2.2, ... , ... , ..., …..

• To which of the two square roots do you think the value of $\sqrt{5}$ is closer?

....

• Fill in the blanks

$(2.1)^2$ = ...
$(2.2)^2$ = ...
………= ....

• According to your above answers, between which two squares does 5 lie?

.... , ...

• Accordingly between which two numbers does $\sqrt{5}$ lie?

.... , ...

• Which of these two numbers is closer to $\sqrt{5}$? ....

• In the same manner find an approximate value for $\sqrt{15}$.

Student's worksheet:

- Distribute the assignment sheets among the students in your group.
- Write down the answers step by step on the assignment sheet itself.
- You may use the calculator to perform simplifications.
- Discuss the answers you obtained within the group.
- Present how you obtained the answers to the class.

Assessment and Evaluation:

- Assessment criteria
  - Finds the two perfect squares which are on either side and closest to a number which is not a perfect square.
  - Writes down to one decimal place, the values between the two whole numbers which are the square roots of the closest perfect squares lying on either side of a given whole number.
  - Finds the first approximation of the square root of a number which is not a perfect square.
  - Uses correct methods of finding approximate values.
  - Works cooperatively within the group.
- Guide the students to the exercise in lesson 2 of the textbook.

Practical Situations:

- Explain that it is necessary to find the square roots of numbers when Pythagoras' relationship is applied.

For your attention…
Development of the lesson:

- Prepare and implement suitable activities relevant to learning outcomes 4 and 5, to find the square root of a number which is not a perfect square to two decimal places and the square root of a decimal number to two decimal places by using the division method.

Assessment and Evaluation:

- Direct the students to the relevant exercise in lesson 2 of the text book.

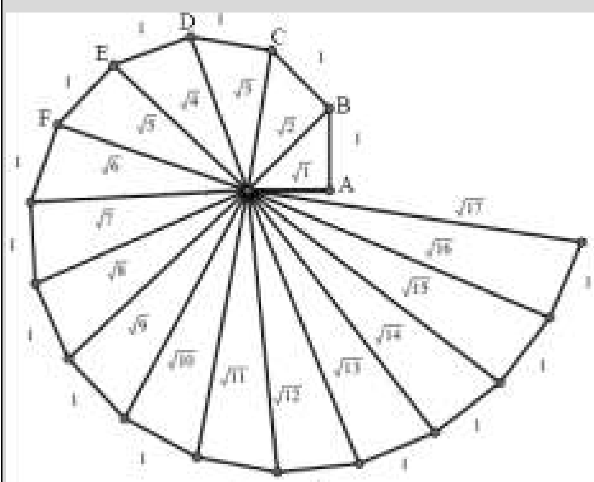
For further reference:

For the teacher only:

- Once 2.2 is obtained as the value for $\sqrt{5}$ according to the assignment,
  $(2.21)^2 = 4.8841$
  $(2.22)^2 = 4.9284$
  $(2.23)^2 = 4.9729$
  $(2.24)^2 = 5.0176$
  The best approximation is given by $2.23 < \sqrt{5} < 2.24$
- Accordingly, the second approximation is 2.24.
- By doing this repeatedly better approximations can be found for the square root of 5.
  $2 < \sqrt{5} < 3$
  $2.2 < \sqrt{5} < 2.3$
  $2.23 < \sqrt{5} < 2.24$

- Finding the square root using Newton's Method.
  This method was discovered by Sir Isaac Newton (1642 – 1727).
  Since $\sqrt{4} < \sqrt{5} < \sqrt{9}$, the value 2 is considered as an approximate value.
  Then $\frac{5}{2} = 2.5$ is obtained.
  Since $2 \times 2 = 4$ and $2.5 \times 2.5 = 6.25$, to find a better approximation,
  $\frac{2+2.5}{2} = \frac{4.5}{2} = 2.25$ is obtained.
  Next $\frac{5}{2.25} = 2.222\ldots$ is obtained.
  Now, $\frac{2.25+2.222}{2} = \frac{4.472\,22}{2} = 2.23611$ is obtained.
  $(2.23611)^2 = 5.00019$
  By repeating this process better approximations can be found.

Archimedes' Spiral



- The length of the hypotenuse of the right angled triangle with the sides including the right angle being of length 1 unit each is $\sqrt{2}$ units.
- The right triangle drawn with this hypotenuse as one side and the other side of length 1 unit has a hypotenuse of length $\sqrt{3}$.
- By similar constructions $\sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}$....etc., can be found. The square root of any positive integer can be found in this manner.

- The curve obtained by joining the points A,B,C,D,E,F, ... is known as Archimedes' spiral.

3. Fractions

Competency 3: Manipulates units and parts of units under the mathematical operations to easily fulfill the requirements of day to day life.

Competency level 3.1: Solves problems involving fractions.

Number of periods: 4

Introduction:
All the topics on fractions have been covered in grades 6 to 9.
That is, the concept of fractions, unit fractions, proper fractions, equivalent fractions, and under the comparison of fractions, fractions with equal denominators, unit fractions, fractions with equal numerators and fractions with related denominators, as well as the addition and subtraction of fractions with equal denominators and related denominators have been covered in grade 6.
Defining mixed numbers and improper fractions, converting between improper fractions and mixed numbers, comparison of unrelated fractions, as well as the addition and subtraction of unrelated fractions and mixed numbers were done in grade 7. The multiplication and division of fractions was covered in grade 8.
Simplification of fractions using the BODMAS rule was done in grade 9. Accordingly, under the grade 10 syllabus, the application of the subject content relevant to fractions learnt up to grade 9 is expected to be covered. Here, it is expected to develop in students the skills of solving problems related to fractions giving prominence to practical applications.

Learning outcomes relevant to competency level 3.1:
1. Analyses instances where fractions are used in day to day life.
2. Solves problems related to day to day life using fractions.

Instructions to plan the lesson:

With the expectation of guiding students towards achieving the learning outcomes 1 and 2, the 4 periods that are allocated should be utilized in a suitable manner, to provide students with the opportunity to develop the skills of solving problems involving fractions.

Instructions for the teacher:
- Guide the students to solve problems involving fractions by considering relevant practical situations.
- Explain how a problem involving fractions given in statement form is analyzed.

For further reference: • http://www.youtube.com/watch?v=MZpULgKhaEU

4. Binomial Expressions

Competency 14: Simplifies algebraic expressions by systematically exploring various methods.

Competency level 14.1: Squares a binomial expression.

Number of periods: 04

Introduction:
In this section, it is expected to study the product of two binomial expressions and how the square of a binomial expression is found.
The product of an expression of the form $(ax + by)(cx + dy)$ can be written as $acx^2 + bcxy + adxy + bdy^2$. By substituting $(-b)$ for b and $(-d)$ for d, the product of $(ax - by)(cx - dy)$ can be obtained.
When an expression such as $(ax + by)$ is squared, the result is the sum of the square of the first term, the square of the second term and twice the product of the two terms, which is $a^2x^2 + b^2y^2 + 2abxy$. When a, b, c and d are given, by substituting various numbers for x and y, the expressions obtained for the product of two binomial expressions and the square of a binomial expression can be verified.
Numbers of which the square cannot easily be found directly, can be obtained easily by writing them as the sum or the difference of two numbers.
Example: $107^2 = (100 + 7)^2$
$96^2 = (100 - 4)^2$

Learning outcomes relevant to competency level 14.1:

1. **Finds the product of two binomial expressions of the form $(ax + by)(cx + dy)$ and simplifies it.**
2. **Verifies the product of two binomial expressions by means of the areas of squares/rectangles.**
3. Expands $(a x + by)^2$.
4. Squares a given binomial expression by considering the relationship between the terms of the expansion of $(a + b)^2$.
5. Verifies the product of two binomial expressions and the square of a binomial expression by substituting values.

Glossary of Terms:

Algebraic expressions	-	වීජීය ප්‍රකාශන	-	அட்சரகணிதக் கோவை
Binomial expressions	-	ද්විපද ප්‍රකාශන	-	ஈருறுப்புக் கோவை
Square	-	වර්ගය	-	வர்க்கம்
Squaring	-	වර්ගායනය	-	வர்க்கித்தல்
Expansion	-	ප්‍රසාරණය	-	விரிவு
Squared	-	වර්ගායිතය	-	வர்க்கித்த
Area	-	වර්ගඵලය	-	பரப்பளவு

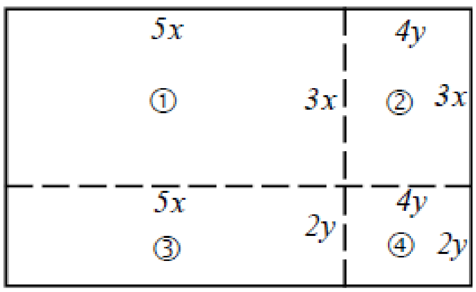
Instructions to plan the lesson:

An exemplar lesson plan together with an exploration activity to guide students towards achieving learning outcomes 1 and 2 relevant to competency level 14.1 is given below.

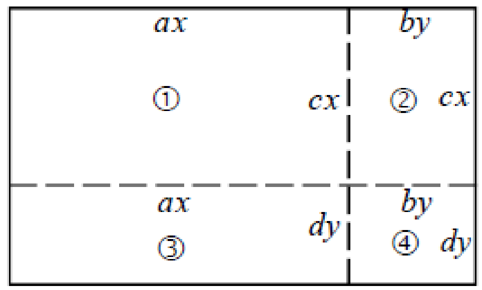
Time: 40 minutes

Quality inputs:

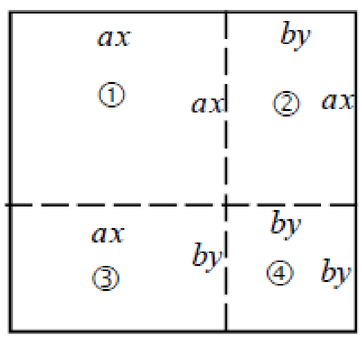
- Prepare four sheets of thick board, each containing one of the following two rectangles and two square.



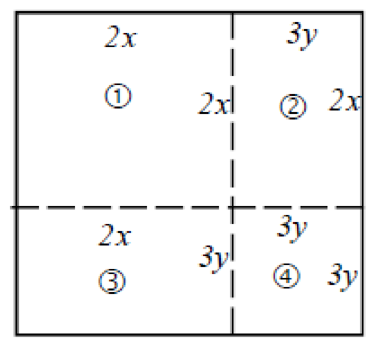
Group I



Group II



Group III



Group IV

- Copies of the student work sheet

Instructions for the teacher:

Approach:

- Elicit from the students, expressions for the area of a square of side m units and a rectangle of length px units and breadth qy units.
- Recall how the expression $(x + y)(a + b)$ is expanded by first writing it as $(x + y)(a + b) = x(a + b) + y(a + b)$.
- Similarly let the students expand products such as $(x + 2)(a - 3)$.

Development of the lesson:

- Distribute the work sheet and the sheets with the figure drawn, among the groups.
- Engage them in the activity.
- After the presentations by the students, discuss how the terms of an expression such as $(ax + by)(cx + dy)$ is obtained through the expansion $ax(cx + dy) + by(cx + dy)$, and then strengthen their knowledge by getting the students to expand such expressions using different values for a, b, c and d.

Student's Worksheet:

- Examine the four figures of squares / rectangles, and record their areas.

  Area of square/rectangle (1) = ...
  Area of the rectangle (2) = ...
  Area of the rectangle (3) = ...
  Area of rectangle/square (4) = ...
  Sum of the areas 7= ....
- Write down algebraic expressions for the length and the breadth of the large rectangle or square made up of the parts (1), (2), (3) and (4).
- Express its area as a product of two binomial expressions in the form ( )( ).
- Now you may notice that the same area has been expressed in two ways. By comparing these, obtain an expression for the above product of two binomial expressions.
- Discuss within the group how the terms in the product of the two binomial expressions are formed using the terms of the two binomial expressions themselves and prepare to present your views to the others.

Assessment and Evaluation:

- Assessment Criteria
  - Expresses the product of two binomial expressions as a sum of algebraic terms by obtaining the same area in two different ways.
  - Builds up a relationship between the terms of two binomial expressions and the terms of the product of the same two binomial expressions.
  - Multiplies two binomial expressions.
  - Works cooperatively within the group.
- Direct the students to the exercises in chapter 2 of the text book.

For your attention…

Development of the lesson:

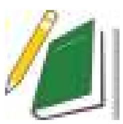
- In relation to learning outcome 3, discuss with the students how $(ax + by)^2$ is expanded by writing it in the form $(ax + by)(ax + by)$.

- With respect to learning outcome 4, guide the students to understand how the square of a binomial expression is found by considering the relationships between the terms of the expansion of $(a + b)^2$. Explain that the expansion is the sum of the squares of the two terms and twice the product of the two terms. Guide them to square expressions such as $(ax + by)$.
- In relation to learning outcome 5, show that it is easy to obtain the squares of numbers by using the expansion of the square of a binomial expression.
  Example: $102^2 = (100 + 2)^2 = 10000 + 400 + 4 = 10404$
- Discuss with the students how the results $(ax + by)(cx + dy) = acx^2 + adxy + bcxy + bdy^2$ and $(ax + by)^2 = a^2x^2 + 2abxy + b^2y^2$ are verified by substituting different values for x and y.

Assessment and Evaluation:

- Direct the students to the relevant exercise in chapter 4 of the text book.

Further reference:



- http://www.youtube. com/watch?v=Sc0e6xrRJYY
- http://www.youtube.com/watch?v=ZMLFfTX615w
- http://www.youtube.com/watch?v=HB48COey2O8
- http://www.youtube. com/watch?v=xjkbR7Gjgjs

5. Congruence

Competency 23: Makes decisions regarding day to day activities based on geometrical concepts related to rectilinear plane figures.

Competency level 23.3: Investigates the requirements for two triangles to be congruent.

Number of periods: 05

Introduction:
In arriving at conclusions related to geometry, knowledge on the congruence of rectilinear figures is applied to a great extent. Figures which are similar in shape and size are said to be congruent.
Figures which are of the same shape but of different sizes are not congruent. They are considered to be similar figures.
Figures which are congruent are equal in area and their corresponding aspects (sides and angles) are also equal to each other.
Two triangles can be shown to be congruent under the conditions SAS/AAS/ SSS and RHS (for a right angled triangle)
The congruence of triangles is used very frequently in the proofs of the theorems and riders in geometry. Identifying congruent triangles and their characteristics, identifying the necessary and sufficient conditions for two triangles to be congruent and proving riders using congruence is expected to be covered in this section.

Learning outcomes relevant to competency level 23.3:

1. **Identifies two plane figures which coincide with each other as congruent figures.**
2. **Identifies the properties of congruent plane figures.**
3. Identifies the conditions that are necessary and sufficient for two triangles to be congruent as SAS, AAS, SSS and RHS.
4. Proves riders using the congruence of triangles.

Glossary of Terms:

Aspects of plane figures	-	තලරූපවල අංග	-	தளவுருக்களின் உறுப்புகள்
Coincide	-	සමපාත වීම	-	ஒன்றுடனொன்று பொருந்துதல்
Corresponding aspects	-	අනුරූප අංග	-	ஒத்த உறுப்புகள்
Congruence	-	අංගසම වීම	-	ஒருங்கிசைவு

Instructions to plan the lesson:

An exemplar lesson plan designed to guide students towards achieving learning outcomes 1 and 2 under competency level 23.3 is given below. This involves a group activity based on the discovery method.

Time: 40 minutes

Quality Inputs:

- Copies of the chart consisting of the figures given in Annex 2
- Copies of the student's worksheet
- Pairs of scissors
- Tissue paper

Instructions for the teacher:
Approach:

- Display the figures in Annex 1 on the chalk board. Introduce the aspects of the plane figures and by considering the figures in the chart, discuss their aspects and how they are named. Emphasize the fact that the aspects of a triangle are its sides and its angles.

Development of the lesson:

- Group the students and issue copies of the handouts and a pair of scissors to each group.
- Engage the students in the activity.
- Once they have completed the activity and presented their discoveries, engage in a discussion.
- Stress that plane figures that can be superimposed are called congruent figures, that the corresponding aspects of congruent figures are equal and that the areas of congruent figures are also equal.
- Show using examples that figures which are of the same shape, but different in size are not congruent. Explain that such figures are called similar figures.
- Show by illustrations that although some figures may appear to be equal in size and shape, we cannot say that they are congruent unless the congruence is formally established.
- Mention that various plane figures can be congruent and that a necessary and sufficient condition for two circles to be congruent is that their radii are equal. Also state that the necessary and sufficient condition for two squares to be congruent is that their side lengths are equal. Draw the attention of the students to the necessary and sufficient conditions for any two plane figures to be congruent.

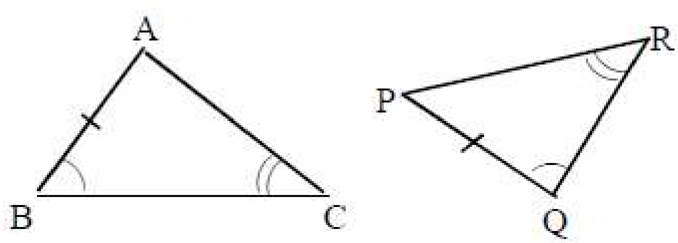
Student's worksheet:

- Guess the figures that are equal in shape and size among the figures in the worksheet provided to you by the teacher and name them using pairs of letters.
- Copy each of the figures in part A onto the tissue paper and determine the pairs of figures that coincide, by placing the figures on the tissue paper on the figures on the given sheet to determine which figures coincide. Select the

pairs which coincide and write them down using pairs of letters.
- Cut out the figures in part B and by placing them on each other, determine the pairs that coincide and name them using pairs of letters.
- Check whether what you guessed and the results of the activity are the same.
- List out the aspects and properties which are equal in the figures that coincided.
- Suggest a name which can be used in common for the set of figures which coincided.
- Prepare to present to the class what you have discovered through the activity.

Assessment and Evaluation:

- Assessment criteria:
    - States that plane figures which coincide are congruent figures.
    - Discovers that the corresponding aspects of congruent figures are equal and that their areas are also equal.
    - In order to determine whether two given plane figures are congruent, correctly places one plane figure on the other to see whether they coincide.
    - Correctly separates out the plane figures which are congruent from a given set of plane figures.
    - Is cooperative and active within the group.
- Provide the students with several pairs of congruent figures and guide them to write the corresponding aspects which are equal to each other.
  Example:



  The two triangles given above are congruent. List out all the pairs of aspects which are equal.
- Guide the students to the relevant exercises in chapter 5 of the text book.

For your attention…
Development of the lesson:

- With respect to learning outcome 3, engage the students in an individual activity to examine the necessary and sufficient conditions for two triangles to be congruent. For this, get the students to cut out triangles of measurements provided by the teacher for each case under which two triangles are congruent, and check whether they coincide.
- Show the students using similar triangles that simply because the corresponding angles of two triangles are equal, we cannot say that they are congruent.

- With respect to learning outcome 4, prepare exercises where the congruence of triangles is used to prove riders.

Assessment and Evaluation:

- Provide the students with exercises involving choosing pairs of congruent triangles from among a given set of triangles and writing the conditions under which they are congruent. Also provide exercises where the students need to write the equal aspects for two triangles to be congruent when the conditions under which they are congruent together with some of the requirements are given. Provide examples of riders proved using the congruence of triangles, and then give the students similar exercises.
- Direct the students to the relevant exercises in chapter 5 of the textbook.

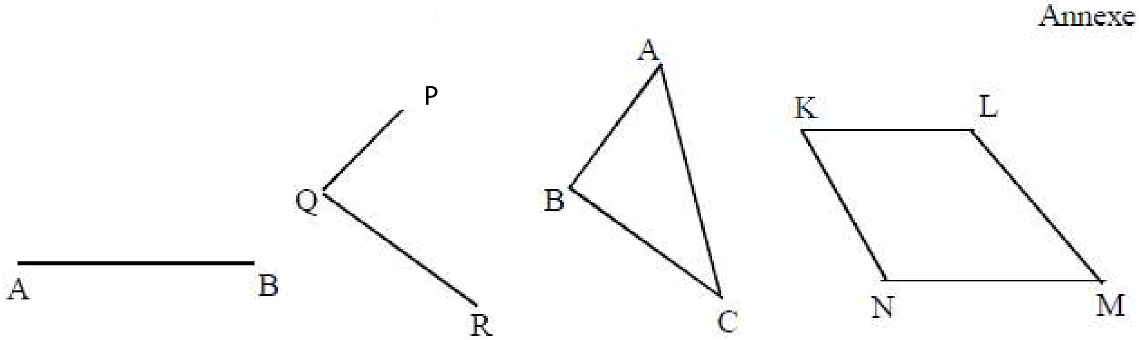
For further reference:

- http://www.youtube.com/watch?v=CJrVOf_3dN0
- http://www.youtube.com/watch?v=8Ld8Csu4sEs
- http://www.youtube.com/watch?v=d5UCZ9hO8X4
- http://www.youtube.com/watch?v=fSu1LKnhM5Q
- http://www.youtube.com/watch?v=fSu1LKnhM5Q

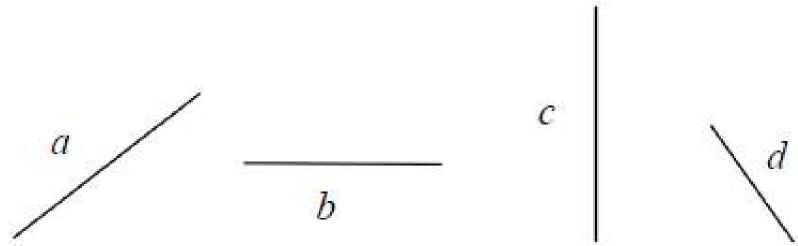
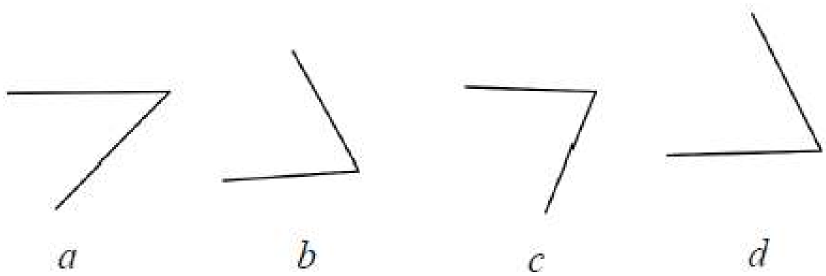
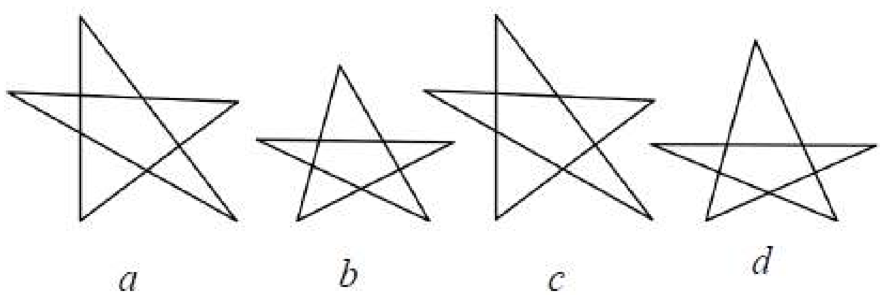
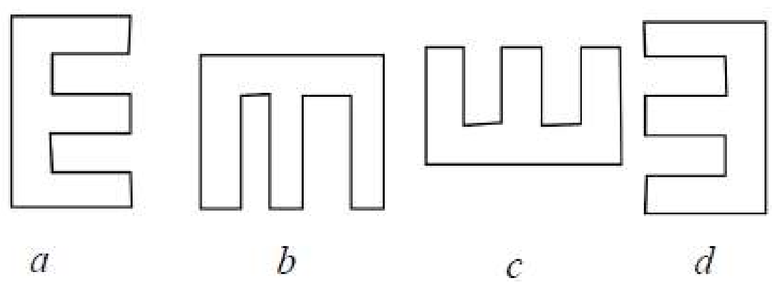
For the teacher only:

- The corresponding sides of congruent rectilinear plane figures are equal while the corresponding sides of similar rectilinear plane figures are proportional.
- Corresponding angles of congruent rectilinear plane figures and similar rectilinear plane figures are equal.
- Congruent plane figures are of equal area but similar plane figures are not.
- Under the SAS case of congruence, the equal angles should necessarily be the included angles. If not, the triangle with the given measurements is not a unique one. (In the triangle *PQR*, *PR* = 7cm, *PQ* = 6cm, $P\hat{R}Q = 50^0$. Construct this triangle and note the condition. Get the students to appreciate the ambiguity)
- When the triangles are congruent under the AAS case, it is a pair of corresponding sides that should be equal. It need not necessarily be the side common to the two angles. The reason for this is, since the sum of the angles of a triangle is 180^0, the remaining pair of angles is necessarily equal.

Annexe 1

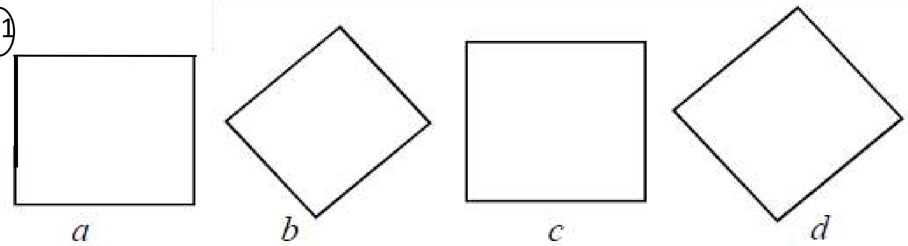


Annexe 2

Part A	① 
	② 
	③ 
	④ 

Part B

1



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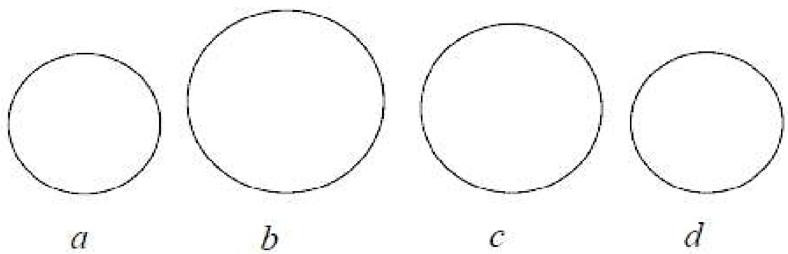

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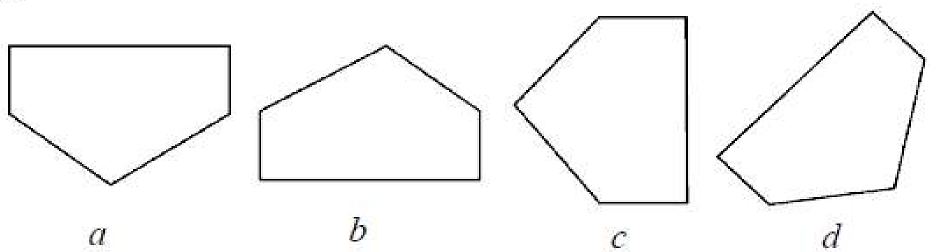

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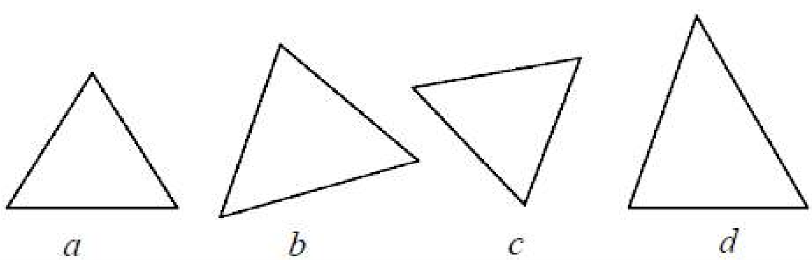

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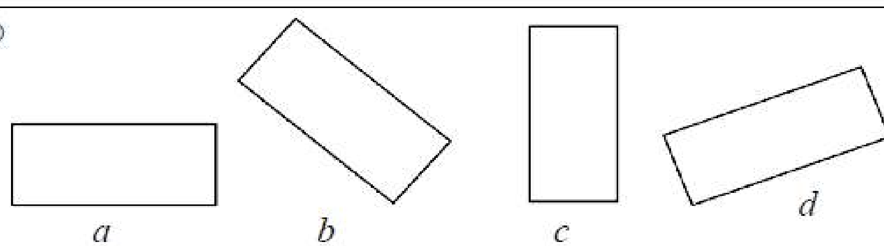

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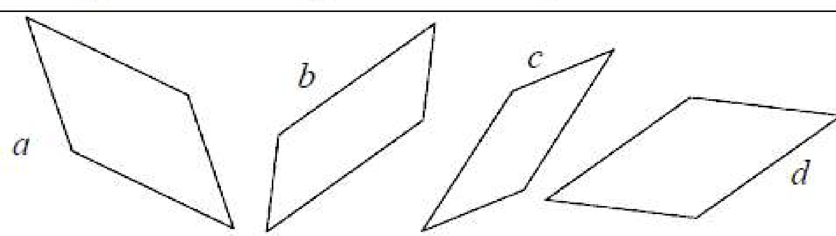


6. Area

Competency 8: Makes use of a limited space in an optimal manner by investigating the area.

Competency Level 8.1: Solves problems related to the area of plane figures containing sectors.

Number of periods: 04

Introduction:
A sector is a part of a circle bounded by two radii and the arc lying between the two radii. The area of a sector varies according to the angle at the centre. If the radius is r and the angle at the centre is θ, then the area A of the sector is given by $A = \frac{\theta}{360} \times \pi r^2$. When finding the area of a compound plane figure containing sectors, the figure has to be first divided into the rectilinear plane figures and sectors which formed the compound figure. The area of each of these parts has to be found separately, and the area of the compound figure is found by adding these areas together. In this section, it is expected to develop a formula for the area of a sector, to find the areas of sectors using the formula, and to find the areas of compound plane figures containing sectors.

Learning outcomes related to competency level 8.1:

1. Develops the relationship $A = \frac{\theta}{360} \times \pi r^2$ for the area A of a sector when the angle at the centre is θ and the radius of the circle is r.
2. Finds the area of a sector using the formula $A = \frac{\theta}{360} \times \pi r^2$.
3. Constructs an algebraic expression for the area of a sector when the measurements of the sector are given as algebraic terms.
4. Solves problems related to the area of compound plane figures which contain sectors of circles.

Glossary of Terms:

Segment of a circle	-	වෘත්ත ඛණ්ඩය	-	வட்டத்தண்டம்
Area	-	වර්ගඵලය	-	பரப்பளவு
Sector of a circle	-	වෘත්තයක කේන්ද්‍රික ඛණ්ඩය	-	வட்டமொன்றின் ஆரைச்சிறை
Radius	-	අරය	-	ஆரை
Arc of a circle	-	වෘත්ත චාපය	-	வட்டவில்
Compound plane figures	-	සංයුක්ත තලරූප	-	கூட்டுத்தளவுருக்கள்

Instructions to plan the lesson:

An exemplar lesson plan to guide students towards achieving learning outcomes 1 and 2 under competency level 8.1 through a group activity based on the discovery method is given below. The students will also engage in an individual activity within the group.

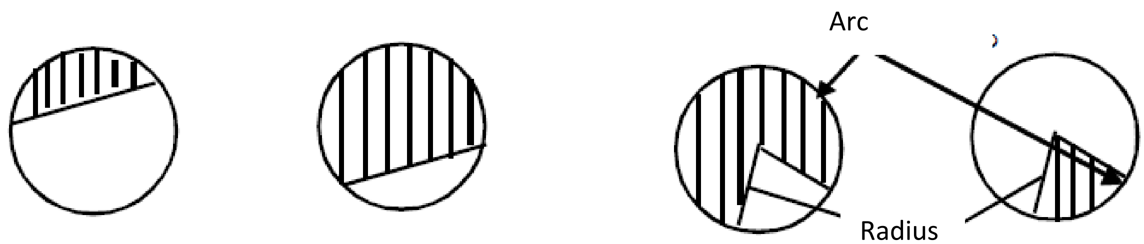
Quality Inputs:

- Circles cut out from demy paper of radius about 7cm - one per child (The radius and the centre shown)
- Pastels and protractors

Instructions for the teacher:

Approach:

- Recall the formulae for the area of a square, a rectangle, a triangle, a parallelogram and a trapezium, and that the area of a circle of radius *r* is given by πr^2.
- Using pictorial examples similar to those given below, explain the difference between sectors and segments of a circle.



Development of the lesson:

- Separate the students in a suitable manner into groups of 4.
- Distribute a circle and a copy of the worksheet to each student and lead them to do the activity.
- Assist the students to find what fraction of the circle the sector is.
- At the end of the activity, give each group an opportunity to share what they discovered.
- Lead a discussion so that the following facts are highlighted.
- Establish the fact that when finding the area of a sector, the angle at the centre could be used to determine what fraction of the whole circle the sector is.
- Emphasize that the ratio of the area of the sector to the area of the whole circle is the same as the ratio of the angle at the centre to the angle around the centre. Stress the fact that if the angle at the centre is θ and the radius is *r* then the area of the sector is given by $\frac{\theta}{360} \times \pi r^2$.

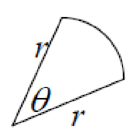
Student's worksheet:

Angles for the sectors

180^0, 90^0, 45^0 and 60^0

- Let each student in your group select one of the given angles.
- Draw the sector with the angle you selected as the angle at the centre, while paying attention to the centre of the circle you have been given and the radius which is marked.
- Shade the sector.
- Find what fraction of the area of the whole figure the area of the sector is by folding your figure.
- Taking the radius of the circle you have been provided with to be *r*, find the area of the shaded sector in terms of the area πr^2 of the circle.
- Record the results of the members of your group in the following table.

Angle at the centre	Area of the sector as a fraction of the area of the circle	$\frac{\text{Angle at the centre of the sector}}{360}$	Area of the sector of radius *r*
180^o 90^o 45^o 60^o 70^o θ			

- Build up a relation between the information in column 2 and column 3.
- Use this relation to write an expression for the area of a sector of a circle of radius *r* with angle at the centre 70^o
- 

Obtain an expression in terms of π , r and θ for the area of
a sector of a circle of radius *r* with angle at the centre θ.

- Prepare to present what you have discovered to the whole class.

Assessment and Evaluation:

- Assessment criteria
  - Expresses the area of a sector as a fraction of the whole circle.
  - States that the area of a sector varies depending on the magnitude of the angle at the centre.
  - Discovers that the area of the sector and the area of the circle are in the same ratio as the corresponding angles at the centre.
  - Builds up a formula for the area of the sector based on the radius and the angle at the centre.

- Finds the area of a sector when the measurements are given.
- Direct the students to the relevant exercises in lesson 6 of the textbook.

For your attention…

Development of the lesson:

- In relation to learning outcome 3, direct the students to find the area of a sector using the formula $A = \frac{\theta}{360} \times \pi r^2$, when the radius and the angle at the centre are given as algebraic terms.
- Discuss various problems involving finding the area of compound plane figures which include sectors of circles.
- Emphasize that when finding the area of a compound plane figure, the areas of the sectors and other plane figures that together form the compound figure need to be found and added together.

Assessment and Evaluation:

- Direct the students to the relevant exercises in chapter 6 of the text book.

For further reference:

- http://www.youtube.com/watch?v=u8JFdwmBvvQ

7. Factors of Quadratic Expressions

Competency 15: Factorizes algebraic expressions by systematically exploring various methods.

Competency Level 15.1: Factorizes a trinomial quadratic expression.

Number of periods: 04

Introduction:
Students have learnt earlier that the factors of a trinomial quadratic expression of the form $x^2 + (a + b)x + ab$ are $(x + a)$ and $(x + b)$, and that the factors of the difference of two squares such as $a^2 - b^2$ are $(a + b)$ and $(a - b)$. The factors of the difference of two squares such as $(x + a)^2 - b^2$ are $(x + a + b)$ and $(x + a - b)$. When we have to find the factors of a trinomial quadratic expression of the type $ax^2 + bx + c$, by finding two factors of ac such that their sum is equal to b, the quadratic expression can be written as a sum of four terms, and thereby factored. The accuracy of the factors of an expression of the type $ax^2 + bx + c$ can be verified by various methods. The following lesson plan is expected to be done with students once they are familiar with finding the factors of the difference of two squares as given in learning outcome 1.

Learning outcomes relevant to competency level 15.1:

1. Finds the factors of the difference of two squares which include algebraic expressions.
2. **Finds the factors of expressions of the form $ax^2 + bx + c$.**
3. **Verifies the accuracy of the factors of an expression of the form $ax^2 + bx + c$ using various methods.**

Glossary of Terms:

Algebraic expressions	වීජීය ප්‍රකාශන	அட்சரகணிதக் கோவைகள்
Difference of two squares	වර්ග දෙකක අන්තරය	இரண்டு வர்க்கங்களின் வித்தியாசம்
Factors	සාධක	காரணிகள்
Trinomial expressions	ත්‍රිපද ප්‍රකාශන	மூவுறுப்புக் கோவைகள்

Instructions to plan the lesson:

An exemplar lesson plan to develop the subject concepts under learning outcomes 2 and 3 related to competency level 15.1, through a group activity of finding the factors of a quadratic expression of the form $ax^2 + bx + c$ is given below.

Time: 40 minutes

Quality Inputs:
- Set of figures made as per annex 1
- Demy papers, platignum pens

- Copies of the student's worksheet

Instructions for the teacher:

Approach:

- Discuss that when we have to find the factors of a trinomial quadratic expression of the form $x^2 + (a + b)x + ab$, by selecting suitable values for a and b, so that the expression can be written as a sum of four terms in the form $x^2 + ax + bx + ab$ and separated into two pairs, the factors can be found as $(x + a)(x + b)$.
- Discuss also how factors of a trinomial quadratic expression of the form $ax^2 + bx + c$ where the coefficient of x^2 is a ($a \neq 0$) are found.

Development of the lesson:

- Supply a copy of the student worksheet and the set of figures as per annex 1 to each group and engage them in the activity.
- After they have participated in the group activity, give them a chance of presenting to the class how the factors of a trinomial quadratic expression are found.
- Discuss how the factors of a trinomial quadratic expression are found with the aid of the laminas, taking into account the students' discoveries.
- Using the results obtained through the activity, discuss that, for a trinomial quadratic expression of the form $ax^2 + bx + c$, by finding two factors of the product ac such that the sum of the factors is b, and writing the given expression as a sum of four terms based on these factors of ac, the given expression can be factorized.

Student's worksheet:

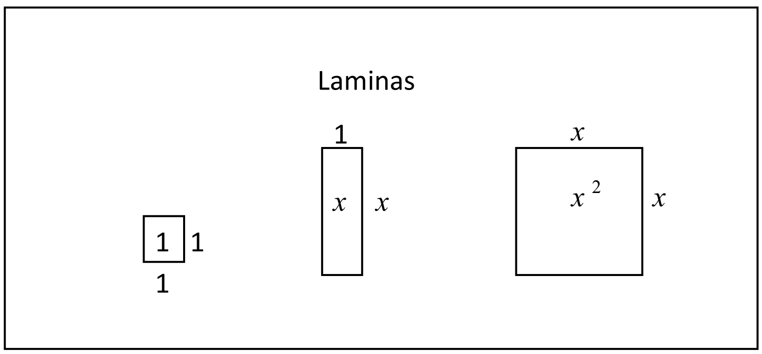
Group A - $2x^2 + 7x + 3$
Group B - $3x^2 + 5x + 3$
Group C - $2x^2 + 7x + 6$
Group D - $3x^2 + 13x + 4$

- Select sufficient laminas so that the areas of the laminas correspond to the terms of the quadratic expression your group received.
- Using all the laminas you have selected construct a rectangle.
- What is the length of the rectangle?
- What is the breadth of the rectangle?
- Obtain an expression for the area of the rectangle using the above length and breadth.
- Write down the relationship between the expression for the area you obtained using the length and breadth and the areas of the laminas used to construct the rectangle.
- Make an enlarged copy of the rectangle on the demy paper, write down the relevant information and then present it to the class.

Evaluation and assessment:

- Assessment criteria
  - Selects laminas to suit the three terms of the trinomial expression.
  - Builds up a relationship between the area of the laminas and the area obtained using the length and breadth of the rectangle.
  - Confirms the factors of a quadratic expression using the areas of the rectangular laminas.
  - Finds the factors of a given quadratic expression.
  - Works cooperatively within the group.
- Direct the students to the relevant exercise in lesson 7 of the textbook.

Annex 1



For further reference:

- http://www.youtube.com/watch?v=h6HmHjkA034
- http://www.youtube.com/watch?v=GMoqg_s4Dl4
- http://www.youtube.com/watch?v=X7B_tH4O-_s

8 Triangles

Competency 23: Applies geometrical concepts related to rectilinear plane figures to arrive at conclusions in day to day activities.

Competency Level 23.1: Formally investigates the sum of the three interior angles of a triangle.

Competency Level 23.2: Investigates the relationship between the exterior angle formed by producing a side of a triangle and the interior opposite angles.

Competency Level 23.4: Formally proves the relationship between the sides and the angles of an isosceles triangle.

Competency Level 23.5: Applies the converse of the theorem on the relationship between the sides and the angles of an isosceles triangle.

Number of periods: 10

Introduction:
It is expected to discuss the relationships between the sides and the angles of a triangle as detailed below. Hence, the theorem that the sum of the three angles of a triangle is 180^o, the theorem that when one side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles, that triangles with two sides equal are known as isosceles triangles, the theorem that if two sides of a triangle are equal then the angles opposite these sides are equal, and its converse that if two angles of a triangle are equal then the two sides opposite them are equal, will be discussed. It is expected that once the students have understood the facts related to competency levels 23.1 and 23.2, the following exemplar lesson plan will be implemented.

Learning outcomes relevant to competency level 23.4:

1. **Identifies the theorem, "If two sides of a triangle are equal, the angles opposite those sides are equal".**
2. **Verifies the theorem, "If two sides of a triangle are equal, the angles opposite those sides are equal".**
3. **Performs calculations using the theorem, "If two sides of a triangle are equal, the angles opposite those sides are equal".**
4. Proves riders using the theorem, "If two sides of a triangle are equal, the angles opposite those sides are equal".
5. Formally proves the theorem, "If two sides of a triangle are equal, the angles opposite those sides are equal".

Glossary of Terms:

Triangle -	ත්‍රිකෝණය	-முக்கோணி
Opposite side -	සම්මුඛ පාදය	- எதிர்ப்பக்கம்
Opposite angle -	සම්මුඛ කෝණය	- எதிர்க்கோணம்
Vertex -	ශීර්ෂය	- உச்சிதிர்க்கோணம்
Bisector -	සමච්ඡේදකය	- இருகூறாக்கி
Theorem -	ප්‍රමේයය	- தேற்றம்
Formal proof -	විධිමත් සාධනය	- முறையான நிறுவல்
Verify –	සත්‍යාපනය	- வாய்ப்புப் பார்த்தல்
Congruence -	අංග සාම්‍යය	- ஒருங்கிசைவு
Isosceles triangle -	සමද්විපාද ත්‍රිකෝණය -	இருசமபக்க முக்கோணி

Instructions to plan the lesson:

An exemplar lesson plan with the aim of guiding students towards achieving learning outcomes 1, 2 and 3 relevant to competency level 23.4 using the lecture-discussion method and an individual activity is given below.

Time: 40 minutes

Quality Inputs:

- A sheet of paper and a pair of scissors per student

Instructions for the teacher:

Approach:

- Show a figure of an isosceles triangle to the class, mention that two sides of the triangle are equal and remind them that such triangles are known as isosceles triangles.
- Discuss the fact that the angle facing a side of a triangle is known as the angle opposite that side, and that the angle between the two equal sides is known as the vertex angle.
- Recall how a triangle is constructed when the lengths of the three sides are given.

Development of the lesson:

- Divide the class into three groups.
- Prepare several sets of measurements of side lengths of isosceles triangles to be given to the groups, where the first group is given a set of side lengths of an acute angled triangle, the second group a set of side lengths of a right angled triangle and the third group a set of side lengths of an obtuse angled triangle.
- By considering the number of students in each group, prepare sufficient sets of side lengths of triangles so that a group of three students would get at least one set.
- Once the students have received the sets of measurements, let them construct the triangle individually. Assist those who require help. Instruct them to name the triangles as ABC, PQR etc (Writing the letters inside the triangle).

- After the triangles are constructed, get the students to cut the triangles out and by folding them to obtain a relationship between the angles.
- Direct the students to measure the angles and to examine whether a relationship exists between them.
- Once the pupil activity is over, elicit from the students that the two angles opposite the two equal sides are equal. Show the students that by folding through the angle formed by the two equal sides, a pair of congruent triangles can be obtained.
- Accordingly, after verifying the theorem that if two sides of a triangle are equal, the angles opposite them are also equal, get them to apply the theorem in calculations by instructing them to find the angles of various isosceles triangles.

Assessment and Evaluation:

- **Assessment Criteria:**
  - Names the angles opposite equal sides of an isosceles triangle.
  - Constructs triangles with given measurements.
  - Finds that the angles opposite equal sides are equal by folding the triangle into two.
  - Engages in calculations using the theorem on isosceles triangles.
  - Makes decisions by comparing one's results with those of others.
- Direct the students to the relevant exercises in lesson 8 of the text book.

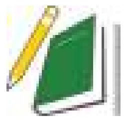
For your attention …
Development of the lesson:

- Once the theorem on isosceles triangles has been verified and used in calculations, direct the students to prove riders and to formally prove the theorem. The students must be clearly advised that in formal proofs, it is required to state reasons step by step.
- Accordingly to Euclid's geometry, this theorem should be proved by constructing the bisector of the vertex angle and showing that the two triangles that are so formed are congruent under the conditions of SAS. This is because, according to Euclid's sequence, this theorem has been proved after presenting the SAS case, before the other congruence cases are presented. However in the sequence of lessons in the school syllabus, this theorem on isosceles triangles comes after all the cases of the congruence of triangles are studied. Also, the bisector of the vertex angle is the perpendicular bisector of the side opposite the vertex. Therefore, this theorem can be proved with the above construction using other cases of congruence as well.
- Direct the students to identify the converse of the theorem on isosceles triangles, to perform calculations involving it, and to prove riders using it.

Assessment and Evaluation:

- Direct the students to the relevant exercises in lesson 8 of the text book.

For further reference:



- http://www.youtube.com/watch?v=7UISwx2Mr4c
- http://www.youtube.com/watch?v=7FTNWE7RTfQ
- http://www.youtube.com/watch?v=ceDV0QBpcMA
- http://www.youtube.com/watch?v=CVKAro3HUxQ

9. Inverse Proportions

Competency 4: Uses ratios to facilitate day to day activities.

Competency level 4.1: Investigates the relationships between quantities using ratios.

Number of periods: 05

Introduction:
A **ratio** is a numerical relationship between two quantities. A proportion is a certain type or relationship between two quantities of different types. In a proportion, if the ratio of two terms of one quantity is equal to the ratio of two corresponding terms of the other, then they are said to be **directly proportional** to each other. Also, the ratio that is obtained using corresponding numerical values of the two quantities is a constant. If two quantities are directly proportional, when one quantity increases then the other too increases. When the product of the quantities is a constant then the quantities are said to be inversely proportional. In this case, the ratio of two values of one quantity is equal to the ratio of the corresponding two values of the other quantity interchanged. Also when one quantity increases, the other quantity decreases. If x and y are two quantities which are inversely proportional to each other, it is written algebraically as $x \propto \frac{1}{y}$. This can also be written as $xy = k$ where k is a constant. Knowledge of proportions can be used to solve day to day problems.

Learning outcomes relevant to competency level 4.1:

1. **Analyses the relationship between two quantities and identifies inverse proportions.**
2. **Identifies that when x and y are two inversely proportional quantities, the proportion is denoted by $x \propto \frac{1}{y}$.**
3. **Solves problems related to inverse proportions by using $xy = k$, where k is a constant.**
4. Solves problems related to work and time using the knowledge on inverse proportions.

Glossary of Terms:

Ratio	- අනුපාතය	- விகிதம்
Proportion	- සමානුපාතය	- விகிதசமன்
Direct proportion	- අනුලෝම සමානුපාතිකය	- நேர்விகித சமன்
Indirecd proportion (Inverse proportion)	- ප්‍රතිලෝම සමානුපාතිකය	- நேர்மாறு விகிதசமன்

Instruction to plan the lesson:

This lesson plan is presented to assist students to achieve learning outcomes 1, 2 and 3 using '**group work**' and the '**lecture-discussion**' method, by getting them to actively engage in constructing information methodically.

Time: 80 minutes

Quality Inputs:

- A poster with the table in Annex 1
- A copy each of the 4 tables in Annex 2
- 4 platignum pens
- Four copies of the student work sheet

Instructions for the teacher:

Approach:

- Exhibit the poster (Annex 1) in the class room.
- With the aid of the poster, recall work done in grade 9 on two quantities which are directly proportional to each other.
- Discuss the fact that, the ratio of two terms in one quantity is equal to the ratio of the corresponding terms of the other.(250 : 500 = 25 : 50), that the ratio obtained by taking only the numerical values (250 : 25, 500 : 50, 750 : 75) is a constant, that if one quantity increases, the other quantity too increases, and that the knowledge of direct proportions can be made use of to easily solve day to day problems.

Development of the lesson:

- Separate the students into 4 groups.
- Supply a copy of one of the tables in annex 2, platignam pens and a copy of the work sheet to each group.
- Engage the students in group work.
- Exhibit the completed tables in the class room and give an opportunity for the students to present their findings.
- Discuss the specific relationship between the two quantities with reference to the completed tables.
- Discuss that, when one quantity increases the other decreases accordingly, the ratio of two terms of one quantity can be obtained by interchanging the corresponding terms of the other quantity, the product of the corresponding terms in the two quantities is a constant, two quantities which have the characteristics described above are said to be inversely proportional to each other, and if *x* and *y* are inversely proportional, we write $x \propto \frac{1}{y}$, and in this case, $xy = k$, where k is a constant, this characteristic of the product of the terms of the two quantities being a constant could be used to solve problems.

Students' work sheet:

- Focus your attention on the table supplied to your group.
- Complete columns 1 and 2 using the information in A. Work mentally and by reasoning.
- When the first quantity in the table gradually increases, how does the other change?
- Complete column 3 with the product of the corresponding values of the two quantities.

- What can be said about the product of the two quantities?
- Write down the ratio of any two terms of the first quantity.
- Find the ratio of the corresponding terms of the other quantity.
- What can you say about the ratios in the two instances?
- Prepare to present and discuss your group's answers and ideas.

Assessment and Evaluation:

- Assessment criteria:
  - Expresses that for two quantities that are inversely proportional, when one increases the other decreases.
  - For a given inverse proportion, finds mentally, the value of one quantity when the value of the other is given.
  - Solves problems related to time and work using the fact that the product of two quantities which are inversely proportional is a constant.
  - Engages in the activity efficiently observing instructions carefully.
  - Arrives at common conclusions by comparing the results of one's group with those of the others.
  - Direct the students to the relevant exercises in lesson 9 of the text book.

For your attention …

Development of the lesson:

- Prepare constructive problems related to inverse proportions, as well as problems on time and work and get the students to work them out.

Evaluation and assessment:

- Direct the students to the relevant exercises in lesson 9 of the textbook.

For further reference:

Annex 01

Amount of sugar	Price (Rupees)
250g 500g 1kg250g 750g	25 50 125 75

Annex 02

Group 1

A → Distance 180 km

Distance travelled in an hour (km)	Time taken for the journey (Hours)	Product of the two quantities
90	…………………	
60	…………………	
45	…………………	
36	…………………	
………………	6	
………………	9	
………………	18	

Group2

- A man needs to dig 2m of a drain in a day.
- Length of the drain is 60 m

Number of men	Number of days	Product of the two quantities
1	………………	
2	………………	
3	………………	
5	………………	
………………	5	
………………	3	
………………	2	

Group 3

A → Distance - 120*km*

Distance travelled in an hour (km)	Time taken for the journey (Hours)	Product of the two quantities
90	…………………	
60	…………………	
45	…………………	
36	…………………	
………………	6	
………………	9	

Group 4

A →
- A man needs to dig 2m of a drain in a day.
- Length of the drain is 180 m

No: of men	No: of days	Product of the two quantities
1	…………………	
2	…………………	
3	…………………	
5	…………………	
………………	5	
………………	3	
………………	2	

10 Data Representation

Competency 28: Investigates the various methods of representing data to facilitate daily work.

Competency level 28.2: Represents data graphically to facilitate comparison and thereby solves problems.

Number of periods: 03

Introduction:
Students have learnt different methods of representing data. When the data is represented in a circle it is known as a pie chart. The different types of information are represented by sectors in a pie chart. In a pie chart, the whole circle which has an angle at the centre of 360^o is used to represent all the data. Therefore, the pie chart is drawn by calculating the angle of the sector representing the different type of data, based on the amount of data being represented. The different sectors are distinguished by either shading them using different colours, or by some other method. A key identifying the different types is also given. It is expected to communicate the information in pie charts and to solve problems related to pie charts under competency level 28.2.

Learning outcomes relevant to competency level 28.2:

1. **Represents a given set of data by a pie chart.**
2. Uses pie charts to communicate information efficiently and effectively.
3. Solve problems related to pie charts.

Glossary of Terms:

Pie charts	- වට ප්‍රස්තාර	- வட்ட வரைபுகள்
Sector of a circle	- වෘත්තයක කේන්ද්‍රික ඛණ්ඩය	- வட்டமொன்றின் ஆரைச்சிறை
Angle at the centre	- කේන්ද්‍ර කෝණය	- மையக்கோணம்
Key	- යතුර	- சாவி

Instructions to plan the lesson:

An exemplar lesson plan to help students achieve learning outcome 1 of competency level 28.2 by using the lecture –discussion method and an individual activity is given below.

Time: 40 minutes

Quality Inputs:

- Figures of circles with sectors marked on them
- Leaflets containing pie charts
- Pie charts from news papers, magazines or drawn by the teacher
- Protractor and A_4 sheets
- Pair of compasses, pastels

Instructions for the teacher:
Approach:

- Recall the different methods such as tables, picture graphs, bar graphs and stem and leaf diagrams previously used by the students to represent data.
- Show examples in newspapers, magazines or drawn by the teacher, of pie charts as another means of representing data.
- Help the students understand through a discussion that communication is facilitated by representing data in a pie chart.
- Recall what sectors are by displaying figures of circles with sectors marked on them.
- Remind the students how angles of given magnitude are drawn and measured using a protractor.

Development of the lesson:

- Discuss how a pie chart is used to represent a given set of data.
- Display the information in the table given below on the blackboard, and mention that a pie chart representing this information needs to be drawn.

Information on the results obtained for mathematics at the 2012 O'level examination by 90 students of a certain school is given below.

Grade	Number of students
A	08
B	15
C	17
S	30
W	20

- Elicit from the students the fact that since 360^{o} represents the total number of students which is 90, the number of degrees which represents one student is $\frac{360^0}{90^0} = 4^0$.
- Direct the students to find the magnitudes of the angle representing the different types accordingly.
- Supply an A_4 sheet to each student.
- Instruct them to draw a circle of radius about 7cm on the A_4 sheet using a pair of compasses.
- Guide the students to draw the angles at the center of the various sectors and to complete the pie chart.
- Instruct them to use different colours to highlight the different types to facilitate communication.
- Explain the importance of a key to present the information clearly. Get the students to prepare the key using the different colours used to denote the different types and to write the relevant information next to them.
- Once the pie-chart is drawn, lead a discussion and highlight the following.
  - Pie charts are useful to compare the whole and its components.

- It facilitates seeing all the information at a glance.
- When data is in the form of fractions, percentages and decimals, they can be represented in a pie chart.
- Drawing the pie chart is easy if the total number of data is a multiple or factor of 360^{o}.

Assessment and Evaluation:

- Assessment criteria
    - Identifies the relevant variables to draw the pie chart according to the different types.
    - Finds the magnitudes of the angles at center of the sectors representing the different types.
    - Compares the components of a pie chart.
    - Expresses each component as a fraction of the whole.
    - Completes the activity successfully following the teacher's instructions.
- Direct the students to the relevant exercises in lesson 10 of the text book.

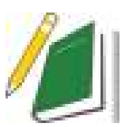
For your attention..
Development of the lesson:

- It is expected that students will be guided to achieve learning outcomes 2 and 3 of competency level 28.2 by developing the skills of interpreting data in a pie chart and solving problems related to pie charts.

Assessment and Evaluation:

- Direct the students to the relevant exercises in lesson 10 of the text book.

For further reference:



- http://www.youtube.com/watch?v=4JqH55rLGKY

11. Least Common Multiple of Algebraic Expressions

Competency 16: Explores the various methods of simplifying algebraic fractions to solve problems encountered in day to day life.

Competency level 16.1: Finds the least common multiple of several algebraic expressions.

Number of periods: 04

> **Introduction:**
> The smallest algebraic expression which can be divided by several algebraic expressions is their Least Common Multiple (LCM)
> Eg: (i) The LCM of $2x$, $9x$, $12xy$ is $36xy$
> (ii) The LCM of $2(x + 1)$, $3(x - 1)$, $10x^2$ is $30x^2(x + 1)(x - 1)$
> The least common multiple can be found either by writing the terms as a product of factors or by the division method. In this section, it is expected to find the least common multiple of several given algebraic expressions.

Learning outcomes relevant to competency level 6.1:

1. **Recognizes that the smallest algebraic expression that can be divided by several algebraic expressions is the least common multiple of those expressions.**
2. Finds the least common multiple of several given algebraic terms.
3. Using factors, finds the least common multiple of several algebraic expressions.
4. Logically decides what the least common multiple of several algebraic expressions is.

Glossary of Terms:

Factors	- සාදක	- காரணிகள்
Algebraic terms	-වීජීය පද	- அட்சரகணித உறுப்புக்கள்
Algebraic expressions	- වීජීය ප්‍රකාශන	- பொதுமடங்குகளுள் சிறியது
Least common multiple	-කුඩාම පොදු ගුණාකාරය	- பொதுமடங்குகளுள் சிறியது
Algebraic fractions	– වීජීය භාග	- அட்சரகணிதப் பின்னங்கள்

Instruction to plan the lesson:

> An exemplar lesson plan based on the lecture-discussion method together with an activity to help students develop the skill of finding the least common multiple of several algebraic terms and to strengthen their understanding of the concepts related to learning outcomes 1 and 2 under competency level 16.1 is given here.

Time: 80 minutes

Quality Inputs

- Leaflets of sets of at most 3 algebraic terms, of at most two variables and index not more than 2, as in Annex 1 (one for each student)
- Leaflets of the least common multiples of all the sets of algebraic terms given, as in Annex 2 (a copy for each student)
- A_4 sheets

Instructions for the teacher:

Approach:

- Recall that the smallest number that is divisible by several numbers is the least common multiple of those numbers.
- Mention that the least common multiple of several numbers can be found by writing each number as a product of its prime factors
- Show that the least common multiple of several numbers can also be found using the division method.
- Start developing the lesson by mentioning that we will attempt to find the least common multiple of several algebraic terms using both these methods.

Development of the lesson:

- Define the least common multiple of several algebraic expressions in the same way that the least common multiple of several numbers is defined, as the smallest algebraic expression which is divisible by all these algebraic expressions.
- Obtain the LCM of 3x, 4xy and 6y as 12xy by expressing these terms as products of their factors and proceeding step by step.
- Establish the fact that the LCM of the above terms is 12xy by using the division method too, proceeding step by step.
- Inquire from the students about the answers obtained using the above two methods and lead a discussion and highlight the following facts: that the LCM of several terms can be found using various methods, that it can be found by writing the terms as products of prime and algebraic factors and taking the LCM to be the product of the highest powers of these factors appearing in the terms, that it can also be found by dividing the terms by the prime and algebraic factors and taking the product of these prime and algebraic factors to be the LCM..
- Engage the students in the following activity to strengthen their knowledge further.

Activity:

- Divide the students into two groups.
- Select one student from each group to record the marks.
- Give each student a copy of the leaflet containing the LCMs and an A_4 paper.
- Direct the questions alternatively to the two groups (The sets of algebraic expressions in the leaflet).
- Direct the questions to the competitors in the two groups in a particular order giving each student a chance to answer.

- Once the question is asked, provide sufficient time for the answer. Inform the students to state the number in Annex 2 which corresponds to the correct least common multiple.
- If the answer is correct award sufficient marks.
- If the student to whom the question was put, fails to answer the question correctly, a member in the same group should be given the chance to answer, in which case a lesser mark is to be awarded for the correct answer.
- If the team fails to furnish the correct answer, the question can be directed to the other team and a reasonable mark can be awarded.

Assessment and Evaluation:

- Assessment criteria
  - Describes what the LCM of several algebraic terms is.
  - Shows that there are many methods of finding the LCM of several terms.
  - Obtains the LCM of given algebraic terms.
  - Uses the chance given correctly.
  - Works cooperatively within the team to achieve victory.
- Direct the students to the relevant exercises in lesson 11 of the textbook.

For your attention..
Development of the lesson:

- Direct the students to find the LCM of a set of expressions using a suitable method. (At most 3 expressions, at most 2 variables and indices not exceeding 2)
- Stress the fact that when finding the LCM of algebraic expressions, each expression should be expressed as a product of its factors of the simplest form.
- Discuss with the students how the LCM of algebraic expressions can be found through reasoning.

Assessment and Evaluation:

- Direct the students to the relevant exercises in lesson 11 of the text book

For further reference:

- http://www.youtube.com/watch?v=FNnmseBlvaY
- http://www.youtube.com/watch?v=MNeNHoCXoGU
- http://www.youtube.com/watch?v=auQU-9KNG74

Annex 01

- Sets of algebraic terms of which the least common multiple is to be found

(i) $2x, 12y, 4xy$
(ii) $3xy, 6y^2, 12x$
(iii) $4x^2, 5y^2, 8xy$
(iv) $6xy, 9y^2, 10x$
(v) $3x^2, 5xy, 4y^2$

Annex 02

- The leaflet with the least common multiples

(vi) $90xy^2$
(vii) $40x^2y^2$
(viii) $12xy^2$
(ix) $12xy$
(x) $60x^2y^2$

12. Algebraic Fractions

Competency 16: Explores the various methods of simplifying algebraic fractions to solve problems encountered in day to day life.

Competency level 16.2: Manipulates algebraic fractions under addition and multiplication.

Number of periods: 04

Introduction :

- Under this section, it is expected to develop the ability of adding and subtracting algebraic fractions with unequal denominators.
- When simplifying such algebraic fractions it is necessary to obtain the relevant equivalent fractions of the given algebraic fractions.
- Before expressing as equivalent fractions, a common denominator of the algebraic fractions needs to be found, and for this, the LCM of the denominators is obtained.
- The simplification can easily be done using the LCM as the common denominator, and finding the equivalent fractions correctly.

Learning outcomes relevant to competency level 16.2:

1. **Explains the necessity of equivalent fractions in the addition and subtraction of algebraic fractions.**
2. **Adds and simplifies algebraic fractions with unequal denominators.**
3. Subtracts and simplifies algebraic fractions with unequal denominators.
4. Simplifies algebraic fractions with unequal denominators.

Glossary of Terms:

Factors - සාධක - காரணிகள்
Algebraic fractions - වීජීය භාග - அட்சரகணிதப்பின்னங்கள்
Least common multiples-කුඩා ම පොදු ගුණාකාරය - பொதுமடங்குகளுள்சிறியது
Denominator - හරය - பகுதி
Numerator - ලවය - தொகுதி
Common denominator - පොදු හරය - பொதுப்பகுதி
Equivalent fractions – තුල්‍ය භාග - சமவலுப்பின்னம்

Instruction to plan the lesson:

An exemplar lesson plan based on the lecture-discussion method, to develop the concepts of adding and simplifying algebraic fractions, relevant to learning outcomes 1 and 2 under competency level 16.2 is given below.;

Time: 40 minutes

Instructions for the teacher:
Approach:

- Discuss the steps to be followed to simplify the sum of two numerical fractions with unequal denominators such as $\frac{2}{3}+\frac{1}{6}$ and $\frac{3}{4}+\frac{5}{7}$.
- Show the need for equivalent fractions in such simplifications.
- Discuss the process of finding the LCM of algebraic expressions such as $3x, 4xy$ and $3x, (2x-3)$.
- Recall how the sums of two algebraic fractions with equal denominators such as $\frac{5x}{6y}+\frac{x}{6y}$ are simplified.

Development of the lesson:

- Write the following problem on the blackboard.

Simplify $\frac{3x}{10y}+\frac{y}{2x}$

- Discuss the fact that equivalent fractions have to be found here, as in the case of numerical fractions with unequal denominators.
- During the discussion, elicit the fact that the least common multiple of $10y$ and $2x$ is $10xy$.
- Taking $10xy$ as the denominator, separately construct the relevant equivalent fractions of $\frac{3x}{10y}$ and $\frac{y}{2x}$ as $\frac{3x^2}{10xy}$ and $\frac{5y^2}{10xy}$.
- Through the discussion, obtain the answer $\frac{3x^2+5y^2}{10xy}$ after simplifying the equivalent fractions by following the steps of adding two fractions with equal denominators.
- In the same manner, obtain the LCM of the denominators relevant to $\frac{5x}{2x+3}+\frac{3}{4x}$ as $4x(2x+3)$.
- Through the discussion, obtain from the students that the sum of these two algebraic fractions is $\frac{20x^2+6x+9}{4x(2x+3)}$.
- After discussing these examples using the lecture-discussion method, explain that when simplifying expressions like those in the above examples, we need to find equivalent fractions (with a common denominator) of the given fractions, that the LCM is used as the common denominator, and that the simplification needs to be done using the equivalent fractions that were obtained.
- Get the students to do several problems to strengthen their understanding of the subject concepts that were learnt.

Assessment and Evaluation:

- Assessment criteria
  - Obtains a common denominator of given algebraic fractions by finding the LCM of the denominators.
  - Accepts that it is necessary to find a common denominator when simplifying algebraic fractions.
  - Finds equivalent fractions based on the common denominator.
  - Adds algebraic fractions with unequal denominators.
  - Completes the assigned task accurately.
- Lead the students to do the exercises in lesson 12 of the textbook.

For your attention..
Development of the lesson:

- Guide the students towards simplifying algebraic fractions such as $\frac{x-1}{x^2+3x}+\frac{x+4}{x^2+x-6}$.
- Use a suitable method to develop the skill of subtracting algebraic fractions in students.
- Assist the students to understand that when simplifying algebraic fractions of the form $\frac{3}{(x-3)}+\frac{2}{(3-x)}$, the common denominator can be found by taking the negative sign in the denominator of the second fraction out of the brackets, and that then the addition is converted into a subtraction.

Assessment and Evaluation:

- Direct the students to the relevant exercises in lesson 12 of the text book.

For further reference:

- http://www.youtube.com/watch?v=7Uos1ED3KHI
- http://www.youtube.com/watch?v=IKsi-DQU2zo
- http://www.youtube.com/watch?v=YO_SwIKGMqQ
- http://www.youtube.com/watch?v=dstNU7It-Ro

13. Percentages

Competency 5: Uses percentages to make successful transactions in the modern world.

Competency level 5.1: Solves problems related to taxes using percentages.
Competency level 5.2: Makes decisions by calculating the interest.

Number of periods: 07

Introduction:
• Taxes are collected in various situations. The amount of the tax is decided by the state. We as citizens are bound to pay taxes. Some of the types of taxes that have to paid in Sri Lanka are Customs duty, Rates, Income Tax and Value Added Tax (VAT). Taxes are calculated as percentages.
• When certain item are imported or exported, a certain percentage of the value of the item is levied as tax. This tax is known as **customs duty** and is collected by Sri Lanka Customs.
• If the income earned by a person annually through employment or a business or personal wealth exceeds a certain amount decided by the government as being exempt from tax, then the person is required to pay a certain proportion of his income as tax. **This is called income tax.**
• The local authority provides certain facilities to the residents within the particular local area. In return, the residents are required to pay a certain percentage of the annual value of the property as taxes to the local authority. This tax is known as **rates**. It can be paid annually or quarterly (every three months).
• A percentage of all expenses incurred related to any item or service is charged and it is known as VAT. Persons who carry out such business or provide such services pay the VAT to the state. Such tax is charged against the customer or the person who receives the service by the person who provides the service. VAT is an indirect tax which falls on the consumer.
• When an amount of money is given as a loan or taken as a loan, an amount is payable above the amount borrowed and such amount is known as **interest**. Interest payable on Rs 100 per year is known as the **annual interest rate**. The interest is charged based on the amount of the loan and is a fixed amount. This rate of interest is known as annual simple interest rate.
• When money is deposited at simple interest, then on a particular deposit, a fixed amount is paid as interest for equal time periods.
• Simple interest is calculated as follows.
Interest = Principal amount of loan × annual rate of interest × number of years.
• Once the students have identified the types of taxes that prevail, and understood how to calculate taxes relevant to competency level 5.1, the following exemplar lesson plan is expected to be implemented.

Learning outcomes relevant to competency level 5.2:

1. **Identifies simple interest as the interest calculated by taking into consideration the initial amount, the time and the interest rate.**
2. **Recognizes that the interest received during equal time periods is the same for a given amount of money.**
3. Calculates the interest for a given amount, for a given period and given interest rate.
4. Solves problems involving finding the interest or the interest rate or the time or the amount, when the necessary information is given.
5. Makes effective decisions on transactions in day to day life by taking into consideration the interest.

Glossary of Terms:

Percentage	- ප්‍රතිශතය	- சதவீதம்
Interest rate	- පොලී අනුපාතිකය	- வட்டி வீதம
Principal	- මුල් මුදල	- முதல்
Tax	- බදු	- வரி
Customs duty	- තීරු බදු	- சுங்க வரி
Rates	- වරිපනම් බදු	- மதிப்பீட்டு வரி
Quarter	- කාර්තුව	- காலாண்டு
Annual value	- වාර්ෂික වටිනාකම	- வருடாந்த பெறுமதி
Value added tax	– වැට් (VAT) බදු	- வற் வரி

Instruction to plan the lesson:

> An exemplar lesson plan based on the lecture-discussion method and an individual activity, to develop in students the subject concepts relevant to learning outcomes 1 and 2 under competency level 5.2 is given below.

Time: 40 minutes

Quality Inputs:

- The poster given in Annex 1
- Copies of the student's work sheet

Instructions for the teacher:
Approach:

- Display the poster in annex 1 and discuss situations in which percentages are used.
- Discuss calculating given percentages, first inquiring what a percentage is.

Development of the lesson:

- Discuss and explain what the principal and the interest are by considering an example such as the following.
  A person who borrowed Rs.1000 had to pay back Rs.1100. This means he had to pay Rs.100 extra as interest.
- Introduce the amount payable as interest on a loan of Rs.100 taken for a year, as the annual interest rate. Explain that when the interest for each period is calculated on the principal only, then it is known as the annual simple rate interest.
- Using the following example, clarify that the interest payable is the same amount for equal periods when money is borrowed from a person who charges simple interest.
  A person borrows Rs. 2 500 at 12% interest.
  Interest for the first year = $2500 \times \frac{12}{100} =$ Rs. 300
  Interest for the second year = $2500 \times \frac{12}{100} =$ Rs. 300
  Same amount is charged every year as shown above.
- Supply a work sheet per student and engage them in the individual activity.
- Highlight the important facts of the lesson by eliciting from the students that by multiplying the principal by the annual interest rate, one year's interest is calculated, that by multiplying the interest for one year by the number of years, the total interest can be calculated, and that the total amount to be paid is obtained by adding the total interest to the principal amount.

Student's work sheet

• Study the questions in (a) and (b) below carefully and record the answers.

(a) Kanthi borrowed Rs. 2000 from a local money lender at an annual simple interest rate of 20%.
(i) What is the interest payable for Rs.100 at the end of the one year?
(ii) What is the interest payable for Rs. 2000 at the end of the year?
(iii) What is the interest payable for Rs. 2000 at the end of two years?
(iv) What is the total amount payable at the end of 2 years with the amount borrowed?

(b) Fill in the following table.

Principal (Rs)	Time (Years)	Annual interest rate	Annual interest	Interest for the loan period	Total amount with principal (Rs)
100	1	5%	5	5	105.00
100	2	5%	……	……	……
2500	3	5%	……	……	……
3000	3	8%	……	……	……
5000	2	3%	……	……	……
4000	5	2.5%	……	……	……
2500	$2\frac{1}{2}$	10%	……	……	……

- Discuss your answers with your mathematics teacher.

Assessment and Evaluation:

- Assessment criteria
  - Identifies the principal, time (period), rate of interest.
  - Calculates the simple interest based on the principal, period and rate of interest.
  - Calculate the total amount (including the loan) based on the principal, period and rate of interest.
  - Accepts that the interest can be calculated for a given period when the principal and the rate of interest are given.
- Direct the students to do the exercises in chapter 13 of the textbook.

Practical situations:

- Discuss situations such as the following where simple interest is used practically.
  - When loans are obtained from banks or other financial organizations.
  - When gold jewellery is pawned, interest is calculated as simple interest.

For your attention…
Development of the lesson:

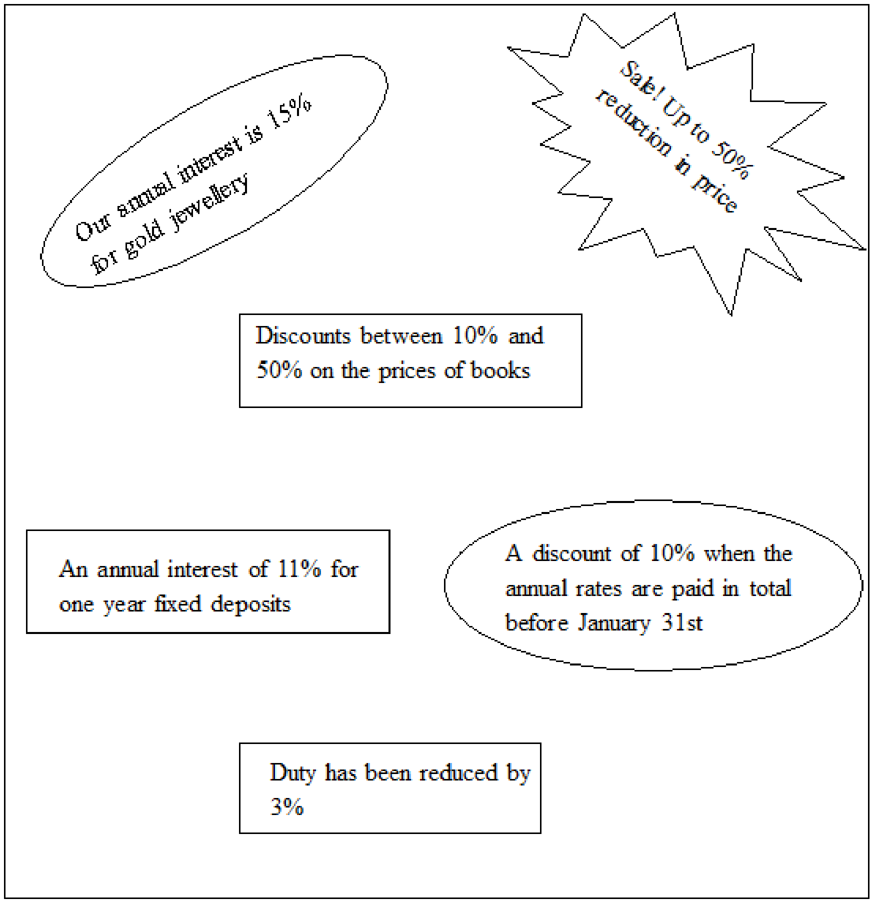
- Discuss how to find the interest, rate of interest, time or principal when the necessary information is available.
- Solve problems of the above type through discussion.
- Prepare an activity to assist students to understand the concepts on taxes, customs duty, rates, income tax etc and implement it.

Assessment and Evaluation:

- Guide the students to do the exercises in lesson 13 of the text book.

For further reference:

Annex 1


Our annual interest is 15% for gold jewellery
Sale! Up to 50% reduction in price
Discounts between 10% and 50% on the prices of books
An annual interest of 11% for one year fixed deposits
A discount of 10% when the annual rates are paid in total before January 31st
Duty has been reduced by 3%

14. Equations

Competency 17: Manipulate methods of solving equations to meet day to day requirements.

Competency level 17.1: Uses linear equations to solve problems.

Number of periods: 08

Introduction :

An equation in which the index of each unknown or variable is 1 is called a linear equation.
If we have just one linear equation which contains two variables, then there is no unique solution. However, if there are two such equations, then it may be possible to find a unique solution which satisfies both the equations. Such a pair of equations is called "simultaneous linear equations".

An equation with one unknown, of the form $ax^2 + bx + c = 0$ where $a \neq 0$, which contains a term of index two of the unknown is called a quadratic equation. There are two solutions to an equation of this form.

It is expected in this section to solve simple equations containing algebraic fractions, simultaneous equations and quadratic equations which come under competency levels 17.1, 17.2 and 17.3.

Plan suitable learning –teaching methodologies and implement them to instill in students the subject concepts relevant to competency levels 17.2 and 17.3.

Learning outcomes relevant to competency level 17.1:

1. **Recognizes that the methods of simplifying algebraic fractions can be used to solve simple equations involving algebraic fractions.**
2. **Solves simple equations involving algebraic fractions.**
3. Expresses the relationship between the data of a given problem by a simple equation involving algebraic fractions and solves it.

Glossary of Terms:

Simple equations - සරල සමීකරණ - எளிய சமன்பாடுகள்
Algebraic fraction - වීජීය භාග - அட்சரகணிதப் பின்னங்கள்
Solution - විසඳුම - விடை
Least common multiple - කුඩා ම පොදු ගුණාකාරය - பொது மடங்குகளுள் சிறியது
Common denominator – පොදු හරය - பொதுப்பகுதியெண் -

Instruction to plan the lesson:

An exemplar lesson plan involving discovery through exploration, to develop the subject concepts relevant to learning outcomes 1 and 2 under competency level 17.1, and to instill in students the skills of solving simple equations involving algebraic fractions is given below.

Time: 40 minutes

Quality Inputs:

- Copies of the student's work sheet - one per group

Instructions for the teacher:
Approach:

- Display on the chalk board, the following simple equation containing an algebraic fraction and the algebraic expression containing an algebraic fraction and clarify the difference between an equation and an expression.

$$\frac{3}{x+1} - \frac{5}{x} + \frac{3}{2x} = 2\,, \qquad \frac{2}{3x} + \frac{1}{x}$$

- Recall how an equation of the form $\frac{3x}{2} - \frac{x}{4} = 5$ is solved.
- Show that when solving equations involving algebraic fractions, if the denominators of the terms are distinct, then the least common multiple of the denominators should be obtained, and recall how the LCM of several algebraic expressions are obtained.
- Recall how expressions containing algebraic fractions are simplified.

Development of the lesson:

- Separate the class into 4 groups.
- Supply a copy of the student's work sheet to each group and engage them in the activity.
- Give the students an opportunity to present their discoveries and explain the following while leading a discussion: When solving an equation involving algebraic fractions, first the two expressions on either side of the equal sign should be simplified such that only one term remains on each side, Then by multiplying both side by the L.C.M. of the denominators (on both side of the equation), an equation with no fractions is obtained. By solving this equation, the solution to the original equation is obtained.

Student's work sheet:

i. $\frac{5}{x+1} - \frac{2}{3} = 1$ ii. $\frac{2}{x+1} - \frac{3}{2(x+1)} = 1$

iii. $\frac{5}{3x} + \frac{2}{x} = 1$ iv. $\frac{2}{x} - \frac{1}{3x} = 5$

- Consider the equation received by your group.
- Find the least common multiple of the denominators of the algebraic fractions on the left side of the equality sign and thereby simplify the left side.
- Convert the denominator to 1, by multiplying both sides of the equation by a suitable value or expression, and then solve the equation.
- Substitute the solution into the equation and examine the accuracy of the solution.
- Prepare to present your findings on solving equations with algebraic fractions to the whole class.

Assessment and Evaluation:

- Assessment criteria
  - Expresses the necessity of reducing the expressions on the two sides of the equality sign of an equation involving algebraic fractions to single terms, when solving such linear equations.
  - Solves equations containing algebraic fractions with numerical denominators.
  - Solves equations containing algebraic fractions with algebraic denominators.
  - Gains satisfaction by solving a problem and verifying the solutions.
  - Learns through experience.
- Direct the students to do the exercises in chapter 14 of the textbook.

For your attention ...
Development of the lesson:

- Prepare an activity or use some other suitable methodology to guide students to construct equations involving algebraic fractions based on given information and to solve them.

Assessment and Evaluation:

- After the students have familiarized themselves with solving equations involving algebraic fractions on one side of the equality sign, help them learn to solve equations with algebraic fractions on both sides of the equality sign.
- Direct the students to do the exercises in chapter 14 of the textbook.

For the special attention of the teacher:

- When solving equations involving algebraic fractions, the students should not be taught to find the LCM of all the denominators on both sides of the equal sign initially. They should first be guided to solve the equation by simplifying the two sides of the equality sign separately. After this has been firmly established, they may be shown how to solve the equation by considering all the denominators.

- Cross multiplication too should not be introduced in the first instance. Allow the students find this out by themselves.

For further reference:

- http://www.youtube.com/watch?v=PPvd4X3Wv5I
- http://www.youtube.com/watch?v=Z7C69xP08d8
- http://www.youtube.com/watch?v=9IUEk9fn2Vs
- http://www.youtube.com/watch?v=bRwJ-QCz9XU
- http://www.youtube.com/watch?v=Yaeze9u6Cv8

15. Parallelograms I

Competency 23: Makes decisions regarding day to day activities based on geometrical concepts related to rectilinear plane figures.

Competency level 23.6: Formally proves the relationships between the sides and the relationships between the angles of parallelograms.

Competency level 23.7: Identifies and uses the relationship between the diagonals of a parallelogram.

Number of periods: 07

Introduction:
- Student have learned that a quadrilateral of which opposite sides are parallel is a parallelogram. Here they will learn more about the properties of parallelograms.
- Identifying the theorem "**In a parallelogram, opposite sides are equal; opposite angles are equal; and each diagonal bisects the area of the parallelogram**", verifying it in different ways, using the theorem in simple calculations, proving riders and the formal proof of this theorem are expected to be done under competency level 23.6.
- Proof of the theorem "**The diagonals of a parallelogram bisect each other**" is not expected, but identifying the theorem, verifying the theorem and using the theorem to prove riders are expected to be done under competency level 23.7.

Learning outcomes relevant to competency level 23.6:
1. **Identifies the theorem, "In a parallelogram, opposite sides are equal; opposite angles are equal; and each diagonal bisects the area of the parallelogram".**
2. **Verifies by various methods the theorem "In a parallelogram, opposite sides are equal; opposite angles are equal; and each diagonal bisects the area of the parallelogram".**
3. Performs simple calculations using the theorem, "In a parallelogram, opposite sides are equal; opposite angles are equal; and each diagonal bisects the area of the parallelogram".
4. Proves riders using the theorem, "In a parallelogram, opposite sides are equal; opposite angles are equal; and each diagonal bisects the area of the parallelogram".
5. Formally proves the theorem, "In a parallelogram, opposite sides are equal; opposite angles are equal; and each diagonal bisects the area of the parallelogram".

Glossary of Terms:

Parallelogram	- සමාන්තරාස්‍රය	- இணைகரம்
Opposite sides	- සම්මුඛ පාද	- எதிர்ப்பக்கங்கள்
Opposite angles	- සම්මුඛ කෝණ	- எதிர்க்கோணங்கள்
Diagonal	–විකර්ණය	- மூலைவிட்டம்
Area	- වර්ගඵලය	- பரப்பளவு
Bisect	- සමච්ඡේදනය කිරීම	- இருகூறாக்கி
Quadrilateral	- චතුරස්‍රය	- நாற்பக்கல்

Instruction to plan the lesson:

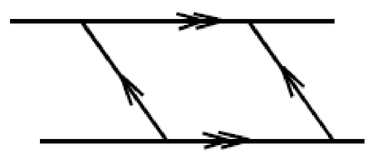
An exemplar lesson plan prepared under the guided discovery method is given below. It is expected that students will be guided towards learning outcomes 1 and 2 of competency level 23.6 through this.

Time: 40 minutes

Quality Inputs:
- Oil papers
- A_4 papers
- Set square
- Straight edge

Instructions for the teacher:
Approach:
- Recall that a parallelogram is a quadrilateral in which opposite sides are parallel.
- Draw a parallelogram on the blackboard and identify the opposite sides, opposite angles and diagonals.
- Recall that congruent triangles are of equal area.
- Illustrate how a parallelogram is drawn as follows.

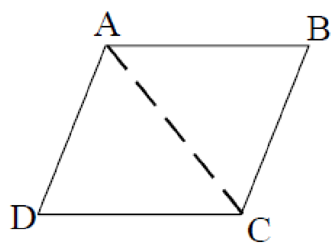
  Draw a pair of parallel lines using a set square and a straight edge. Draw another pair of parallel lines so that they intersect the initial pair of lines.



- Illustrate and introduces the theorem that "**In a parallelogram, opposite sides are equal; opposite angles are equal; and each diagonal bisects the area of the parallelogram**".
- Group the students suitably and provide each student with an A_4 paper and an oil paper.
- Distribute the student work sheet and engage the students in the activity.
- Let them present their discoveries.
- Based on their presentations, state that the opposite sides of a parallelogram are equal, the opposite angles are equal and that each diagonal bisects the area of the parallelogram. Mention also that through the activity, the theorem has been verified, but not formally proved, and that the formal proof of the theorem will be discussed in a future lesson.

Student's work sheet:

- Distribute the A_4 papers and the oil papers among the members of the group.
- Draw a parallelogram on the oil paper as shown by the teacher. (This must be done by all the students)
- Name the parallelogram so drawn as ABCD.
- Mark the points A, B, C, D on the A_4 sheet using a pin and copy the parallelogram onto it. Name it $A'B'C'D'$.
- By placing the parallelogram drawn on the oil paper over the one drawn on the A_4 sheet, find the relationship between
  - (i) the sides, and
  - (ii) the angles, and record them.
- By drawing the parallelogram ABCD on an A_4 paper which has been folded into two and cutting along the sides of the parallelogram, obtain two copies of it.



- Draw the diagonal AC in one parallelogram, cut this parallelogram into two along AC and by placing one over the other see whether the two triangles that are obtained are congruent.
- Cut the other parallelogram along the diagonal BD and see whether the two triangles that are obtained in this case too are congruent.
- Present to the class the following discoveries made by you.
  - Relation between the sides
  - Relation between the angles
  - The areas of the two parts separated by the diagonals

Assessment and Evaluation:

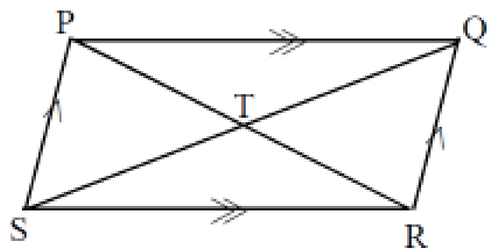
- Assessment criteria
  - Draws a parallelogram using various methods.
  - Describes the properties of a parallelogram.
  - Verifies by an activity, the theorems that "In a parallelogram, opposite sides are equal; opposite angles are equal; and each diagonal bisects the area of the parallelogram".
  - Works within the group in cooperation.
- Direct the students to do the relevant exercises in lesson 15 of the text book.

For your attention …

Development of the lesson:

- Prepare problems involving calculations and proofs of riders to suit learning outcomes 3, 4 and 5 relevant to competency level 23.6 and discuss them with the students.

- An activity can be prepared as shown below, by drawing a parallelogram and cutting out the triangles to verify the theorem relevant to competency level 23.7.



It can be shown that PT = TR and QT = TS by cutting out the triangles PTS and QTR and showing that they are congruent.

.

Assessment and Evaluation:

- Direct the students to the relevant exercises in chapter 15 of the text book.

For further reference

- http://www.youtube.com/watch?v=LhrGS4-Dd9I
- http://www.youtube. com/watch?v=oIV1zM8qlpk
- http://www.youtube. com/watch? v=TEr J-Yr67BI
- http://www.youtube.com/watch?v=_QTFeOvPcbY

16. Parallelograms II

Competency 23: Makes decisions regarding day to day activities based on geometrical concepts related to rectilinear plane figures.

Competency level 23.8: Identifies and uses the necessary conditions on the sides of a quadrilateral for it to be a parallelogram.

Competency level 23.9: Identifies and uses the necessary conditions on the angles of a quadrilateral for it to be a parallelogram.

Competency level 23.10: Identifies and uses the fact that a quadrilateral with certain special characteristics is a parallelogram.

Number of periods: 09

Introduction:
- It is expected under competency levels 23.8. 23.9 and 23.10 to show that certain quadrilaterals are parallelograms based on special characteristics.
- The conditions that should be satisfied for a quadrilateral to be a parallelogram can be stated as follows
- If the opposite sides of a quadrilateral are equal, then it is a parallelogram.
- If the opposite angles of a quadrilateral are equal, then it is a parallelogram.
- If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.
- If a pair of opposite sides of a quadrilateral are parallel and equal, then it is a parallelogram.

Learning outcomes relevant to competency level 23.10:

1. **Identifies the theorem, "If the diagonals of a quadrilateral bisect each other then it is a parallelogram".**
2. **Verifies the theorem, "If the diagonals of a quadrilateral bisect each other then it is a parallelogram".**
3. Proves riders using the theorem, "If the diagonals of a quadrilateral bisect each other then it is a parallelogram".
4. Identifies the theorem, "If a pair of opposite sides of a quadrilateral are equal and parallel, the quadrilateral is a parallelogram".
5. Proves riders using the theorem, "If a pair of opposite sides of a quadrilateral are equal and parallel, the quadrilateral is a parallelogram".

Glossary of Terms:

Quadrilateral	- චතුරස්‍රය	- நாற்பக்கல்
Parallelogram	- සමාන්තරාස්‍රය	- இணைகரம்
Opposite sides	- සම්මුඛ පාද	- எதிர்ப்பக்கங்கள்

Opposite angles	- සම්මුඛ කෝණ	- எதிர்க் கோணங்கள்
Diagonal	- විකර්ණය	- மூலைவிட்டம்
Set square	- විහිත චතුරස්‍රය	- மூலைமட்டம்
Intersection	- ඡේදනය	- இடைவெட்டு
Bisecting	- සමච්ඡේදනය	- இருகூறாக்கி

Instruction to plan the lesson:

An exemplar lesson plan based on the lecture-discussion method, together with a pair-wise activity to help students achieve learning outcomes 1 and 2 under competency level 23.10 is given below.

Time: 40 minutes

Quality Inputs:

- Enlarged copy of the student's work sheet

Instructions for the teacher:

Approach:

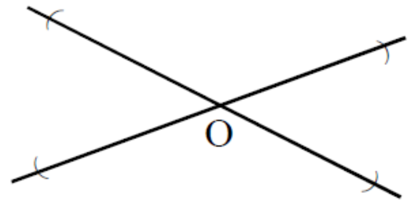
- By presenting a figure of a parallelogram, recall that a quadrilateral of which pairs of opposite sides are parallel is a parallelogram. Also recall what a diagonal is.
- Discuss methods of finding out whether the two lines are parallel (e.g., Using a set square - Obtaining lines perpendicular to a given line, by folding a piece of paper at some points on the given line, and marking where they meet the other line and then examining whether the distances are the same, or seeing whether the corresponding angles are equal or the alternate angles are equal)

Development of the lesson:

- Display an enlarged copy of the student's worksheet so that all the students can observe it.
- Get the students to work individually on the activity.
- Permit the students to discuss the activity with the student sitting next to him/her.
- At the end of the activity, based on their findings highlight that the diagonals of the quadrilateral bisect each other and that as the opposite sides of the quadrilateral are parallel, it is a parallelogram.
- Get the students to check the other characteristics of a parallelogram.
- Finally introduce the theorem that if the diagonals of a quadrilateral bisect each other then the quadrilateral is a parallelogram.

Student's work sheet:

- Draw two intersecting lines.
- Name the point of intersection as O.
- Take any length to the pair of compasses and mark two arcs of equal length on either side of O on one line.
- Repeat this for the other line, taking a length different from the length taken earlier.
- Join the points so marked in order to obtain a quadrilateral.
- Examine whether the opposite sides are parallel.
- Compare your findings with that of a friend of yours. Suggest a suitable name for the quadrilateral you have now got.



Assessment and Evaluation:

- Assessment criteria
    - Constructs two lines which bisect each other.
    - Checks whether two lines are parallel.
    - Arrives at the conclusion that if the diagonals of a quadrilateral bisect each other then the quadrilateral is a parallelogram.
    - By drawing the diagonals of a given quadrilateral states whether the quadrilateral is a parallelogram.
    - Co-operates to arrive at conclusions through generalization.
- Direct the students to the relevant exercises in lesson 16 of the textbook.

For your attention …
Development of the lesson:

- Competency levels 23.8 and 23.9 are based on the conditions that need to be satisfied for a quadrilateral to be a parallelogram. Thus an activity could be planned to construct a quadrilateral with opposite sides equal and show that it is a parallelogram. Similarly plan and implement activities to show that if the opposite angles of a quadrilateral are equal, then it is a parallelogram and that if a pair of opposite sides are parallel and equal, then it is a parallelogram.

Assessment and Evaluation:

- Direct the students to the relevant exercises in chapter 16 of the textbook.

For further reference:

- http://www.youtube.com/watch?v=GDcVdBAnBdU

17. Sets

Competency 30: Manipulates the principles related to sets to facilitate daily activities.

Competency level 30.1: Uses methods of denoting sets to facilitate problem solving.

Competency level 30.2: Solves problems using sets.

Number of periods: 08

Introduction :
- Methods of writing sets are known as set notation.
- Methods of denoting a set.
1. Describing a set within a pair of braces { }.
2. Writing the elements within a pair of braces.
3. Writing the elements of the set in a Venn diagram.
4. Expressing a set in set generating form.
- Sets of which the number of elements can be stated numerically are **finite sets.** If A and B are finite sets, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.
- Set of which the number of elements cannot be stated numerically are infinite sets.
- There are three operations on sets. The intersection of sets, union of sets, and the complement of a set.
- The set containing all the elements of A and of B is the union of A and B. Thus the union of the two sets can be written as $A \cup B = \{x: x \in A \text{ or } x \in B\}$
- The set containing the elements common to both the sets A and B is the intersection of A and B. This set can be written as $A \cap B = \{x: x \in A \text{ and } x \in B\}$.
- The set containing elements which belong to the universal set, but not to A is known as the complement of A. It can be written as $A' = \{x; x \in \varepsilon \text{ and } x \notin A\}$
- It is expected to solve problems using Venn diagrams and the relation $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ in this chapter.
- It is expected that the following lesson plan will be implemented after the students have understood the relationship between $n(A), n(B), n(A \cap B)$ and $n(A \cup B)$ for two finite sets A and B.

Learning outcomes relevant to competency level 30.2:
1. Expresses $n(A \cup B)$ in terms of $n(A)$, $n(B)$ and $n(A \cap B)$, when A and B are two finite sets.
2. **Represents two finite sets in a Venn diagram.**
3. **Illustrates in a Venn diagram, a region relevant to a given set operation.**
4. Describes in words, regions in a Venn diagram that contain information relevant to set operations.
5. Solves problems related to two sets using Venn diagrams.
6. Solves problems related to two finite sets using the formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

Glossary of Terms:

Set	- කුලකය	- தொடை
Finite set	- පරිමිත කුලකය	-முடிவுள்ள தொடை
Number of elements	- කුලකයක අවයව සංඛ්‍යාව	-மூலகங்களின் எண்ணிக்கை
Elements	- අවයව	-மூலகங்கள்
Set notation	- කුලක අංකනය	- தொடைக் குறிப்பீடு
Set operations	- කුලක කර්ම	- தொடைச் செய்கைகள்
Venn diagram	- වෙන් රූපය	- வெண்வரிப்படம்
Set generating form	- කුලක ජනන ස්වරූපය	- தொடைப்பிறப்பாக்கி வடிவம்
Subset	- උපකුලකය	- தொடைப்பிரிவு
Disjoint sets	- වියුක්ත කුලකය	- மூட்டற்ற தொடை
Joint set	- ඡේදන කුලකය	- மூட்டுள்ள தொடை

Instruction to plan the lesson

> Through this exemplar lesson plan, it is expected to develop the subject concepts related to learning outcomes 2 and 3 under competency level 30.2, using the lecture-discussion method to instill in students the skill of identifying and marking a region relevant to a set operation in a Venn diagram and to further strengthen this skill through an individual activity.

Time: 40 minutes

Quality Inputs:

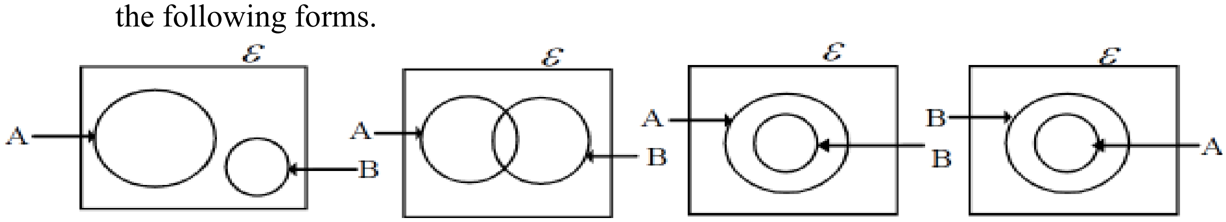
- Copies of the student's work sheet

Approach:

- Lead a discussion and describe the intersection of two sets, union of the two sets and complemented of a set by considering two sets with their elements listed.
- Discuss about subsets and disjoint sets.
- Continue the discussion by inquiring how two finite sets can be represented in a Venn diagram.

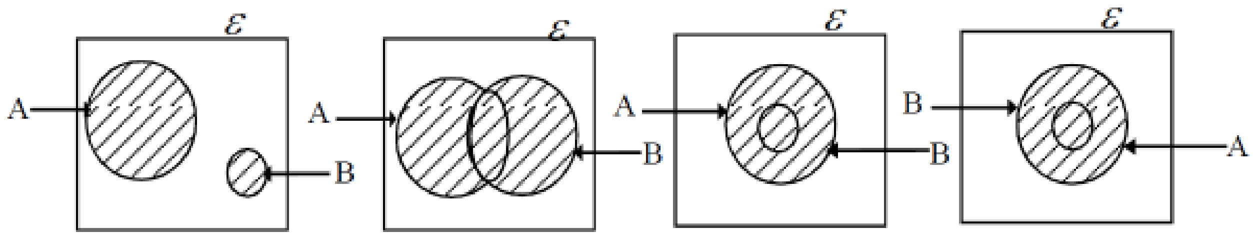
Development of the lesson:

- Discuss the fact that when we represent two sets in a Venn diagram, we obtain one of the following forms.



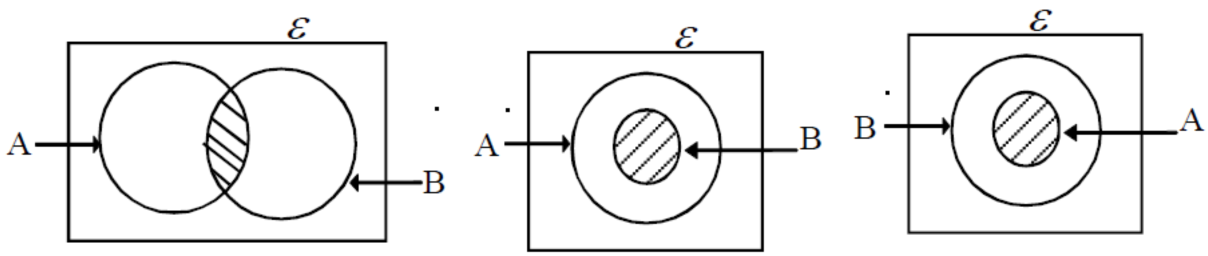
Disjoint sets | When they are intersecting/not disjoint | When B is a subset of A | When A is a subset of B

- Instruct the students to draw these Venn diagrams in their exercise books.
- Using Venn diagrams establish that when two sets are disjoint, then the intersection is the null set, that when the intersection is not the null set, then there are common elements in the intersection and that when all the elements of one set are contained in another set then it is a subset.
- Lead a discussion to establish that there are three set operations - **union of two sets, intersection of two sets and the complement of a set**.
- Use Venn diagrams such as those given here to illustrate the union of two sets.



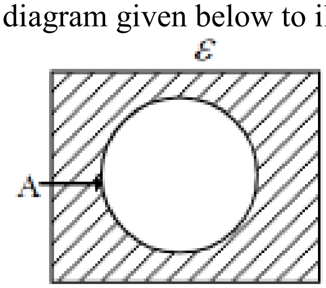
- Describe the union of the two sets A and B as the set consisting of all the elements in both the sets, shade the region showing the union, and explain that the union of the two sets A and B is denoted by $A \cup B$ using symbols.

- Use Venn diagrams such as those given below to illustrate the intersection of two sets.

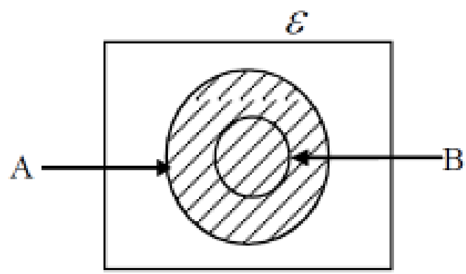


- Describe the intersection of the two sets A and B as the set consisting of the elements common to both A and B, and illustrate the relevant region in a Venn diagram as shown above. Explain that the intersection of A and B is denoted by $A \cap B$ using symbols.

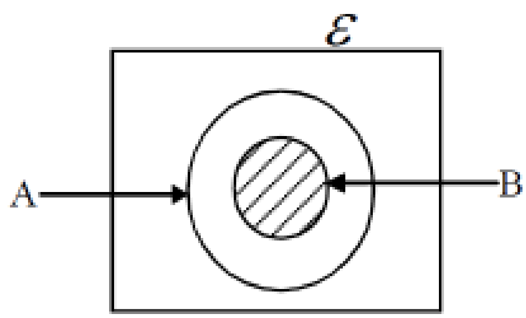
- Use the Venn diagram given below to illustrate the complement of a set.



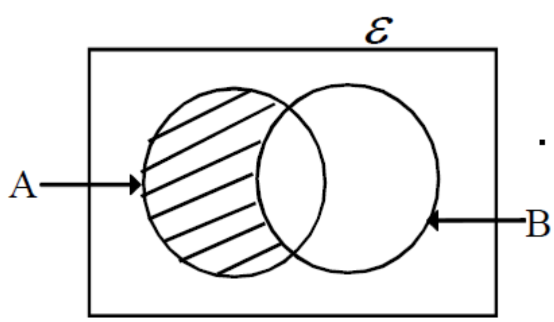
- Describe that the complement of the set A is the set of elements which are not in A, but are in the universal set. Explain that this is denoted by A' using symbols.
- How the union is represented when B is a subset of A can be explained using the Venn diagram given below.



- Use the following Venn diagram to explain how the intersection of the sets A and B is represented when B is a subset of A.



- Explain how the set consisting of the elements which belong to set A but not to set B is represented by using the following Venn diagram.

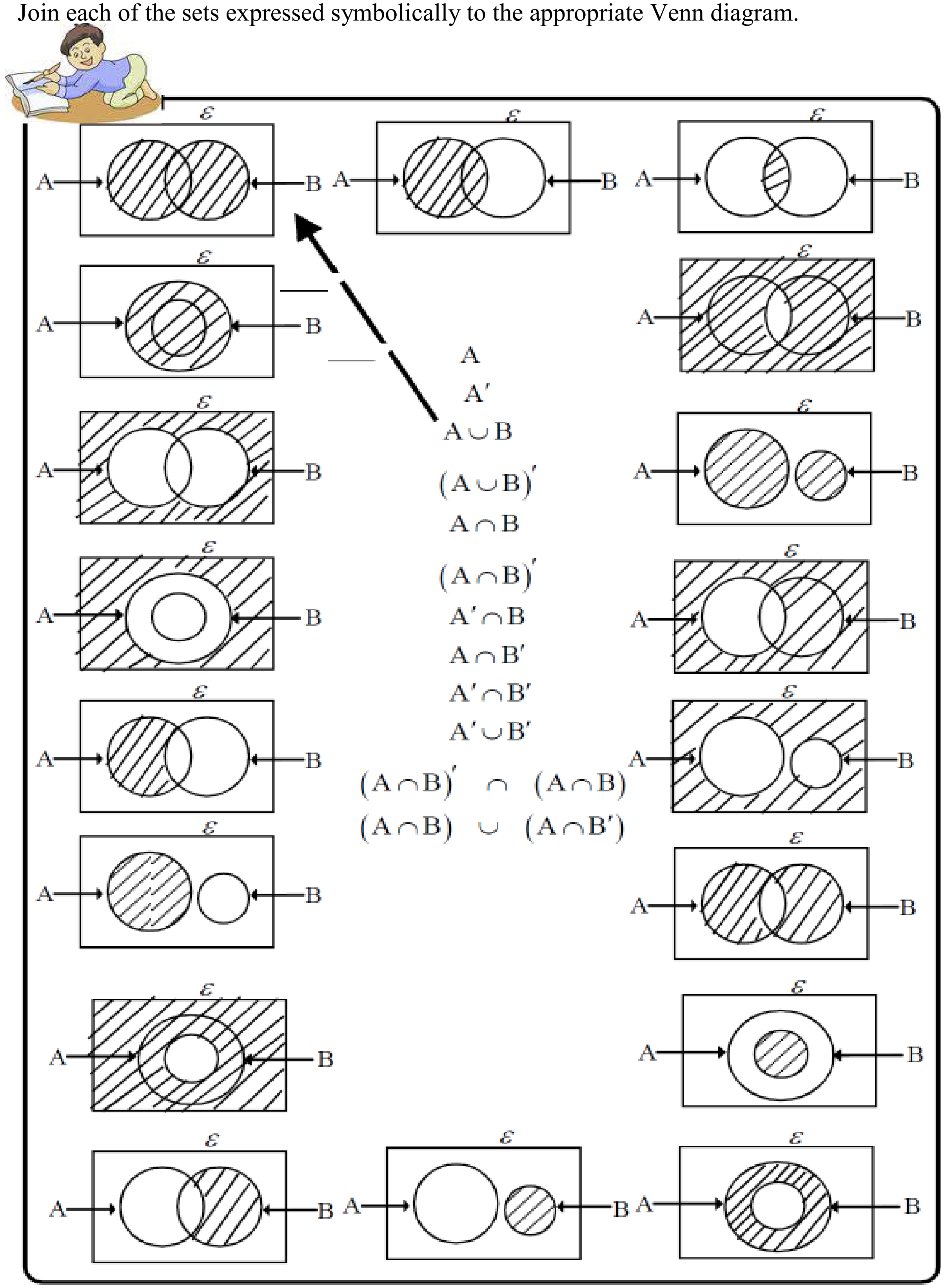


- Explain that this set is written as $A \cap B'$ using symbols.
- Discuss with the students and shade the area denoting $(A \cap B)' \cap (A \cup B)$ in a Venn diagram.
- Distribute a copy of the student's work sheet to each student and get them to join each Venn diagram to the appropriate set, after examining the illustrations, to strengthen their understanding of these concepts.
- Once the students have completed the task, discuss their answers.

- At the end of the lesson, emphasize that the union, intersection and complement are set operations and that they can be illustrated in Venn diagrams by shading the relevant region.

Student's work sheet

Join each of the sets expressed symbolically to the appropriate Venn diagram.



Assessment and Evaluation:

- Assessment criteria
  - Represents two finite sets in a Venn diagram.
  - Describes the set operations union, intersection and complement.
  - Illustrates the union, intersection and complement of sets in Venn diagrams.
  - Matches shaded regions of Venn diagrams with the relevant set operations.
  - Completes the assignment within the stipulated time.
- Direct the students to the relevant exercises in lesson 17 of the text book.

For your attention …

Development of the lesson:

- Get the students to explain in words the shaded regions of the Venn diagrams in the student's work sheet.
- Get the students to solve problems related to two finite sets using Venn diagrams and the formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

Assessment and Evaluation:

- Guide the students to the relevant exercises in chapter 17 of the textbook.

For further reference:

18. Logarithms I

Competency 6: Uses logarithms and calculators to easily solve problems in day to day life.

Competency level 6.1: Analyses the relationship between indices and logarithms.

Competency level 6.2: Uses the laws of logarithms to determine products and quotients of numbers.

Number of periods: 05

Introduction:

The number we get when we multiply a number by itself several times overcan be written as a power. $2 \times 2 \times 2 = 2^3$ and $2^3 = 8$. Here, 2^3 is a power. Its base is 2 and its index is 3. This index is known as the logarithm of the number expressed as a power. The base for this logarithm is 2.

The logarithm of 8 to the base 2 is 3. This is written as $\log_2 8 = 3$. It is important to state what the base is. This is because different numbers can have the same logarithm value under different bases and the same number can have different logarithm values under different bases.

e.g., $\log_2 8 = 3$, $\log_5 125 = 3$, $\log_2 64 = 6$, $\log_4 64 = 3$, $\log_8 64 = 2$.

If $a^b = x$, the index b can be written in general as $b = \log_a x$.

Accordingly,

(i) by substituting $x = a^b$ in $b = \log_a x$, we obtain, $b = \log_a a^b$. That is, $\log_a a^b = b$.

(ii) since $a^1 = a$, we obtain $1 = \log_a a$. That is, $\log_a a = 1$.

(This result can also be obtained by substituting $b = 1$ in (i) above.)

(iii) Since $a^0 = 1$, $0 = \log_a 1$. That is, $\log_a 1 = 0$.

(This result can also be obtained by substituting $b = 0$ in (i) above)

The laws of logarithm can be written as follows:

- $\log_a(pq) = \log_a p + \log_a q$
- $\log_a\left(\frac{p}{q}\right) = \log_a p - \log_a q$

In this section, it is expected to discuss about the relationship between indices and logarithms and the application of the laws of logarithms in finding products and quotients using logarithms.

Learning outcomes under competency level 6.1:

1. **Describes the logarithm of a number in terms of the base, when the number is expressed in index form.**
2. **Converts an expression in index form to logarithm form and an expression in logarithm form to index form.**

Glossary of Terms:

Indices - දර්ශක - சுட்டிகள்
Base - පාදය - அடி
Power - බලය - வலு
Logarithms - ලඝුගණක - மடக்கை

Instructions to plan the lesson:

A demonstration lesson based on the lecture-discussion method to develop the concepts relevant to learning outcomes 1 and 2 under competency level 6.1 and a game to further establish these concepts is given here.

Time: 40 minutes

Instructions for the teacher:
Approach:

- Display the table given here on the chalk board.
- Lead a discussion and fill in the table with input from the students.
- State that powers and the number equal to each power are given in the table. Get the students to state the powers and the corresponding numerical values.
- Elicit from the students how the numbers that appear in a power are named and remind them about the terms base and index.

Number (a)	Base			
	a^0	a^1	a^2	a^3
1	$1^0=1$	$1^1=1$	....	....
2	$2^0=1$	$2^1=2$	....	....
3	....	$3^1=3$	$3^2=9$	....
4	....	....	....	....
5	....	....	....	....
6	....	....	....	....
10	....	....	....	....

- Direct the students to write $\frac{1}{8}$ using the index notation.
- Guide the students to the following discussion by mentioning that, since $\frac{1}{8} = \frac{1}{2^3}$, we obtain $\frac{1}{8} = 2^{-3}$, and that for fractions such as $\frac{1}{8}$ we obtain negative powers. Inquire from them whether expressions in index form such as 2^3 can be converted to another form.

Development of the lesson:

- State that logarithms were first introduced by the mathematician John Napier.
- Mention that the index of an expression of the form $2^3 = 8$ is expressed as $3 = \log_2 8$.
- Thereby show that an expression in index form can be transformed into logarithm form.
- Stress the importance of stating the base in logarithmic expressions by using the examples

$4^2 = 16$ and $2^4 = 16$.

- Show using examples that the expression $a^x = y$ in index form is transformed into $log_a y = x$ in logarithmic form for negative indices too.
- Organize the following game to establish the knowledge on converting an expression in index form to logarithmic form and vice versa and ensure that all the students participate in the game.

- Display this chart on the chalk board.
- Explain how the game is conducted.
- Decide on the number of students who can participate in the game, based on the number of students in the class. (It will be good if all could participate.)
- The game consists of two parts.
- The first part is to convert an expression given in index form into an expression in logarithm form. The second part is to convert expressions in logarithm form to index form.
- To save time, get the students to come to the chalk board.

1 Player	2 Expression in index form	3 Whether correct or not (√ or ×)	4 Expression in logarithmic form	5 Whether correct or not (√ or ×)
1	A			B
2	C			D
3	E			
4				
5				
6				

- To begin, the teacher should write an expression such as $2^5 = 64$ in cage A.
- The first player should then mark √ in column 3 if the expression is correct or write the correct expression in this column if it is wrong.
- Then the first player should write the logarithmic form corresponding to the correct expression in index form in column 4, and leaving the cage B blank, write an expression in index form in cage C of the second column.
- The next player then comes in and marks in cage B, whether the logarithmic expression written by the first player in column 4 is correct or not (√ or ×) and then writes an expression in index form in cage E
- The game is continued in this manner.

Assessment and Evaluation:

- Assessment criteria
  - Explains the relationship between logarithms and indices.
  - Converts expressions in index form with positive indices into logarithmic form.
  - Converts expressions in logarithmic form into expressions in index form.
  - Trains to face challenges successfully.
  - Strengthens the knowledge gained by participating in the game.
- Guide the students to the relevant exercises in lesson 18 of the text book.

For yourattention..

Development of the lesson:

- Plan and implement a suitable lesson for the subject content under competency level 6.2.

Assessment and Evaluation:

- Guide the students to the relevant exercises in chapter 18 of the text book.

For further reference:

- http://www.youtube.com/watch?v=Z5myJ8dg_rM
- http://www.youtube.com/watch?v=fyshrv6YDVY
- http://www.youtube.com/watch?v=eTWCARmrzJ0
- http://www.youtube.com/watch?v=vtStuLV-HvQ
- http://www.youtube.com/watch?v=mQTWzLpCcW0

19. Logarithms II

Competency 6: Uses logarithms and calculators to easily solve problems in day to day life.

Competency level 6.3: Simplifies numerical expressions using the logarithm table.

Competency level 6.4: Uses the calculator to solve mathematical problems.

Number of periods: 05

Introduction :
The ability to read logarithms of numbers greater than 1 and simplify expressions which contain numbers greater than 1, as well as the ability to use the scientific calculator are expected to be developed under competency levels 6.3 and 6.4.
The logarithm of a number between 1 and 10 to the base ten is a number between 0 and 1. A table prepared by John Napier is used to find the logarithm of a number between 1 and 10 to the base ten.
The logarithm of a number between 1 and 10 given to 3 decimal places can be read using this table. To find the logarithm of a number greater than 10, the number should first be expressed as the product of a number between 1 and 10 and an integer power of 10, that is in scientific notation.
When a number is written in scientific notation, the index of the power of 10 is the characteristic of the logarithm of the number.
The logarithm of a number can be read from the table and the number relevant to any logarithm can also be found from the table. This is known as the antilogarithm.
As the logarithm of a number is the index of the power of 10 equal to the number, when numbers are multiplied or divided, we can use the laws of indices and the laws of logarithms to find the value of the product or the value of the quotient.
In the scientific calculator, different keys are introduced for various operations. The value of a numerical expression can be found easily using a scientific calculator and the accuracy of a simplification of a numerical expression including decimals done using logarithms can be verified using the calculator.

Learning outcomes under competency level 6.3:

1. **Using the logarithm table, finds the logarithm of numbers greater than 1.**
2. Using the logarithm table, multiplies and divides numbers greater than 1.
3. Using the logarithm table, simplifies expressions involving the multiplication and division of numbers greater than 1.

Glossary of Terms:

Index	- දර්ශකය	- சுட்டி
Power	- බලය	- வலு
Base	- පාදය	- அடி
Table of logarithm	- ලඝුගණක වගුව	- மடக்கை அட்டவணை
Scientific notation	- විද්‍යාත්මක අංකනය	- விஞ்ஞான முறைக் குறிப்பீடு
Characteristic	- පූර්ණාංශය	- முழு எண் பெறுமானம்
Mantissa	- දශමාංශය	- தசமக்கூட்டு
Anti logarithm	- ප්‍රතිලඝුගණකය	- முரன்மடக்கை
Mean difference	- මධ්‍යන්‍ය අන්තරය	- இடை வித்தியாசம்

Instructions to plan the lesson:

A demonstration lesson to develop the concepts relevant to learning outcome 1 under competency level 6.3 based on the lecture-discussion method is given here.

Time: 80 minutes

Quality Inputs:
- An enlarged copy of the table given under **developing the lesson**.
- Part of the table of logarithms enlarged.

Instructions for the teacher:
Approach:

- Recall what has been learnt about writing a number in index form as well as converting an expression given in index form into logarithmic form.
- Get the students to convert several expressions given in index form into logarithmic form and vice versa and discuss these conversions with them.
- Recall how a number is expressed in scientific form as well as the laws of indices and the laws of logarithms.

Development of the lesson:

- Present the following
  $10^0 = 1 \rightarrow \log_{10} 1 = 0.$
  $10^1 = 10 \rightarrow \log_{10} 10 = 1$
  to the class and elicit the fact that the logarithm of 1 to the base 10 is 0 and the logarithm of 10 to the base 10 is 1.
- Thereby establish the fact that logarithm to base 10 of a number between 1 and 10, for

example 5, is a number between 0 and 1

- Explain that therefore, in order to find the logarithm of 5 to the base 10, 5 should first be written as a power of 10 and that the index would then be $\log_{10} 5$.
- Introduce the table of logarithms stating that in order to find values like this, we use the table of logarithms discovered by John Napier.
- Make the students aware that this table contains logarithms to base 10 of numbers from 1 to 9.999.
- Enlighten the students of the fact that the table contains the indices of numbers written as powers of 10.
- Mention that lg is used to denote logarithm to the base 10. That is, that $\log_{10} x$ is also written as $\log x$ and as $\lg x$.

Num-	Logarithm										Mean difference								
ber	0	1	2	3	4	5	6	7	8	9									
1.0	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
1.1	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
1.2	0792	0828	0864	0839	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
1.3											1	2	3	4	5	6	7	8	9
1.4																			
..																			
..																			
..																			
..																			
9.9																			

- Display an enlarged table of logarithms and explain how the logarithm of a number is read.
- Using an example, show that the logarithm of a number with only one decimal place can be read from the zero column of the relevant row.
- Now show how the logarithm of a number such as 1.26 is read.
- Discuss with the students and obtain from the table, the logarithm of several numbers between 1 and 10 which contain 2 decimal places.
- Clarify that when numbers between 1 and 10 contain 3 decimal places, for the third decimal value we have to read the mean difference column and add the relevant amount to the number in the table corresponding to the second decimal place.
- Show how the logarithm of a number with 4 digits is read by using the example $\lg 1.264 = 0.1004 + 0.0014$.
- Now get the students to find the logarithms of given numbers. Help the weak students to find the logarithm. Thereby establish how the logarithms of numbers between 1 and 10 are read from the table.
- Explain how the logarithm of a number greater than 10 is found. We can read from the

table logarithms of numbers between 1 and 10 only. Show them that any number greater than 10 should first be written as product of a number between 1 and 10 and a power of 10.

- Show that for example, when 12.6 is written in scientific notation as 1.26×10^1, lg 12.6 = lg 10 + lg 1.26 = 1 + 0.1004 =1.1004
- Here the characteristic of lg 12.6 is 1 and the mantissa part is 0.1004.
  Explain that the logarithm of any number greater than 1 can be found in this manner.
- Once the students are familiar with finding the logarithms of numbers, a competition to read the logarithms could be arranged by dividing the students into two groups.
  (An enlarged logarithm table can be used and the questions can be put to them. Marks could be awarded by querying how the answers were obtained.)

Assessment and Evaluation:

- Assessment Criteria
  - Writes a number greater than 1 in scientific notation.
  - Accepts that the logarithm table contains the indices of the powers of 10 of each number between 1.0 and 9.999 expressed as a power of 10.
  - Reads and expresses the logarithm of numbers between 1 and 10.
  - Explains separately the characteristic and mantissa part of the logarithm of a number greater than 10.
  - Writes the logarithm of a number greater than 10 by identifying the characteristic and the mantissa.
- Direct the students to the relevant exercises in lesson 19 of the text book.

For your attention..

Development of the lesson:

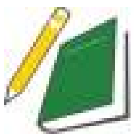
- Explain that the logarithm table has been used to simplify expressions for a long time, and that though the use of the logarithm table has reduced after the calculator was invented, in countries such as Sri Lanka, it is still used to do simplifications.
- Once the students are familiar with reading the logarithms of numbers greater than 1, guide them to find antilogarithms.
- First train them to find the antilog by presenting logarithms of numbers between 1 and 10 containing 2 decimal places.
- Then explain that when finding the antilogarithm of a number which is not in the table, by reading the value less than and closest to this number and considering the difference, the antilogarithm could be found.
- Explain that when finding products and quotients using logarithms, both the laws of indices and the laws of logarithms may be used.
- Discuss with the students how the calculator is used to simplify expressions

involving decimal numbers.

Assessment and Evaluation:

- Once the students are familiar with performing multiplications and divisions using the logarithm table, the answers obtained may be verified using a calculator.
- Direct the students to the relevant exercises in lesson 19 of the text book.

For further reference:



20. Graphs

Competency 20 : Easily communicates the mutual relationships that exist between two variables by exploring various methods.

Competency level 20.1: Determines the nature of the linear relationship between two variables.

Competency level 20.2: Graphically analyzes the mutual quadratic relationships between two variables.

Competency level 20.3: Analyzes the characteristic of a quadratic function by observation.

Number of periods: 09

Introduction

It is expected to analyze linear graphs of functions of the form $y = mx + c$, and the graphs of quadratic functions of the form $y = ax^2$ and $y = ax^2 + b$. Here a and b are non-zero.

The graph of a function is the representation of the ordered pairs corresponding to the function on the Cartesian coordinate plane. When a function is graphed, the independent variable is represented by the x-axis and the dependant variable is represented by the y-axis.

The graph of a function of the form $y = mx + c$ is a straight line. m is the gradient and c is the intercept of the straight line. The gradient is the slope of the straight line. The gradient (m) can be negative, positive or zero. Depending on the value of the gradient, we can decide the nature of the angle formed by the graph with the positive x-axis.

The point at which the graph intersects the y-axis can be decided by the intercept.

The graph of a quadratic function of the form $y = ax^2$ or $y = ax^2 + b$ (where $a, b \neq 0$) takes the shape of a parabola. If the value of a is positive, then the graph has a minimum value and if a is negative then the graph has a maximum value, both given by b. This minimum/maximum point is the turning point of the graph. The axis of symmetry of the graph is the straight line parallel to the y-axis which passes through the turning point. Therefore, the axis of symmetry of a curve which is the graph of a function of the form $y = ax^2$ or $y = ax^2 + b$ is always $x = 0$. That is, the curve is symmetric about the y-axis.

It is expected that the following specimen lesson plan will be implemented after the subject matter related to competency level 20.1 are established.

Learning outcomes relevant to competency level 20.2:

1. **Calculates the y values corresponding to several given x values of a function of the form $y = ax^2$ or $y = ax^2 + b$.**
2. **Draws the graph of a function of the form $y = ax^2$ or $y = ax^2 + b$ for a given domain.**
3. **Finds the maximum/minimum value, equation of the axis of symmetry and the coordinates of the turning point of a function of the form $y = ax^2$ or $y = ax^2 + b$, using its graph.**
4. **Using the graph, finds the interval of values of x corresponding to a given interval of values of the function, for a function of the form $y = ax^2$ or $y = ax^2 + b$.**
5. **Finds the roots of the equation $y = 0$ by considering the graph of the function y which is of the form $y = ax^2$ or $y = ax^2 + b$.**
6. Determines a function similar to a function of the form $y = ax^2$ or $y = ax^2 + b$ by considering its graph

Glossary of Terms:

Gradient	- අනුක්‍රමණය	- படித்திறன்
Coordinates	- ඛණ්ඩාංක	- ஆள்கூறுகள்
Straight line	- සරල රේඛාව	- நேர்கோடு
Maximum/minimum values	- උපරිම/අවම අගය	- உயர்இழிவுப் பெறுமானம்
Graph	- ප්‍රස්තාරය	- வரைபு
Axis of symmetry	- සමමිති අක්ෂය	- சமசீர் அச்சு
Turning point	- හැරුම් ලක්ෂ්‍යය	- திரும்பற்புள்ளி
Quadratic function	- වර්ගජ ශ්‍රිත	- இருபடிச்சார்பு
Interval	- ප්‍රාන්තරය	- ஆயிடை

Instructions to plan the lesson:

It is expected to cover the first five learning outcomes from the 6 learning outcomes under competency level 20.2. The guided discovery method will be used as the learning - teaching methodology to develop this lesson.

Time: 80 minutes

Quality Inputs:
- Demy papers/Bristol board
- Platignum pens

Instructions for the teacher:

Approach:

- Recall for a graph of the form $y = mx + c$, how the coordinate plane is drawn, how the coordinates are found and how the graph is drawn.
- Recall how integers can be substituted into the relevant equation in order to find the coordinates to draw the graph of $y = ax^2$ or $y = ax^2 + b$.

Development of the lesson:

- Enlighten the students on how an interval of suitable values is selected for x, how the coordinates are found and how a smooth curve is drawn in relation to the graph of $y = ax^2$ or $y = ax^2 + b$. Explain about the axis of symmetry and the turning point of the graph.
- Separate the students into 6 groups, distribute the work sheet and give the equation of the function each group is supposed to graph.
- Provide the groups with the necessary quality inputs.
- While the groups are engaged in the activity, guide them as required on how to draw a smooth curve, how to find the turning point and how to identify the axis of symmetry.
- Give each group an opportunity to present their findings to the entire class.
- When the presentations are all done, taking into account the findings by the students, enlighten them as to how a smooth curve of a quadratic function is drawn, about the maximum / minimum values, the axis of symmetry, and the roots of the equation y = 0 relative to the given function.
-

Student's work sheet:

Group	A	B	C	D	E	F
Quadratic function	$y = x^2$	$y = -x^2$	$y = 3x^2$	$y = x^2 - 4$	$y = -2x^2 + 3$	$y = -x^2 + 2$

- Focus your attention on the quadratic function assigned to you.
- Obtain seven (x, y) coordinates for the function in the range -3 to 3 (including both values)
- Mark the points you have found on the coordinate plane, and draw a smooth curve (graph). Ask the teacher for help if required.
- Mark the maximum/minimum point of the graph drawn by you.
- Write down the coordinates of the maximum/minimum point as well as the maximum/minimum value.
- Draw the axis of symmetry and write down its equation.
- Write down the roots of the equation $y = 0$ with the aid of the graph.
- Present to the class your graph drawn on the demy paper according to the instructions of the teacher, and the answers written down in relation to your graph.

Assessment and Evaluation:

- Assessment criteria
    - Draws a smooth graph of the function clearly and accurately, by obtaining the coordinates relevant to the function.
    - Finds the turning point of the graph and writes down its coordinates.
    - Obtains the maximum/minimum value of the function.
    - Draws the axis of symmetry and writes down its equation correctly.
    - Finds the correct roots of the equation y = 0.
- Direct the students to the relevant exercise in lesson 20 of the textbook.

Practical Situations:

- Discuss about the following situations in which the shape of the graph of a quadratic equation is used practically.
- The path of a projectile under gravity takes the shape of a parabola. Accordingly, since the shape of a quadratic function is also a parabola, the graphs we draw are a part of an orbital path.

For your attention…
Development of the lesson:

- Select suitable teaching – learning methodologies for concept development relevant to competency level 20.3 and learning outcome 6 under competency level 20.2.

Assessment and Evaluation:

- Direct the students to the relevant exercise in lesson 20 of the textbook.

For further reference:

- http://www.youtube.com/watch?v=hXP1Gv9IMBo
- http://www.youtube.com/watch?v=8XffLj2zvf4
- http://www.youtube.com/watch?v=R948Tsyq4vA
- http://www.youtube.com/watch?v=Kk9IDameJXk
- http://www. youtube. com/watch?v=jTC ZfMMcHBo

21. Rate

Competency 12: Manages time to fulfill the needs of the world of work.

Competency level 12.1: Manages time to perform daily duties efficiently.

Number of periods: 05

Introduction:

Defining speed, writing the relationship between distance, speed and time, drawing the distance-time graph and finding its gradient, i.e., distance/time gives the speed, time and speed, the rate at which a liquid flows etc., will be discussed in this chapter. The distance-time graph has been introduced newly to the syllabus. Only motion under uniform speed is discussed here.

It is expected that the following lesson plan will be implemented after the subject content relevant to learning outcomes 1 and 2 under competency level 12.1 is established.

Learning outcomes relevant to competency level 12.1:

1. Recognizes that the rate of change of distance with respect to time is **speed**.
2. Writes down the relationship between distance, time and speed.
3. **Represents the information on distance and time by a graph.**
4. **Recognizes that the speed is given by the gradient of a distance-time graph.**
5. Solves problems related to distance, time and speed.
6. Solves problems related to volume and time. (Including liquids flowing through pipes)
7. Performs daily tasks efficiently using speed and rate.

Glossary of Terms:

Distance - දුර - தூரம்
Time - කාලය - நேரம்
Speed - වේගය - கதி
Rate - ශීඝ්‍රතාව - வீதம்
Gradient - අනුක්‍රමණය - படித்திறன்
Distance time graph - දුර කාල ප්‍රස්තාර - தூரநேர வரைபு

Instructions to plan the lesson:

A demonstration lesson suitable for the learning - methodology based on the guided discovery and discussion method is given below with the aim of developing the subject concepts relevant to learning outcomes 3 and 4 under competency level 12.1

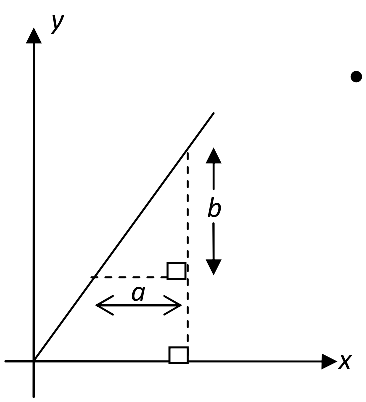
Time: 40 minutes

Quality Inputs:

- Student's work sheets
- Graph paper
- Straight edges

Instructions for the teacher:
Approach:

- Make the students aware of the fact that speed is the rate at which the distance changes with respect to time, that is, $speed = \frac{distance}{time}$, that the units of speed varies depending on the units used to measure the distance and the time. Recall accordingly that the units of speed are ms^{-1}, cms^{-1}, kmh^{-1}.



- Recall that by marking the coordinates corresponding to the points on a line such as $y = 2x$ on a coordinate plane and joining them, a straight line is obtained, and that the gradient of this straight line is given according to the figure by the value $\frac{b}{a}$.

Development of the lesson:

- Divide the class in a suitable manner into groups of 4 – 6 students.
- Provide each student with one sheet of graph paper.
- Give each group a copy of the student's work sheet.
- Select and provide each group with the set of data (table).
- By showing the relationship between the gradient of the graph and the speed calculated by each group, establish the fact that, if the speed takes a constant value, it is said to be uniform, and that the speed could be obtained by the gradient of the distance-time graph.

Student's worksheet:

- Given below are several tables containing the data noted down on the distance travelled and the time taken by several vehicles while travelling with a uniform speed.
- Engage in the activity using the data in the table given to you by your teacher.

I Motor bicycle

Time (s)	1	2	3	4	5	6	7	8
Distance (m)	5	10	15	20	25	30	35	40

II Motor car

Time (s)	1	2	3	4	5	6	7	8
Distance (m)	20	40	60	80	100	120	140	160

III Bus

Time (s)	1	2	3	4	5	6	7	8
Distance (m)	10	20	30	40	50	60	70	80

IV Three wheeler

Time (s)	1	2	3	4	5	6	7	8
Distance (m)	8	16	24	32	40	48	56	64

- According to the set of data your group was assigned, by considering the distance travelled by the end of each time unit (1, 2, 3, .. seconds) , calculate the speed of the vehicle.
- Thus discuss within the group whether the speed is uniform or not.
- By considering your set of data, draw the coordinate axes using a suitable scale.
- Graph distance versus time.
- Calculate the gradient of the graph.
- See whether there is a relationship between the speed obtained by calculation and the gradient of the graph.

Assessment and Evaluation:

- Assessment Criteria
  - Selects a suitable scale
  - Marks the points correctly and draws the graph.
  - Finds the gradient and calculates the speed.
  - Builds up a relationship between the gradient and the speed.
  - Arrives at a common conclusion by comparing the results of each member in the group.
- Direct the students to the relevant exercise in lesson 21 of the text book.

For further reference:

- http://www.youtube.com/watch?v=mt6Nq0dzFjo
- http://www.youtube.com/watch?v=hAy_bavEVCQ

22. Formulae

Competency 19: Explores the methods by which formulae can be applied to solve problems encountered in day to day life.

Competency level 19.1: Investigates the methods by which formulae can be used to solve problems.

Number of periods: 03

Introduction
A formula is a relationship between quantities. We represent it in the form of an equation here. When a certain quantity is expressed in terms of other quantities, then it is known as the subject of the formula.
In a formula where squares of terms or square roots of terms are involved, any quantity can be made the subject. In formulae where squares or square roots are involved, the value of an unknown quantity can be found by substituting values of the other quantities.
It is expected to discuss the above subject matter relevant to competency level 19.1 in this chapter.

Learning outcomes relevant to competency level 19.1:

1. Changes the subject of a formula which contains squares and square roots.
2. Finds the value of a given term by substituting the given values into a formula which contains squares and square roots.
3. Uses formulae to solve problems.

Glossary of Terms:

Formulae	- සූත්‍රය	- சூத்திரம்
Subject	- උක්තය	- எழுவாய
Quantity	- රාශිය	- கணியம
Square	- වර්ගායිතය	- வர்க்கம்
Square root	- වර්ගමූලය	- வர்க்கமூலம்
Substitution	–ආදේශ කිරීම	- பௌதீக நிகழ்வு

Instructions to plan the lesson:

A demonstration lesson aimed at building the concepts relevant to learning outcome 1 under competency level 19.1, using a group activity is given below.

Time: 40 minutes

Quality Inputs:

- Copies of the student's worksheet.
- Sets of cards as given in the annex 1

Instructions for the teacher:
Approach:

- Recall that a formula is a relationship between quantities by taking the following formulae as examples.
  (i) $P = ma$ (ii) $v = u + ft$ (iii) $T = a + (n - 1)d$
- Remind the students that the subject of the three formulae, $P = ma$, $v = u + ft$ and $T = a + (n - 1)d$ are respectively P, v and T.
- By getting the students to change the subject of one of the above formulae, strengthen their knowledge of the subject of a formula.
- Explain that $\sqrt{x^2} = x$ and that $\left(\sqrt{x}\right)^2 = x$.
- Direct the students to the following activity after discussing an example on making a selected quantity the subject of a formula which contains a term which is a square or a square root.

Development of the lesson:

- Divide the class into 3 or 6 groups depending on the number of students.
- Distribute copies of the student work sheet and the cards to each group and engage them in the activity.
- After they have finished, allow them to present the answers with reasons, while verifying their accuracy.
- Lead a discussion and highlight the fact that a given quantity in a formula which contains a square or a square root term can be made the subject and that in doing so, the inverse operations need to be performed.

Student's Worksheet:

	Group 1	Group 2	Group 3
Part I	u in $v^2 = u^2 + 2fs$	l in $T = 2\pi\sqrt{\frac{l}{g}}$	R in $A = \pi(R^2 - r^2)$
Part II	r in $V = \pi r^2 h$	v in $C = \frac{1}{2}mv^2$	v in $F = \frac{mv^2}{gr}$

- Pay attention to the formulae in part I and the set of cards which your group received.

- Carefully observe the steps in the set of cards your group received.
- Discussing the steps be followed in order to make the given term the subject of the formula in part I, arrange the cards.
- Similarly discuss within the group the steps to be followed to make the term given in Part II the subject of the relevant formula.

Assessment and Evaluation:

- Assessment criteria
  - Identifies the steps to be taken, when a given quantity is to be made the subject.
  - Accepts that when a given term in a formula is to be made the subject, the inverse operations must be performed.
  - Makes a given quantity the subject of a given formulae.
  - Respects the ideas of others within the group.
  - Works cooperating with each other within the group.
- Direct the students to the relevant exercises in lesson 22 of the text book.

Practical situations:

- Discuss with the students situations where calculations are facilitated by using formulae.
  E.g. The area of circle (A) can be found using $A = \pi r^2$ and the volume of a cylinder V can be found using $V = \pi r^2 h$.

For your attention…
Development of the lesson:

- Direct the students to find the value of a given quantity in a formula which includes squares and square roots, by substituting values for the remaining quantities in the formula. Get them to practice this using several examples.
- Discuss with the students how formulae can be used to solve problems.

Assessment and Evaluation:

- Direct the students to the relevant exercise in lesson 22 of the textbook.

For further reference:

Annex 01

Set 01	Set 02	Set 03
$v^2 = u^2 + 2fs$	$\sqrt{\frac{l}{g}} = \frac{T}{2\pi}$	$R^2 - r^2 = \frac{A}{\pi}$
$u^2 + 2fs = v^2$	$\frac{l}{g} = \left(\frac{T}{2\pi}\right)^2$	$R^2 = \frac{A}{\pi} + r^2$
$u^2 + 2fs - 2fs = v^2 - 2fs$	$2\pi\sqrt{\frac{l}{g}} = T$	$\pi\left(R^2 + r^2\right) = A$
$u^2 = v^2 - 2fs$	$\frac{2\pi\sqrt{\frac{l}{g}}}{2\pi} = \frac{T}{2\pi}$	$R^2 - r^2 + r^2 = \frac{A}{\pi} + r^2$
$\sqrt{u^2} = \sqrt{v^2 - 2fs}$	$\left(\sqrt{\frac{l}{g}}\right)^2 = \left(\frac{T}{2\pi}\right)^2$	$\frac{\pi\left(R^2 + r^2\right)}{\pi} = \frac{A}{\pi}$
$u = \sqrt{v^2 - 2fs}$	$\frac{l}{g} \times g = \left(\frac{T}{2\pi}\right)^2 \times g$	$\sqrt{R^2} = \sqrt{\frac{A}{\pi} + r^2}$
	$l = g\left(\frac{T}{2\pi}\right)^2$	$R = \sqrt{\frac{A}{\pi} + r^2}$

23. Arithmetic Progressions

Competency 2: Makes decisions for future requirements by investigating the various relationships between numbers.

Competency level 2.1: Identifies arithmetic progressions and solves related problems.

Competency level 2.2: Investigates the various behavioral patterns of arithmetic progressions.

Number of periods: 07

Introduction :
When numbers are arranged to form a pattern it is known as a number sequence. When the difference between two consecutive terms of a sequence is a constant then that sequence is known as an arithmetic progression. The difference between two consecutive terms is known as the **common difference.** The first term is denoted as a and the common difference as d. The nth term of an arithmetic progression is denoted by T_n. This is given by the formula $T_n = a + (n - 1)d$. The sum of the first n terms of an arithmetic progression is denoted by S_n, last term is denoted by l . Since the sum of the first n terms of an arithmetic progression is given by $S_n = \frac{n}{2}(a + l)$ and $l = T_n = a + (n - 1)d$, the formula $S_n = \frac{n}{2}\{2a + (n - 1)d\}$ can be developed for the sum, and thus two formulae for the sum are obtained.. The above formulae can be used to find a given term of a given arithmetic progression, the sum of a given number of terms, which term of an arithmetic progression a given term is, and the number of terms in a given sum. Day to day problems can be solved by using the knowledge on arithmetic progressions.
It is expected that the activity given below will be given to students once they have understood the subject matter related to the learning outcomes relevant to competency level 2.1 and the first three learning outcomes of competency level 2.2 well.

Learning outcomes relevant to competency level 2.2:

1. Develops the formulae $S_n = \frac{n}{2}\{2a + (n - 1)d\}$ and $S_n = \frac{n}{2}\{a + l\}$ for the sum of the first n terms of an arithmetic progression.
2. Finds the sum of the first n terms of an arithmetic progression using the formulae.
3. Using the formulae, finds the number of terms in a progression, when the sum of the first n terms is given.
4. Solves problems related to arithmetic progressions.

Glossary of Terms:

Arithmetic progression - සමාන්තර ශ්‍රේඪී - கூட்டல் விருத்தி
Number sequence - සංඛ්‍යා අනුක්‍රම - எண் தொடர்
First term of an arithmetic progression - සමාන්තර ශ්‍රේඪීයක පළමු පදය - ஒரு கூட்டல் விருத்தியின் முதலாம்

உறுப்பு
Common difference - පොදු අන්තරය - பொதுவித்தியாசம்
n^{th} term - n වන පදය - ஆம் உறுப்பு
Consecutive terms of a number sequence - සංඛ්‍යා අනුක්‍රමයක අනුයාත පද - ஓர் எண்தொடரில்

அடுத்துகள் உறுப்புகள்

Instructions to plan the lesson:

This lesson plan has been proposed to achieve learning outcome 4 which is to easily solve problems in day to day life using the knowledge on arithmetic progressions. This should be implemented after developing a formula for the sum of the first n terms of an arithmetic progression and learning how to use it. It is expected to use the method of **problem based learning** here.

Time: 40 minutes

Instructions for the teacher:
Approach:

- Recall the formulae learnt during the lesson on arithmetic progression.
- Discuss simple examples to show that we can use the knowledge on arithmetic progressions to solve day to day problems.

Development of the lesson:

- Group the students as necessary and distribute copies of the work sheets.
- Get the students to do the activity.
- Once the students' proposals have been prepared, let them present them to the class.
- Assess the validity and suitability of the proposals made by the students.
- Explain further by citing practical situations that the knowledge on arithmetic progressions could be used to solve day to day problems easily.

Student's worksheet:

- A drama has been planned to be staged in the outdoor theatre of a school. The organising committee has announced that the following conditions must be met when the seating arrangement is planned.

Conditions

* Seats should be provided for an audience of at least 400 people.
* There should be more than 3 but less than 8 seats in the first row for special invitees.
* Each row must contain 3 seats more than the number of seats in the preceding row.
* The maximum number of seats that could be in a row is 50.

- Your group has been given the task of proposing a suitable seating arrangement.
- Organize your proposal according to the following guide lines.
  * How many seats should be placed in the first row?
  * How many rows should there be?
  * How many seats should be placed in the last row?
  * What is the total number of seats?
  * How many vacant seats will there be after 420 people take their seats?
- Check whether your proposal satisfies all the conditions.
- What mathematical subject content did you consider when preparing your proposal easily and efficiently?
- Prepare to present your proposal to the whole class.

Assessment and Evaluation:

- Assessment criteria
  - Identifies the problem.
  - Select a suitable methodology to solve the problem.
  - Collects the necessary and sufficient data to solve the problem.
  - Analyzes data to check how the seats could be arranged under the given conditions.
  - Presents the proposal to arrange the seats in an attractive manner.
- Direct the students to the relevant exercises in lesson 23 of the text book.

For your attention…

Development of the lesson:

- After the students have learnt how to directly use the formula to find the sum of the first n terms of an arithmetic progression, they should be directed to tackle problems which involve solving pairs of simultaneous equations using the formula to find the n^{th} term of an arithmetic progression, i.e., $T_n = a + (n-1)d$ and the formulae to find the sum of the first *n* terms of an arithmetic progression, namely $S_n = \frac{n}{2}(a + l)$ and $S_n = \frac{n}{2}\{2a + (n-1)d\}$.

Assessment and Evaluation:

- Direct the students to the relevant exercises in lesson 23 of the text book.

For further reference:

- http://www.youtube.com/watch?v=Uy_L8tnihDM
- http://www.youtube.com/watch?v=cYw4MFWsB6c

24. Algebraic Inequalities

Competency 18: Analyzes the relationships between various quantities related to real-life problems.

Competency level 18.1: Solves problems in daily life related to the inequality of two quantities.

Number of periods: 06

Introduction :
Solving inequalities such as $x + a \gtreqless b \ (a, b \in \mathbb{Z})$ and $ax \gtreqless b \ (a \neq 0)$, and representing the solutions, both integral and not, on the number line have been discussed in grade 9. Writing the integral solutions, and representing the interval of solutions on the number line for inequalities of the form $ax + b \gtreqless c \ (a, b, c \in \mathbb{Z}, \ a \neq 0)$, representing inequalities of the form $x \gtreqless a, y \gtreqless b$ and $y \gtreqless x$ on a coordinate plane and solving day to day problems on inequalities are expected to be done in this lesson.

Learning outcomes relevant to competency level 18.1:

1. **Writes down the integral solution set of the inequalities** $\boldsymbol{ax + b < c \ ; \ ax + b > c \ ; ax + b \leq c \ ; \ ax + b \geq c.}$
2. Represents the solutions of the inequalities $ax + b < c$; $ax + b > c$; $ax + b \leq c$; $ax + b \geq c$ on a number line.
3. Represents inequalities of the form $x < a, x > a, x \leq a, x \geq a$ on a coordinate plane.
4. Represents inequalities of the form $y < b, y > b, y \leq b, y \geq b$ on a coordinate plane.
5. Represents inequalities of the form $y < x, y > x, y \leq x, y \geq x$ on a coordinate plane.
6. Recognizes that inequalities can be used to represent information in day to day life.
7. Uses inequalities to solve problems in day to day life.

Glossary of Terms:

Inequalities	- අසමානතා	- சமனிலிகள்
Number line	- සංඛ්‍යා රේඛාව	- எண்கோடு
Algebraic Inequalities	- වීජීය අසමානතා	- அட்சரகணிதச் சமனிலிகள்
Solutions	- විසඳුම්	- தீர்வுகள்
Solution set	- විසඳුම් කුලකය	- தீர்வுத் தொடைகள்

Instructions to plan the lesson:

This demonstration lesson is prepared with the aim of developing the subject concepts relevant to learning outcome 1 of competency level 18.1 through a group activity based on solving algebraic inequalities.

Time: 40 minutes

Quality inputs:

- Copies of the student's worksheet
- Demy paper, platignum pens

Instructions for the teacher:

Approach:

- Discuss using examples, the fact that if a positive number or a negative number is added to both sides of an inequality, the inequality remains unchanged.
- Discuss using examples, the fact that if both sides of an inequality are multiplied or divided by a positive number, the inequality remains unchanged.
- Discuss using examples the fact that, if both sides of an inequality are multiplied or divided by the same negative number, the inequality changes.
- Discuss solving inequalities of the form $x + a \gtreqless b \; (a, b \in \mathbb{Z})$ and $ax \gtreqless b \; (a \neq 0)$, and representing the integral solutions as well as ranges of solutions on the number line.
- Engage the students in the activity inquiring how inequalities of the form $ax + b < c$; $ax + b > c$; $ax + b \leq c$; $ax + b \geq c$ are solved.

Development of the lesson:

- Group the students in a suitable manner, give each group a copy of the work sheet and engage the students in the activity.
- Once the students have finished the group activity, lead a discussion and highlight how mathematical operations are used to solve inequalities of the type $ax + b \gtreqless c$. Discuss the fact that the inequality remains unchanged when a positive number or a negative number is added to both sides as well as when both sides are multiplied or divided by the same positive number, and that when both sides are multiplied by the same negative number, the inequality changes. Also discuss the fact that when the solution set is considered, if the equality sign is there along with the inequality sign, then the value of the equality is included in the solution set.

Student's worksheet:

Group A $\rightarrow$ $2x + 3 < 11$

Group B $\rightarrow$ $3x - 5 \geq 10$

Group C $\rightarrow$ $6 - 4x < -10$

Group D $\rightarrow$ $9 \geq 4 - \frac{x}{2}$

- Solve the inequality assigned to your group, showing each step clearly.
- Write within brackets the mathematical operation done in each step.
- Write down the set of integers you got as solutions.
- On a demy paper, write down how you solved the inequality and present it to the class.

Assessment and Evaluation

- Assessment Criteria
    - Solves the given inequality step by step.
    - Describes the mathematical operations used in each step.
    - Writes down the correct solution set.
    - Explains how the inequality was solved.
    - Works within the group cooperating with each other.
- Direct the students to the relevant exercises in lesson 24 of the text book.

For your attention

Development of the lesson:

- In the exercises to strengthen the knowledge on solving inequalities, pay attention to the changes in the signs of a, b and c and the changes in the locations of the terms in inequalities of the form
  $ax + b < c;\ \ b + ax < c;\ ax - b \geq c;\ \ b - ax \leq c;$
  $ax + b \leq -c; c \leq ax + b;\ c > b - ax$
- Discuss with the students using a suitable method, about the interval of solutions that is obtained when inequalities of the above types are solved.
- Discuss using a suitable method, how inequalities of the following forms are represented on a coordinate plane.
  $x < a,\ x > a,\ x \leq a,\ x \geq a,\ y > b,\ y < b,\ y \leq b,\ y \geq b,\ y < x, y > x,$
  $y \geq x,\ y \leq x.$

Assessment and Evaluation:

- Direct the students to the relevant exercises in lesson 24 of the text book.

For further reference:

- http://www.youtube.com/watch?v=y7QLay8wrW8
- http://www.youtube.com/watch?v=SgKBBUFaGb4
- http://www.youtube.com/watch?v=XOAn5z8mkvI

25. Frequency Distributions

Competency 28: Facilitates daily work by investigating the various methods of representing data.

Competency level 28.1: Extends frequency tables to easily communicate data.

Competency 29: Makes predictions after analyzing data by various methods to facilitate daily activities.

Competency level 29.1: Uses representative values to interpret data.

Number of periods: 10

Introduction:

There are two types of data, continuous data and discrete data. Those who are present on a certain day; the number of fruits in a bunch of banana, are example of discrete data, while length, mass, time etc. are continuous data. Data is grouped for easy handling. Under competency level 28.1, it is expected to identify types of data and to study mid-values of grouped frequency distributions.

Among the measures of central tendency, the mode, the median and the mean are frequently used representative values. It is expected under competency level 29.1 to discuss interpreting data using these representative values.

In finding the mean of grouped data, the **mid value** of a class interval is taken as the value representing that particular class interval. It is assumed that all the data within the group are represented by the mid value. When the mid value of a group is *x*, the corresponding frequency is *f* and the symbol Σ denotes the sum, the mean is given by $\frac{\Sigma fx}{\Sigma f}$. Here Σf is the sum of the frequencies which is the number of data values. When finding the mean of ungrouped data we may sometimes come across many large scores in the frequency distribution. In such situations, finding Σfx can be a difficult task. Therefore a certain score is assumed to be the mean. The difference between this number and each score is recorded using either a positive or a negative sign depending on whether it is greater or less than the assumed mean. These values are known as **deviations**. Deviations are obtained by subtracting the assumed mean from each score, and are recorded under the column *d*. When the assumed mean is A and the deviation is *d*, the mean of the frequency distribution can be written as $A + \frac{\Sigma fd}{\Sigma f}$ .The means of grouped frequency distributions too can be found using the above formula by considering the mid-value of one of the class intervals as the assumed mean. Finding the deviation is facilitated by using the mode or the median as the assumed mean.

It is expected that the following lesson plan will be implemented after the subject concepts under competency level 28.1 are established.

Learning outcomes relevant to competency level 29.1:

1. **Calculates the mean of a grouped set of data using the mid-value.**
2. Calculates the mean of a grouped set of data using the assumed mean.
3. Identifies the easiest method of finding the mean of a grouped set of data.
4. Expresses the advantages/disadvantages of calculating the mean as the central tendency measurement to interpret data.
5. Recognizes that the mean can be used to numerically estimate daily requirements.
6. Makes predictions for daily requirements by using the mean.

Glossary of Terms:

Data	- දත්ත	- தரவு
Continuous data	- සන්තතික දත්ත	- தொடர்மாறி
Discrete data	- විවික්ත දත්ත	- பின்னகமாறி
Grouped data	- සමූහිත දත්ත	- கூட்டமாக்கப்பட்ட தரவுகள்
Mid value	- මධ්‍ය අගය	- நடுப்பெறுமானம்
Mean	- මධ්‍යන්‍යය	- இடையைப்
Assumed mean	- උපකල්පිත මධ්‍යන්‍යය	- எடுகொண்ட இடையைப்
Deviation	- අපගමනය	- விலகல்
Measure of central tendency	- කේන්ද්‍රික ප්‍රවණතා මිනුම්	-

Instructions to plan the lesson:

A demonstration lesson plan to develop the subject concepts relevant to competency level 29.1 using a guided discovery method is given below.

Time: 40 minutes

Quality inputs:

- Copies of the student's work sheets
- Demy papers
- Marker pens

Instructions for the teacher:
Approach:

- Discuss the fact that the mode, median and mean are used as measures of central tendency of frequency distributions, and citing various situations and examples, show that depending on the distribution, the mode, median or mean could be the suitable representative value.
- Show using examples that the mean of a set of data is obtained by dividing the sum of all the values by the number of values.

- Explain that, since the actual values of the data in a class interval are not known for a grouped frequency distribution, the mid-value of the interval is used as the representative value of all the data values within that interval. Show how the mid-value is found using examples.

Development of the lesson:

- Group the students in a suitable manner and give each group a copy of the student's work sheet.
- Let the groups present their findings.
- After the activity is completed, taking into consideration the findings of the groups, lead a discussion on how the mean of a grouped frequency distribution is found using the mid-values.
- Highlight the following facts at the discussion.
    - For a grouped frequency distribution, the product (fx) of the mid-value (x) of a class interval and the relevant frequency (f) gives the sum of the data values in that interval.
    - The total sum of the data values in all the class intervals of a grouped frequency distribution is given by the sum of the values in the fx column. This value is denoted by Σfx.
    - The total frequency of a grouped frequency distribution, which is the total number of data values, is obtained by taking the sum of the frequencies of the class intervals. This value is denoted by Σf.
    - The mean of the distribution can be calculated using the relationship Mean $= \frac{\Sigma fx}{\Sigma f}$.

Student's worksheet:

- Study the grouped frequency distribution given below.

  Information on the quantity in kilogrammes of beans brought to a vegetable collecting centre on a certain day by some farmers is as follows.

Mass of Beans (Kg)	4-8	8 – 12	12-16	16 - 20	20 - 24
Number of farmers	3	7	15	11	4

- Copy the following table onto the demy paper you have received.

Class interval	Mid-value (x)	Frequency (f)	fx

- Complete the class interval column using the given information.
- Complete the mid-value column (x) relevant to the class intervals.

- Complete the frequency column (f) using the given information.
- Complete the fx column by multiplying the respective numbers in the frequency column and the mid-value column.
- Discuss and determine how many farmers supplied beans to this centre.
- Calculate the total mass of beans supplied to this centre on that day.
- Thereby calculate in kilogrammes, the mean mass of beans supplied by a farmer on that day.
- Present the discoveries made by your group to the entire class.

Assessment and Evaluation:

- Assessment Criteria
  - For a given grouped frequency distribution, uses the mid-value of a particular class interval as the representative value of the data in that interval.
  - Calculates the mid-value of each class interval and completes the mid-value column.
  - Completes the fx column.
  - Calculate the mean using the mid values.
  - Works within the group appreciating the contributions made by others.
- Direct the students to the relevant exercises in lesson 25 of the text book.

For your attention..

Development of the lesson:

- Plan a suitable methodology and use it to help students to calculate the mean of a grouped frequency distribution using the assumed mean, to identify the easiest method to find the mean of a grouped frequency distribution, and to express the advantages / disadvantages of using that method.
- Plan a suitable method to give students the opportunity to estimate daily requirements numerically using the mean and also to predict future requirements using the mean.

Assessment and Evaluation:

- Direct the students to the relevant exercises in lesson 25 of the text book

For further reference:

26. Chords of a Circle

Competency 24: Thinks logically to make decisions based on geometric concepts related to circles.

Competency level 24.1: Identifies and applies the theorem on the relationship between a chord and the centre of a circle.

Competency level 24.2: Applies the converse of the theorem on the relationship between a chord and the centre of a circle.

Number of periods: 06

Introduction:

The straight line segment joining two points on a circle is a chord. A chord divides the circle into two segments. Many such chords can be drawn to a given circle. Their lengths may differ. The chord of greatest length is a diameter. If a circular piece of paper is folded such that two parts of a chord drawn on it overlap each other, the crease passes through the centre.

The theorem that **the line joining the midpoint of a chord of a circle to the centre is perpendicular to the chord** is a theorem on the chords of a circle.

The perpendicular from the centre of a circle to a chord bisects the chord, is the converse of the above theorem.

It is expected to discuss the above theorem and its converse in this chapter.

Learning outcomes relevant to competency level 24.1:

1. Identifies the theorem that **the line joining the midpoint of a chord of a circle to the centre is perpendicular to the chord.**
2. Verifies the theorem that **the line joining the midpoint of a chord of a circle to the centre is perpendicular to the chord.**
3. Performs calculations using the theorem that **the line joining the midpoint of a chord of a circle to the centre is perpendicular to the chord.**
4. Proves riders using the theorem that the line joining the midpoint of a chord of a circle to the centre is perpendicular to the chord.
5. Formally proves the theorem that the line joining the midpoint of a chord of a circle to the centre is perpendicular to the chord.

Glossary of Terms:

Chord	- ජ්‍යාය	- நாண்
Mid point	- මධ්‍ය ලක්ෂ්‍යය	- நடுப்புள்ளி
Centre	- කේන්ද්‍රය	- மையம்
Theorem	- ප්‍රමේයය	- தேற்றம்
Converse	- විලෝමය	- மறுதலை
Proof	- සාධනය	- நிறுவல்

Instructions to plan the lesson:

A specimen lesson plan which involves an individual activity based on exploration is given below, in order to build the concepts relevant to learning outcomes 1, 2 and 3 under competency level 24.1.

Time: 40 minutes

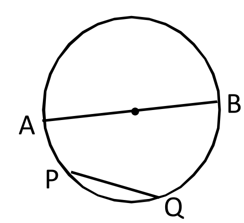
Quality Inputs:

- Circles of different radii with the centre marked, cut out from paper (one per student)
- Copies of the student work sheet

Instructions for the teacher:
Approach:

- Display a circle on the chalk board and inquire about the centre, radius, diameter and chord.
- Discuss with the students and mark each of them on the circle.
- Draw a figure like this on the chalk board and obtain from the students the words to fill the blanks in the two sentences given below.

AB is a .... of the circle with centre O
PQ is a .... of the circle with centre O



- Explain that the line joining two points on a circle is a chord, that the chord which passes through the centre is a diameter and that the longest chord is the diameter.

- Recall Pythagoras' relation which is a relationship between the sides of a right angled triangle, and engage the students in the activity mentioned below.

Development of the lesson:

- Prepare every student to participate in the activity.
- Distribute among the students, copies of the work sheet (one per student) and engage them in the activity.
- Give the students a chance to present their findings to the class. State that the line joining the midpoint of a chord to the centre of the circle is perpendicular to the chord, that what they have discovered is a theorem which can be confirmed by measurement, by reasoning and by using a corner in the shape of a right angle .Once the students have grasped the facts, explain that by joining two radii to the pair of perpendicular lines, we get two right angled triangles and that by using Pythagoras' relation we can calculate certain lengths as required.

Student's work sheet:

- Name the centre of the circle you have drawn as O.
- Fold the circle such that you get a chord which is not a diameter. Draw a line along the fold and name it AB. Mark the midpoint of AB and name it P. Join P to the centre of the circle. Check the magnitude of the angles OPA and OPB, by a suitable method.
- Confirm by other ways what you have found out about the two angles OPA and OPB.
- Fill in the blank of the sentence given below.

  The line joining the midpoint of a chord to the centre of a circle is .... to the chord.

- Write this sentence in your exercise book and draw a figure that suits the sentence.
- Find out what type of triangles are obtained by drawing the line OA and OB on the figure you have drawn in your exercise book.
- If AB is 6cm and OP= 4cm, find the length of AP and the length of BP. Measure the radius of the circle.
- Prepare yourself to present what you have discovered to the class.

Assessment and Evaluation:

- Assessment criteria
  - Investigates the angle formed by a chord and the line joining the midpoint of the chord to the centre of the circle.
  - States that the line joining the midpoint of a chord to the centre of the circle is perpendicular to the chord,
  - Verifies that the line joining the midpoint of a chord to the centre of the circle is

perpendicular to the chord.
- Uses the fact that the line joining the midpoint of a chord to the centre of the circle is perpendicular to the chord in calculations.
- Obtains important results by using the given hints.

- Direct the students to the relevant exercises in lesson 26 of the text book.

Practical situations:

- Discuss with the students about situations like the one given below where this theorem is used practically.
- When a part of a circular shaped item is unearthed, say in some archeological investigation, the centre can be found using the facts of the theorem.

For your attention..
Development of the lesson:

- Get the students to prove riders using the theorem that **the line joining the midpoint of a chord to the centre of the circle is perpendicular to the chord**.
- Explain to the students how the theorem that **the line joining the midpoint of a chord of a circle to the centre is perpendicular to the chord** is formally proved using the two triangles identified in the exploration.
- By planning a suitable activity, get the students to identify the theorem that **the perpendicular drawn from the centre of a circle to a chord bisects the chord**.
- Using a suitable method, guide the students to verify the theorem that **the perpendicular drawn from the centre of a circle to a chord bisects the chord**.
- Also plan suitable lessons to prove riders and do calculations using the theorem that **the perpendicular drawn from the centre of a circle to a chord bisects the chord.**

Assessment and Evaluation:

- Direct the students to the relevant exercises in lesson 26 of the text book.

For further reference:

27. Constructions

Competency 27: Analyses according to geometric laws the nature of the locations in the environment.

Competency level 27.1: Uses the knowledge on the basic loci to determine locations.

Competency level 27.2: Constructs triangles using the given data.

Competency level 27.3: Constructs rectilinear plane figures involving parallel lines by considering the relationships between the angles related to parallel lines.

Introduction:

It is expected in this section to discuss the construction of the four basic loci under competency level 27.1, the construction of triangles under competency level 27.2 and the constructions of plane figures related to parallel lines relevant to competency level 27.3.

- Knowledge of loci is used in constructions. Constructions can be used to identify the locus of a point, and to verify theorems of geometry. Only a pair of compasses and a straight edge are used in constructions.
- Construction of the four basic loci is discussed in this section.

1. Constructing the locus of a point moving at a constant distance from a fixed point.
2. Constructing the locus of a point moving equidistant from two given points.
3. Constructing the locus of a point moving at a constant distance from a straight line.
4. Constructing the locus of a point moving equidistant from a pair of intersecting straight lines.

- Under the construction of triangles, the following are discussed. Constructing a triangle when the lengths of the three sides are given, when the lengths of two sides and the magnitude of the included angle are given, when the magnitude of two angles and the length of one side are given.
- Other plane figures can also be constructed by constructing these triangles.
- Under the construction of quadrilaterals related to parallel lines, when the lengths of two adjacent sides and the magnitude of the included angle are given, or when the perpendicular distance between a pair of parallel sides and the lengths of two adjacent sides are given, the parallelogram can be constructed. This, and the construction of a trapezium when the measurements are given, as well as obtaining various measurements by constructing other plane figures with the given measurements are discussed in this section.

- It is important to emphasize that constructions done according to given data are facilitated by adhering to the following steps.

1. Drawing a rough diagram using the given data.
2. Inserting data in the rough diagram.
3. Identifying the geometric relationships.
4. Deciding the order of construction.
5. Constructing the geometric figure.

- It is expected that the lesson plan given below will be carried out once the students develop the ability to construct the 4 basic loci under competency level 27.1.

Learning outcomes relevant to competency level 27.2:

1. **Constructs a triangle with a straight edge and a pair of compasses when the lengths of the three sides are given.**
2. **Constructs a triangle with a straight edge and a pair of compasses when the lengths of two sides and the magnitude of the included angle are given.**
3. Constructs a triangle with a straight edge and a pair of compasses when the magnitude of two angles and the length of a side are given.
4. Constructs various plane figures by constructing triangles.

Glossary of Terms:

Point	- ලක්ෂ්‍යය ලක්‍ෂ්‍යය	- மையம்
Locus	- පථය	- ஒழுக்கு
Straight line	- සරල රේඛාව	- நேர்கோடு
Parallel lines	- සමාන්තර රේඛා	- சமாந்தரக் கோடுகள்
Adjacent sides	- බද්ධ පාද	- அடுத்துள்ள பக்கங்கள
Perpendicular height	- ලම්බ උස	- செங்குத்துயரம்
Intersection	- ඡේදනය	- இடைவெட்டு
Equilateral	- සමපාද	- சமபக்க
Triangle	- ත්‍රිකෝණය	- முக்கோணி
Quadrilateral	- චතුරස්‍රය	- நாற்பக்கல்
Parallelogram	- සමාන්තරාස්‍රය	- இணைகரம்
Trapezium	- ත්‍රපීසියම	- அரியம்

Instructions to plan the lesson:

Given below is a demonstration lesson prepared as an individual activity to be done step by step to develop in students the concepts under learning outcomes 1 and 2 relevant to competency level 27.2. (It is very important that the teacher demonstrates the steps of the construction on the chalk board with a large pair of compasses and a straight edge.)

Time: 40 minutes

Quality Inputs:

- Pair of compasses
- Straight edge

Instructions for the teacher:

Approach:

- Discuss in brief about the instruments, a pair of compasses and a straight edge which are used in constructions and how they are used.
- On the chalkboard, show how a line segment of given length is constructed using a pair of compasses and a straight edge.
- Demonstrate on the chalk board how angles of 60^0, 90^0, 30^0, 45^0, 75^0 etc., are constructed and get the students to construct them.
- Recall the fact that the sum of the lengths of any two sides of a triangle should be greater than the third side and show by examples that this condition should be satisfied in the construction of a triangle too.

Development of the lesson:

- Use the following example to develop in students the skill of constructing a triangle when the lengths of the three sides are given.
  Triangle ABC with sides AB = 5cm, AC = 6cm and BC = 7cm (or the teacher could give the lengths as he/she wishes). Guide the students through the steps given below.
- Inquire whether you can draw the triangle with the given measurements.
- Thereafter get the students to draw a rough diagram and insert the data.
- Direct the students to do the construction following the demonstration by the teacher (it is essential that the teacher uses the pair of compasses and straight edge correctly).
  Step 1: Draw a line segment on the board using the straight edge. Mark point A on it.
  Step 2: Taking the length AB as radius and with A as centre, draw an arc to cut the line segment drawn earlier. Name the point of intersection as B.
  Step 3: Taking the length AC as radius and A as centre draw an arc.
  Step 4: With B as centre and radius equal to BC, draw an arc to cut the arc in step 3.
  Step 5: Name the point of intersection as C. Join AC and BC.
  Now, triangle ABC is the required triangle.
- Construction of a triangle when two sides of a triangle and the included angle are given should also be done as before with the teacher's guidance. You may use the example given below.
  Construct the triangle PQR such that PQ = 6cm, PQR< = 60^0, PR = 4cm

  Adhere to the following steps.
  Step1: Drawing a sketch and marking the data
  Step2: Constructing the line segment PQ
  Step3: Constructing the angle PQR
  Step4: Marking PR
  Step5: Joining RQ

Assessment and Evaluation:

- Assessment criteria
  - Uses the straight edge and the pair of compasses correctly.
  - Engages in the construction following instructions.
  - Constructs the triangle when the three sides are given.
  - Constructs the triangle when two sides and the included angle are given.
  - Completes the task with patience and planning.
- Direct the students to the relevant exercises in lesson 27 of the text book.

For your attention..

Development of the lesson:

- With a teacher demonstration, guide the students to construct the triangle when the lengths of two sides and the magnitude of the included angle are given, to develop the skills relevant to learning outcome 3 under competency level 27.2.
- Give the students the opportunity to develop their skills in constructions by making them construct various plane figures as expected under learning outcome 4.
- Plan and carry out a suitable method with a teacher demonstration, for the construction of a quadrilateral with parallel sides relevant to competency level 27.3

Assessment and Evaluation:

- Direct the students to the relevant exercises in lesson 27 of the text book.

For further reference:

28. Surface Area and Volume

Competency 8: Makes use of a limited space in an optimal manner by investigating the area.

Competency level 8.2: Investigates the surface area of cylinders.

Competency level 8.3: Investigates the surface area of prisms.

Competency 10: Makes the maximum use of space by working critically with volume.

Competency level 10.1: Has an awareness about the volumes of cylinders.

Competency level 10.2: Has an awareness about the volumes of prisms.

Number of periods: 10

Introduction:

- A solid cylinder consists of two plane faces and a curved surface part as well as a circular cross-section.
- The circumference of a circle of radius r is $2\pi r$ while its area is πr^2
- The surface of a cylinder can be separated into two circular parts and a rectangular part. The length of one side of the rectangular part is the circumference of the circular part while the length of the other side is the height of the cylinder.
- The area of the curved surface of a cylinder is given by $2\pi rh$, where r is the radius of the base and h the height.

  The total surface area is given by $2\pi r^2 + 2\pi rh$.
- In a solid, if parallel cross sections are equal in all respects then the solid is said to be of **uniform cross-section**.
- A right triangular prism has 5 faces consisting of 2 triangular faces equal in shape and size and 3 rectangular faces.
- The sum of the areas of all the faces of a right triangular prism is the total surface area of the prism.
- The space occupied by a solid is the volume of the solid. The volume of a solid of uniform cross-section is the product of the area of the cross section and the height.
- The volume of a right circular cylinder of base radius r and height h is $\pi r^2 h$.
- The volume of a right triangular prism is obtained by length $\times$ area of triangular cross section.
- It is expected to discuss the above subject matter relevant to competency levels 8.2, 8.3, 10.1 and 10.2 in this section.

Learning outcomes relevant to competency level 8.2:

1. Builds up the formula $A = 2\pi r^2 + 2\pi rh$ for the surface area of a right circular cylinder of base radius r and height h .
2. Calculates the surface area of a right circular cylinder using the formula $A = 2\pi r^2 + 2\pi rh$.
3. Solves problems related to the surface area of a right circular cylinder.

Glossary of Terms:

Cylinder	- සිලින්ඩරය සිලින්ඩරය	- உருளை
Prism	- ප්‍රිස්මය	- அரியம்
Uniform Cross Section	- ඒකාකාර හරස්කඩ	- சீரான குறுக்குவெட்டு
Area	- වර්ගඵලය	- பரப்பளவு
Volume	- පරිමාව	- கனவளவு
Circular	- වෘත්තාකාර	- வட்டமான
Triangular	- ත්‍රිකෝණාකාර	- முக்கோணமான

Instructions to plan the lesson:

A specimen lesson plan to guide students to achieve learning outcome 1 under competency level 8.2, using the discovery method is given below.

Time: 40 minutes

Quality Inputs:

- A cylinder made for demonstration (This should be such that the two circular faces can be removed and the curved surface can be displayed as a rectangle)
- Copies of the student work sheet

Instructions for the teacher:
Approach:

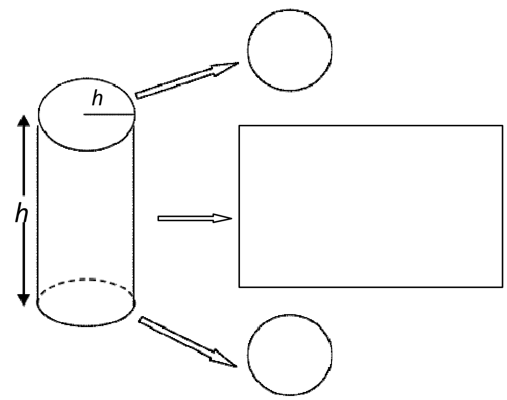
- Recall that the area of a rectangle is found by multiplying the length by the breadth, tha the area of a circle is given by πr^2 and the circumference by $2\pi r$.
- Separate the cylinder into two circles and a rectangle, show it to the students and discuss about the surface parts of a cylinder.
- Draw the shapes of these surface parts on the black board.
- Thereby highlight the fact that the cross-section of the cylinder is uniform and that the two circular faces are equal to each other.
- While conducting the discussion, show that the length of one side of the rectangle which formed the curved surface of the cylinder is the circumference of the circular

face and that the length of the other side is the height of the cylinder.

Development of the lesson:

- Group the students in a suitable manner and give each group a copy of the work sheet.
- Engage the students in the activity.
- After completing the activity, asking for responses from the students, highlight the fact that the area of the curved surface of a cylinder of base radius r and height h with a uniform cross section is $2\pi rh$, that the total surface area of the two circular faces is $2\pi r^2$ and that the total surface area of the cylinder is $2\pi r^2 + 2\pi rh$.

Student's work sheet:



- The figure shows a cylinder of radius r and height h.
- Find the area of a circular face in terms of r.
- Build up an expression for the area of the two circular faces in terms of r.
- Find the circumference of the base in terms of r.
- Find an expression in terms of r for the length of the rectangle.
- Discuss what the breadth of the rectangle is.
- Find an expression for the area of the rectangle.
  Find the area of the curved surface of the cylinder in terms of r and h.
- Build up an expression for the total surface area of the cylinder.

Assessment and Evaluation:

- Assessment criteria
  - Recognizes that the surface of a cylinder is composed of two equal circular faces and a curved surface which when opened out is rectangular in shape.
  - Accepts that the length of the rectangle is equal to the circumference of the base.
  - Writes the correct expression for the area of the curved surface of a cylinder.
  - Develops the correct expression for the total surface area of a cylinder.
  - Works within the group with mutual cooperation.

- Direct the students to the relevant exercises in lesson 26 of the text book.

For your attention..

Development of the lesson:

- Use a suitable methodology to give students the opportunity to find the total surface area of a right circular cylinder and to solve related problems using the formula $A = 2\pi r^2 + 2\pi rh$, when the total surface area is A.
- Organise and implement an activity to identify the shapes of the faces of a right triangular prism and to find its total surface area, which are the subject concepts under competency level 8.3.
- Organise and implement an activity to develop the formula $\pi r^2 h$ for the volume of a cylinder of height h and base radius r, and using it to find the volume, which are the subject concepts under competency level 10.1.
- Organise and implement an activity to build up a formula for the volume of a right prism with a triangular cross-section, and develop the skills of calculating the volume, which are the subject concepts of competency level 10.2.

Assessment and Evaluation:

- Direct the students to the relevant exercises in lesson 28 of the text book.

For further reference:

29. Probability

Competency 31: Analyzes the likelihood of an event occurring to predict future events.

Competency level 31.1: Analyses the mutual relationships between events.

Competency level 31.2: Illustrates the occurrences of a compound event pictorially.

Number of periods: 08

Introduction:

- The probability of an event which takes place definitely is 1 and that of an event which never takes place is zero (0) .When it cannot be stated definitely whether an event would take place or not, then such an event is known as a random event. The probability of such an event taking place lies between 0 and 1.
- The set containing all the outcomes of a random experiment is known as the **sample space.**
- A subset of the sample space of a random experiment is called an **event.**
- If the number of elements of the sample space is n, then the total number of events is 2^n.
- If the probabilities are equal, then the events are said to be equally probable.
- If an event in a sample space of a random experiment cannot be separated into two or more events it is known as a **simple event.**
- When an event in a sample space can be divided into two or more simple events, it is known as a compound event.
- If A is an event in a sample space S, and the probability of A occurring is P(A), then $(A) = \frac{n(A)}{n(S)}$. Here $n(A)$ is the number of elements in the expected event and $n(S)$ is the number of elements in the sample space.
- If A means the event that takes place, the event that A does not take place is the complement of event A and is denoted by $A^{/}$. Thus $P(A^{/}) = 1 - P(A)$.
- If A and B are two events of the sample space S, and if event B does not take place when the event A takes place, then the two events A and B are said to be **mutually exclusive events.** There are no elements common to A and B in this case. That is, $A \cap B = \varnothing$. Thus, if the events have no common elements, that is, when the intersection is the null set, the two events are mutually exclusive.
- When A and B are mutually exclusive events, $P(A \cup B) = P(A) + P(B)$. Thus when A and B are mutually exclusive events, $P(A \cap B) = 0$.
- Two events that have common elements are known as events which are not mutually exclusive. Then $A \cap B \neq \varnothing$.

- If two events A and B are not mutually exclusive, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
- When two or more events are considered, if the occurrence or non-occurrence of one event does not affect the occurrence or non-occurrence of the other events, then these events are said to be **independent events**.
- For example, if an unbiased cube shaped die is thrown and at the same time a coin is tossed, then getting 6 on the die and 'head' on the coin are independent events.
- The sample space of an experiment of the above type can be represented on a grid. Only two stages can be represented on a grid. However there is no limit to the stages that can be represented in a tree diagram. But, in this curriculum we restrict tree diagrams to two stages.
- The grid (the point graph) or the tree diagram can be used to easily solve problems related to probability .
- It is expected to discuss the subject matter mentioned under competency levels 31.1 and 31.2 in this section.

Learning outcomes relevant to competency level 31.1:

1. **Identifies simple events and compound events.**
2. **Expresses that if A is an event in a sample space S, the probability of A occurring is $P(A) = \frac{n(A)}{n(S)}$.**
3. **Identifies the complement of an event.**
4. **States that if the complement of the event A is A' then $P(A') = 1 - P(A)$.**
5. Describes mutually exclusive events citing examples.
6. Describes events which are not mutually exclusive citing examples.
7. Finds the probability of a compound event which consists of events that are not mutually exclusive using the formula $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Glossary of Terms:

Probability	- සම්භාවිතාව	- நிகழ்தகவு
Random events	- අහඹු සිදුවීම්	- எழுமாற்று நிகழ்ச்சிகள்
Random experiment	- සසම්භාවී පරීක්ෂණ	- எழுமாற்று பரிசோதனைகள்
Events	- සිද්ධි	- நிகழ்ச்சிகள்
Simple events	- සරල සිද්ධි	- எளிய நிகழ்ச்சிகள்
Compounds events	- සංයුක්ත සිද්ධි	- கூட்டு நிகழ்ச்சிகள்
Equally likely	- සමසේ භව්‍ය	- சம நேர்தகவுடைய நிகழ்ச்சிகள்
Mutually exclusive events	- අන්‍යොන්‍ය වශයෙන් බහිස්කාර සිද්ධි	- புறுநீங்கலான நிகழ்ச்சிகள்
Independent events	- ස්වායත්ත සිද්ධි	- சாரா நிகழ்ச்சிகள்
Sample space	- නියැදි අවකාශය	- மாரிவெளி
Tree diagram	- රුක් සටහන	- மரவரிப்படம்
Dependent events	- පරායත්ත සිද්ධිසම්භාවිතාව	- சார்ந்த நிகழ்ச்சிகள்

Instructions to plan the lesson:

An exemplar lesson based on the lecture-discussion method to inculcate the subject concepts relevant to the learning outcomes 1, 2, 3 and 4 under competency level 31.1, is given below.

Time: 40 minutes

Quality Inputs:

- Copies of the question paper, designed to establish the knowledge gained from the lesson, one copy per student

Instructions for the teacher:
Approach:

- Recall what has been learnt in previous grades regarding random experiments, unbiased experiments, biased experiment, success factor, experimental probability, theoretical probability and sample space.

Development of the lesson:

- Elicit from the students the sample space of the experiment of throwing an unbiased coin once, and ask them what the subsets of the sample space are. State that these subsets are known as the events of the experiment.
- Explain to the students that events that cannot be further divided are simple events and that events that can be divided further are compounds events. Give examples.
- Mention that if A is an event in the sample space S, then $P(A) = \frac{n(A)}{n(S)}$.

$$P(A) = \frac{number\ of\ elements\ in\ the\ required\ set}{number\ of\ elements\ in\ the\ sample\ space}$$

- Present simple questions like those given below to obtain the probability using the formula, and get the students to answer the questions.
- Explain that an event not taking place is known as the complement of the event. Mention that the complement of A is denoted by A′.
- Explain that P(A′) = 1 - P(A) using examples.
- Obtain examples of the complement of events from the students.
- Get the students to answer the problems in the assignment given at the end of the lesson to establish what they learnt.

Assessment and Evaluation:

- Assessment criteria
  - Selects the simple events and compound events from a given set.
  - Finds the probability of a given event.

- Expresses the complement of a given event.
- Explains the relationship between the probabilities of an event and its complement.
- Completes the work sheet during the allocated time.

- Direct the students to the relevant exercises in lesson 29 of the text book.

Question paper to establish the subject content

(1) Join the corresponding statements with reference to a random experiment where an unbiased die with the numbers 1 to 6 written on the faces is thrown once.

	A	B
(i)	Probability of getting a multiple of 3	{Obtaining 5}
(ii)	Probability of getting a number less than 7	1
(iii)	Probability of getting a prime number	{Getting a factor of 6}
(iv)	Simple event	$\frac{1}{3}$
(v)	Compound event	$\frac{1}{2}$

(2) If the expression is correct mark √ and if not mark × in the box.

(i) The probability of getting a vowel when a letter is picked at random from a set of letters of the English alphabet is $\frac{51}{263}$. ☐

(ii) The coach says that in a volleyball match between the teams A and B, the probability of A winning is $\frac{1}{3}$. Then the probability of B winning is also $\frac{1}{3}$. ☐

(iii) The event of obtaining an even number and the event of obtaining an odd number when a fair die with the faces marked 1 to 6 is thrown are not complements of each other. ☐

(iv) It was found that 3% of the products of a certain factory are faulty. ☐

Thus the probability of an item produced in that factory not being faulty is 97%.

(v) The two events mentioned in (iv) above are compliments of each other.

For your attention….

Development of the lesson:

- Plan and implement a suitable method to achieve the learning outcomes 5, 6 and 7 under competency level 31.1.
- Use a suitable method to develop the concepts under competency level 31.2.

Assessment and Evaluation:

- Direct the students to the relevant exercises in lesson 29 of the text book.

For further reference:

30. Angles in a Circle

Competency 24: Thinks logically to make decisions based on geometrical concepts related to circles.

Competency level 24.3: Formally proves and applies the relationships between the angles that are subtended by an arc of a circle.

Competency level 24.4: Solves problems using the relationships between the angles in a circle.

Number of periods: 08

Introduction:
Students have learnt about the various aspects of a circle earlier. They have also learnt the theorem about the chords of a circle. Identifying the theorem that the angle subtended at the centre of a circle by an arc is equal to twice the angle subtended on the circumference by the same arc, verifying this theorem and using it to perform calculations and to prove riders and finally proving it formally is discussed under competency level 24.3. For this purpose it is important to identify the minor arc and the major arc by marking two points say A and B, on a circle with centre O. It is also important to identify the angles subtended by the minor arc and the major arc at the centre and the remaining part of the circle.
Similarly, under competency level 24.4, identifying the theorem that the angles in the same segment are equal, verifying the theorem and using it in calculations and to prove riders is discussed. Identifying the theorem that the angle in a semi-circle is a right angle, verifying it, and using it in calculations and to prove riders are also expected.

Learning outcomes relevant to competency level 24.1:

1. **Identifies the theorem that the angle subtended at the centre of a circle by an arc is equal to twice the angle subtended on the circumference by the same arc.**
2. **Verifies the theorem that the angle subtended at the centre of a circle by an arc is equal to twice the angle subtended on the circumference by the same arc.**
3. **Uses the theorem that the angle subtended at the centre of a circle by an arc is equal to twice the angle subtended on the circumference by the same arc to perform calculations.**
4. Proves riders using the theorem that the angle subtended at the centre of a circle by an arc is equal to twice the angle subtended on the circumference by the same arc.
5. Formally proves the theorem that the angle subtended at the centre of a circle by an arc is equal to twice the angle subtended on the circumference by the same arc.

Glossary of Terms:

Angles in a circle - වෘත්තයක කෝණ - வட்டமொன்றில் உள்ள கோணங்கள்
Angle subtended at the centre - කේන්ද්‍රයේ ආපාතිත කෝණය - வட்டமொன்றின் மையத்தில் அமையும் கோணங்கள
Centre - කේන්ද්‍රය -மையம்
Arc - චාපය - வில்
Chord - ජ්‍යායය - நாண்
Semicircle - අර්ධ වෘත්තය - அரைவட்டம்
Segment - වෘත්ත ඛණ්ඩය - துண்டம்
Sector - කේන්ද්‍රික ඛණ්ඩය - ஆரைச்சிறை
Angle in a semicircle - අර්ධ වෘත්තයක කෝණය - அரைவட்டமொன்றில் அமையும் கோணங்கள
Angles in the same segment - එකම ඛණ්ඩයේ කෝණ - ஒரேதுண்டக் கோணங்கள்

Instructions for the lesson:

A demonstration lesson which uses the discovery method to guide the students to achieve learning outcomes1, 2 and 3 under competency level 24.3, is given below.

Time: 40 minutes

Quality Inputs:
- A peg board as shown in Annex 1
- Copies of the student work sheet (one per student)
- Copies of the circle in Annex 2 (one copy per pair of students)

Instructions for the teacher:
Approach:
- Using a large circle drawn on the chalk board, inquire from the students as to what the centre, radius, diameter, arc and segment are, and ask the students to mark these on the circle.
- Using the same circle or a different one, begin the lesson by introducing the minor arc, major arc, minor segment and major segment.

Development of the lesson:
- Prepare a peg board as shown in Annex 1.
- Introduce the angle subtended by an arc at the centre and on the remaining part of the circle by using rubber bands.
- Illustrate several angles which are angles subtended by an arc on the remaining part of

the circle.

- Make the students aware of the minor arc and the major arc using the same arc.
- Introduce the angles subtended by the major arc and the minor arc, using different arcs.

- Divide the students into 6 groups so that there is an even number of students in each group.
- Distribute a copy of the worksheet in Annex 2 to each group.
- Distribute a copy of annex 3 to each pair of students.
- Assign each group a pair of points from the given pairs.
- While the students are engaged in the activity, check whether the angle subtended at the center has been calculated correctly. If not guide the students to get the right value.
- Let each group present their findings.
- Taking into account all the findings, discuss with the students that the angle subtended by an arc at the centre is twice that subtended on the remaining part of the circle.
- Join any two points A and B on the circle to the centre O, as well as to a point on the remaining part of the circle, and show how the above relationship is obtained using other geometric relationships.

Student worksheet:

Pairs of points
A and C
A and D
A and E
A and F
A and G
A and H

(i) Consider the pair of points assigned to your group.

(ii) Join each of the points assigned to you which are marked on the circle, to the centre.

The circle has been separated into 9 equal length arcs.

(iii) What is the magnitude of the angle formed by the lines joining the two points to the centre?

(iv) Join these two points to a point which has been marked on the remaining part of the circle.

(v) Carefully study the two triangles you obtained by joining the point you selected on the circle to the centre.

(vi) What type of triangles did you obtain?

(vii) Calculate the magnitude of the angle that is subtended by the selected arc on the remaining part of the circle, by using the subject facts you

have learnt earlier.
(viii) Compare the answers you obtained for (iii) and (vii) above, and find a relationship between them.
(ix) Present your findings to the class.

Assessment and Evaluation :

- Assessment criteria
  - Identifies the angle subtended at the centre and the remaining part of the circle with reference to two points marked on the circle.
  - Marks the angles formed at the centre and on the circle by the minor arc and the major arc.
  - Identifies that the angle formed at the centre by an arc is twice the angle formed on the remaining part of the circle.
  - Performs calculations based on the above relationship.
  - Works in mutual cooperation within the group.
- Direct the students to the relevant exercise in lesson 30 of the text book.

For your attention...

Development of the lesson:

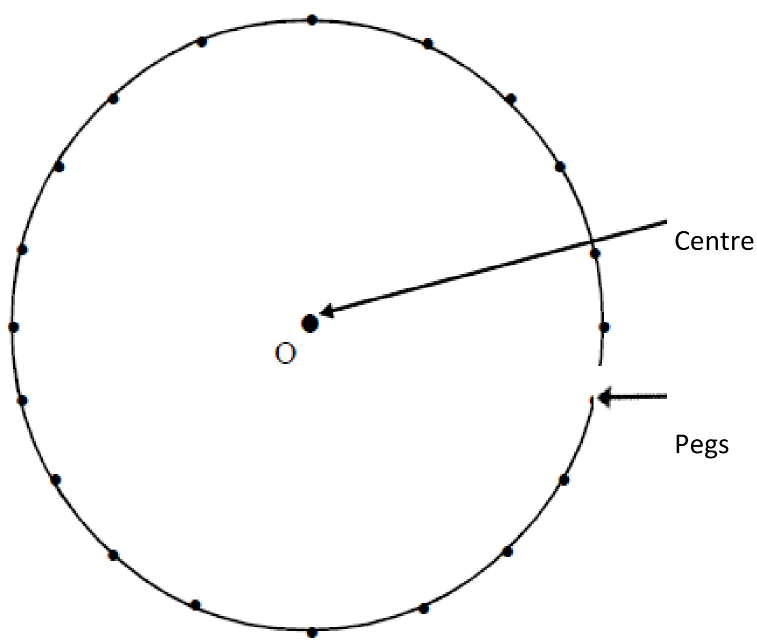
- Get the students to do various exercises to establish the knowledge gained on the theorem that the angle subtended at the centre of a circle by an arc is equal to twice the angle subtended on the circumference by the same arc.
- Guide the students to prove riders using this theorem.
- Once this theorem has been established, explain with the relevant steps how it is proved.
- Plan a suitable lesson to introduce and verify the theorem that the angles in the same segment are equal, and implement it.
- After this, guide the students to perform calculations and to prove riders using the theorem.
- Once this theorem has been established in students, formally prove the theorem.
- In the same manner, introduce and verify the theorem that the angle in a semi-circle is a right angle. Plan and implement a lesson to guide students to perform calculations and prove riders using this theorem.

Assessment and Evaluation:

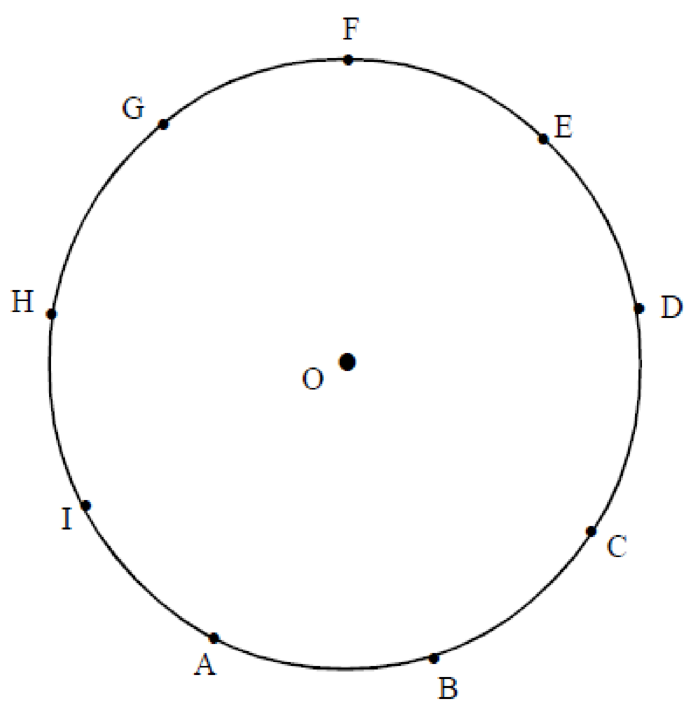
- Direct the students to the relevant exercise in lesson 30 of the text book.

For further reference:

Annex 01



Annex 02



31. Scale Diagrams

Competency 13: Uses scale diagrams in practical situations by exploring various methods.

Competency level 13.1: Investigates the various locations in the environment using scale diagrams.

Number of periods: 05

Introduction :

Students have studied about drawing scale diagrams with reference to positions in the horizontal plane, in grade 8. It is expected to discuss drawing scale diagrams of positions in the vertical plane in this section.

The concept of scale diagrams is used in practical life to plan buildings and to draw plans of lands. It is also used when drawing maps of Sri Lanka and other countries.

Identifying the angle of elevation and the angle of depression and describing the position of an object using these angles, as well as drawing scale diagrams is expected to be done in this chapter.

Learning outcomes relevant to competency level 13.1:

1. **Identifies the angle of depression.**
2. **Identifies the angle of elevation.**
3. **Describes the location of an object in terms of the angle of depression and the angle of elevation.**
4. Draws scale diagrams to represent the information on measurements in a vertical plane.
5. Describes locations in the environment using scale diagrams.
6. Solves problems related to scale diagrams of vertical planes.

Glossary of Terms:

Angle of elevation	- ආරෝහණ කෝණය	- ஏற்றக்கோணம்
Angle of depression	- අවරෝහණ කෝණය	- இறக்கக்கோணம்
Line of sight	- දෘෂ්ටි රේඛාව	- பார்வைக்கோடு
Scale drawings	- පරිමාණ රූප	- அளவிடை வரைபு
Vertical plane	- සිරස් තලය	- நிலைகுத்து தளம்
Horizontal plane	- තිරස් තලය	- கிடை தளம
Clinometer	- ආනති මානය	- கோணமானி

Instructions to plan the lesson:

A specimen lesson plan based on the lecture discussion method together with a group activity to develop in students the subject concepts relevant to learning outcomes 1, 2 and 3 under competency level 13.1 is given below.

Time: 80 minutes

Quality Inputs:

- An enlarged poster showing a practical situation similar to what is given in annex 1 illustrating the angle of elevation and the angle of depression.
- Clinometer - made as described in annex 2, one per group.
- A list of locations in the school environment equal to the number of groups, such that each when illustrated contains one angle of elevation and one angle of depression.

Instructions for the teacher:

Approach:

- Explain that the ground is considered as the horizontal plane and that any other plane which is perpendicular to the ground is considered as a vertical plane.
- Recall using examples that bearings were used in grade 8 to describe a location with respect to a given position in the horizontal plane.
- Confront the students with a situation where some way of expressing the position of an object (or point) in the vertical plane using angles becomes necessary. (e.g. the position occupied by a bird perched on the top of a vertical post)

Development of the lesson:

- Display the enlarged poster (annex 1) in front of the students.
- Query about what is in the picture. Taking their responses into consideration, describe that the picture illustrates how a person observes a bird flying above his eye level and a fish swimming below his eye level.
- Explain that here the **eye level** is considered as the horizontal plane.
- Explain to the students that, as the observer looks at different objects, the position of his eye is the **place of observation**. Similarly the bird and the fish are being observed and hence are the **points of observation**.
- Explain that the line from the place of observation to the point of observation is known as the **line of sight.**
- Also explain that when we observe an object which is above the horizontal plane (eye level) the angle between the line of sight and the eye level is known as the **angle of elevation.** Mark the angle of elevation on the poster.

- Explain that when we observe an object which is below the horizontal plane (eye level) the angle made between the line of sight and the eye level is known as the **angle of depression.** Mark the angle of depression also on the poster.
- Lead a discussion with examples of angles of depression and the angles of elevation in practical situations, and illustrate them on the chalk board.
- Show the clinometer to the students and explain how it is used and how the angle of observation is measured.
- Group the students as required.
- To each group, assign a location in the school environment where an angle of elevation and an angle of depression can be observed (e.g. Observing some object or point on the roof, observing from an upper floor some object on the ground)
- Provide each group with a clinometer and a sheet of A_4 paper.
- Get the students to measure the angle of elevation and the angle of depression using the clinometer and to illustrate the locations pictorially on the paper , to describe the positions and to note down the magnitudes of the angles.
- Explain to the students how the positions above the eye level and below the eye level are described, by using the students' illustrations.
- (e.g. Himashi observes a flower pot on the ground from a position on the second floor, at an angle of depression of 45^0; Saman observes the flag staff from a position on the ground at an angle of elevation of 30^0)
- In conclusion, state that the angle between the eye level and the line of sight when an object above the eye level is observed is known as the **angle of elevation** and that the angle between the eye level and the line of sight when an object below eye level is observed is known as **the angle of depression**. Also mention how the position of an object relative to another position is described in terms of the angle of elevation or angle of depression.

Assessment and Evaluation :

- Assessment criteria
  - Identifies separately the angle of elevation / depression of a vertical position.
  - Accepts that the eye level should be considered as the horizontal level when the angle of elevation / depression is measured.
  - Measure the angle of elevation/depression at situations with respect to a point of observation.
  - Describes positions of objects using angles of elevation/depression.
  - Works within the group with mutual cooperation.
- Direct the students to the relevant exercises in lesson 31 of the text book.

For your attention...

Development of the lesson:

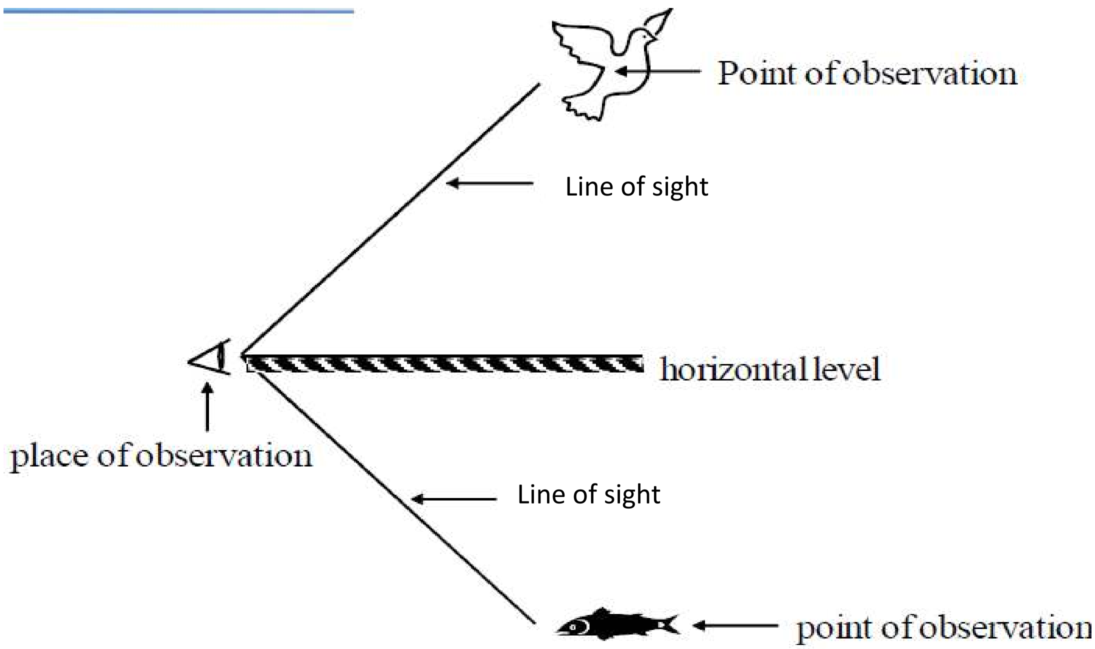
- Plan a suitable methodology to help improve the skills relevant to learning outcomes 4, 5 and 6.

Assessment and Evaluation:

- Direct the students to the relevant exercises in lesson 31 of the text book.

For further reference:

Annex 01



Annex 02

Clinometer