MATHEMATICS

Grade 7

Part - I

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The National Anthem of Sri Lanka

Sri Lanka Matha
Apa Sri Lanka Namo Namo Namo Namo Matha
Sundara siri barinee, surendi athi sobamana Lanka
Dhanya dhanaya neka mal palaturu piri jaya bhoomiya ramya
Apa hata sepa siri setha sadana jeewanaye matha
Piliganu mena apa bhakthi pooja Namo Namo Matha
Apa Sri Lanka Namo Namo Namo Namo Matha
Oba we apa vidya
Obamaya apa sathya
Oba we apa shakthi
Apa hada thula bhakthi
Oba apa aloke
Apage anuprane
Oba apa jeevana we
Apa mukthiya oba we
Nava jeevana demine, nithina apa pubudukaran matha
Gnana veerya vadawamina regena yanu mana jaya bhoomi kara
Eka mavakage daru kela bevina
Yamu yamu vee nopama
Prema vada sema bheda durerada
Namo, Namo Matha
Apa Sri Lanka Namo Namo Namo Namo Matha

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ආනන්ද සමරකෝන්

ஒரு தாய் மக்கள் நாமாவோம்
ஒன்றே நாம் வாழும் இல்லம்
நன்றே உடலில் ஓடும்
ஒன்றே நம் குருதி நிறம்

அதனால் சகோதரர் நாமாவோம்
ஒன்றாய் வாழும் வளரும் நாம்
நன்றாய் இவ் இல்லினிலே
நலமே வாழ்தல் வேண்டுமன்றோ

யாவரும் அன்பு கருணையுடன்
ஒற்றுமை சிறக்க வாழ்ந்திடுதல்
பொன்னும் மணியும் முத்துமல்ல - அதுவே
யான்று மழியாச் செல்வமன்றோ.

ஆனந்த சமரக்கோன்
கவிதையின் பெயர்ப்பு.

Being innovative, changing with right knowledge
Be a light to the country as well as to the world.

Message from the Hon. Minister of Education

The past two decades have been significant in the world history due to changes that took place in technology. The present students face a lot of new challenges along with the rapid development of Information Technology, communication and other related fields. The manner of career opportunities are liable to change specifically in the near future. In such an environment, with a new technological and intellectual society, thousands of innovative career opportunities would be created. To win those challenges, it is the responsibility of Sri Lankan Government and myself, as the Minister of Education, to empower you all.

This book is a product of free education. Your aim must be to use this book properly and acquire the necessary knowledge out of it. The government in turn is able to provide free textbooks to you, as a result of the commitment and labour of your parents and elders.

Since we have understood that the education is crucial in deciding the future of a country, the government has taken steps to change curriculum to suit the rapid changes of the technological world. Hence, you have to dedicate yourselves to become productive citizens. I believe that the knowledge this book provides will suffice your aim.

It is your duty to give a proper value to the money spent by the government on your education. Also you should understand that education determines your future. Make sure that you reach the optimum social stratum through education.

I congratulate you to enjoy the benefits of free education and bloom as an honoured citizen who takes the name of Sri Lanka to the world.

Akila Viraj Kariyawasam
Minister of Education

Foreword

The educational objectives of the contemporary world are becoming more complex along with the economic, social, cultural and technological development. The learning and teaching process too is changing in relation to human experiences, technological differences, research and new indices. Therefore, it is required to produce the textbook by including subject related information according to the objectives in the syllabus in order to maintain the teaching process by organizing learning experiences that suit to the learner needs. The textbook is not merely a learning tool for the learner. It is a blessing that contributes to obtain a higher education along with a development of conduct and attitudes, to develop values and to obtain learning experiences.

The government in its realization of the concept of free education has offered you all the textbooks from grades 1-11. I would like to remind you that you should make the maximum use of these textbooks and protect them well. I sincerely hope that this textbook would assist you to obtain the expertise to become a virtuous citizen with a complete personality who would be a valuable asset to the country.

I would like to bestow my sincere thanks on the members of the editorial and writer boards as well as on the staff of the Educational Publications Department who have strived to offer this textbook to you.

W. M. Jayantha Wickramanayaka,
Commissioner General of Educational Publications,
Educational Publications Department,
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2019.04.10

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Message of the Boards of Writers and Editors

This textbook has been compiled in accordance with the new syllabus which is to be implemented from 2016 for the use of grade seven students.

We made an effort to develop the attitude "We can master the subject of Mathematics well" in students.

In compiling this textbook, the necessity of developing the basic foundation of studying mathematical concepts in a formal manner was specially considered. This textbook is not just a learning tool which targets the tests held at school. It was compiled granting due consideration to it as a medium that develops logical thinking, correct vision and creativity in the child.

Furthermore, most of the activities, examples and exercises that are incorporated here are related to day to day experiences in order to establish mathematical concepts in the child. This will convince the child about the importance of mathematics in his or her daily life. Teachers who guide the children to utilize this textbook can prepare learning tools that suit the learning style and the level of the child based on the information provided here.

Learning outcomes are presented at the beginning of each lesson. A summary is provided at the end of each lesson to enable the child to revise the important facts relevant to the lesson. Furthermore, at the end of the set of lessons related to each term, a revision exercise has been provided to revise the tasks completed during that term.

Every child does not have the same capability in understanding mathematical concepts. Thus, it is necessary to direct the child from the known to the unknown according to his / her capabilities. We strongly believe that it can be carried out precisely by a professional teacher.

In the learning process, the child should be given ample time to think and practice problems on his or her own. Furthermore, opportunities should be given to practice mathematics without restricting the child to just the theoretical knowledge provided by mathematics.

Our firm wish is that our children act as intelligent citizens who think logically by studying mathematics with dedication.

Boards of Writers and Editors

Contents

1 Symmetry

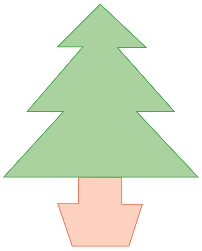
By studying this lesson you will be able to

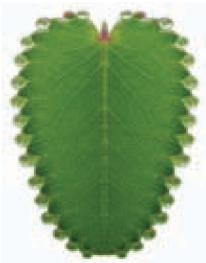
- identify plane figures with bilateral symmetry,
- draw the axes of symmetry of a bilaterally symmetric figure, and
- create bilaterally symmetric figures on square ruled paper.

1.1 Bilateral Symmetry

A figure of a blue quadrilateral shaped card is given here. By folding this figure along the dotted line, we obtain two parts that coincide on each other.

A few figures having two parts which coincide with each other when folded along a certain line are shown below.





Many of the objects in the environment have the property that they can be divided into two equal parts. Most creations too have this property which helps preserve their beauty. Let us learn more about plane figures and laminas with a plane figure as the boundary, that have this property.

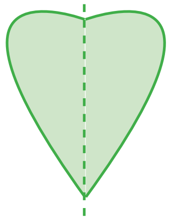


Figure 1

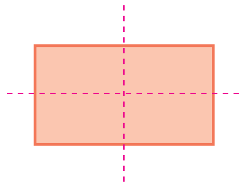


Figure 2

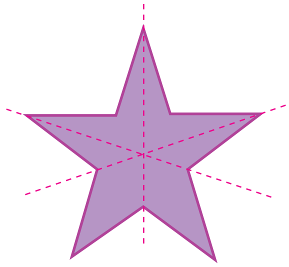


Figure 3

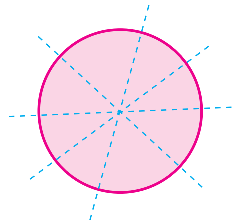


Figure 4

In figure 1, there is only one line that divides the figure into two equal parts which coincide. However, each of the figures 2, 3 and 4, has more than one line that divides the figure into two parts which coincide.

Activity 1

Step 1 - Trace this figure onto a paper and cut along the border of the figure.

Step 2 - Fold the figure so that you get two equal parts which coincide as shown in figure 2.

Step 3 - Draw a dotted line along the fold. Now paste the figure in your exercise book.

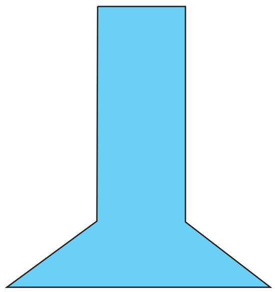


Figure 1

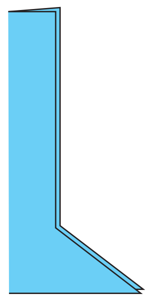
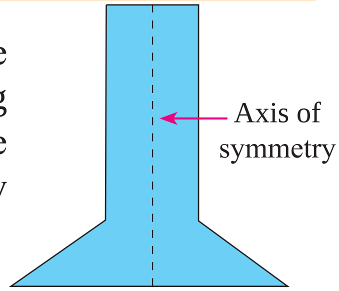


Figure 2

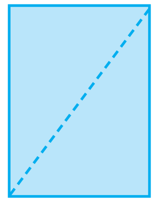
If a plane figure can be folded along a straight line so that you get two parts which coincide, then that plane figure is defined as **a bilaterally symmetric plane figure**. The line of folding is defined as **an axis of symmetry** of the figure.

In the above activity you must have drawn the dotted line shown in the figure as the line along the fold. This line is an "axis of symmetry of the figure". This bilaterally symmetric figure has only one axis of symmetry.



In a bilaterally symmetric figure, the two parts on either side of an axis of symmetry are of the same shape and of the same area.

The figure depicts a rectangle with a dotted line drawn across it. This line divides the rectangle into two equal parts. However if we fold the rectangle along the dotted line, the two parts will not coincide. Therefore this line is not an axis of symmetry of the figure.

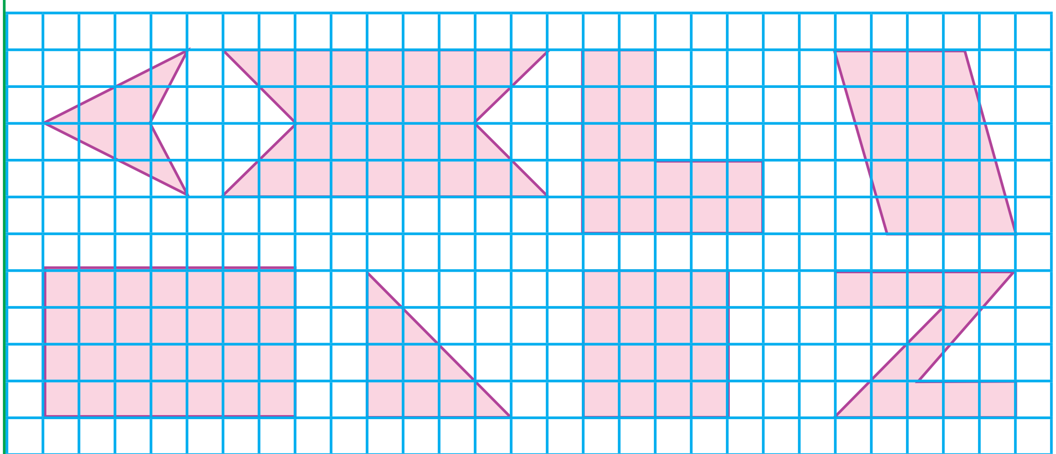


A line through a plane figure which divides it into two parts of the same shape and of the same area which do not coincide with each other is not an axis of symmetry of the figure.

1.2 Drawing axes of symmetry

Activity 2

Step1 - Copy the figures given below on a piece of paper and cut out each lamina.



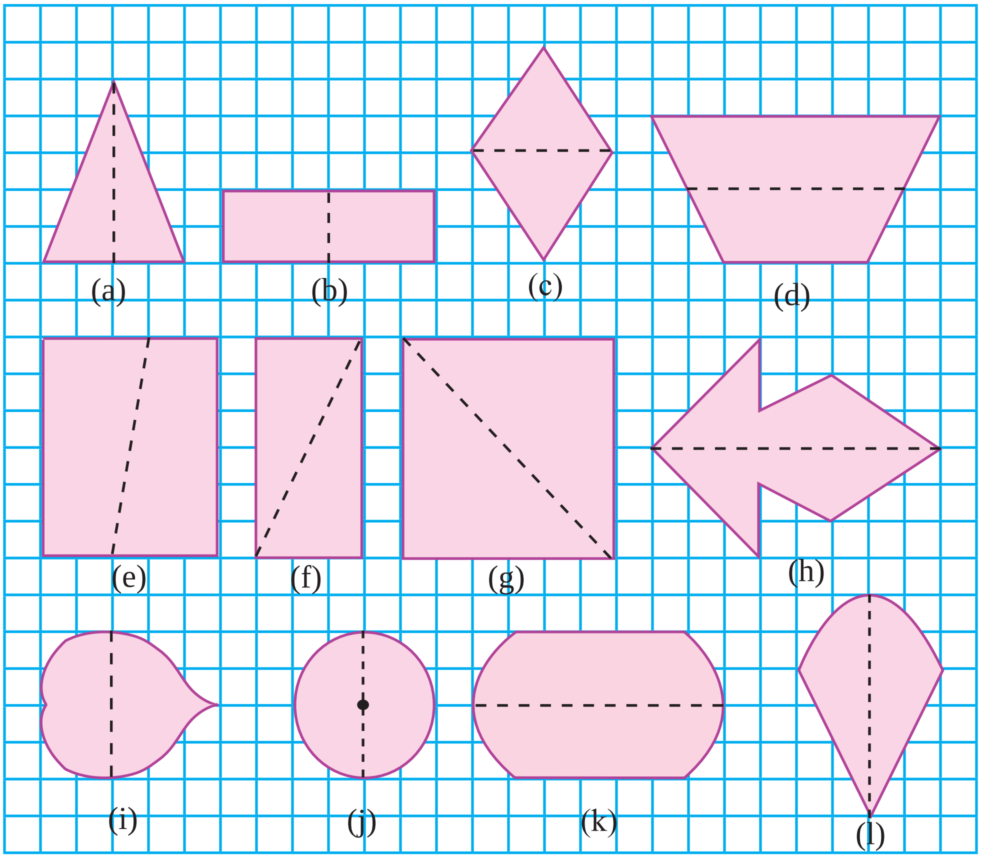
Step 2- Find the bilaterally symmetric figures from the figures that were cut out.

Step 3 - Draw all the axes of symmetry of the figures with bilateral symmetry.

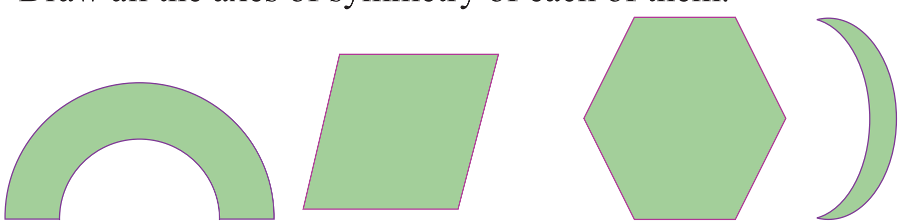
Step 4 - Paste all the figures having axes of symmetry in your exercise book. Near each figure, write its number of axes of symmetry.

Exercise 1.1

(1) From the following, choose the bilaterally symmetric figures with a correctly drawn axis of symmetry and write down the corresponding letters.



(2) (i) Cut out laminas of the following shapes using a tissue paper. Draw all the axes of symmetry of each of them.



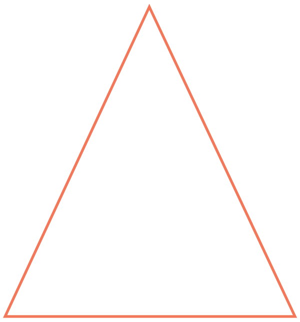
(ii) Paste all the figures having axes of symmetry in your exercise book.

(3) (i) Cut laminas of the following shapes using paper. Draw all the axes of symmetry of each of them.

A - Rectangular shape

B - Triangular shape with two sides of equal length

A



B

(ii) Write the number of axes of symmetry in each of the above figures.

(iii) Create another symmetric figure by joining two figures of the shapes given in A and B above and paste it in your exercise book.

(4) Write the statements below in your exercise book. Mark a ✓ in front of the correct sentences and a × in front of the incorrect ones.

(i) In a bilaterally symmetric figure, the two parts on either side of an axis of symmetry are equal in shape and in area.

(ii) There are bilaterally symmetric figures having more than one axis of symmetry.

(iii) The number of axes of symmetry in a circular lamina is greater than the number of axes of symmetry in a square.

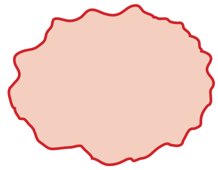
(iv) The maximum number of axes of symmetry in a bilaterally symmetric figure is one.

(v) If a bilaterally symmetric figure which has at least two axes of symmetry is cut along one axis and divided into two equal parts, then each of these parts too will be bilaterally symmetric.

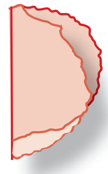
1.3 Creating plane figures having bilateral symmetry

Activity 3

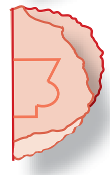
Step 1 - Get a piece of paper of any shape and a pair of scissors.



Step 2 - Fold the paper into two.

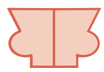


Step 3 - Draw any figure of your choice such that it contains a part of the line of folding and is limited to the area where the two portions overlap (see diagram).



Step 4 - Cut out the figure you drew.

Step 5 - Unfold the figure.



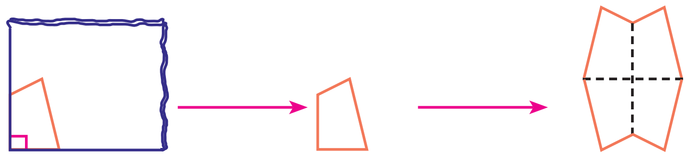
At the end of the above activity you obtain a bilaterally symmetric figure. Its axis of symmetry is the initial line along which you folded the paper.

Activity 4

Step 1 - Take another piece of paper and fold it twice so that you obtain a right angled corner.

Step 2 - Now draw a figure on this paper so that it includes the right angled corner and such that it is limited to the region where the four portions of paper overlap.

Cut the figure and unfold it. You will obtain a bilaterally symmetric figure with two axes of symmetry, where the axes of symmetry are the two lines along which you folded the paper.

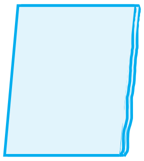


Step 3 - Cut out other symmetric figures in this manner.

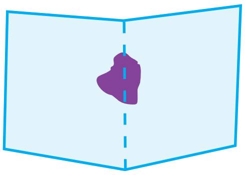
Activity 5

Step 1 - Get a paper and some paint.

Step 2 - Fold the paper into two parts.

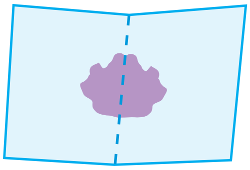


Step 3 - Now unfold the paper. On the side that is folded in, place a drop of paint so that it lies on the line of folding.



Step 4 - Now fold the paper back and press with your hand.

Step 5 - Unfold the paper.



At the end of this activity you will obtain a bilaterally symmetric figure as in the given diagram.

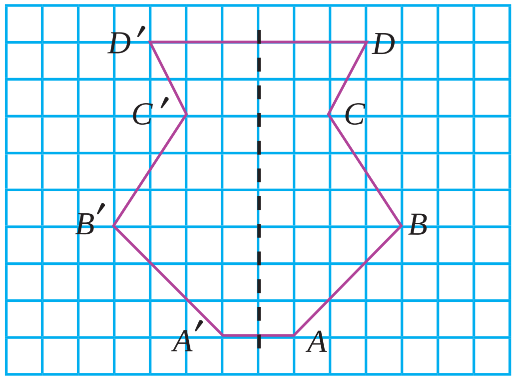
Step 6 - Following the above steps, obtain different bilaterally symmetric figures by changing the amount of paint used or by pressing down in different directions.

Assignment

- Create various bilaterally symmetric plane figures by cutting out folded paper as well as by placing drops of paint on folded paper as done in the previous activities.
- Prepare an attractive wall decoration using the symmetric figures that you created.

1.4 Drawing bilaterally symmetric plane figures

Let us consider the symmetric plane figure given below which has been drawn on a square ruled paper.



The axis of symmetry of this figure is the vertical line indicated by the dotted line. The points at which the straight line segments of a rectilinear plane figure meet are defined as the **vertices** of the plane figure. Usually the vertices are named using capital letters of the English alphabet.

The vertices A, B, C and D are on the right side of the axis of symmetry of the figure. Let us consider where the points A', B', C' and D' are located on the left side of the axis of symmetry.

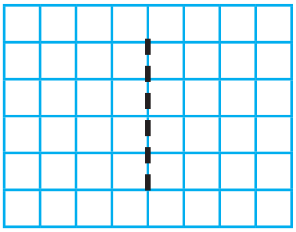
The point A' is located at a distance from the axis of symmetry which is equal to the distance from A to the axis of symmetry, on a horizontal line which passes through A. A' is defined as **the vertex corresponding to** A.

Similarly, B', C' and D' are defined as the vertices corresponding to B, C and D respectively.

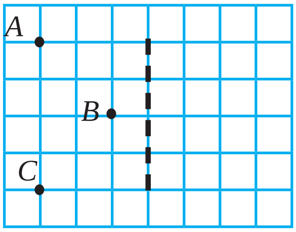
Let us consider how a bilaterally symmetric figure is drawn on a square ruled paper (or grid) by identifying corresponding vertices.

Activity 6

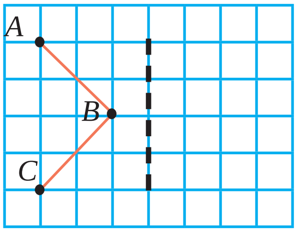
Step 1 - As indicated in the figure, select a vertical line on the grid and mark it with a dotted line.



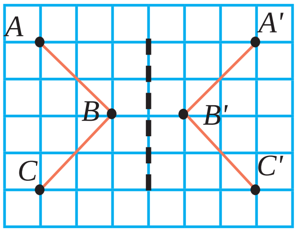
Step 2 - Select three points of intersection of vertical and horizontal lines on the grid which lie on the left side of the dotted line. Name these points as A, B and C respectively.



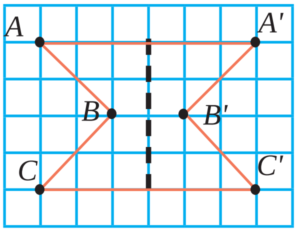
Step 3 - Join the points A and B, and the points B and C using straight line segments.



Step 4 - On the right side of the dotted line, mark on the grid, the points corresponding to the above points. Name them A', B' and C'. Join the points A' and B', and the points B' and C' using straight line segments.



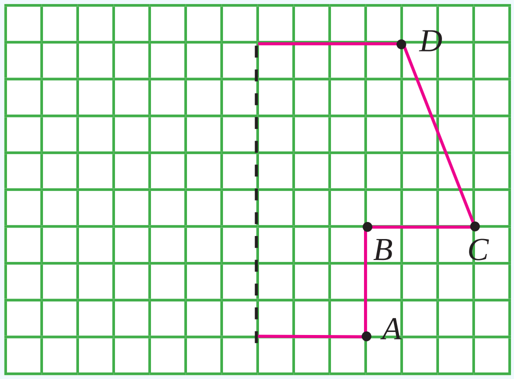
Step 5 - Join the points A and A', and the points C and C' using straight line segments.



Now you have obtained a bilaterally symmetric rectilinear plane figure with the dotted line as its axis of symmetry and the marked points as vertices.

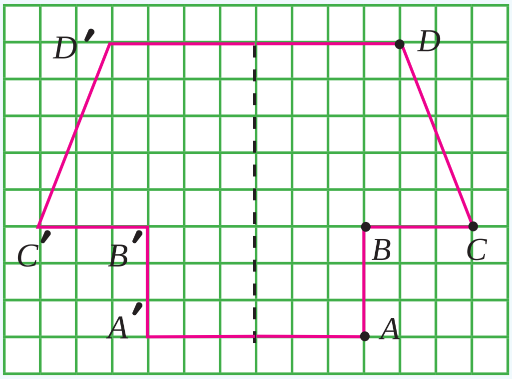
Let us consider how symmetric figures can be drawn by using the above properties.

Example 1



Complete the bilaterally symmetric figure such that the dotted line in the diagram is its axis of symmetry.

The distance from A and B to the axis of symmetry is equal to the length of three squares.



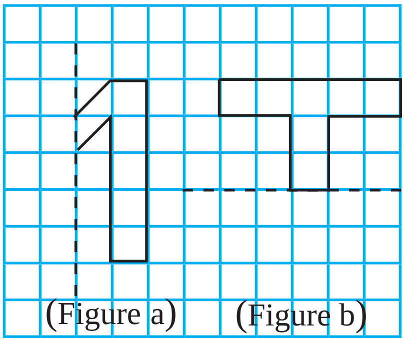
Therefore, let us mark the points A' and B' such that the distance from it to the axis of symmetry is also equal to the length of three squares, By similarly marking the points C' and D' such that the distance from C' to the axis of symmetry is equal to the length of 6 squares, and the distance from D' to the axis of symmetry is equal to the length of 4 squares as shown in the figure, and joining the points as indicated, we obtain a bilaterally symmetric figure.

Exercise 1.2

(1) (i) Copy figure *a* in your square ruled exercise book.

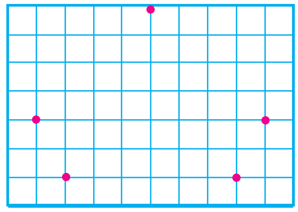
(ii) The dotted line indicates the axis of symmetry. Place a mirror on this line and observe the bilaterally symmetric figure.

(iii) Draw and complete the bilaterally symmetric figure.

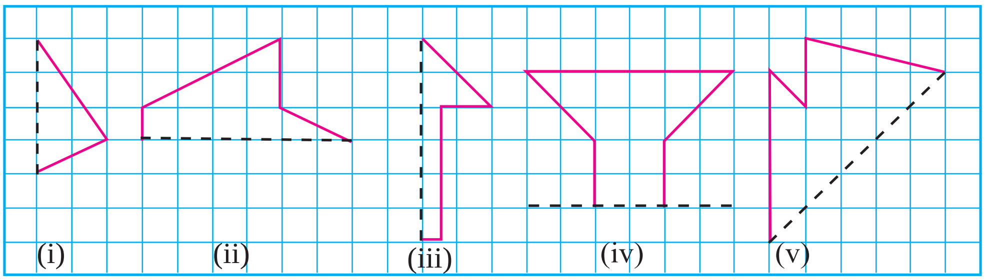


(iv) Repeat the above steps for figure *b* and complete the bilaterally symmetric figure.

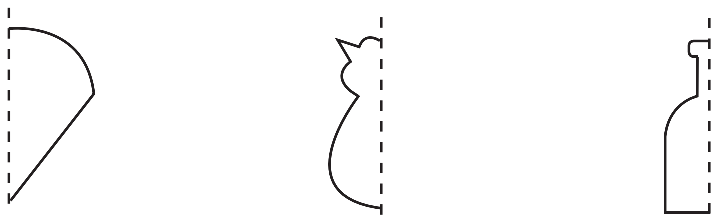
(2) Draw a bilaterally symmetric figure with the points marked on the grid as vertices and identify its axis of symmetry.



(3) Copy each of the figures given below in your exercise book. Complete the figures so that you obtain a bilaterally symmetric figure in each case.



(4) Trace each of the figures given below on a tissue paper and copy them in your exercise book.



(i) (ii) (iii)

Now turn the tissue paper on the dotted line. Draw the other half of each of the figures to obtain bilaterally symmetric figures.

(5) (i) Draw three bilaterally symmetric figures on a square ruled paper such that each figure has only one axis of symmetry.

(ii) Draw the axis of symmetry of each of the above figures.

(6) (i) Draw two bilaterally symmetric figures on a square ruled paper such that each figure has only 2 axes of symmetry.

(ii) Draw the axes of symmetry of each figure.

Summary

- If a plane figure is divided into two equal parts which coincide with each other when folded along a particular line, then that figure is defined as a bilaterally symmetric figure.
- The line of folding described above is an axis of symmetry of the figure.

2 Sets

By studying this lesson you will be able to

- identify sets,
- identify the elements of a set,
- write a set by listing the elements that belong to the set,
- write a set in terms of a common property of the elements of the set so that the elements can be clearly identified, and
- represent a set by a Venn diagram.

2.1 Introduction to Sets

The figure shows the types of vegetables that a certain vendor has for sale. The only types of vegetables that the vendor has are carrots, beans, pumpkins and ladies fingers. Accordingly, we can state with certainty whether the vendor has a certain type of vegetable for sale or not.

What has been given above is a collection of several items. Such a collection can be called a group. In our day to day life we have to make decisions on groups, that is, on such collections of items.

Let us consider the following groups.

- The districts that belong to the Southern Province of Sri Lanka
- The odd numbers between 1 and 10
- The vowels in the English alphabet
- The types of birds that are endemic to Sri Lanka, that have been identified by the year 2014
- The students who sat the Grade Five Scholarship Examination in 2014

The items that belong to these groups too can be clearly identified.

A group consisting of such items that can be clearly identified is called a **set**.

Various types of items can belong to a set. Numbers, physical objects, living beings and symbols too can belong to a set. A set can be expressed by writing down all the items in a certain group or by giving a common property or several common properties by which the items in the group can be clearly identified.

It can be stated with certainty whether a particular item belongs or does not belong to a set which has thus been specified.

The items that belong to a set are defined as its elements.

Accordingly, the district of Galle belongs to the set consisting of the districts of the Southern Province, while neither the district of Gampaha nor the district of Kalutara belongs to this set.

Three more examples of sets are given below.

- The set consisting of the even numbers between 1 and 10
- The set consisting of the symbols a, d, g, 5, 2
- The set consisting of the vehicles that were registered in Sri Lanka in 2014

The elements that belong to the above sets can be clearly identified.

Let us now consider the following.

- The tall students in a class
- Popular singers of Sri Lanka

The items that belong to such groups cannot be clearly identified since the common properties given above are subjective and debatable.

Therefore a set cannot be identified by considering such properties.

Exercise 2.1

(1) Place a ✓ next to each of the expressions which clearly define a set, and a × next to those which do not clearly define a set.

(i) Those who obtained more than 100 marks in the Grade 5 Scholarship examination held in 2013
(ii) Talented singers
(iii) Districts of Sri Lanka
(iv) Beautiful flowers
(v) Numbers between 0 and 50 which are multiples of 6
(vi) People who are fortunate

2.2 Writing a set

Let us now learn two methods of writing a set.

- **Writing a set by listing the elements of the set within curly brackets**

A set can be expressed by writing the elements of the set separated by commas, within curly brackets, when it is possible to list all the elements of the set.

The set consisting of the elements 9, 1, 3 is written as {9, 1, 3}.

➤ When writing a set in this form, the order in which the elements appear within the curly brackets is not important.

Thus, the above set can be written as {1, 3, 9} or {9, 3, 1} or {1, 9, 3} etc.

The set consisting of the elements a, b, d, 9, 1, 3 can be written as {1, 3, 9, a, b, d} or {1, a, 3, b, 9, d} or {a, 1, 3, b, 9, d} etc.

➤ Capital letters of the English alphabet are usually used to name sets.

Let A be the set of even numbers between 0 and 10. Then it can be written as follows. $A = \{2, 4, 6, 8\}$

Let B be the set of letters of the word "integers". Let us express B by writing its elements within curly brackets.
B = {i, n, t, e, g, r, s}.

Here the element “e” is written just once.

That is, even if an element appears several times within a group, it is written only once when it is written as an element of a set.

- **Writing a set by specifying common properties of its elements by which the elements of the set can be clearly identified**

A set can be expressed by writing a common property or common properties of the elements within curly brackets.

The set consisting of the even numbers between 1 and 10 can be written as {Even numbers between 1 and 10}.

The set consisting of the types of birds endemic to Sri Lanka that have been identified by the year 2014 can be written as {Types of birds endemic to Sri Lanka that have been identified by the year 2014}.

Since there are a large number of such types of birds, it is difficult to write this set by listing all the different types within curly brackets.

The set consisting of all odd numbers greater than 0, can be written as{Odd numbers greater than 0}.

Although this set cannot be expressed by writing down all its elements within curly brackets, it can be written as {1, 3, 5, 7, …}

If the elements of a set are in a certain order, when writing the set, the first few elements can be written, and to indicate the remaining elements an ellipsis (three periods) can be used within the curly brackets, after the first few elements.

Accordingly, the set of positive integers can be written as {1, 2, 3, 4, … }.

The set consisting of the types of birds endemic to Sri Lanka that have been identified by 2014 cannot be written in this manner.

Example 1

(i) Write the set A = {Prime numbers between 0 and 15} by writing all the elements that belong to A within curly brackets.

(ii) Are 1 and 17 elements of the set A?

(i) $A = \{2, 3, 5, 7, 11, 13\}$

(ii) Since 1 is not a prime number and 17 is a prime number which is greater than 15, they do not belong to A. Therefore they are not elements of A.

Example 2

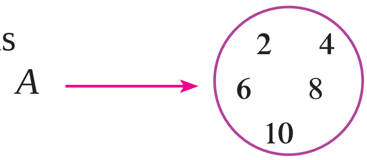
B = {The positive integers that are multiples of 3}. Write the elements of B within curly brackets.

$B = \{3, 6, 9, 12, 15, 18, ...\}$

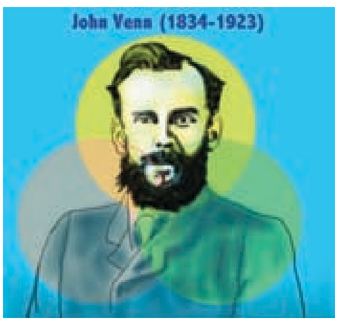
2.3 Representing a set by a Venn diagram

Let us write down the elements of the set A = {Even numbers from 1 to 10}.
$A = \{2, 4, 6, 8, 10\}$.

Let us represent this set by a closed figure as shown.



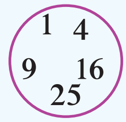
When a set is represented in the above manner by a closed figure, such a figure is defined as a **Venn diagram.** The elements of the set are written inside the closed figure. Expressing a set in this manner as a closed figure is defined as, **representing a set by a Venn diagram.**



This method of representing a set by a figure was introduced by the English mathematician **John Venn**. Therefore such a closed figure is called a **Venn diagram**.

Example 1

A set P has been represented here by a Venn diagram. P →



(i) Write down the set P by writing its elements within curly brackets.

(ii) Write P in terms of a common property by which the elements of P can be clearly identified.

(i) $P = \{1, 4, 9, 16, 25\}$

(ii) P = {Square numbers from 1 to 25}

Example 2

A is the set of positive whole numbers from 1 to 9.

(i) Write down the set A in terms of a common property of its elements.

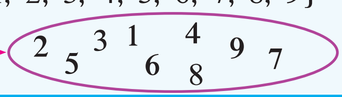
(ii) Write down the set A by listing its elements.

(iii) Represent the set A by a Venn diagram.

(i) A = { Positive whole numbers from 1 to 9}

(ii) $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

(iii) A →



Exercise 2.2

(1) (a) Express each of the following sets by writing all the elements of each set within curly brackets.

(i) A = {Days of the week}

(ii) B = {Prime numbers between 0 and 10}

(iii) C = {Multiples of 4 between 0 and 25}

(iv) D = {Letters of the word "diagram"}

(v) E = {Districts of the western province}

(vi) F = {Digits of the number 21 412 }

(vii) G = {Multiples of 6 from 1 to 10}

(b) For the sets defined above, state whether the following statements are true or false.

(i) "Saturday" is an element of A. (ii) "p" is an element of D.

(iii) All the elements of C are even numbers.

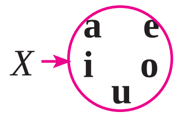
(iv) Any multiple of 3 from 1 to 10 is an element of G.

(2) Express each of the following sets in a different form by writing all the elements of each set within curly brackets.
Represent each of these sets by a Venn diagram too.
(i) $P = \{$Prime numbers less than 10$\}$
(ii) $Q = \{$Colours of a rainbow$\}$
(iii) $R = \{$Letters of the word "number"$\}$
(iv) $S = \{$Whole numbers between 0 and 7$\}$
(v) $T = \{$Districts of the Southern Province$\}$

(3) $K = \{4, 8, 12, 16, 20\}$

(i) Represent the set K by a Venn diagram.
(ii) Write down the set K in terms of a common property of its elements by which the elements can be clearly identified.

(4) The set X has been represented by a Venn diagram here.



(i) Express the set X in a different form by writing the elements of X within curly brackets.
(ii) Write down the set X in terms of a common property of its elements by which the elements can be clearly identified.

(5) Represent the set of multiples of 5 between 6 and 25.
(i) by writing down a common property by which the elements of the set can be clearly identified,
(ii) by writing all the elements of the set within curly brackets,
(iii) by a Venn diagram.

Summary

- A group of items that can be clearly identified is defined as a set.
- The items in a set are called its elements.
- A set can be expressed by writing the elements of the set separated by commas within curly brackets.
- An element of a set is written just once when the set is expressed in terms of its elements.
- A set can be expressed by writing a common property or common properties of the elements by which the elements can be clearly identified, within curly brackets.
- A set can be represented by a Venn diagram.

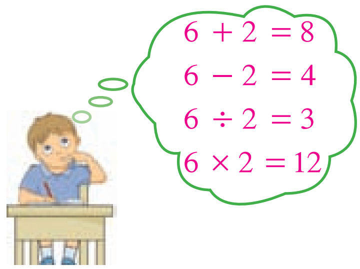
3 Operations on whole numbers

By studying this lesson you will be able to
- identify the convention used in simplifying numerical expressions, and
- simplify numerical expressions consisting of whole numbers.

3.1 Mathematical operations on two whole numbers

Addition, multiplication, subtraction and division are symbolized by +, ×, – and ÷ respectively.

You have already learnt how to add and multiply two whole numbers.



Further, you know how to subtract one whole number from another, and how to divide one whole number by another.

Here each mathematical operation was performed only once.

3.2 The order in which mathematical operations in a numerical expression are performed

Consider the expression $3 + 7 \times 5$.

This is a numerical expression with three whole numbers and two operations.

Here + and × are defined as the operations of this expression.
The order in which the operations appear is + first and then ×.

If we consider the expression $15 \div 3 - 2$, the order in which the operations appear is ÷ first and then –.

Example 1

Write down the operations of the expression $12 \times 2 - 5 \times 3$ in the order in which they appear.

The order in which the operations appear is $\times$, $-$ and $\times$.

Exercise 3.1

(1) For each of the following numerical expressions, write down the mathematical operations in the order in which they appear.

(i) $5 + 3 + 2$ (ii) $6 \times 3 - 6$ (iii) $10 - 8 \div 2 \times 3$

(iv) $11 \times 2 + 5 - 2$ (v) $24 \div 6 + 6 \div 3$

3.3 Simplifying numerical expressions

• Simplifying expressions involving only addition

Let us simplify the expression $8 + 7 + 2$ in two different ways.

Let us add 8 and 7 first, and then add 2 to the result. This yields the answer 17.

$8 + 7 + 2 = 15 + 2 = 17$

Adding 7 and 2 first, followed by adding 8 to the result also yields the answer 17.

$8 + 7 + 2 = 8 + 9 = 17$

• Simplifying expressions involving only multiplication

Let us simplify the expression $5 \times 2 \times 3$ in two different ways.

Multiplying 5 by 2 first, and then multiplying the result by 3 yields the answer 30. $5 \times 2 \times 3 = 10 \times 3 = 30$

Multiplying 2 by 3 first, and then multiplying the result by 5 also yields the answer 30.

$5 \times 2 \times 3 = 5 \times 6 = 30$

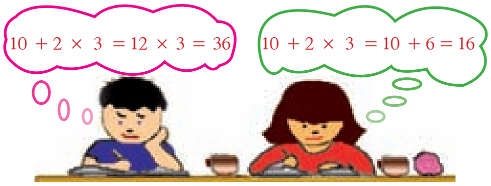
Thus, if either addition or multiplication is the only operation in a numerical expression, then irrespective of the order in which the operations are performed, the result obtained is the same.

Exercise 3.2

(1) Simplify each of the following expressions.

(i) $12 + 5 + 8$ (ii) $5 \times 8 \times 3$ (iii) $7 + 3 + 2 + 6$ (iv) $2 \times 5 \times 4 \times 3$

3.4 Further simplification of numerical expressions



Let us simplify the expression $10 + 2 \times 3$. Let us compare the answers we obtain when we simplify $10 + 2 \times 3$ by performing the operations in two different orders.

Let us first add 10 to 2 and then multiply the answer by 3.
$10 + 2 \times 3 = 12 \times 3 = 36$.

Now let us multiply 2 by 3, and then add 10 to it.
$10 + 2 \times 3 = 10 + 6 = 16$

Therefore, it is clear that when numbers simplify such numerical expressions which involve more than two terms and several operations, we may end up with different answers, depending on the order in which we perform the operations.

This emphasises the need for a convention when simplifying expressions involving two or more operations.

Let us consider below the convention used when simplifying such expressions.

- First perform all divisions (÷) and multiplications (×), working from left to right.
- Then perform all additions (+) and subtractions (–), working from left to right.

Only the operations of addition and multiplication appear in the expression $10 + 2 \times 3$. According to the above convention, multiplication should be performed first.

$10 + 2 \times 3 = 10 + 6 = 16$

Also, if only subtraction (–) and addition (+), or only division (÷) and multiplication (×) appear in a numerical expression, simplification is done from left to right in the order that the operations appear.

Simplifying expressions involving only addition and subtraction

Let us simplify the expression $10 - 7 + 2$.

Here the order in which the operations appear from left to right is – first and then +.

When simplifying $10 - 7 + 2$, first 7 is subtracted from 10 and then 2 is added to the result.

$\therefore\ 10 - 7 + 2 = 3 + 2 = 5$

Another example is: $6 + 7 - 2 = 13 - 2 = 11$

Simplifying expressions involving only multiplication and division

Let us deal with the expression $36 \div 6 \times 3$ in a similar way.
Here the order in which the operations appear from left to right is ÷ first and then ×.

Let us first divide 36 by 6, and then multiply the answer by 3.
We then obtain $36 \div 6 \times 3 = 6 \times 3 = 18$

Another example is: $36 \times 6 \div 3 = 216 \div 3 = 72$

Simplifying expressions in which only subtraction (–) or division (÷) appears several times.

When simplifying expressions involving only subtraction (–) or division (÷), the order in which the operations are performed is from left to right.

Consider the expression $10 - 3 - 2$, where the operation of subtraction is applied twice. When the expression $36 \div 6 \div 3$ is considered, division is applied twice.

Let us simplify these expressions.

Now let us subtract 3 from 10, and then subtract 2 from the result.
$10 - 3 - 2 = 7 - 2 = 5$.
Let us divide 36 by 6, and then divide the answer by 3.We then obtain,
$36 \div 6 \div 3 = 6 \div 3 = 2$.

Example 1

Simplify $7 - 4 + 5$.

$7 - 4 + 5 = 3 + 5$
$= 8$

Example 2

Simplify $80 \div 10 \times 5$.

$80 \div 10 \times 5 = 8 \times 5$
$= 40$

Example 3

Simplify $4 \times 6 \div 3$.

$4 \times 6 \div 3 = 24 \div 3$
$= 8$

Example 4

Simplify $25 + 10 - 7$.

$25 + 10 - 7 = 35 - 7$
$= 28$

Example 5

Simplify $28 \div 2 - 3$.

$28 \div 2 - 3 = 14 - 3$
$= 11$

Example 6

Simplify $50 - 10 \times 3$.

$50 - 10 \times 3 = 50 - 30$
$= 20$

Example 7

Simplify $18 \times 5 - 62$.

$18 \times 5 - 62 = 90 - 62$
$= 28$

Example 8

Simplify $50 - 10 \div 2$.

$50 - 10 \div 2 = 50 - 5$
$= 45$

Example 9

Simplify $5 + 6 \div 3 + 2$.

$5 + 6 \div 3 + 2 = 5 + 2 + 2$
$= 9$

Example 10

Simplify $2 \times 12 \div 3 \times 5$.

$2 \times 12 \div 3 \times 5 = 24 \div 3 \times 5$
$= 8 \times 5 = 40$

Exercise 3.3

(1) Place a ✓ next to the correct statements and a × next to the incorrect statements.

(i) $8 - 5 + 2 = 1$ (ii) $12 \times 3 - 11 = 25$
(iii) $7 + 18 \div 6 = 10$ (iv) $5 \times 6 \div 3 + 7 = 3$

(2) Simplify the following expressions.

(i) $10 \times 4 + 17$ (ii) $8 \times 3 + 5$ (iii) $14 \div 7 \times 5$
(iv) $448 + 12 \div 3$ (v) $7 \times 200 + 108$ (vi) $8 \times 9 - 61$
(vii) $100 - 7 \times 8$ (viii) $195 - 12 \times 10 \div 5$ (ix) $7 + 5 \times 37 + 278$

• Simplifying expressions with brackets

If we want to subtract 2 from 3 first, and then subtract the result from 10, we write it as $10 - (3 - 2)$, with $3 - 2$ within brackets. This emphasises that the operation within brackets has to be done first.

That is, $10 - (3 - 2) = 10 - 1 = 9$.

Consider the following example.

A practical examination in music is held over six days. Each day, twelve candidates participate in the morning session, while 8 participate in the afternoon session. Let us find the total number of candidates.

Number of candidates in each morning session $= 12$
Number of candidates in each afternoon session $= 8$
Total number of candidates during the six days $= (12 + 8) \times 6$
$= 20 \times 6 = 120$

Observe that the usage of brackets has been necessary in deriving the correct answer.

The convention followed when simplifying expressions involving whole numbers and the operations $+$, $-$, $\times$, $\div$, and brackets is as follows.

- First perform any calculations inside brackets.
- Then perform all multiplications and divisions, working from left to right.
- Finally perform all additions and subtractions, working from left to right.

Example 1

Simplify $20 \div (12 - 7)$.

$20 \div (12 - 7) = 20 \div 5 = 4$

Example 2

Simplify $5 \times (10 + 12) \div 11$.

$$5 \times (10 + 12) \div 11 = 5 \times 22 \div 11$$
$$= 110 \div 11 = 10$$

Example 3

Simplify $8 + 5 \times (10 + 2) \div 3 - 4$

$$\begin{aligned} 8 + 5 \times (10 + 2) \div 3 - 4 &= 8 + 5 \times 12 \div 3 - 4 \\ &= 8 + 60 \div 3 - 4 \\ &= 8 + 20 - 4 \\ &= 28 - 4 \\ &= 24 \end{aligned}$$

Example 4

The pencils in five boxes, each of which contains 12 pencils, are divided equally among 4 students. Write down an expression for the number of pencils a single student receives, and simplify it.

$(12 \times 5) \div 4 = 60 \div 4 = 15$

The number of pencils each child receives is 15.

Example 5

Nimal plucked 47 mangoes from a tree in his garden. He kept 18 in his possession, and sold the rest to his neighbour at Rs. 9 per fruit. Write down an expression for the total amount of money Nimal earned in rupees by selling the mangoes, and simplify it.

$(47 - 18) \times 9 = 29 \times 9 = 261$

This can also be written as $9 \times (47 - 18)$ or as $9\ (47 - 18)$, omitting the multiplication symbol.

The total amount earned by selling the mangoes is 261 rupees.

Example 6

The taxi fare for the first kilometre is Rs. 50. It is Rs. 42 for each kilometre above the first. Write down an expression for the amount paid by a passenger who enjoyed a ride of 12 kilometres. Simplify your expression.

$50 + 42\ (12 - 1) = 50 + 42 \times 11 = 50 + 462 = 512$

The total amount paid is 512 rupees.

Exercise 3.4

(1) Simplify the following expressions.

(i) $(12 + 8) - 15$ (ii) $35 - (14 + 9)$ (iii) $7\ (12 - 7)$

(iv) $108 + 3\ (27 - 13)$ (v) $24 \div (17 - 5)$ (vi) $3\ (5 + 2) \times 8$

(vii) $31 + (16 \div 4)$ (viii) $73 - (8 \times 9)$ (ix) $(19 \times 10) + 38$

(x) $475 - (30 \div 6)$

(2) An international call to a certain country costs Rs. 7 for the first minute, and Rs. 4 per minute thereafter. Write down an expression in rupees for the cost of a 10 minutes long international call. Simplify your expression.

(3) Write down an expression for the number of two-litre bottles that can be filled with a fruit drink made from 8 litres of water and 4 litres of fruit juice.

(4) Simplify the following expressions.

(i) $30 \div 10 \times 5$ (ii) $40 \times 10 \div 5$ (iii) $400 - 20 \times 10$

(iv) $30 \div (10 \times 3)$ (v) $(40 \div 10) \times 8$ (vi) $3 + 7 \times 5$

(vii) $6 \div 2 + 7$ (viii) $(24 \times 3) \div 8$ (ix) $24 \div (3 \times 4)$

(x) $3 + 6 \times (5 + 4) \div 3 - 7$ (xi) $10 + 8\ (11 - 3) \times 4 - 4$

Summary

- The convention followed when simplifying expressions involving whole numbers and the operations $+$, $-$, $\times$, $\div$ and brackets is as follows.
  - First perform any calculations inside brackets.
  - Then perform all multiplications and divisions, working from left to right.
  - Finally perform all additions and subtractions, working from left to right.

4 Factors and Multiples
(Part I)

By studying this lesson, you will be able to
- examine whether a whole number is divisible by 3, 4, 6 or 9.

4.1 Examining whether a number is divisible by 3, 4, 6 or 9

It is important to know the divisibility rules when solving problems related to factors and multiples.

If a certain whole number can be divided by another whole number without remainder, then the first number is said to be divisible by the second number. We then identify the second number as a factor of the first number.

$6 \div 2 = 3$ with remainder 0. Therefore, 2 is a factor of 6.

$6 \div 4 = 1$ with remainder 2. Therefore, 4 is not a factor of 6.

One way to find factors of numbers quickly is to use tests of divisibility.

The divisibility rules you learnt in grade 6 are as follows.

- If the ones place digit of a number is divisible by 2, then that number is divisible by 2.
- If the ones place digit of a number is either 0 or 5, then that number is divisible by 5.
- If the ones place digit of a number is 0, then that number is divisible by 10.

• The digital root

Now let us learn about the digital root of a number.

The digital root of a number is calculated by adding up all the digits of that number (and adding the digits of the sums if necessary), until a single digit from 1 to 9 is left. That single digit is defined as the **digital root** of the relevant number.

Let us see how the digital root of a number is found by considering the following example.

Let us find the digital root of 213. Let us add the digits of 213.

$2 + 1 + 3 = 6$

Then the digital root of 213 is 6.

The digital root of 242 = $2 + 4 + 2 = 8$

Let us find the digital root of 68.

$6 + 8 = 14$. Let us add the digits of 14. $1 + 4 = 5$.

$\therefore$ The digital root of 68 is 5.

It is possible to identify certain properties of a number by considering its digital root.

• Examining whether a number is divisible by 9

Let us do the following activity. Our goal is to identify a rule to examine whether a number is divisible by 9 or not.

Activity 1

Complete the following table and answer the given questions.

Number	Digital root	Remainder when divided by 9	Is the number divisible by 9?	Is 9 a factor?
45				
52				
134				
549				
1323				
1254				
5307				

(i) What are the digital roots of the numbers which are divisible by 9, in other words, the numbers of which 9 is a factor.

(ii) Using your answer to the previous part, suggest a method (other than division) to test whether a number is divisible by 9.

- If the digital root of a whole number is divisible by 9, then that number is divisible by 9. That is 9 is a factor of that number.

• Examining whether a number is divisible by 3

Let us do the following activity. Our goal is to identify a rule to examine whether a number is divisible by 3 or not.

Activity 2

Complete the following table and answer the given questions.

Number	Digital root of the number	Is the digital root of the number divisible by 3?	Is the number divisible by 3?	Is 3 a factor?
15				
16				
24				
28				
210				
241				
372				
1269				

(i) What values do you get as the digital roots of the numbers which are divisible by 3, in other words, the numbers of which 3 is a factor?

(ii) Does 3 always divide the digital root of the numbers which are divisible by 3?

(iii) Is every number of which the digital root is indivisible by 3, indivisible by 3?

If the digital root of a whole number is divisible by 3, then that number is divisible by 3. That is 3 is a factor of that number.

Exercise 4.1

(1) Without dividing, select the numbers which are divisible by 9.

504, 652, 567, 856, 1143, 1351, 2719, 4536

(2) Without dividing, select and write down the numbers which are divisible by 3.

81, 102, 164, 189, 352, 372, 466, 756, 951, 1029

(3) 3 divides the number 65□. Suggest two digits suitable for the empty space.

(4) Pencils were brought to be distributed among Nimal's friends on his birthday Party. The number of pencils was less than 150, but close to it. Nimal observed that each friend could be given 9 pencils. What is the maximum number of pencils that may have been brought?

(5) The following quantities of items were brought to make gift packs to be given to the winners of a competition.

131 exercise books	130 pencils
128 platignum pens	131 ballpoint pens

If each gift pack should contain 3 units of each item, write down the minimum extra amounts needed from each item.

• Examining whether a number is divisible by 6

You have learnt previously that, if the ones place digit of a number is zero or an even number, then that number is divisible by 2. You have also learnt how to determine whether a number is divisible by 3. Do the following activity to examine whether a number is divisible by 6.

Activity 3

Complete the following table and answer the given questions.

Number	Is the number divisible by 2 ?	Is the number divisible by 3 ?	Is the number divisible by 6 ?	Is 6 a factor?
95				
252				
506				
432				
552				
1236				

(i) Are all numbers which are divisible by 6, divisible by 2 also?
(ii) Are all numbers which are divisible by 6, divisible by 3 also?
(iii) Are all numbers which are divisible by 6, divisible by both 2 and 3?

(iv) Suggest a suitable method to identify the numbers which are divisible by 6, in other words, the numbers of which 6 is a factor.

If a number is divisible by both 2 and 3, then it is divisible by 6.

That is 6 is a factor of that number.

• Examining whether a number is divisible by 4

In order to identify a rule to determine whether a number is divisible by 4, do the following activity.

Activity 4

Complete the following table and answer the given questions.

Number	Is the ones place digit divisible by 4?	Is the number formed by the last two digits divisible by 4?	Is the number divisible by 4?	Is 4 a factor?
36				
259				
244				
600				
658				
1272				
4828				

(i) Is the ones place digit of every number which is divisible by 4, divisible by 4?

(ii) Is the digital root of every number which is divisible by 4, divisible by 4?

(iii) Which of the above properties should be used to determine whether a number is divisible by 4?

If the last two digits of a whole number consisting of two or more digits is divisible by 4, then that number is divisible by 4. That is 4 is a factor of that number.

Exercise 4.2

(1) From the following, select and write down the numbers

(i) Which are divisible by 6.
(ii) Which are divisible by 4.

162, 187, 912, 966, 2118, 2123, 2472, 2541, 3024, 3308, 3332, 4800

(2) Write the following numbers in the appropriate column of the table given below. (A number may be written in both column (i) and column (iii).)

348, 496, 288, 414, 1024, 1272, 306, 258, 1008, 6700

(i)	(ii)	(iii)	(iv)
Numbers of which 4 is a factor	Reason for your selection	Numbers of which 6 is a factor	Reason for your selection

(3) The number 62☐6 is divisible by both 4 and 6. Find the suitable digit for the empty space.

(4) A drill team arranges themselves in the following manner. On one occasion they form lines consisting of 3 members each and on another occasion lines consisting of 4 members each. They also make circles of 9 members each. If the drill team must have more than 250 members, use the divisibility rules to find the minimum number of members that could be in the team.

(5) Determine whether 126 is divisible by 2, 3, 4, 5, 6, 9 and 10.

Summary

Number	Divisibility Rule
2	If 2 divides the ones place digit of a whole number, then that number is divisible by 2.
3	If the digital root of a whole number is divisible by 3, then that number is divisible by 3.
4	If the last two digits of a whole number consisting of two or more digits is divisible by 4, then that whole number is divisible by 4.
5	If the ones place digit of a whole number is either 0 or 5, then that number is divisible by 5.
6	If a whole number is divisible by both 2 and 3, then it is divisible by 6.
9	If the digital root of a whole number is 9, then that number is divisible by 9.
10	If the ones place digit of a whole number is 0, then that number is divisible by 10.

4

Factors and Multiples
(Part II)

By studying this lesson, you will be able to
- find the factors of a whole number,
- write the multiples of a whole number,
- write the prime factors of a whole number,
- find the highest common factor (HCF) of a whole number and
- find the least common multiple (LCM) of a whole number.

4.2 Factors and multiples of a whole number

You learnt in grade six, how to find the factors and multiples of a whole number. Let us recall what you learnt.

Let us find the factors of 36.

Let us factorize 36 by expressing it as a product of two whole numbers.

$$36 = 1 \times 36$$
$$36 = 2 \times 18$$
$$36 = 3 \times 12$$
$$36 = 4 \times 9$$
$$36 = 6 \times 6$$

When a whole number is written as a product of two whole numbers, those two numbers are known as **factors** of the original number.

Therefore, the factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18 and 36.

Let us factorize 126, using the method of division.

$$\begin{array}{r|l} 2 & \underline{126} \\ & \ \ 63 \end{array}$$

Since the number 126, can be divided by 2 without remainder, 2 is a factor of 126.
Since, $2 \times 63 = 126$, we obtain that 63 is also a factor of 126.

$$\begin{array}{r|l} 3 & \underline{126} \\ & \ \ 42 \end{array} \qquad \begin{array}{r|l} 6 & \underline{126} \\ & \ \ 21 \end{array} \qquad \begin{array}{r|l} 7 & \underline{126} \\ & \ \ 18 \end{array} \qquad \begin{array}{r|l} 9 & \underline{126} \\ & \ \ 14 \end{array} \qquad \begin{array}{r|l} 14 & \underline{126} \\ & \ \ \ \ 9 \end{array}$$

$2 \times 63 = 126$ $\qquad$ $3 \times 42 = 126$ $\qquad$ $6 \times 21 = 126$
$7 \times 18 = 126$ $\qquad$ $9 \times 14 = 126$ $\qquad$ $14 \times 9 = 126$

Therefore, the factors of 126 are 1, 2, 3, 6, 7, 9, 14, 18, 21, 42, 63 and 126.

Note:
The divisibility rules can be used to determine whether a given number is divisible by another number or not.

Now let us consider how multiples of a whole number are found.

Let us compute the multiples of 13.
This can be done by multiplying 13 by whole numbers.

$13 \times 1 = 13$ $\quad$ $13 \times 2 = 26$ $\quad$ $13 \times 3 = 39$ $\quad$ $13 \times 4 = 52$

13, 26, 39 and 52 are a few examples of multiples of 13. Note that 13 is a factor of each of them. Therefore, any number of which 13 is a factor, is a multiple of 13.

Exercise 4.3

(1) Factorize.

(i) 150 (ii) 204 (iii) 165 (iv) 284

(2) Write down the ten factors of 770 below 100.

(3) (i) Write five multiples of 36.
(ii) Write five multiples of 112.
(iii) Write five multiples of 53 below 500.

(4) 180 chairs in an examination hall have to be arranged such that each row has an equal number of chairs. If the minimum number of chairs that should be in a row is 10 and the maximum that could be in a row is 15, find how many possible ways there are to arrange the chairs.

4.3 Prime factors of a whole number

You have already learnt that whole numbers greater than one with exactly two distinct factors are called prime numbers.

Let us recall the prime numbers below 20.

They are 2, 3, 5, 7, 11, 13, 17 and 19.

Let us identify the prime factors of 36. We learnt above that the factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18 and 36.

There are only two prime numbers among them, namely, 2 and 3. These are the prime factors of 36.

Let us find the prime factors of 60.
The factors of 60 are 1, 2, 3, 4, 5, 6, 12, 15, 20, 30 and 60.
The prime factors among them are 2, 3 and 5.

The prime numbers among the factors of a number are its prime factors.

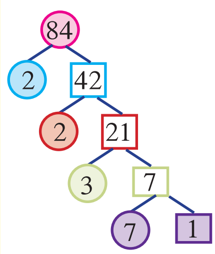
Any whole number which is not a prime number can be expressed as a product of its prime factors.

A method of finding the prime factors of a whole number using the method of division and writing the number as a product of its prime factors is described below.

Let us find the prime factors of 84 and write it as a product of its prime factors.

- Here 84 has been divided by 2, the smallest prime number.
- Division by 2 is continued, until a number which is not divisible by 2 is obtained.
- When this result is divided by the next smallest prime, which is 3, the result 7 is obtained. When this is divided by the prime number 7, the answer obtained is 1.
- In this manner, we continue dividing by prime numbers until the answer 1 is obtained.

$$\begin{array}{r|l} 2 & 84 \\ \hline 2 & 42 \\ \hline 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$



Accordingly, the prime factors of 84 are 2, 3 and 7, which are the numbers by which 84 was divided.

Now, to write 84 as a product of its prime factors, write it as a product of the prime numbers by which it was divided.

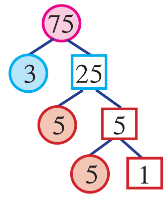
$84 = 2 \times 2 \times 3 \times 7$

Let us write 75 as a product of its prime factors. Let us divide 75 by prime numbers.

- Since 75 is not divisible by 2, we divide it by 3, the next smallest prime number.
- The result 25 is not divisible by 3.
- When 25 is divided twice by 5 which is the next smallest prime number, the result is 1.

Accordingly, when 75 is written as a product of its prime factors we obtain $75 = 3 \times 5 \times 5$.

$$\begin{array}{r|l} 3 & 75 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$



- When finding the prime factors of a whole number, the number is divided by the prime numbers which divide it without remainder, starting from the smallest such prime number, until the answer 1 is obtained.
- The prime numbers which divide a number without remainder are its prime factors.
- To write a number as a product of its prime factors, write it as a product of the prime numbers by which it was divided.

Example 1

Write 63 as a product of its prime factors.

$$\begin{array}{r|l} 3 & 63 \\ \hline 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

Since 63 is not divisible by 2, division is started with 3. The result 21 is again divided by 3. Then we obtain 7 which is not divisible by 3. We divide this by 7 to obtain 1.

Therefore, 63 written as a product of its prime factors is

$63 = 3 \times 3 \times 7$.

Exercise 4.4

(1) Find the prime factors of each of the following numbers.

(i) 81 (ii) 84 (iii) 96

(2) Express each of the following numbers as a product of its prime factors.

(i) 12 (ii) 15 (iii) 16 (iv) 18 (v) 20
(vi) 28 (vii) 59 (viii) 65 (ix) 77 (x) 91

4.4 Finding the factors of a number by considering its prime factors

Suppose we need to find the factors of 72.
Let us start by writing 72 as a product of its prime factors.

$$\begin{array}{r|l} 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$\begin{aligned}
72 &= 2 \times 2 \times 2 \times 3 \times 3 &&= 1 \times 72 \\
72 &= 2 \times 2 \times 2 \times 3 \times 3 &&= 2 \times 36 \\
72 &= 2 \times 2 \times 2 \times 3 \times 3 &&= 4 \times 18 \\
72 &= 2 \times 2 \times 2 \times 3 \times 3 &&= 8 \times 9 \\
72 &= 2 \times 2 \times 2 \times 3 \times 3 &&= 24 \times 3
\end{aligned}$$

The factors of a whole number (which are not its prime factors or 1) can be obtained by taking products of 2 or more of its prime factors.

2, 36, 4, 18, 8, 9, 24 and 3 are eight factors of 72. 1 and 72 are also factors of 72.

1, 2, 3, 4, 8, 9, 18, 24, 36 and 72 are ten factors of 72.

Exercise 4.5

(1) Find six factors of each of the following numbers by considering their prime factors.

(i) 20 (ii) 42 (iii) 70 (iv) 84 (v) 66 (vi) 99

4.5 Highest Common Factor (HCF) (Greatest Common Divisor (GCD))

Let us now consider what the highest common factor (HCF) of several numbers is and how it is found.

Let us find the highest common factor of the numbers 6, 12 and 18.

- Write down the factors of these numbers as follows.

Factors of 6: 1, 2, 3 and 6
Factors of 12: 1, 2, 3, 4, 6 and 12
Factors of 18: 1, 2, 3, 6, 9 and 18

- Circle and write the factors common to all three numbers.

  The factors which are common to 6, 12 and 18 are, 1, 2, 3 and 6.
- The largest number among the common factors is the **highest common factor** of these numbers.

We observe that the largest or the greatest of these common factors is 6. Therefore, 6 is the **highest common factor** of 6, 12 and 18.

Thus, the highest common factor of 6, 12 and 18 is 6, which is the largest number by which these three numbers are divisible.

- Given two or more numbers, the largest number which is a factor of all of them is known as their highest common factor (HCF).
- Accordingly, the highest common factor is the largest number by which all the given numbers are divisible.
- If 1 is the only common divisor of several numbers, then the highest common factor of these numbers is 1.

Finding the highest common factor by writing each number as a product of its prime factors

Let us find the highest common factor of 6, 12 and 18.

- Let us write each number as a product of its prime factors.

$$
\begin{aligned}
6 &= 2 \times 3 \\
12 &= 2 \times 2 \times 3 \\
18 &= 2 \times 3 \times 3
\end{aligned}
$$

$$
\begin{array}{r|l}
2 & 6 \\ \hline
3 & 3 \\ \hline
 & 1
\end{array}
\qquad
\begin{array}{r|l}
2 & 12 \\ \hline
2 & 6 \\ \hline
3 & 3 \\ \hline
 & 1
\end{array}
\qquad
\begin{array}{r|l}
2 & 18 \\ \hline
3 & 9 \\ \hline
3 & 3 \\ \hline
 & 1
\end{array}
$$

$6 = 2 \times 3$ $\quad$ $12 = 2 \times 2 \times 3$ $\quad$ $18 = 2 \times 3 \times 3$

- The highest common factor is obtained by taking the product of the prime factors which are common to all three numbers.

2 and 3 are the common prime factors of 6, 12 and 18.
Thus, the HCF of 6, 12 and 18 is $2 \times 3 = 6$.

• Finding the highest common factor by the method of division

Let us find the highest common factor of 6, 12 and 18.

$$
\begin{array}{r|lll}
2 & 6, & 12, & 18 \\ \hline
3 & 3, & 6, & 9 \\ \hline
 & 1, & 2, & 3
\end{array}
$$

- Write the numbers as shown.
- Since all these numbers are divisible by 2, divide each of them by 2 individually.
- The result is 3, 6 and 9. Since 3, 6 and 9 are divisible by 3, the next smallest prime number, divide them by 3 and write the result below the respective numbers.
- The result is 1, 2 and 3. Since there isn't a prime number which divides all of 1, 2 and 3 without remainder, the division is stopped here.
- The HCF is obtained by multiplying the numbers by which division was done.

Thus the HCF of 6, 12 and 18 is $2 \times 3 = 6$.

When using the method of division to find the HCF,
- keep dividing all the numbers by the prime numbers which divide all the numbers without remainder.
- then multiply all the divisors and obtain the HCF of the given numbers.

The HCF of any set of prime numbers is 1.

Example 1

Find the highest common factor of 72 and 108.

Method I

The factors of 72 are 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36 and 72.
The factors of 108 are 1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54 and 108.

When the factors common to both these numbers are selected we obtain 1, 2, 3, 4, 6, 9, 12, 18 and 36.

Since the greatest of these common factors is 36, the highest common factor of 72 and 108 is 36.

Method II

Let us write 72 and 108 as products of their prime factors.

$$\begin{array}{r|l} 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 108 \\ \hline 2 & 54 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$72 = 2 \times 2 \times 2 \times 3 \times 3$$
$$108 = 2 \times 2 \times 3 \times 3 \times 3$$

The prime factors which are common to both the numbers 72 and 108 are 2, 2, 3 and 3.

Accordingly, the HCF of 72 and 108 $\Big\} = 2 \times 2 \times 3 \times 3 = 36$

Method III

Find the HCF of 72 and 108.

$$\begin{array}{r|l} 2 & 72,\ 108 \\ \hline 2 & 36,\ \ 54 \\ \hline 3 & 18,\ \ 27 \\ \hline 3 & 6,\ \ \ 9 \\ \hline & 2,\ \ \ 3 \end{array}$$

Since there isn't another prime number which divides both 2 and 3 without remainder, stop the division here.

The HCF of 72 and 108 $\Big\} = 2 \times 2 \times 3 \times 3$
$= 36$

The HCF 36 of 72 and 108 can also be described as the largest number that divides them both without remainder.

Example 2

(1) Three types of items were brought in the following quantities to be offered at an almsgiving.

30 cakes of soap

24 tubes of toothpaste

18 bottles of ointment

Parcels were made such that each contained all three items. Every parcel had the same number of items of a particular type. What is the maximum number of parcels that can be made accordingly? Write down the quantity of each item in a single parcel.

Every parcel should contain the same number of items of a particular type. To find the maximum number of parcels that can be made, we need to find the largest number which divides 30, 24 and 18 without remainder.

Therefore, let us find the HCF of 30, 24 and 18.

$$30 = 2 \times 3 \times 5$$
$$24 = 2 \times 2 \times 2 \times 3$$
$$18 = 2 \times 3 \times 3$$

HCF of 30, 24 and 18 = $2 \times 3 = 6$.

The maximum number of parcels that can be made = 6

The number of cakes of soap in a parcel = $30 \div 6 = 5$

The number of tubes of toothpaste in a parcel = $24 \div 6 = 4$

The number of bottles of ointment in a parcel = $18 \div 6 = 3$

Exercise 4.6

(1) Fill in the blanks to obtain the HCF by writing down all factors of the given numbers.

(i) Factors of 8 are ...., ..., ... and .....

Factors of 12 are ...., ..., ..., ..., ..., and .....

Factors common to 8 and 12 are ...., ..., and .....

$\therefore$ The HCF of 8 and 12 is .... .

(ii) 54 written as a product of its prime factors = 2 × ..... × 3 × .....

90 written as a product of its prime factors = ... × 3 × ..... × 5.

72 written as a product of its prime factors = 2 × 2 × ..... × ..... × .....

∴ The HCF of 54, 90 and 72 = ... × ..... × .....

= .....

(2) Find the HCF of each pair of numbers by writing down all their factors.

(i) 12, 15 (ii) 24, 30 (iii) 60, 72
(iv) 4, 5 (v) 72, 96 (vi) 54, 35

(3) Find the HCF of each pair of numbers by writing each number as a product of its prime factors.

(i) 24, 36 (ii) 45, 54 (iii) 32,48 (iv) 48, 72 (v) 18, 36

(4) Find the HCF by any method you like.

(i) 18, 12, 15 (ii) 12, 18, 24 (iii) 24, 32, 48 (iv) 18, 27, 36

(5) A basket contains 96 apples and another basket contains 60 oranges. If these fruits are to be packed into bags such that there is an equal number of apples in every bag and an equal number of oranges too in every bag and no fruits remain after they are packed into the bags, what is the maximum number of such bags that can be prepared?

4.6 Least Common Multiple (LCM)

Now let us consider what is meant by the least common multiple of several numbers and how it is found.

As an example, let us find the least common multiple of the numbers 2, 3 and 4.

- List the multiples of the given numbers.

Several multiples of the numbers 2, 3 and 4 are given in the following table.

Multiples of 2	2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26
Multiples of 3	3, 6, 9, 12, 15, 18, 21, 24
Multiples of 4	4, 8, 12, 16, 20, 24, 28

☛ Circle and write down the common multiples.

You will observe that the common multiples of the three numbers listed here are 12 and 24.

Further, if we continue to write the common multiples of 2, 3 and 4, we will obtain 12, 24, 36, 48, 60 etc

☛ The smallest of the common multiples of several numbers is called the **least common multiple** (LCM) of these numbers.

The smallest or the least of the common multiples 12, 24, 36, 48, 60, … of the numbers 2, 3 and 4 is 12.

Therefore, 12 is the least common multiple of 2, 3 and 4.

In other words, the smallest number which is divisible by 2, 3 and 4 is the least common multiple of 2, 3 and 4.

The least common multiple of several numbers is the smallest positive number which is divisible by all these numbers.

Note

- The HCF of several numbers is equal to or smaller than the smallest of these numbers
- The LCM of several numbers is equal to or larger than the largest of these numbers.
- The HCF of any two numbers is smaller than the LCM of the two numbers.

• Finding the LCM of several numbers by considering their prime factors

Let us see how the LCM of several numbers is found by considering their prime factors.

Let us find the LCM of 4, 12 and 18.

- Let us write each number as a product of its prime factors.

$$4 = 2 \times 2 = 2^2$$
$$12 = 2 \times 2 \times 3 = 2^2 \times 3^1$$
$$18 = 2 \times 3 \times 3 = 2^1 \times 3^2$$

- Let us select the greatest power of each prime factor.

There are two distinct prime factors, namely, 2 and 3. When the factors of all three numbers are considered,

the power of 2 with the largest index = 2^2
the power of 3 with the largest index = 3^2.

- The LCM of the given numbers is the product of these greatest powers.

  Therefore, the LCM of 4, 12 and 18 $= 2^2 \times 3^2$
  $= 2 \times 2 \times 3 \times 3$
  $= 36$

• Finding the LCM by the method of division

Let us find the LCM of 4, 12 and 18.

$$\begin{array}{r|lll} 2 & 4, & 12, & 18 \\ \hline 2 & 2, & 6, & 9 \\ \hline 3 & 1, & 3, & 9 \\ \hline & 1, & 1, & 3 \end{array}$$

- Write these numbers as shown.
- Since all these numbers are divisible by 2, divide each of them by 2 individually.
- We get 2, 6 and 9 as the result. No prime number divides all of them without remainder. However, 2 divides both 2 and 6 without remainder. Divide 2 and 6 by 2, and write the results below the respective numbers. Write 9 below 9.
- Since 3 and 9 are divisible by 3, the next smallest prime number, divide them by 3 and write the results below the respective numbers. Now observe that we cannot find at least two numbers which are divisible by the same number. Therefore, the division is stopped here.
- Multiply all divisors and all numbers left in the last row. The product gives the LCM of the given numbers.

  Accordingly, the LCM of 4, 12 and 18 $= 2 \times 2 \times 3 \times 1 \times 1 \times 3 = 36$

Note
When using the method of division to find the LCM, keep dividing if there remain at least two numbers, divisible by another and obtain the LCM of the given numbers as above.

Let us find the LCM of 4, 3 and 5.

Here, we do not have at least two numbers which are divisible by a common number which is greater than 1.

Therefore, the LCM of 4,3 and 5 $= 4 \times 3 \times 5$
$= 60$

Example 1

Find the LCM of 8, 6 and 16.

Method I

Let us write each number as a product of its prime factors as follows.

$$\begin{aligned} 8 &= 2 \times 2 \times 2 &&= 2^3 \\ 6 &= 2 \times 3 &&= 2^1 \times 3^1 \\ 16 &= 2 \times 2 \times 2 \times 2 &&= 2^4 \end{aligned}$$

The distinct prime factors of these three numbers are 2 and 3. Here, the maximum number of times that 2 appears is four. The maximum number of times that 3 appears is one.

Therefore, the LCM of 8, 6 and 16 $= 2^4 \times 3$
$= 2 \times 2 \times 2 \times 2 \times 3$
$= 48$

Method II

$$\begin{array}{r|lll} 2 & 8, & 6, & 16 \\ \hline 2 & 4, & 3, & 8 \\ \hline 2 & 2, & 3, & 4 \\ \hline & 1, & 3, & 2 \end{array}$$

Since we cannot find another prime number by which at least two of the three numbers 1, 3, 2 are divisible, we stop the division here.

The LCM of 8, 6 and 16
$= 2 \times 2 \times 2 \times 1 \times 3 \times 2 = 48$

Example 2

(1) Two bells ring at intervals of 6 minutes and 8 minutes respectively. If they both ring together at 8.00 a.m., at what time will they ring together again?

To find when both bells ring together again, it is necessary to find at what time intervals the bells ring together.

The first bell rings once every 6 minutes. 6, 12, 18, 24, …
The second bell rings once every 8 minutes. 8, 16, 24, …
Therefore, the two bells ring together again after 24 minutes.

The answer could be obtained by finding the LCM too.
Since the bells ring together at intervals of the LCM of 6 minutes and 8 minutes, to find out when the two bells ring together again, the LCM of these two numbers needs to be found.

$\therefore$ let us find the LCM of 6 and 8.

$$\begin{array}{r|l} 2 & 6,\ 8 \\ \hline & 3,\ 4 \end{array}$$

The LCM of 6 and 8 = $2 \times 3 \times 4 = 24$.
Therefore, the two bells ring together again after 24 minutes.
The bells ring together initially at 8.00 a.m.
Therefore, the bells ring together again at 8.24 a.m.

Exercise 4.7

(1) Find the LCM of each of the following triples of numbers.

(i) 18, 24, 36 (ii) 8, 14, 28 (iii) 20, 30, 40

(iv) 9, 12, 27 (v) 2, 3, 5 (vi) 36, 54, 24

(2) At a military function, three cannons are fired at intervals of 12 seconds. 16 seconds and 18 seconds respectively. If the three cannons are fired together initially, after how many seconds will they all be fired together again?

Miscellaneous Exercise

(1) Without dividing, determine whether the number 35 343 is divisible by 3, 4, 6 and 9.

(2) Fill in the blanks.

(i) The HCF of 2 and 3 is .... .

(ii) The LCM of 4 and 12 is .... .

(iii) The HCF of two prime numbers is .... .

(iv) The LCM of 2, 3 and 5 is .... .

(3) Find the HCF and LCM of 12, 42 and 75.

(4) It is proposed to distribute books among 45 students in a class, such that each student receives no less than 5 books and no more than 10 books. Find all possible values that the total number of books that need to be bought can take, if all the students are to receive the same number of books and no books should be left over.

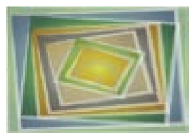
Summary

- Prime numbers among the factors of a number are called the prime factors of that number.
- Given two or more numbers, the largest of their common factors is called the highest common factor (HCF) of these numbers. Thus, the largest number which divides a list of numbers is their HCF.
- Given two or more numbers, the smallest of their common multiples is called the least common multiple (LCM) of these numbers. Thus, the smallest positive number which is divisible by a list of numbers is their LCM.

Ponder

(1) The length and breadth of a rectangular piece of cloth are 16 cm and 12 cm respectively. If this piece is to be cut without any waste into equal sized square pieces, what is the maximum possible side length of such a square piece?

(2) The length and the breadth of a floor tile are 16 cm and 12 cm respectively. What is the side length of the smallest square shaped floor area that can be tiled using such tiles, if no tile is to be cut?



(3) The circumference of the front and a back wheel of a tricycle are 96 cm and 84 cm respectively. What is the minimum distance the tricycle must move, for the front wheel and back wheels to complete full revolutions at the same instant?

(4) What is the smallest number which is greater than 19 that has a remainder of 19, when divided by 24, 60 and 36?

5 Indices

By studying this lesson you will be able to

- write a number in index form, as a power having a prime number as the base,
- identify powers that have an algebraic symbol as the base,
- expand powers that have an algebraic symbol as the base and
- find the value of an algebraic expression by substituting positive integers for the unknowns.

Indices

Index notation is used to write a number which is multiplied repeatedly, in a concise way. Let us recall what has been learnt thus far about indices.

$2 \times 2 \times 2$ is written as 2^3 using indices.
That is, $2 \times 2 \times 2 = 2^3$.

In 2^3, 2 is defined as the base and 3 is defined as the index. 2^3 is read as "two to the power 3".

2^3 Index Base

The value of 2^3 is 8. Therefore, the number 8 can be written as 2^3 in index notation.

When the index is a positive integer, it denotes how many times the number in the base is multiplied by itself.

Product	Number of times 3 is multiplied by itself	Index notation
3×3	2	3^2
$3 \times 3 \times 3$	3	3^3
$3 \times 3 \times 3 \times 3$	4	3^4
$3 \times 3 \times 3 \times 3 \times 3$	5	3^5
$3 \times 3 \times 3 \times 3 \times 3 \times 3$	6	3^6

You have learnt these facts in Grade 6. Do the following exercise to recall what you have learnt thus far about indices.

Review Exercise

(1) Expand each of the following as a product and find the value of the given expression.

(i) 3^2 (ii) 5^4 (iii) $2^2 \times 3$ (iv) $6^2 \times 5^2$

(2) Write down each of the following products using index notation.

(i) $4 \times 4 \times 4$ (ii) $7 \times 7 \times 7 \times 7$

(iii) $2 \times 2 \times 3 \times 3$ (iv) $3 \times 3 \times 5 \times 3 \times 5$

(3) Fill in the blanks in the following table.

Number	Index Notation	Base	Index	How the index notation is read
25	5^2	5	2	Five to the power two
343	....	7	....	....
....	....	....	....	Six to the power three

(4) Write the number 16

(i) using index notation with base 2.

(ii) using index notation with base 4.

5.1 Expressing a number in index notation with a prime number as the base

Let us write 8 in index notation with a prime number as the base.

Let us write 8 as a product of its prime factors.

$$\begin{array}{r|l} 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

$8 = 2 \times 2 \times 2$

8 in index notation $= 2^3$

Now let us express the number 40 in index notation with prime numbers as the bases of the powers.

Let us write 40 as a product of prime numbers.

$$\begin{array}{r|l} 2 & 40 \\ \hline 2 & 20 \\ \hline 2 & 10 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$40 = 2 \times 2 \times 2 \times 5$

When this is written in index notation we obtain $40 = 2^3 \times 5$.

That is, 40 can be expressed as a product of powers with prime numbers as bases, in the form $40 = 2^3 \times 5$.

Do the following to express a number as a product of powers with prime numbers as bases.

- Start by dividing the number by the smallest prime number which divides it without remainder,
- Continue dividing the result by the prime numbers which divide it without remainder, in increasing order of the prime numbers, until the answer 1 is obtained.
- Write the number as a product of powers of these primes, where the index is the number of times division by that prime is done.

Example 1

Write down the number 36 as a product of powers with prime numbers as bases.

$$\begin{array}{r|r} 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$36 = 2 \times 2 \times 3 \times 3$

$36 = 2^2 \times 3^2$

Example 2

Write down the number 100 as a product of powers with prime numbers as bases.

$$\begin{array}{r|r} 2 & 100 \\ \hline 2 & 50 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$100 = 2 \times 2 \times 5 \times 5$

$100 = 2^2 \times 5^2$

Exercise 5.1

(1) (i) Write 25 in index notation with 5 as the base.
(ii) Write 64 in index notation with 2 as the base.
(iii) Write 81 in index notation with 3 as the base.
(iv) Write 49 in index notation with 7 as the base.

(2) Write each of the following numbers as a product of powers with prime numbers as bases.
(i) 18 (ii) 24 (iii) 45 (iv) 63 (v) 72

5.2 Powers with an algebraic symbol as the base

We have learnt about powers with a number as the base. Let us now consider instances when the base is an algebraic symbol.

$2 \times 2 \times 2 = 2^3$

$5 \times 5 \times 5 = 5^3$

We can in a similar manner write $x \times x \times x = x^3$.

The base of x^3 is x and the index is 3.

x^3 Index, Base

Further,

$a \times a$ and $m \times m \times m \times m$ can be expressed as powers as $a \times a = a^2$ and $m \times m \times m \times m = m^4$ with an algebraic symbol as the base.

$2^1 = 2$. Accordingly, a can be written as $a = a^1$ in index notation.
The product of 2 and 3 is written as 2×3.
The product of a and b can be written as $a \times b$.
$a \times b$ can be expressed as ab or ba.
Accordingly $3 \times a \times b$ can be expressed as $3ab$.

Further, $m \times m \times m \times n \times n = m^3 \times n^2$.

$m^3 \times n^2$ which is also equal to $n^2 \times m^3$, can be expressed as m^3n^2 or as n^2m^3.

When two powers are connected with a multiplication sign, if the bases of both the powers are not numerical values, then it is not necessary to include the multiplication sign.

Example 1

Write down each of the following expressions using index notation.

(i) $p \times p \times p$ (ii) $x \times x \times y \times y \times y$
(iii) $2 \times 2 \times a \times a \times a$ (iv) $m \times 3 \times m \times 3 \times 3$

(i) $p \times p \times p = p^3$ (ii) $x \times x \times y \times y \times y = x^2 \times y^3 = x^2 y^3$
(iii) $2 \times 2 \times a \times a \times a = 2^2 \times a^3 = 2^2 a^3$
(iv) $m \times 3 \times m \times 3 \times 3 = 3^3 \times m^2 = 3^3 m^2$

Example 2

Expand and write each of the following expressions as a product.

(i) m^3 (ii) $p^2 q^3$ (iii) $5^2 x^3$

(i) $m^3 = m \times m \times m$ (ii) $p^2 q^3 = p \times p \times q \times q \times q$
(iii) $5^2 x^3 = 5 \times 5 \times x \times x \times x$

Exercise 5.2

(1) Write down each of the following expressions using index notation.

(i) $x \times x \times x \times x$ (ii) $a \times a \times a$ (iii) $m \times m \times m \times n \times n \times n$

(iv) $7 \times 7 \times 7 \times p \times p$ (v) $y \times y \times y \times y \times 7 \times 7 \times 7$

(2) Expand and write each of the following expressions as a product.

(i) a^2 (ii) $2p^2$ (iii) $2^3 m^2$ (iv) $3^2 x^3$ (v) $x^3 y^3$

5.3 Finding the value of a power by substitution

Let us consider expressions in index notation with bases which are unknowns. By substituting values for the unknown bases, the value of an expression in index notation can be found. In this lesson, only positive integers are substituted.

Let us find the value of the expression x^3 when $x = 2$.

Method I

By substituting the value 2 for x we obtain,

$x^3 = 2^3$
$= 2 \times 2 \times 2$
$= 8$

Method II

$x^3 = x \times x \times x$

By substituting the value 2 for x we obtain,

$x^3 = 2 \times 2 \times 2$
$x^3 = 8$

Example 1

Find the value of each of the following expressions when $x = 5$.

(i) x^3

Method I

$x^3 = 5^3$
$= 5 \times 5 \times 5$
$= 125$

Method II

$x^3 = x \times x \times x$
$= 5 \times 5 \times 5$
$= 125$

(ii) $3x$

$3x = 3 \times x$
$= 3 \times 5$
$= 15$

Example 2

Find the value of each of the following expressions when $a = 3$ and $b = 5$.

(i) $a^2 b$ (ii) $2a^3 b^2$

(i) a^2b

$a^2b = a \times a \times b$

Substituting $a = 3$ and $b = 5$

we obtain,

$a^2b = 3 \times 3 \times 5$

$= 45$

(ii) $2a^3b^2$

$2a^3b^2 = 2 \times a \times a \times a \times b \times b$

Substituting $a = 3$ and $b = 5$

we obtain,

$2a^3b^2 = 2 \times 3 \times 3 \times 3 \times 5 \times 5$

$= 1350$

Exercise 5.3

(1) Find the value of each of the following expressions by substituting $x = 3$.

(i) x^4 (ii) $3x^2$ (iii) $5x^3$

(2) Find the value of each of the following expressions by substituting $a = 3$.

(i) $2a^2$ (ii) 2^2a^2 (iii) $7a^2$

(3) Find the value of each of the following expressions by substituting $x = 1$ and $y = 7$.

(i) x^2y^3 (ii) $2x^3y$ (iii) $3xy^2$

(4) Find the value of each of the following expressions by substituting $a = 2$ and $b = 7$.

(i) a^2b (ii) ab^2 (iii) a^3b^2 (iv) $3a^2b^2$

Summary

- An expression of an unknown term multiplied repeatedly can be expressed as a power with the unknown term as the base and the number of times the term is multiplied as the index.

a to the power three $\rightarrow a^3$

Index

Base

- When two powers are connected with a multiplication sign, if the bases of both the powers are not numerical values, then it is not necessary to include the multiplication sign.
- A value can be obtained for an expression in index notation with an unknown base, by substituting a number for the unknown term.

6 Time

By studying this lesson you will be able to

- identify months, years, decades, centuries and millenniums as units of time
- identify a leap year,
- identify the relationships between units of time, and
- add and subtract units of time.

6.1 Units of time

You have already learnt that the units seconds, minutes , hours and days are used to measure time.

You have also learnt to find the time it takes to do different daily activities.

Now, let us learn more on the units of measuring time - months, years, decades, centuries and millenniums.

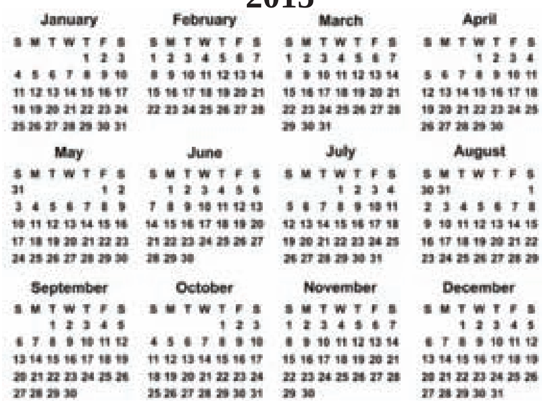
• Months and years

If we want to calculate the time taken for an event which commences on a particular date and ends on another date, in terms of days, weeks or months, we can do so by looking at a calendar.

A calendar is made up of the units days, weeks and months. You will see that there are 12 months in a calendar.

The calendar of year 2015 is shown below. The table shows the number of days in each month.

2015



The months having 31 days	The months having 30 days	The months having 28 days
January	April	February
March	June	
May	September	
July	November	
August		
October		
December		

The calendar of a particular year provides information on a period of a year, starting from the first of January and ending on the thirty first of December of that year.

According to the year 2015 calendar, the total number of days in the year is 365. There are 365 days in a year which is not a leap year. We will be studying about leap years later.

- ☞ The day 2015-08-01 means, the time period from 00:00 on 2015-08-01 to 24:00 on 2015-08-01.
- ☞ The time at which a particular day ends ,is the time at which the next day starts. So the time 24:00 on 2015-08-01 is the same as the time 00:00 on 2015-08-02.
- ☞ The year 2015 means, the time period from 00:00 on 2015-01-01 to 24:00 on 2015-12-31.

Note :
The international convention for measuring years is by considering the year of the birth of Jesus.

BC and AD are commonly used to count years in time. Jesus Christ's birth is used as the starting point to count years that existed before (BC) and after (AD) he was born.

Decades

A time period of ten years is considered as a decade. Let us consider 1948.

The first year in the decade that contains the year 1948 is 1941, and the last year in that decade is 1950.

The time period from AD 1 to AD 10 is called the first decade.
The time period from AD 11 to AD 20 is called the second decade.
The time period from AD 1811 to AD 1820 is called the hundred and eighty second decade.
The time period from AD 1951 to AD 1960 is called the hundred and ninety sixth decade.
The time period from AD 2011 to AD 2020 is called the two hundred and second decade.

That is, the time period from time 00:00 on 1941-01-01 to time 24:00 on 1950-12-31 is a decade. This decade is identified as the 195th decade.

Centuries

A time period of a hundred years is called a century.

AD 1 to AD 100 is the first century.
AD 101 to AD 0200 is the second century.
AD 1801 to AD 1900 is the nineteenth century.
AD 1901 to AD 2000 is the twentieth century.
AD 2001 to AD 2100 is the twenty first century.

The time period from 00:00 on 2001-01-01 to 24:00 on 2100-12-31 is the twenty first century.

A Millennium

A time period of a 1000 years is known as a millennium. According to the calendar, at this moment we are living in the third millennium.

The time period from AD 1 to 1000 is the first millennium.
The time period from AD 1001 to 2000 is the second millennium.

Example 1

(i) To which millennium does AD 1505 belong? Second millennium

(ii) To which century does AD 1505 belong ? 16th century

(iii) To which decade does AD 1505 belong? Hundred and fifty first decade.

Exercise 6.1

(1) Write down the decade to which each one of the following years belongs.

(i) AD 1856 (ii) AD 1912 (iii) AD 1978 (iv) AD 2004

(2) Write the first date and the last date of the 22nd century.

(3) Write down the century to which each one of the following years belongs.

(i) AD 1796 (ii) AD 1815 (iii) AD 1956 (iv) AD 2024

6.2 Leap year

The calendar of 2016 is given below. Consider the number of days in each month . How does this differ from the calendar of 2015 ?

The months having 31 days	The months having 30 days	The months having 29 days
January	April	February
March	June	
May	September	
July	November	
August		
October		
December		

There are 29 days in the month of February. So the total number of days in 2016 is 366.

Any year in which there are 29 days in the month of February has 366 days in total. Such a year is defined as a leap year.

If a number that denotes a year is divisible by 4 but is not a multiple of 100, then that year is a leap year. However years which are denoted by numbers that are multiples of 100 become leap years only if they divisible by 400.

Example 1

Is the year 2000 a leap year?

Since $2000 = 100 \times 20$, 2000 is a multiple of 100.
Since $2000 \div 400 = 5$, 2000 is divisible by 400.
So year 2000 is a leap year.

Example 2

Is the year 1900 a leap year?
1900 is a multiple of 100.
1900 is not divisible by 400.
$\therefore$ 1900 is not a leap year.

Example 3

Is the year 2008 a leap year?
2008 is not a multiple of 100.
$2008 \div 4 = 502$, 2008 is divisible by four .
$\therefore$ 2008 is a leap year.

Example 4

Is 2010 a leap year?
2010 is not a multiple of 100.

The number that is formed by the last two **digits** of 2010, that is 10, is not divisible by 4. Therefore by the rules of divisibility, 2010 is also not divisible by 4.

Hence, 2010 is not a multiple of 4. Therefore 2010 is not a leap year.

Note : Any year which is not a multiple of 4 is not a leap year.

• Further units of time

60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day
There are months consisting of 28 days, 29 days, 30 days and 31 days.

However a time period of 30 days is considered as a month.
12 months = 1 year
365 days = 1 year
366 days = 1 leap year

A time period given in years can be represented in days, by multiplying it by 365.

A time period given in years can be represented in months, by multiplying it by 12.

Note :
We consider 30 days as a month. However, because a year consists of 12 months, you should not think that the number of days in a year is 360 (12×30 days). A year consists of 365 days.

Example 1

(i) Indicate 280 days in months and days.

$$\begin{array}{r|l} & 9 \\ \hline 30 & 280 \\ & 270 \\ \hline & 10 \end{array}$$

Therefore, 280 days is 9 months and 10 days.

Example 2

(i) Indicate 3 years in months.
(ii) Indicate 3 years in days.

(i) 3 years $= 3 \times 12$ months
$= 36$ months

(ii) 3 years $= 3 \times 365$ days
$= 1095$ days

6.2 Exercise

(1) Choose the leap years from the years given below.

(i) AD 1896 (ii) AD 1958 (iii) AD 1960
(iv) AD 1400 (v) AD 1600 (vi) AD 2016

(2) (a) Indicate the days given below, in months and days.

(i) 225 days (ii) 100 days (iii) 180 days

(b) How many months are there in 5 years? How many days are there in 5 years?

(3) A bus which makes 4 trips a day, runs continuously for 6 months daily. Find the total number of trips it makes during this period.

(4) A patient has to take 3 tablets per day for a period of 2 months. How many tablets are required for this purpose?

(5) A person exercises for 1 hour every day.

(i) How many hours does he spend exercising during a year which is not a leap year?

(ii) Indicate this time in days.

(6) A person puts a minimum of 5 Rupees in a till every day. Find the least amount of money he would collect during each time period below.

(i) 6 months (ii) A leap year

6.3 Calculations related to time

A certain school was in session as follows during a certain year.

The first term consisted of 3 months and 6 days, the second term consisted of 3 months and 8 days, and the third term consisted of 3 months and 3 days.

Months	Days
3	6
3	8
+3	3
9	17

Let us express the time period that the school was in session that year, in months and days.

For this we need to add the above time periods to find the total time period.

So the school was in session for 9 months and 17 days.

Example 1

A teacher served for 5 years 6 months and 23 days in a school in the Eastern province and for 6 years 8 months and 15 days in a school in the Central province. He served the rest of his career in a school in the Southern province.

(i) Find in total how long he served in the Eastern and Central provinces.
(ii) If he served for 28 years, 2 months and 2 days in total, then find how long he served in the school in the Southern province.

(i)

	Years	Months	Days
	5	6	23
+	6	8	15
			8

Let us add the days in the "days column"
23 days + 15 days = 38 days
38 days = 1 month + 8 days
Let us write the 8 days in the "days column". Let us carry the 1 month to the "months column".

	Years	Months	Days
	5	6	23
+	6	8	15
	12	3	8

1 month + 6 months + 8 months = 15 months = 1 year and 3 months
Let us write the 3 months in the "months column".
Let us carry the year to the "years column".

1 year + 5 years + 6 years =12 years.

The total service of the teacher in the Eastern and Central provinces is 12 years, 3 months and 8 days.

(ii)

	Years	Months	Days
	28	2	2
–	12	3	8
			24

Let us subtract the days in the "days column".
Since 2<8 let us take a period of one month, that is 30 days from the "months column" and add it to the "days column".
Then, 30 days + 2 days = 32 days.
32 days - 8 days = 24 days.
Let us write the 24 days in the "days column".

	Years	Months	Days
	28	2	2
–	12	3	8
	15	10	24

Now, in the months column we have to subtract 3 months from the remaining 1month. Since this cannot be done, let us carry a period of 1 year, that is 12 months, from the "years column" to the "months column". Then,
12 months + 1 month = 13 months
13 months – 3 months = 10 months
Let us write the 10 months in the " months column".

When 12 years are deducted from the remaining 27 years in the "years column" we get 15 years.

So the amount of time the teacher spent in the school in the Southern province is 15 years, 10 months and 24 days.

Example 2

Sunitha's date of birth is 2008-05-06.

(i) What is her age on 2016-08-24?

(ii) Nimal is younger to her by 3 years, 6 months and 3 days. Find Nimal's date of birth.

(i) The date on which we want to know the age = 2016-08-24

Sunitha's date of birth = 2008-05-06

Let us find Sunitha's age on 2016-08-24.

	Years	Months	Days
	2016	8	24
−	2008	5	6
	8	3	18

Sunitha's age is 8 years, 3 months and 18 days.

(ii) Nimal's date of birth is the ninth of November, 2011.

	Years	Months	Days
	2008	5	6
+	3	6	3
	2011	11	9

Exercises 6.3

(1) Do the following additions.

(i)

	Months	Days
	8	18
+	2	11

(ii)

	Months	Days
	8	22
+	2	16

(iii)

	Years	Months	Days
	12	6	21
+	3	2	19

(iv)

	Years	Months	Days
	8	9	19
+	2	6	23

(2) Do the following subtractions.

(i)

	Months	Days
	6	23
−	3	15

(ii)

	Months	Days
	6	18
−	2	24

(iii)

	Years	Months	Days
	3	6	15
−	2	4	18

(iv)

	Years	Months	Days
	2	8	12
−	1	2	15

(3) Dileepa's date of birth is 2003-09-07.
Sithumini's date of birth is 2000-02-04.

(i) Find how old Dileepa and Sithumini are today.
(ii) Find how much older Sithumini is to Dileepa,
(a) using their ages,
(b) using their dates of birth.

(4) Below are the service periods of two teachers in a certain school.

	Date he started work in the school	**The date he was transferred from this school**
Mr. Iqbal	2001 - 07 - 13	2015 - 11 - 22
Mr. Subhairudeen	1997 - 03 - 20	2012 - 01 - 10

(i) Find the period of service of each teacher. Who has served longer in this school?
(ii) How many more years has the teacher has served than the other one ?

(5) Shashika's date of birth is 2014-08-13. Aheli is 1 year, 8 months and 25 days older to her. What is Aheli's date of birth?

(6) A school was opened on 1928-03-26.
(i) When is the school's centennial anniversary?
(ii) How many days are there to the centennial anniversary date from today?

(7) Amila participated in Agricultural training progammes in Japan and China. He stayed in Japan from 2012-02-13 to 2014-07-27 and in China from 2014-12-17 to 2015-10-05. Find the total time he spent in Japan and China.

Miscellaneous Exercise

(1) A person borrows a certain amount of money. He has to pay the debt in equal installments once every month, for 10 years. The first installment was paid on 2016-01-01. Find the date on which he has to pay the final installment.

(2) Below are the age limits for participants in an inter-house sportsmeet of a certain school.
Under 11 games – Age should be less than 11 years on 2016-03-31.
Under 13 games – Age should be less than 13 years and greater than or equal to 11 years on 2016-03-31.

(3) Under 15 games – Age should be less than 15 years and greater than or equal to 13 years on 2016-03-31.
Under 17 games – Age should be less than 17 years and greater than or equal to 15 years on 2016-03-31.

The dates of birth of several students are given below.

Name	**Birthday**
Vanthula	2005-12-08
Hashan	2002-05-17
Hasintha	2000-01-16

Find which age group each student qualifies to participate in.

Summary

- A time period of 10 years is defined as a decade.
- A time period of 100 years is defined as a century.
- A time period of 1000 years is defined as a millennium.
- If a number that denotes a year is divisible by 4 but is not a multiple of 100, then that year is a leap year. However years which are denoted by numbers that are multiples of 100 become leap years only if they divisible by 400.

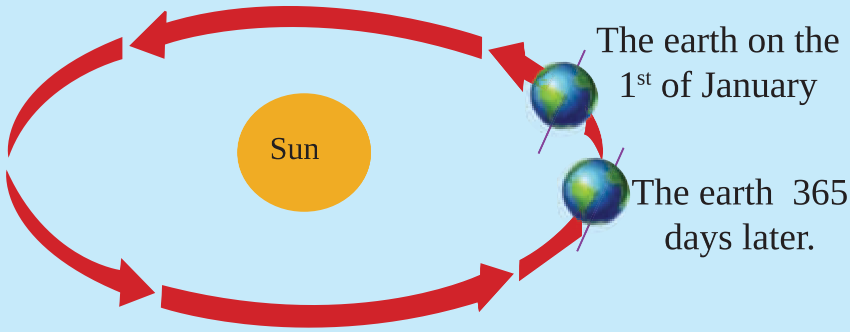
Ponder

(1) A person was born on 2002-09-23 at 9.32 a.m. Find for how long he has lived in years, days, hours and minutes when it is 12 noon of 2015-06-05.

(2) A certain person lived for 20591 days. Find his age in years, months and days at the time he passed away.

Extra Knowledge –More on leap years

• Why Do We Have Leap Years?



In any year which is not a leap year there are 365 days.
A year is defined as the time it takes for the Earth to orbit around the sun once.

However, the exact time it takes for the earth to orbit around the sun is 365 days 5 hours 48 minutes and 46 seconds. This is about $365\frac{97}{400}$ days. So when we say a year has 365 days, we have neglected a time period of 5 hours, 48 minutes and 46 seconds (**which is little less than 1/4 day**). Four of these periods added together is approximately one day. We add this as an extra day to the calendar once every four years. It is added to the month of February. This is how we get a leap year.

A leap year has an extra day. Due to the decision to add an extra day to the calendar once every four years, 3 additional days get included in each 400 year period.

Although an extra day is added once every four years, only 23 hours 15 minutes and 4 seconds should actually be added once every four years. Therefore, due to the decision of adding an extra day once every four years, there are approximately 3 additional days that are included in each period of 400 years.

Therefore three days need to be removed from every 400 year period.

To do this, an extra day is not added to the month of February for the first three century years. (A century year is a year ending in 00)

Non-century years are leap years if they are multiples of four.

7 Parallel Straight Lines

By studying this lesson you will be able to
- identify parallel straight lines,
- identify that the gap between a pair of parallel straight lines is the perpendicular distance, that is, the shortest distance between the two lines,
- examine whether a given pair of straight lines is parallel or not by using a straight edge and a set square,
- draw parallel lines using a straight edge and a set square, and
- draw rectilinear plane figures containing parallel lines using a straight edge and a set square.

7.1 Straight line segment

Activity 1

(1) Draw a straight line using a straight edge. Name this straight line *l*.

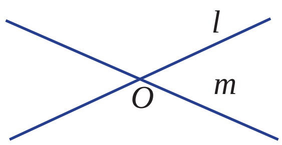
(2) Mark the two points *A* and *B* on the straight line *l* as shown in the figure.

The portion *AB* of the straight line *l* is defined as the straight line segment *AB*. The two points *A* and *B* are defined as **the two end points of the straight line segment** *AB*.

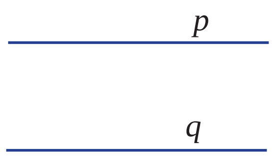
The convention is to use capital letters of the English alphabet to name straight line segments.

7.2 Parallel straight lines

Examine the two pairs of straight lines given below which are drawn on the same plane.



The straight lines l and m intersect each other at O.



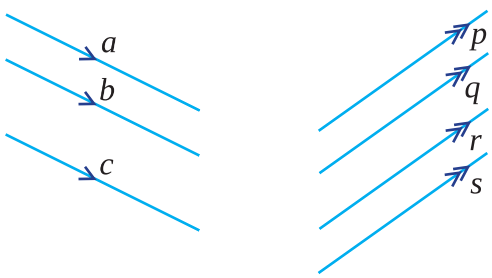
The straight lines p and q do not intersect each other.

Two straight lines which do not intersect each other are called **parallel straight lines**.

Accordingly, the two straight lines p and q are parallel, while the two straight lines l and m are not parallel.

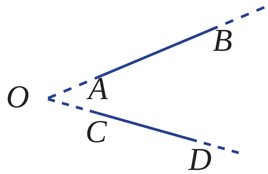
When several straight lines do not intersect each other, they are defined as straight lines which are parallel to each other.

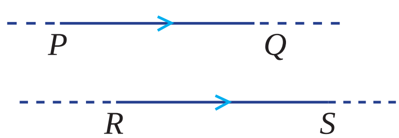
To indicate that several lines are parallel to each other, arrowheads are drawn on the straight lines in the same direction and sense, as shown in the figure.



Accordingly, in the above figure, a, b and c are parallel to each other and p, q, r and s are parallel to each other.

Let us check whether each of the following pairs of straight line segments are parallel to each other or not.





The two straight lines on which the straight line segments *AB* and *CD* lie, intersect at *O*. However the two straight lines on which the straight line segments *PQ* and *RS* lie, do not intersect.

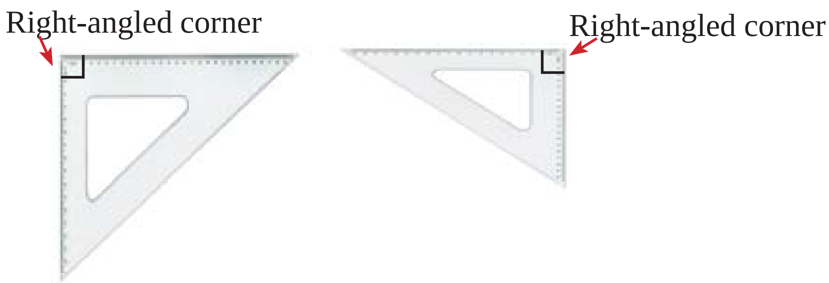
Accordingly, *PQ* and *RS* are parallel straight line segments while *AB* and *CD* are not.

We indicate the fact that *PQ* and *RS* are parallel straight line segments using the notation "*PQ* // *RS*".

7.3 Perpendicular distance

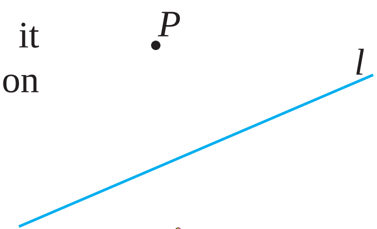
- **The perpendicular distance from a point to a straight line**

The following is a figure of set squares. Let us consider how the perpendicular distance from a point to a straight line is found using a set square.

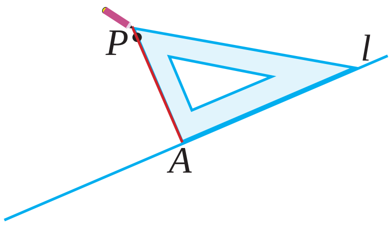


Activity 2

(1) Draw a straight line and name it *l*. Mark a point *P* which is not on *l*.

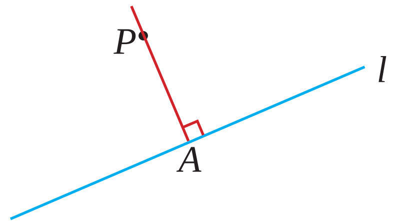


(2) As shown in the figure, place the set square such that one edge which forms the right angled corner lies on *l* and the other edge passes through the point *P*.



(3) Mark the point *A* on *l* as indicated and join *AP*.

The angle marked at A is a right angle. **We say that the straight line segment AP is perpendicular to l.**



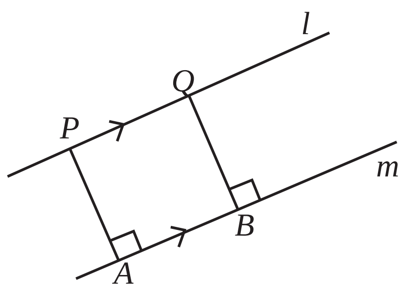
(3) Observe that the point on l which is closest to P is A. Measure AP.

The length of the straight line segment AP is defined as the **perpendicular distance** from the point P to the straight line l. The length of AP is the **shortest distance** from the point P to l.

• The perpendicular distance between two parallel straight line

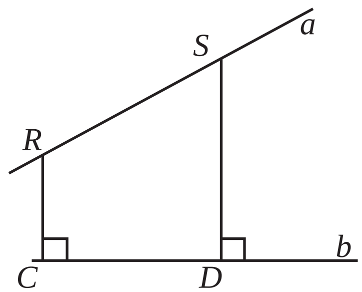
The perpendicular distances from the two points P and Q that lie on the line l to the straight line m are equal to each other. That is, $PA = QB$.

$\therefore l$ and m are two parallel straight lines.



However, the perpendicular distances from the two points R and S on a to the straight line b are unequal. That is, $RC \neq SD$.

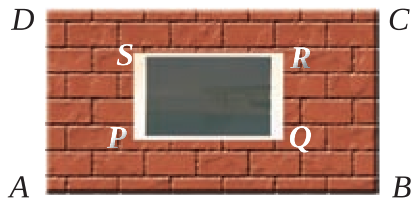
$\therefore$The straight lines a and b are not parallel to each other.



- The shortest distance from every point on a straight line to a parallel straight line is a constant. This constant distance is defined as the **perpendicular distance** between the two parallel straight lines.

  This perpendicular distance is also defined as the **gap** between the two parallel straight lines.
- Straight lines which lie on the same plane and which are a constant distance from each other are parallel to each other.

The figure given below depicts a wall of a room and a window in the wall. Since the wall is rectangular in shape, the opposite edges are parallel.



- That is, the horizontal edges which are represented by the straight line segments *AB* and *DC* are parallel to each other.
- Similarly, the vertical edges which are represented by the straight line segments *AD* and *BC* are parallel to each other.
- The straight line segments *PQ* and *SR*, represent the horizontal edges of the window. They are parallel to each other.
- The straight line segments *PS* and *QR*, represent the vertical edges of the window. They are parallel to each other.

There are several locations in the environment where such parallel edges can be observed.

- The horizontal panels of a ladder
- The beams of a roof
- The straight line segments of a 100 m running track

are some examples.

Exercise 7.1

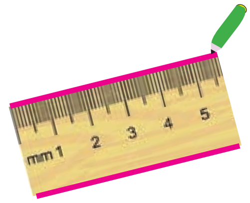
(1) Write down the names of two objects that can be observed in the classroom that have parallel edges.

(2) Write down the names of two objects in your day to day environment that have parallel edges.

(3) Name four locations where parallel lines can be observed in architectural designs.

(4) Describe several arrangements and tasks which involve parallel straight lines.

7.4 Drawing parallel lines using a straight edge and a set square



As shown in the figure, place the ruler on a page of your exercise book and draw two straight lines along the edges of the ruler. Now you have obtained a pair of parallel straight lines.

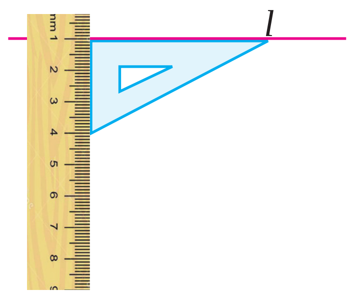
• Drawing a straight line parallel to a given straight line using a straight edge and a set square

Activity 3

(1) Draw a straight line using a straight edge and name it l.

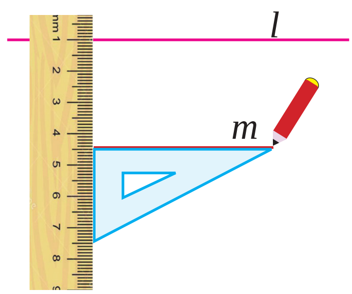
(2) Place the set square such that one edge which forms the right angled corner lies on the straight line l.

As shown in the figure, place the straight edge such that it touches the other edge which forms the right angled corner of the set square.

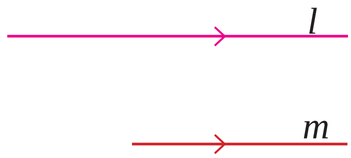


(3) Keeping the straight edge fixed, move the set square along the straight edge.

(4) Stop moving the set square and draw a straight line along the edge which forms the right angled corner and is not touching the straight edge.

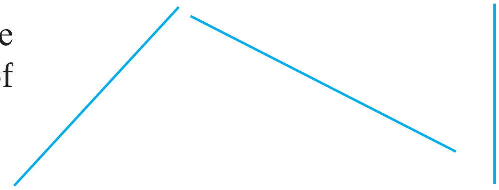


(5) Name this straight line m.



Now you have obtained a straight line m which is parallel to the straight line l.

➢ Copy the straight lines in the figure and draw a line parallel to each of them.

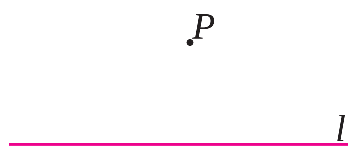


Only one line can be drawn parallel to a given line on a plane, through a point on the plane which does not lie on the given line.

• Drawing a straight line parallel to a given straight line, through a point which is not on the straight line, using a straight edge and a set square

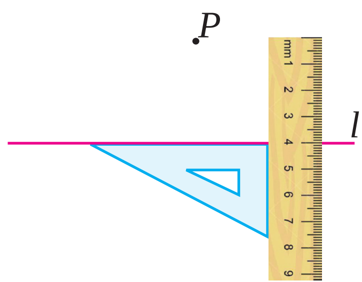
Activity 4

(1) As indicated in the figure, name a point P that does not lie on the straight line l.



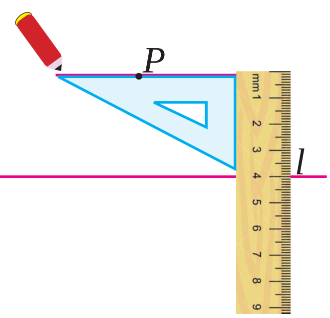
(2) Place the set square such that one edge which forms the right angled corner lies on the straight line l.

As shown in the figure, place the straight edge such that it touches the other edge which forms the right angled corner of the set square.



(3) Keeping the straight edge fixed, move the set square along the straight edge.

(4) When the edge of the set square which was lying on the straight line l touches the point P, draw a straight line along it.



Now you have obtained a straight line through the point P, which is parallel to the straight line l.

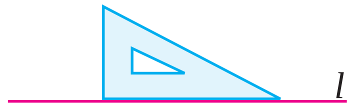
- **Drawing a line parallel to a straight line at a given distance from the straight line, using a ruler and a set square**

Activity 5

Let us draw a straight line which is parallel to the straight line *l* through a point at a distance of 2.5 cm above it.

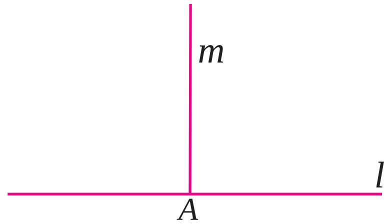
(1) Draw a straight line *l* as shown in the figure.

(2) Place the set square such that one edge which forms the right angled corner lies on the straight line *l*.



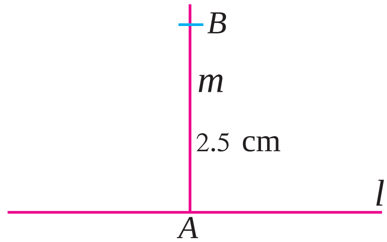
(3) Draw a straight line along the edge which forms the right angled corner and does not lie on the straight line *l*.

Name this straight line *m*.



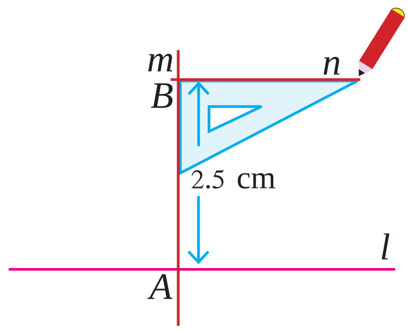
(4) Name the point at which the straight line *m* meets the straight line *l* as *A*.

(5) Mark the point *B* on the straight line *m*, at a distance of 2.5 cm from *A*.



(6) Place the set square such that the right angled corner coincides with *B* and one of the edges which forms the right angled corner lies on *m*, and draw the line *n* along the other edge which forms the right angled corner.

Now you have obtained a straight line *n* which is parallel to the straight line *l* and which lies at a distance of 2.5 cm from *l*.



(7) Draw in a similar manner, the straight line which is parallel to the straight line *l* and which lies 2.5 cm below *l*.

Exercise 7.2

(1) (i) Draw a straight line segment of length 6 cm and name it AB.

(ii) Mark a point P which does not lie on the straight line segment.

(iii) Draw a straight line passing through P parallel to AB, using a ruler and a set square.

(iv) Find the gap between the two straight lines by using a straight edge and a set square.

(2) (i) Draw a straight line segment. Name it PQ.

(ii) Mark a point A below PQ such that the perpendicular distance from A to PQ is 4.8 cm.

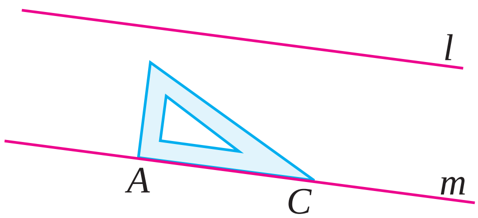
(iii) Draw a straight line segment which passes through A and is parallel to PQ.

7.5 Examining whether two straight lines are parallel

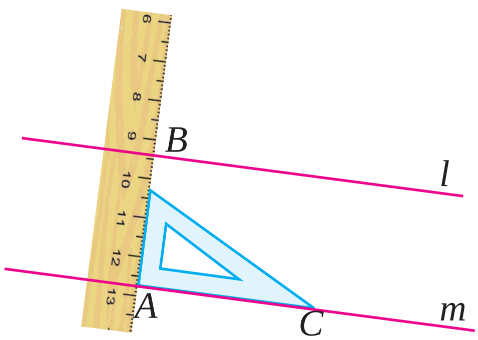
To determine whether two straight lines in the same plane are parallel or not, it is necessary to check whether the perpendicular distances from any two points on one line to the other line are equal or not.

Activity 6

Let us examine whether the two straight lines l and m are parallel.

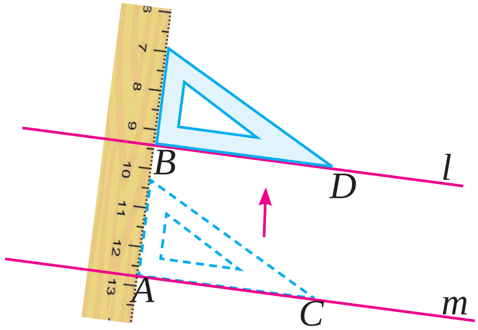


(1) As shown in the figure, place the set square such that one edge which forms the right angled corner lies on the straight line m.



(2) Place the straight edge such that it touches the other edge which forms the right angled corner of the set square as indicated in the figure. Name the point at which the straight edge meets the line l as B.

(3) Keeping the straight edge fixed, move the set square along the straight edge as shown in the figure, until the right angled corner coincides with the point *B* on the straight line *l*.



(4) Check whether the edge of the right angled corner which was initially on the straight line *m*, now coincides with the straight line *l*.

If it coincides, then the perpendicular distances from the two points B and D to the straight line *m* will be equal, and hence *l* and *m* are two parallel straight lines.

If it does not coincide, then the straight lines *l* and *m* are not parallel to each other.

7.6 Drawing rectilinear plane figures using a set square and a straight edge

Activity 7

(1) Draw a rectangle of length equal to the length of 6 squares and breadth equal to the length of 4 squares on your square ruled exercise book.

(2) Establish that the distance between the two longer sides of the rectangle is a constant value by counting squares. Confirm this by measuring the distance between the two longer sides using a ruler.

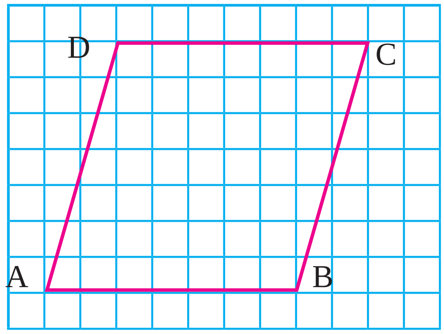
- If the distance is a constant value, then the straight line segments which represent the two longer sides of the rectangle are parallel to each other.
- It can be seen similarly that the two straight line segments drawn to represent the shorter sides of the rectangle are also parallel to each other.

Activity 8

(1) Draw the straight line segments AB and DC on a square ruled paper such that their lengths are equal to the length of 7 squares.

(2) Complete the figure $ABCD$ by drawing the straight line segments AD and BC.

(3) Using a set square and a straight edge, show that AD and BC are parallel to each other and find the gap between them.



Activity 9

(1) Draw a straight line segment and mark the points A and B on it such that AB = 6 cm.

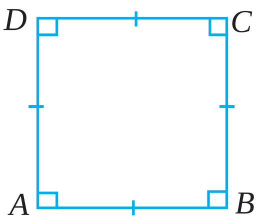
(2) Using a set square, draw two straight lines through the points A and B, perpendicular to the given line.

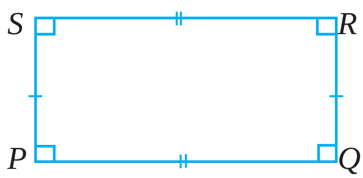
(3) Mark the points C and D such that AD = 6 cm and BC = 6 cm.

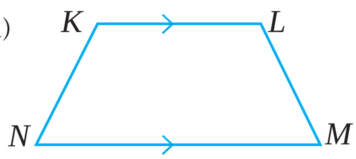
(4) Complete the figure $ABCD$ using a straight edge. What is the name given to a quadrilateral of the shape $ABCD$?

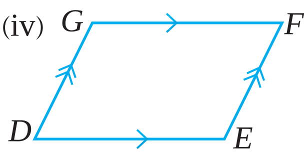
Exercise 7.3

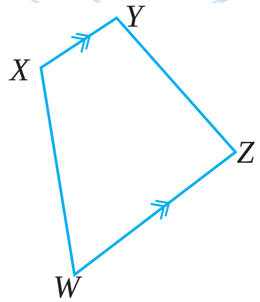
(1) Draw each of the following figures using a straight edge and a set square.

(i) 

(ii) 

(iii) 

(iv) 

(v) 

(2) Write down for each of the above figures whether each pair of opposite sides is parallel or not.

(3) Using a straight edge and a set square,
  (i) draw a square of side length 5 cm.
  (ii) draw a rectangle of length 8 cm and breadth 5 cm.

(4) (i) Draw a straight line segment *AB* such that *AB* = 6 cm.
  (ii) Draw the straight line segment *BC* such that if forms an obtuse angle with *AB* at *B*.
  (iii) Draw a straight line through *C* parallel to *AB* in the direction of *A*.
  (iv) Mark the point *D* on this straight line such that *CD* = 6 cm. Join *AD* to obtain the parallelogram *ABCD*.

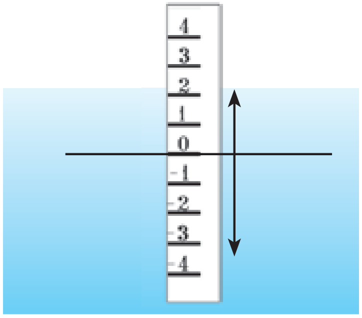
Summary

- Two straight lines in a plane which do not intersect each other are called parallel straight lines.
- Two straight lines in a plane which are at a constant distance from each other are parallel to each other.
- The gap between two parallel straight lines is a constant.

8 Directed Numbers

By studying this lesson you will be able to
- identify what directed numbers are,
- add integers using the number line, and
- add directed numbers without using the number line.

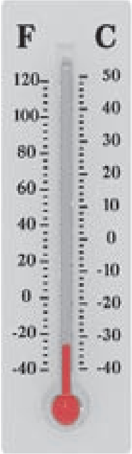
8.1 Identifying directed numbers



The figure given here represents an indicator that is used to measure the water level of a reservoir from which water is distributed to a certain city.

The usual water level of the reservoir has been marked as 0 (zero), and the indicator has been calibrated such that the gaps between the numbers above the 0 limit and below the 0 limit are equal.

Thereby it can be observed whether the water level of the reservoir is above or below 0 (the usual level). Here, by calibrating the indicator in opposite directions, a correct perception of the water level of the reservoir is obtained.



Similarly, thermometers that are used to measure the temperature of the environment are calibrated in opposite directions from 0^{o} C, to indicate temperatures that are greater than 0^{o} C and temperatures that are less than 0^{o} C.

The thermometer in the figure has been calibrated with the values 10, 20, 30, … in one direction to indicate the temperatures that are greater than 0^{o} C, and with the values –10, –20, –30, … in the opposite direction to indicate the temperatures that are less than 0^{o} C.

Let us now consider the number line given below.

The positive whole numbers marked to the right of the position indicating zero on the number line are defined as positive integers and the negative whole numbers marked to the left of the position indicating zero are defined as negative whole numbers.

{…, –3, –2, –1, 0, 1, 2, 3, …} is the set consisting of all the integers.

Any positive number can be marked on the above number line to the right of the position indicating and any negative number can be marked to the left of the position indicating 0, taking into consideration the magnitude of the number.

All the numbers that are written with a positive or negative sign to indicate not only their magnitude but also one of two directions which are opposite to each other are defined as **directed numbers**.

Accordingly, numbers such as $+4,\ +\frac{3}{4},\ +5.7,\ -10,\ -\frac{1}{3}$ and -3.2 are directed numbers. $+4$ is read as positive four and $-\frac{1}{3}$ is read as negative one third.

Note

- When a sign is not written in front of a number, it is considered as a positive number.

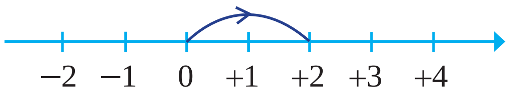
8.2 Adding directed numbers which are integers by using the number line

Let us consider adding directed numbers which are positive integers by using the number line.

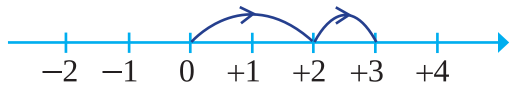
- **The sum of two positive integers**

Let us find the value of $(+2) + (+1)$ using the number line.

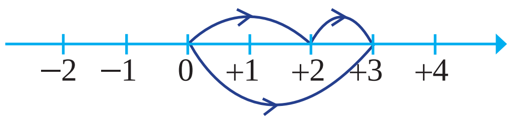
First, starting from 0, let us go two units towards the right along the number line.



Next, from this point, let us go one unit towards the right along the number line.



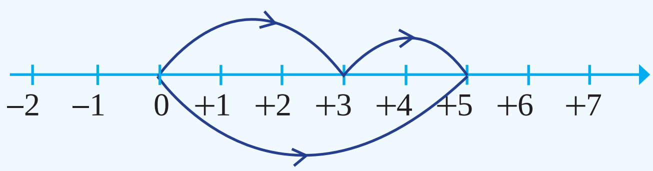
The directed number denoted by the position at which we finally stop is the answer.



$(+2) + (+1) = (+3)$

Example 1

Find the value of $(+3) + (+2)$ using the number line.



The final position is five units to the right of 0.

$\therefore\ (+3) + (+2) = (+5)$

Exercise 8.1

Find each of the following sums using the number line.

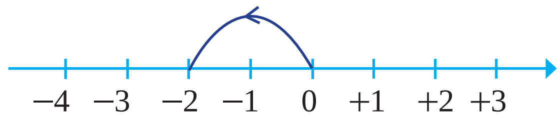
(i) $(+2) + (+3)$ (ii) $(+3) + (+3)$ (iii) $(+4) + (+1)$ (iv) $(+5) + (+3)$

• The sum of two negative integers

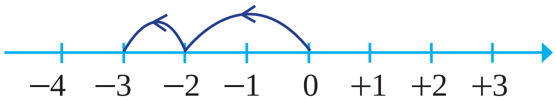
Let us consider adding directed numbers which are negative integers by using the number line.

Let us find the value of (–2) + (–1) using the number line.

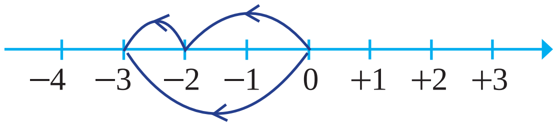
First, starting from 0, let us go two units towards the left along the number line.



Next, from this point, let us go one unit towards the left along the number line.



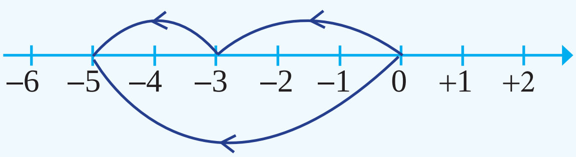
The directed number denoted by the position at which we finally stop is the answer.



(–2) + (–1) = (–3)

Example 1

Find the value of (–3) + (–2) using the number line.



The final position is five units to the left of 0.

$\therefore$ (–3) + (–2) = (–5)

Exercise 8.2

Find the value using the number line.

(i) (– 4) + (–1) (ii) (– 2) + (– 2) (iii) (– 2) + (– 3)

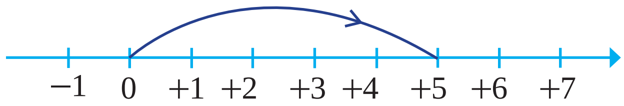
(iv) (– 1) + (– 3) (v) (– 3) + (– 3) (vi) (– 4) + (– 2)

• The sum of a positive integer and a negative integer

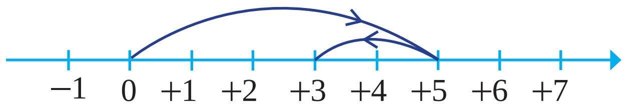
Now let us consider adding a positive integer and a negative integer.

Let us find the value of $(+5) + (-2)$ using the number line.

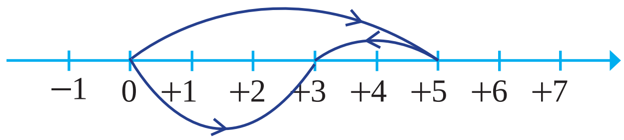
First, starting from 0, let us go five units towards the right along the number line.



Next, from this point, let us go two units towards the left along the number line.



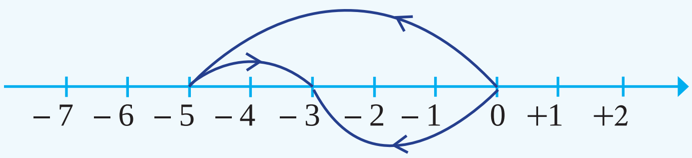
The directed number denoted by the position at which we finally stop is the answer.



$(+5) + (-2) = (+3)$

Example 1

Find the value of $(-5) + (+2)$ using the number line.



$(-5) + (+2) = (-3)$

Since the final position is three units to the left of 0, the number (–3) relevant to this position is the answer.

Exercise 8.3

Find the value using the number line.

(i) $(+3) + (-1)$ (ii) $(-4) + (+6)$ (iii) $(-7) + (+2)$

(iv) $(+2) + (-5)$ (v) $(+1) + (-1)$ (vi) $(-3) + (+3)$

8.3 Adding integers without using the number line

- **Finding the sum of two integers**

Let us consider the examples related to adding two positive integers that were studied in the previous section.

Using the number line we obtained previously that,

$(+2) + (+1) = (+3)$ and
$(+3) + (+2) = (+5)$.

$$(+2) + (+1) = (+3) \qquad\qquad (+3) + (+2) = (+5)$$
$$2 + 1 = 3 \qquad\qquad 3 + 2 = 5$$

- When adding two positive integers, add the two numbers without considering the signs.
- Place the positive sign in the final answer.

Let us now reconsider the examples related to adding two negative integers that were studied in the previous section.

Using the number line we obtained previously that

$(-2) + (-1) = (-3)$ and
$(-3) + (-2) = (-5)$.

Let us consider $(-2) + (-1) = (-3)$

- Without considering the signs of the two directed numbers, obtain their sum.

$$2 + 1 = 3$$

- Then write the answer with the negative sign. Therefore the answer is -3.

When adding two negative directed numbers, add the two numbers without considering the negative sign and then write the answer with the negative sign.

Example 1

Simplify:

(i) $(+4) + (+6)$ (ii) $(+11) + (+3)$ (iii) $(-5) + (-2)$ (iv) $(-4) + (-1)$

(i) $(+4) + (+6) = (+10)$ (ii) $(+11) + (+3) = (+14)$

(iii) $(-5) + (-2) = (-7)$ (iv) $(-4) + (-1) = (-5)$

Exercise 8.4

Simplify.

(i) $(+3) + (+8)$ (ii) $(-7) + (-3)$ (iii) $(+12) + (+4)$

(iv) $(-9) + (-16)$ (v) $(-20) + (-13)$ (vi) $(+17) + (+13)$

(vii) $(-11) + (-29)$ (viii) $(+2) + (+8)$ (ix) $(-3) + (-10)$

• Finding the sum of a positive integer and a negative integer

Using the number line we obtained previously that,

$(+5) + (-2) = (+3)$ and

$(-5) + (+2) = (-3)$.

We can find the sum of a positive integer and a negative integer as follows.

Let us consider $(-8) + (+5)$.

- Without considering the signs of the two directed numbers, obtain their difference. $8 - 5 = 3$
- From the two directed numbers (-8) and $(+5)$, the number which is further away from 0 on the number line is (-8). Its sign is negative.
- Therefore the answer is -3.

$(-8) + (+5) = (-3)$

When adding two directed numbers of opposite signs (positive and negative), obtain their difference without considering the signs, and write the answer with the sign of the directed number which is further away from 0 on the number line.

Example 1

Simplify $(+8) + (-3)$

$8 - 3 = 5$

From the two directed numbers $(+8)$ and (-3), the number which is further away from 0 on the number line is $(+8)$. Its sign is positive.

$(+8) + (-3) = (+5)$

Example 2

Simplify $(+4) + (-10)$

$10 - 4 = 6$

From the two directed numbers $(+4)$ and (-10), the number which is further away from 0 on the number line is (-10). Its sign is negative.

$(+4) + (-10) = (-6)$

Exercise 8.5

(1) Evaluate the following.

(i) $(+7) + (-2)$	(ii) $(-10) + (+4)$	(iii) $(-3) + (+6)$
(iv) $(-5) + (+9)$	(v) $(-11) + (+4)$	(vi) $(-4) + 0$
(vii) $(+9) + (-8)$	(viii) $(+7) + (-15)$	(ix) $(+5) + (-6)$
(x) $(-7) + (+5)$	(xi) $(+8) + (-10)$	(xii) $(-9) + (+4)$

8.4 Adding directed numbers

We have so far considered the addition of directed numbers which are integers. Now let us consider the addition of any two directed numbers.

The methods that were used above to add integers are used here too.

Example 1

Add the following directed numbers.

(i) $(+\frac{1}{2}) + (+\frac{1}{2})$

Without considering the signs of the two directed numbers, obtain their sum.

$$\frac{1}{2} + \frac{1}{2} = 1$$

Place the positive sign in the final answer.

$$(+\frac{1}{2}) + (+\frac{1}{2}) = +1$$

(ii) $(-\frac{2}{7}) + (-\frac{4}{7})$

Without considering the signs of the two directed numbers, obtain their sum.

$$\frac{2}{7} + \frac{4}{7} = \frac{6}{7}$$

Place the negative sign in the final answer.

$$(-\frac{2}{7}) + (-\frac{4}{7}) = (-\frac{6}{7})$$

(iii) $(+ 7.2) + (+ 1.3) = (+ 8.5)$

(iv) $(- 6.9) + (+ 2.5) = (- 4.4)$

Exercise 8.6

Evaluate the following.

(i) $(+\frac{3}{5}) + (+\frac{1}{5})$ (ii) $(-\frac{4}{7}) + (-\frac{1}{7})$ (iii) $(+\frac{2}{3}) + (+\frac{1}{3})$

(iv) $(- 2) + (-\frac{1}{2})$ (v) $(- 8.1) + (- 1.3)$ (vi) $(- 3.6) + (- 1.8)$

(vii) $(+ 4) + (- 2.5)$ (viii) $(-5) + (- 3.7)$ (ix) $(-\frac{4}{8}) + (-\frac{3}{8})$

(x) $(-2.6) + (+ 6.5) + (-4.3)$ (xi) $(+ 5.7) + (- 3.9) + (+ 1.4)$

Miscellaneous Exercise

(1) Fill in the blanks.

(i) $(+ 8) + (- 1) = (.....)$ (ii) $(+ 11) + (- 12) = (.....)$

(iii) $(- 4) + (- 11) = (.....)$ (iv) $(-\frac{7}{9}) + (-\frac{5}{9}) = (.....)$

(v) $(-\frac{8}{11}) + (-\frac{3}{11}) = (.....)$ (vi) $(+ 8.95) + (+ 2.97) = (.....)$

(vii) $(- 5.81) + (- 2.25) = (.....)$ (viii) $(- 6.57) + (+ 11.21) = (.....)$

(ix) $(-\frac{4}{13}) + (-\frac{7}{13}) = (.....)$ (x) $(+ 3.52) + (- 2.51) = (.....)$

(2) The ground floor of a building has been named Floor 0 and the floors above it have been named 1, 2, 3, … respectively, while the floors below it have been named -1, -2, -3, ... respectively.

(i) If a person in Floor 7 climbs up a further 5 floors, which floor will he be in?

(ii) If a person in Floor -1 descends a further 2 floors, which floor will he be in?

(iii) If a person in Floor 8 descends 3 floors, which floor will he be in?

(iv) If a person in Floor 2 descends 4 floors, which floor will he be in?

(3) The temperature at 6.00 a.m. in Moscow on a certain day was recorded as -4.7^{o} C, while the temperature at 4.00 p.m. on the same day was increased by 12^{0} C. Find the temperature in Moscow at 4.00 p.m.

Summary

- All numbers that are written with a positive or negative sign to indicate not only their magnitude but also one of two directions which are opposite to each other are called directed numbers.
- When adding two directed numbers of the same sign, add the numbers without considering the sign, and then include the sign with the answer.
- When adding two directed numbers of opposite signs (positive and negative), obtain their difference without considering the signs, and write the answer with the sign of the directed number which is further away from 0 on the number line.

9

Angles

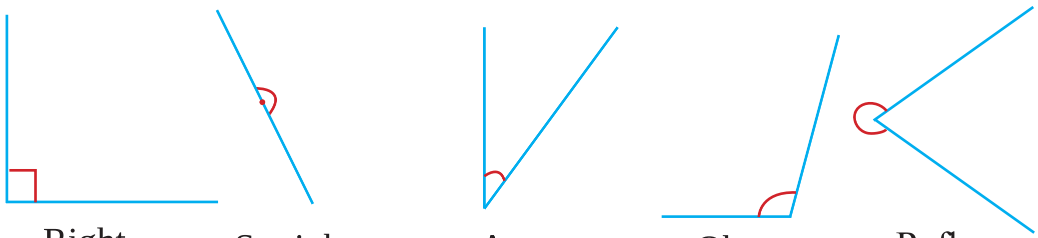
By studying this lesson you will be able to

- identify the dynamic or static nature of an angle,
- name angles,
- measure and draw angles using the protractor, and
- classify angles based on their magnitude.

9.1 Angles

You learnt in grade 6 that an angle is created when two straight line segments meet each other.

Below are a few types of angles we identified.



Right angle | Straight angle | Acute angle | Obtuse angle | Reflex angle

Do the following review exercise to recall the facts you have learnt about angles.

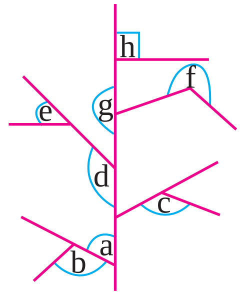
Review Exercise

(1) Choose the figures that are angles and write down the corresponding letters.

(a) (b) (c) (d)

(2) Identify the angles in the figure below and complete the table.

Angle	Type of Angle	Angle	Type of Angle
a		e	
b		f	
c		g	
d		h	

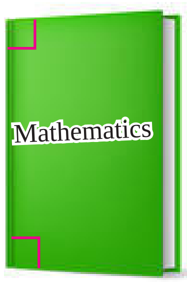


(3) Draw an angle of each type on a square ruled paper. Write the type of angle next to the corresponding figure.
Acute angle, Right angle, Obtuse angle, Straight angle, Reflex angle

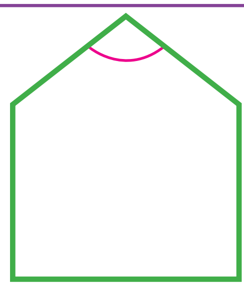
9.2 The dynamic or static nature of an angle

Let us investigate more on angles.

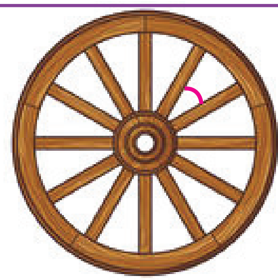
If we observe our surroundings, we can identify many angles. A few examples are given below.



Angle between the edges of a cover of a book



Angle between two parts of the roof.

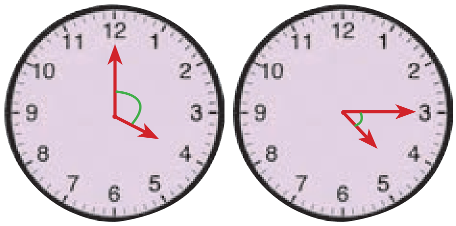
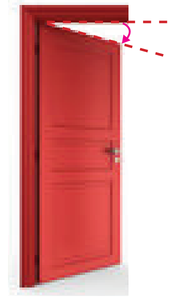


Angles between the adjacent spokes of a wheel

A common property of the above angles is that their magnitude does not change.

- If the magnitude of an angle does not change, then it is static in nature.
- So the angles in the above figures are static in nature.
- Note that the magnitude of the angle between two spokes of a wheel does not change, even when the wheel is turning.

Let us now consider some situations that involve rotation.

		
The angle between the hour hand and the minute hand of a clock changes in magnitude with time. The figure shows this angle at 4 p.m. and 4.15 p.m.	The angle between the two blades of a pair of scissors changes when it is used for cutting.	The angle between the top edge of a door and a door frame changes when the door is being opened or closed.

In the examples given above, let us consider the **arms** of the angle involved.

We see that there is a rotation of both arms or of one of them. Therefore, the magnitude of the angle changes. This is the **dynamic nature** of such an angle.

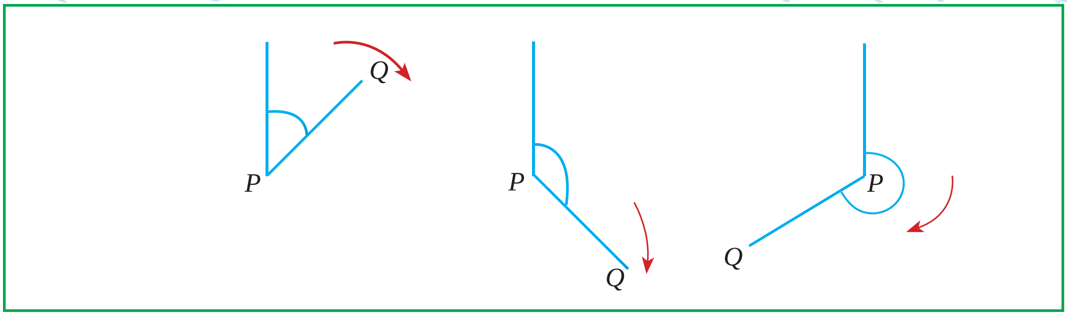
Let us understand the dynamic nature of an angle further by doing the following activity.

Activity 1

Step 1 - Take a fresh green ekel and bend it into two parts at the centre, taking care not to break it.

Step 2 - Overlap the two parts of the ekel and place it on a table. Hold one part tightly on the table.

Step 3 - In your exercise book, draw several situations that are obtained by rotating the other part on the table.
A few such situations that can be obtained are shown below.



- You can see that the magnitude of the angle between the two parts of the ekel changes. That is, this angle is dynamic in nature.
- When both parts of the ekel are rotated too the magnitude of the angle between the two parts changes.

A rotation which is in the same direction as the rotation of the arms of a clock is defined as a clockwise rotation. A rotation which is in the opposite direction to that of the rotation of the arms of a clock is defined as an anticlockwise rotation.

Exercise 9.1

(1) (i) Write down 3 instances where you can observe angles which are dynamic in nature in your surrounding environment.

(ii) Write down 3 instances where you can observe angles which are static in nature in your surrounding environment.

(2) (i) Give an example of an angle which is static in nature where the positions of the arms of the angle are fixed.

(ii) Give an example of an angle which is static in nature where there is a change in the positions of the arms of the angle.

(iii) Give an example of an angle which is dynamic in nature where there is a change in the position of only one arm of the angle.

(iv) Give an example of an angle which is dynamic in nature where there is a change in the positions of both arms of the angle.

9.3 Naming Angles

Let us now consider how angles are named.

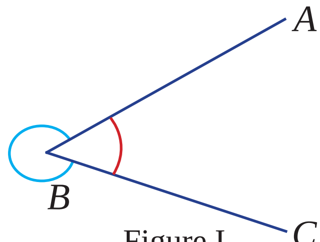
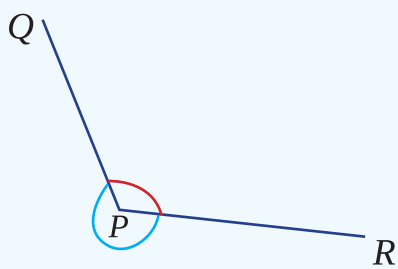


Figure I

- In Figure I, two angles have been created by the straight line segments AB and BC meeting.
- The straight line segments AB and BC are defined as the "arms of the angle". The point B where AB and BC meet is defined as the "vertex of the angle".
- The magnitude of the angle which is indicated in red is less than that of a straight angle; that is, less than the magnitude of two right angles.
- The magnitude of the angle indicated in blue is greater than that of a straight angle.
- The angle indicated in red is named as angle ABC and is written as , $A\hat{B}C$ or $C\hat{B}A$
- Here we write the letter which indicates the vertex in the middle and the other two letters beside it.
- The angle indicated in blue is named as the reflex angle ABC and is written as reflex angle $A\hat{B}C$ or reflex angle $C\hat{B}A$.
- In some books angle ABC is written as $\measuredangle ABC$.

Example 1

Draw the angles with the straight line segments PQ and PR as their arms. Name the two angles.



Since P is common to both arms, P is the vertex of the angles. Therefore the angle indicated in red is $Q\hat{P}R$ and the angle indicated in blue is the reflex angle $Q\hat{P}R$.

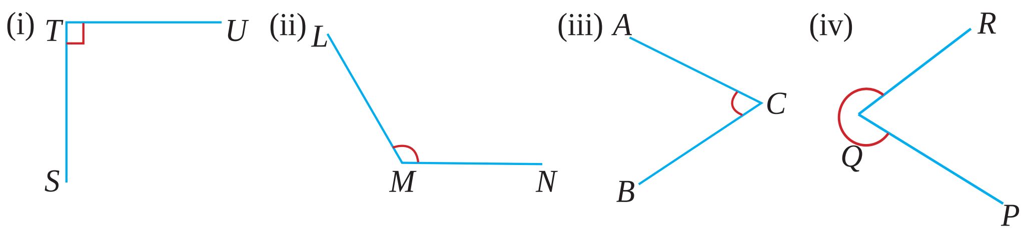
Example 2

Write down the vertex and the arms of $D\hat{E}F$.

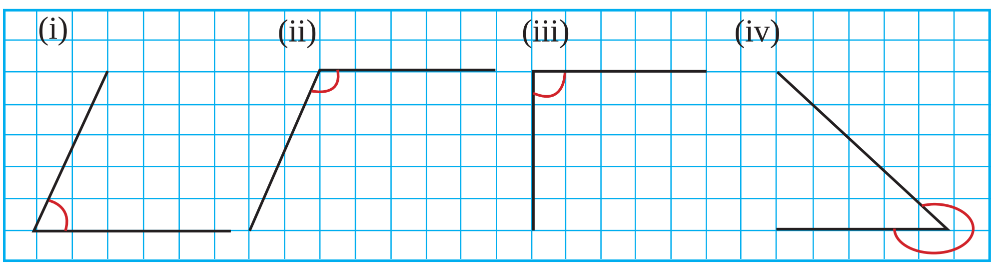
Since the letter in the middle of $D\hat{E}F$ is E, the vertex of the angle is E and the arms of the angle are ED and EF.

Exercise 9.2

(1) Write down the arms and the vertex of each of the angles given below.



(2) Copy each of the angles given below and name them using letters of the English alphabet.



(3) Draw and name an angle of your choice on a square ruled paper.

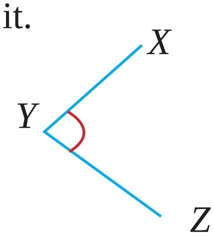
(4) Draw an obtuse angle with arms XY and YZ on a square ruled paper.

(5) Draw an angle and name it $D\hat{E}F$. Name its arms and its vertex.

(6) Draw a reflex angle and name it.

(7) Draw a right angle on a square ruled paper and name it.

(8) Prabath has written the angle in the figure as $X\hat{Y}Z$. Sumudu has written it as $Z\hat{Y}X$. Kasun says that both Prabath and Sumudu are correct. Do you agree with Kasun? Explain your answer.



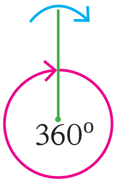
9.4 Measuring angles

There are standard units and instruments to measure distance, mass, time and the volume of a liquid. You learnt about these in grade 6.

Now let us learn about a standard unit and an instrument used to measure angles.

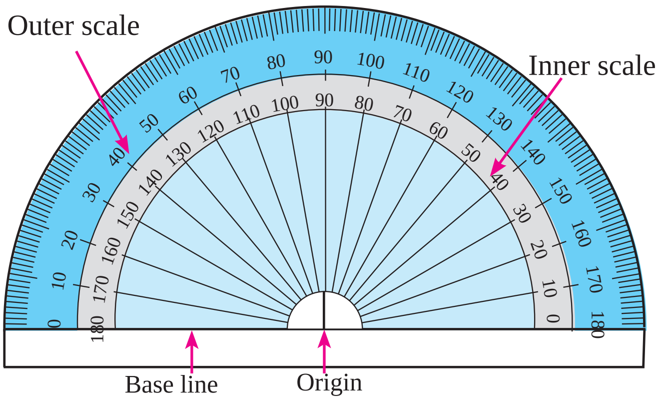
The standard unit used to measure angles is **degrees**. One degree is written as 1°.

The angle that is formed when a straight line segment completes one full circle by rotating about a point is 360°.



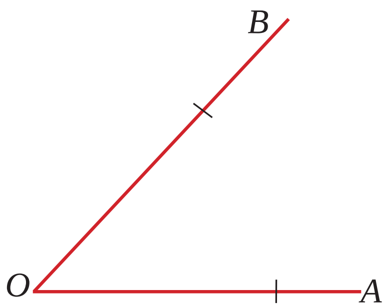
The instrument used to measure angles is made of one half of a full circle. It is called a "**protractor**". The figure of a protractor is shown below. It is numbered from 0° to 180° clockwise and anticlockwise. The line indicated by 0 - 0 is called the "base line".

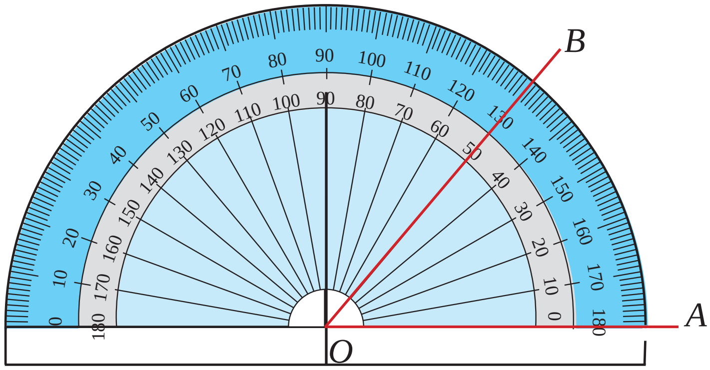
There are two scales indicated in the protractor. They are the inner scale and the outer scale.



The long line segments on the outer scale are marked as 0, 10, 20, …, 180. The gap between every pair of long line segments is again divided into 10 similar parts using short line segments. As indicated in the figure, the magnitude of the angle between two long line segments is 10°.

Let us now see how we can use the protractor to measure the angle $A\hat{O}B$ in the figure.





Place the protractor on the figure such that the origin and the base line coincide with the vertex O and the arm OA respectively.

Then the arm OB coincides with the line indicated by 50^{o} in the inner scale (Note that OA coincides with 0^{o} on this scale). Therefore the magnitude of the angle $A\hat{O}B$ is 50^{o}, and we write $A\hat{O}B = 50^{o}$.

By observing this figure, we see that an angle of 1^{o} is a small angle which is difficult to draw.

Activity 2

Step 1 - In your exercise book, draw an angle similar to the one in Figure I using a ruler.

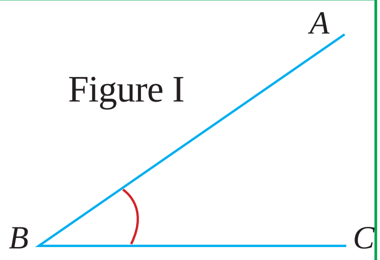


Figure I

Step 2 - Measure the magnitude of the angle drawn, and write it inside the space margined by AB, BC and the red arc.

Step 3 - Draw an angle similar to the one in Figure II below, measure the magnitude of the angle and write it down as done in step 2.

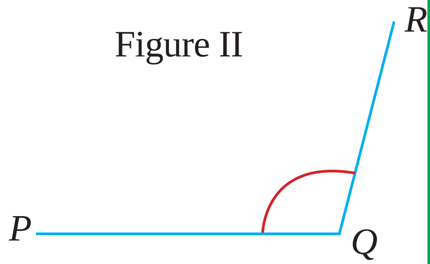
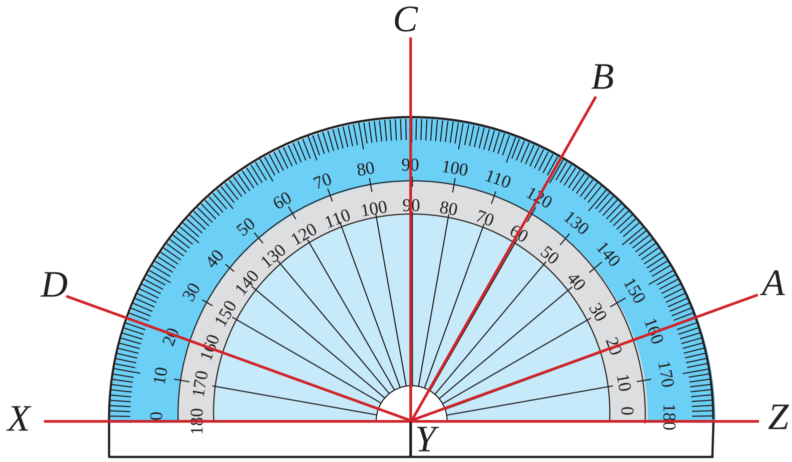


Figure II

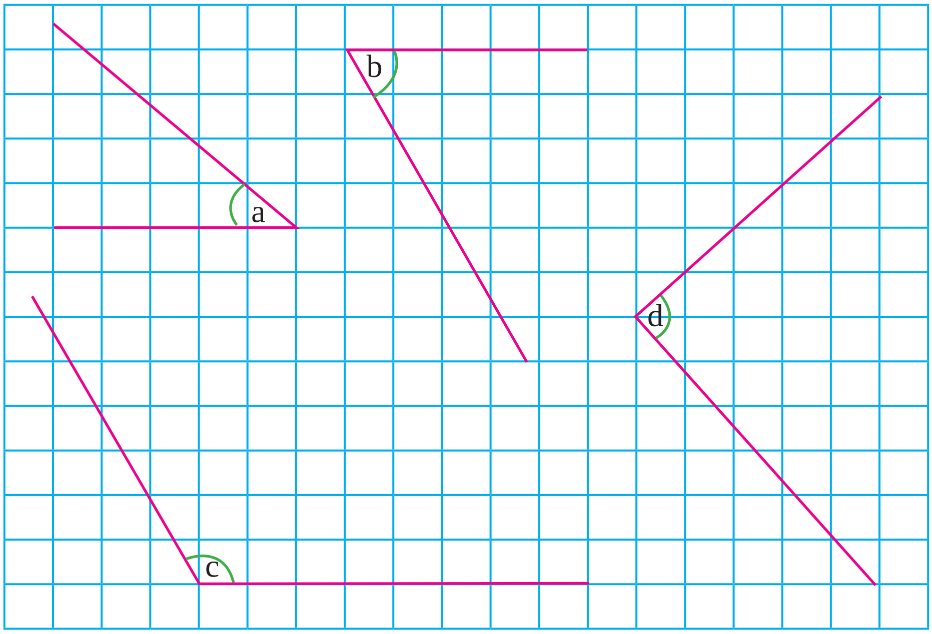
Exercise 9.3

(1) Write down the magnitude of each angle using the given figure.

(i) $X\hat{Y}Z$ (ii) $Z\hat{Y}A$ (iii) $X\hat{Y}C$ (iv) $B\hat{Y}Z$
(v) $X\hat{Y}B$ (vi) $C\hat{Y}Z$ (vii) $X\hat{Y}A$ (viii) $Z\hat{Y}D$

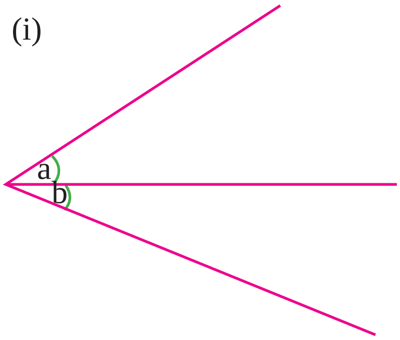


(2) Draw each of the angles below on a square ruled paper. Measure and write the magnitude of each angle.

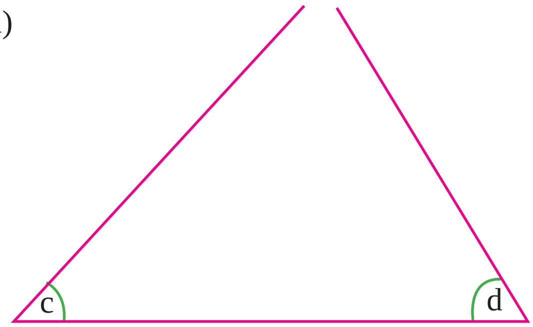


(3) Draw the following figures in your exercise book. Measure and write down the magnitude of each of the angles indicated by the English letters.

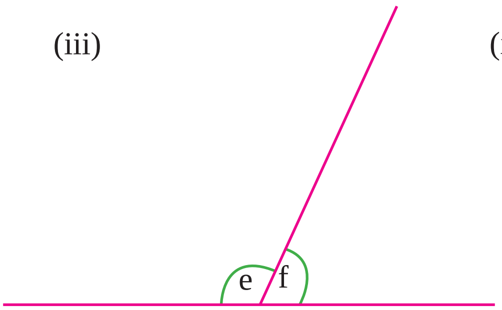
(i)



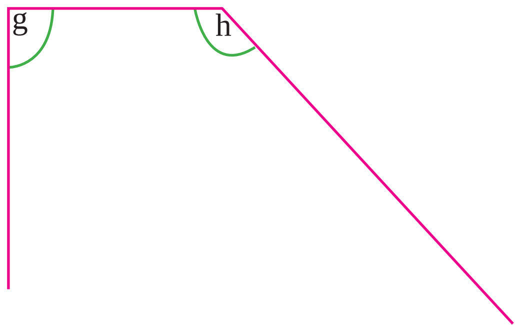
(ii)



(iii)



(iv)



9.5 Drawing angles with given magnitude

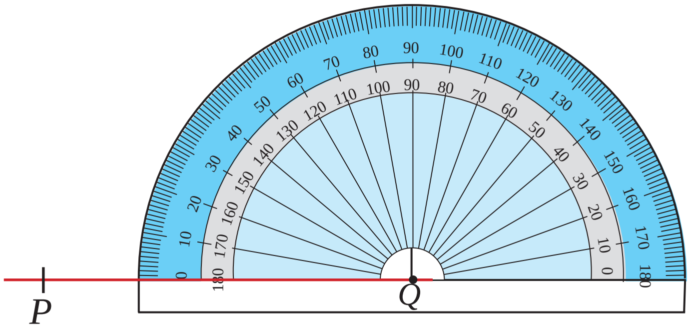
Let us now draw angles when their magnitude is given.

Activity 3

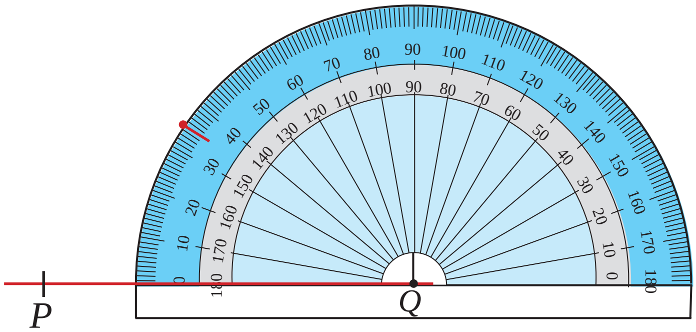
Performing the steps given below, draw the angle $P\hat{Q}R = 35^{o}$

Step 1 - Using the ruler, draw a straight line segment and name it PQ.

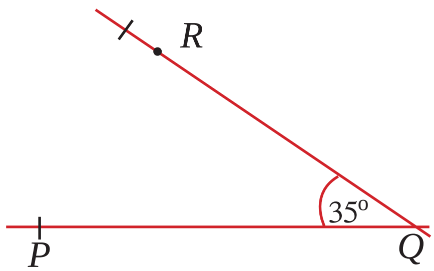
Step 2 - Since the vertex of the angle is Q, place the protractor so that its origin and the base line coincide with Q and PQ respectively.



Step 3 - Now find 35^{o} in the outer scale. Place a dot mark on the paper at 35^{0}.

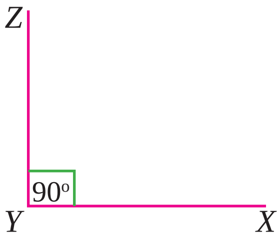


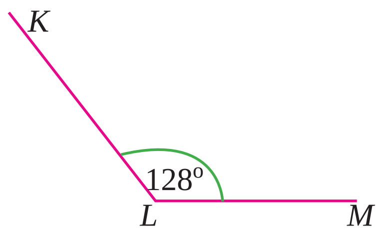
Step 4 - Remove the protractor. Name the dot marked in step 3 as R. Now draw a straight line from Q to R. Write the magnitude of the angle $P\hat{Q}R$ as 35^{0}.



As above, draw the following:

(i) $X\hat{Y}Z = 90^{o}$ (ii) $K\hat{L}M = 128^{o}$





Exercise 9.4

(1) Draw the following angles.

(i) $A\hat{B}C = 48^\circ$ (ii) $P\hat{Q}R = 90^\circ$ (iii) $K\hat{L}M = 130^\circ$ (iv) $X\hat{Y}Z = 28^\circ$

(2) (i) Draw a straight line segment and name it PQ.

(ii) Draw the arm PR such that $Q\hat{P}R = 82^\circ$.

(iii) Draw the arm QS such that $P\hat{Q}S = 43^\circ$.

(3) (i) Draw any triangle you like and name it ABC.

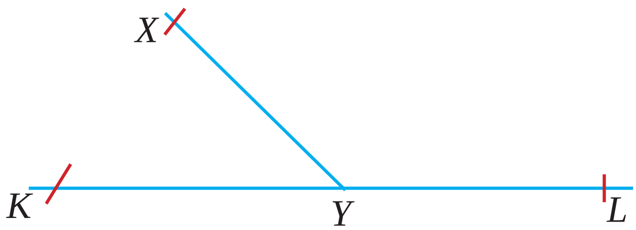
(ii) Measure $A\hat{B}C$, $B\hat{C}A$ and $C\hat{A}B$ and write their magnitudes separately.

(iii) Obtain the value of $A\hat{B}C + B\hat{C}A + C\hat{A}B$ using the measured values.

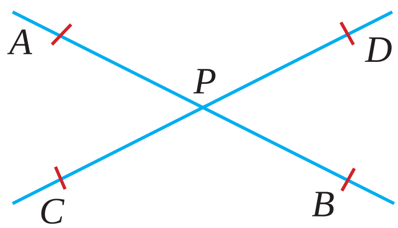
(4) (i) Draw two straight line segments KL and XY so that they meet each other at Y.

(ii) Measure and write down the magnitudes of $K\hat{Y}X$ and $X\hat{Y}L$

(iii) Obtain $K\hat{Y}X + X\hat{Y}L$.



(5) (i) As given in the figure, draw two straight line segments AB and CD so that they intersect each other.

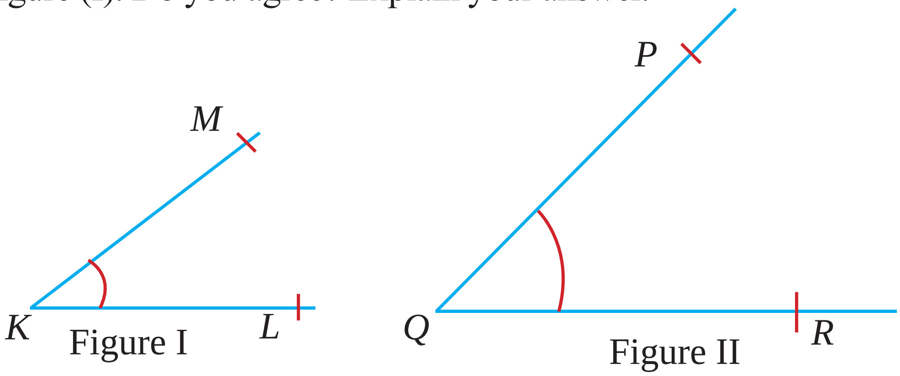


(ii) Measure and write the magnitudes of $A\hat{P}C$, $C\hat{P}B$, $B\hat{P}D$ and $D\hat{P}A$ separately.

(iii) Write the relationship between $A\hat{P}C$ and $B\hat{P}D$.

(iv) Write the relationship between $A\hat{P}D$ and $C\hat{P}B$.

(6) Dasun says that the angle in Figure (II) is larger than the angle in Figure (I). Do you agree? Explain your answer.

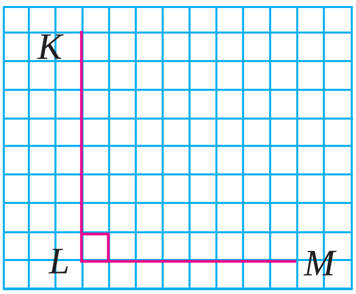


9.6 Classification of angles

We learnt in grade 6 to classify angles using a right angle. Magnitude of a right angle is 90°. We can classify angles by comparing them with 90°.

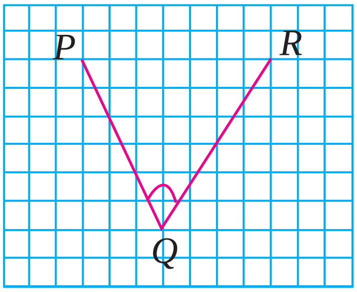
Right Angles

Any angle of magnitude 90° is called a "right angle". $K\hat{L}M$ is a right angle.



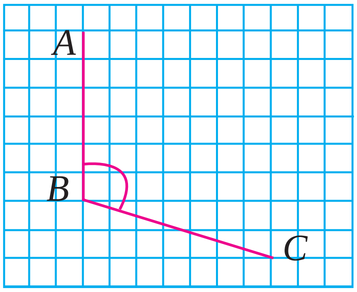
Acute Angles

Any angle of magnitude less than 90° is called an "acute angle". $P\hat{Q}R$ is an acute angle.



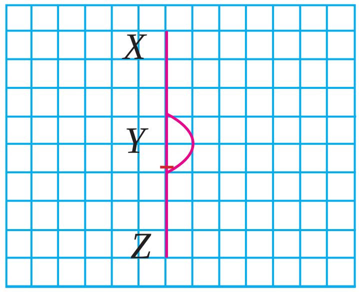
Obtuse Angles

Any angle of magnitude greater than 90° but less than 180° (that is an angle between 90°and 180°) is called an "obtuse angle". $A\hat{B}C$ is an obtuse angle.



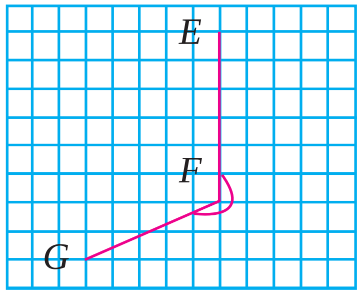
Straight Angles

Any angle of magnitude 180° is called a "straight angle". $X\hat{Y}Z$ is a straight angle.



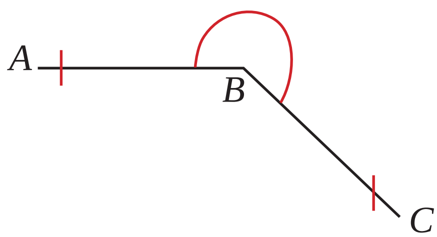
Reflex Angles

Any angle of magnitude between 180°and 360° is called a " reflex angle". $E\hat{F}G$ is a reflex angle.



9.7 Measuring and Drawing Reflex angles

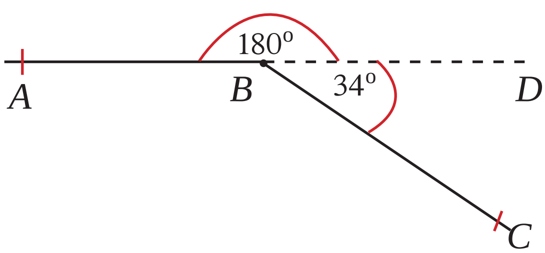
The figure shows the reflex angle $A\hat{B}C$.This angle cannot be measured directly using a protractor. So let us see how we can measure this reflex angle.



Method I :-

Let us use the ruler to extend AB and obtain the straight angle $A\hat{B}D$.

That is, $A\hat{B}D = 180^{o}$.

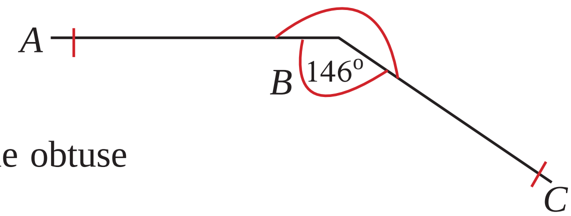


Now let us measure $D\hat{B}C$ using the protractor. We will then obtain $D\hat{B}C = 34^{\circ}$.

Since the reflex angle $A\hat{B}C = A\hat{B}D + D\hat{B}C$
the reflex angle $A\hat{B}C = 180^{\circ} + 34^{\circ} = 214^{\circ}$.

Method II :-

Measure the obtuse angle $A\hat{B}C$.
It is equal to146°.
Since the reflex angle $A\hat{B}C$ + the obtuse angle $A\hat{B}C = 360^{\circ}$



The reflex angle $A\hat{B}C = 360^{\circ} - 146^{\circ}$
$= 214^{\circ}$

Let us now see how to draw reflex angles.

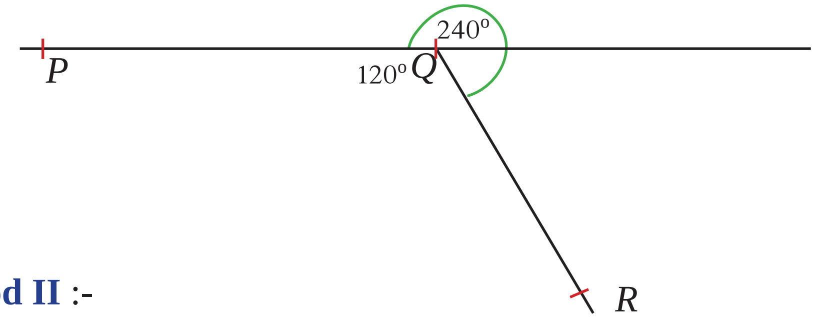
Activity 4

Draw the reflex angle $P\hat{Q}R = 240^{\circ}$ according to the following steps.

Step 1 - Draw the straight line segment PQ.

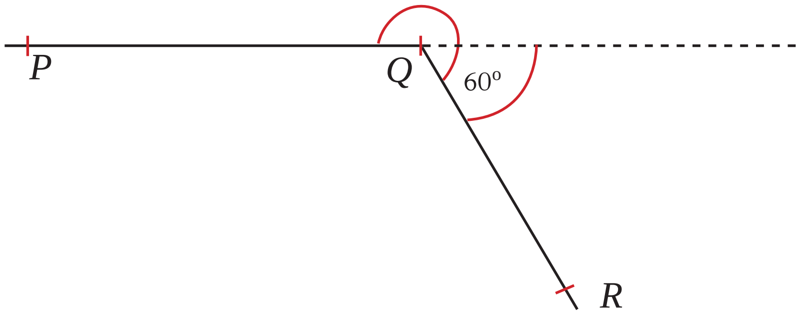
Step 2 - Calculate the magnitude of the obtuse angle $P\hat{Q}R$.
$P\hat{Q}R = 360^{\circ} - 240^{\circ} = 120^{\circ}$

Step 3 - Draw the angle $P\hat{Q}R = 120^{\circ}$. Now mark the reflex angle 240°.



Method II :-

Step 4 - By drawing an angle of 60° (That is, 240° – 180°) on the straight angle appropriately, we can obtain the reflex angle 240°.



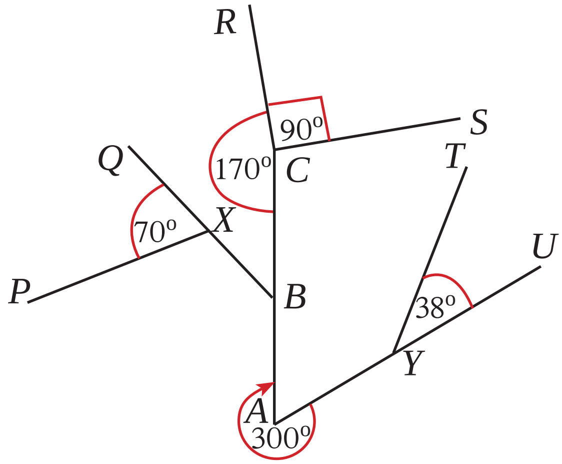
Exercise 9.5

(1) Copy the two groups (a) and (b) in your exercise book. Join each angle and its type with a straight line.

Group (a) (Magnitude of the angle)	Group (b) (Type of angle)
18°	Straight angle
135°	Right angle
180°	Acute angle
255°	Obtuse angle
90°	Reflex angle

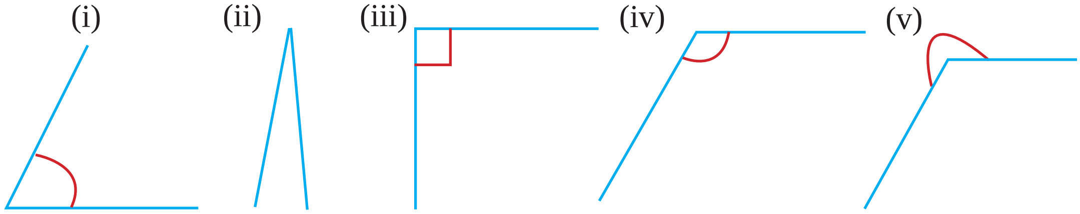
(2) Using the information in the figure, write down the type of each of the angles given below.

(i) $P\hat{X}Q$ (ii) $B\hat{C}R$ (iii) $S\hat{C}R$ (iv) $T\hat{Y}U$ (v) $B\hat{A}Y$



(3) Choose and write down the most appropriate magnitude for each of the angles below, from the values given in brackets.

(i) (ii) (iii) (iv) (v)



(25°, 65°, 10°) (1°, 80°, 15°) (50°, 90°, 180°) (360°, 120°, 180°) (185°, 240°, 350°)

(4) Draw the following reflex angles using the protractor.

(i) $A\hat{B}C = 300°$ (ii) $P\hat{Q}R = 195°$ (iii) $M\hat{N}O = 200°$

(iv) $K\hat{L}M = 243°$ (v) $X\hat{Y}Z = 310°$

Summary

- The standard unit used to measure angles is degrees. One degree is written as 1°.
- Any angle of magnitude less than 90° is called an "acute angle".
- Any angle of magnitude 90° is called a "right angle".
- Any angle of magnitude greater than 90° but less than 180° (that is an angle between 90° and 180°) is called an "obtuse angle".
- Any angle of magnitude 180° is called a "straight angle".
- Any angle of magnitude between 180° and 360° is called a "reflex angle".

Revision Exercise - I

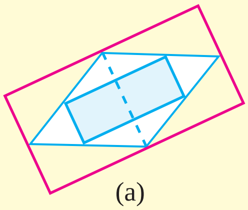
(1) (a) Simplify the following.

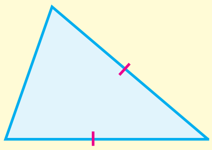
(i) $15 + 13 + 12$	(ii) $18 - 12 + 6$	(iii) $9 + 6 - 8$
(iv) $8 \times 7 - 12$	(v) $7 \times 3 + 5$	(vi) $24 - 18 \div 3$
(vii) $15 + 18 \div 3$	(viii) $16 + 5 \times 3$	(ix) $15 - 9 \div 3$

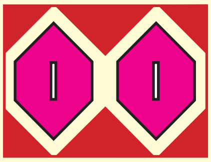
(b) Hasintha says "when we simplify $91 - 35 \div 7$, we get 8 as the answer". Explain why Hasintha's answer is incorrect.

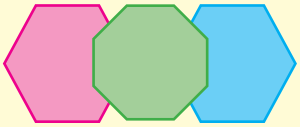
(2) (i) What is a bilaterally symmetric plane figure?

(ii) Write the number of axes of symmetry in each of the symmetric figures given below.


(a)

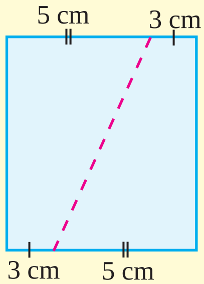

(b)


(c)


(d)

(iii) Draw the following symmetric figures in your square ruled exercise book. Draw their axes of symmetry and name them.
(a) A rectilinear plane figure with only one axis of symmetry
(b) A rectilinear plane figure with only two axes of symmetry
(c) A rectilinear plane figure with more than two axes of symmetry

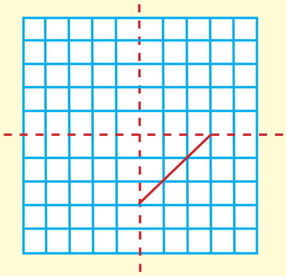
(iv)

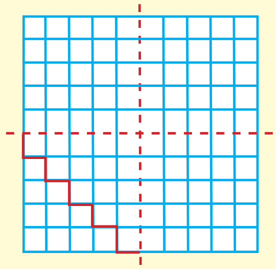


If the plane figure is cut along the dotted line, then it will be divided into two parts which coincide with each other.

Is the figure bilaterally symmetric about this line? Explain your answer giving reasons.

(v) Copy each of the following figures onto a square ruled paper and complete each figure such that the two dotted lines become axes of symmetry of the completed figure.


(a)


(b)

(3) (i) Set A is given below by listing its elements.

$A = \{2, 3, 5, 7\}$

Write A using a common property of its elements.

(ii) Re-write P = {factors of 12} by listing its elements.

(iii) Let A = {multiples of 3 that lie between 8 and 20}

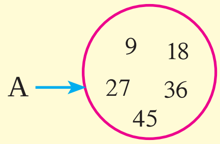
(a) Write A by listing its elements.

(b) Represent A in a Venn diagram.

(iv) Write the set represented by the Venn diagram,

(a) using a common property of its elements,

(b) by listing its elements.



(4) (i) Write the factors of 44.

(ii) Write the prime factors of 44.

(iii) Write 56 as a product of its prime factors.

(iv) Find the highest common factor of 18, 30, 42.

(v) Find the least common multiple of 18, 30, 42.

(5) (i) What is the digital root of 522?

(ii) Using the digital root, explain why 522 is divisible by 3.

(iii) Using the digital root explain why 522 is divisible by 9.

(iv) How do we find out without dividing a number whether it is divisible by 4 or not?

(v) $\boxed{4}$ $\boxed{3}$ $\boxed{2}$ $\boxed{1}$ are four numbers written on four cards. How many numbers which are divisible by 4 can be made using all these cards? Write down all such numbers.

(vi) If the number $53\square$ which has 3 digits is divisible by 9, then what is the digit in the units place?

(vii) If the number $53\square$ which has 3 digits is divisible by 6, then what is the digit in the units place?

(6) (a) (i) Find the value of 6^2.

(ii) Write all the factors of the number corresponding to the value found in (i).

(iii) There are only two prime factors among the factors written in (ii). Write down three more numbers where each of them has only two prime factors.

(iv) Write each of the three numbers as a power of a prime number.

(b) (i) Expand $a^2\ b^3$.

(ii) Evaluate $x^3\ y^2$ when $x = 5$ and $y = 4$.

(7) Write whether the following statements are true or false.

(i) Any multiple of 2 has only one prime factor.

(ii) Any number which can be written as a power of 2, has 2 as its only prime factor.

(iii) Any multiple of 3 has only one prime factor.

(iv) Any number which can be written as a power of 3, has only one prime factor.

(v) Any number which can be written as a power of 5, has 5 as its only prime factor.

(vi) The highest common factor of any two positive integers is less than or equal to their least common multiple.

(vii) The highest common factor of any two distinct prime numbers is 1.

(viii) The highest common factor of 12 and 13 is 1.

(8) (i) Explain, giving reasons whether AD 1892 is a leap year or not.

(ii) Explain, giving reasons whether AD 2100 is a leap year or not.

(iii) To which decade does the year AD 2100 belong to?

(9) (a) Add the following.

(i)

	years	months	days
	3	6	19
+	2	8	20

(ii)

	years	months	days
	16	9	21
+	7	3	9

(b) Subtract the following.

(i)

	years	months	days
	6	8	12
−	4	5	20

(ii)

	years	months	days
	5	7	19
−	2	9	25

(10) The fifth birthday of a child fell on 2002 - 08- 26. His mass was 20 kg and 700 g on that day.

(i) When was his birthday?

(ii) On his eighth birthday his mass was 30kg and 600g. What is the increase in his mass during the three years?

(iii) What was his age on 2012 - 03 - 25?

(iv) On 2012 - 03 - 25, the mass of the child was 12kg and 800g more than his mass on his fifth birthday. Find the mass of the child on 2012 - 03 - 25.

(11) (a) Using the number line, determine each of the following sums.

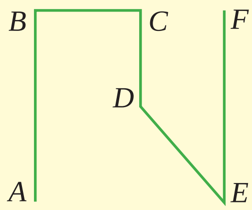
(i) $(-6) + (-4)$ (ii) $(-5) + (+5)$ (iii) $(+8) + (-9)$

(b) Simplify

(i) $(+4) + (-10)$ (ii) $(-9) + (+5)$ (iii) $(-8) + (-5)$

(iv) $(+\frac{1}{4}) + (+\frac{1}{4})$ (v) $(-\frac{2}{7}) + (-\frac{3}{7})$ (vi) $(-1.76) + (+0.36)$

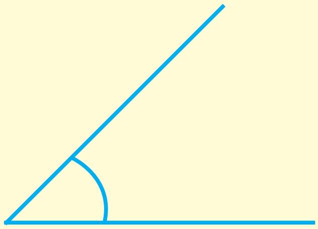
(12) (a)



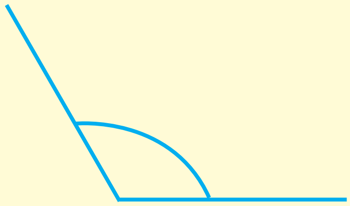
Complete the table given below by considering a person whose journey starts at A and ends at F.

	Two roads travelled	Name the angle between the two roads	Name the vertex and the arms of the angle between the two roads	Name the type of angle between the two roads when classified according to its magnitude
(i)	A to C through B	....	....	....
(ii)	B to D through C	....	....	....
(iii)	C to E through D	....	....	....
(iv)	D to F through E	....	....	....

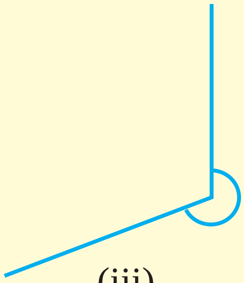
(b) Measure the magnitude of each angle given below using a protractor and write it down.



(i)



(ii)

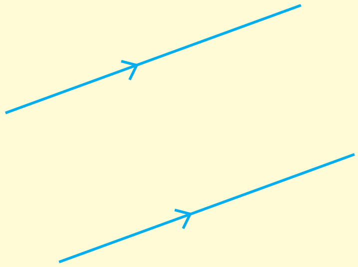


(iii)

(c) Draw the following angles using the protractor and the ruler.

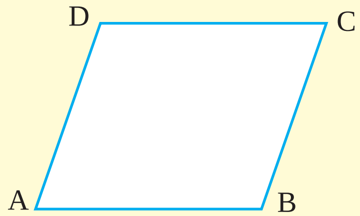
(i) $A\hat{B}C = 65^{\circ}$ (ii) $P\hat{Q}R = 130^{\circ}$ (iii) $M\hat{N}R = 145^{\circ}$

(13) (i) Two parallel lines are shown below. How far apart are they?



(ii) (a) Draw a straight line segment and name it *XY*.
(b) Mark a point *A* which is a distance of 4.8 cm from *XY*.
(c) Draw a straight line segment through the point *A* parallel to *XY*.

(iii) (a) Draw the parallelogram ABCD.



Draw parallel lines to diagonal AC throught B and D.

(14) (i) Nimal's birthday is 2002 -11 -25. Find Nimal's age in years, months and days on 2016 - 08 - 20.

(ii) Write the time that has elapsed between 12:35 of 2015 - 01 - 01 and 19:20 of 2015 - 02 - 05 in days, hours and minutes.

10 Fractions

(Part I)

By studying this lesson you will be able to

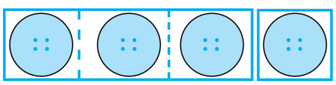
- identify mixed numbers and improper fractions, and
- convert a mixed number into an improper fraction and an improper fraction into a mixed number.

10.1 Fractions

Let us take the area bounded by the figure given below as one unit.

This unit has been divided into five equal parts, of which two have been coloured. As we have learnt previously, the amount coloured is $\frac{2}{5}$.

Similarly, if we take the four buttons shown below as one unit, we known that an amount of 3 buttons is $\frac{3}{4}$ of the amount of buttons there are.



Of the 25 children in a class, 13 are girls. When the number of girls in the class is written as a fraction of the total number of children in the class we obtain $\frac{13}{25}$. Here the total number of children in the class, that is 25, has been taken as one unit.

When a fraction is written numerically in this manner, the number below the line is called the **denominator** and the number above the line is called the **numerator**.

$$\frac{3 \leftarrow \text{Numerator}}{4 \leftarrow \text{Denominator}}$$

Numbers such as $\frac{2}{3}, \frac{3}{4}$ and $\frac{2}{5}$ which are smaller than one but larger than zero are called **proper fractions**. In a proper fraction, the numerator is always smaller than the denominator.

Proper fractions such as $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$, with the numerator equal to 1, are defined as **unit fractions**.

Any fraction can be written in terms of its corresponding unit fraction.

For example,

$\frac{2}{3}$ is two $\frac{1}{3}$ s.

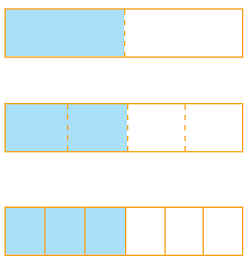
$\frac{5}{17}$ is five $\frac{1}{17}$ s.

Next let us recall what we have learnt about equivalent fractions.

$\frac{1}{2}$

$\frac{2}{4}$

$\frac{3}{6}$



Let us consider these three figures. The amounts that have been coloured in these figures are equal. That is, the fractions $\frac{1}{2}, \frac{2}{4}$ and $\frac{3}{6}$ that are represented by these figures are equal to each other.

That is, $\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$.

We learnt in grade 6 that such fractions, which have denominators which are different to each other and numerators too which are different to each other, but which represent the same number are defined as equivalent fractions.

A fraction equivalent to a given fraction can be obtained by multiplying both the numerator and the denominator of the given fraction by a whole number (other than 0). Two such examples are given below.

$$\frac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$$

$$\frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}$$

A fraction equivalent to a given fraction can also be obtained by dividing both the numerator and the denominator of the given fraction by a whole number (other than 0) which divides the numerator and the denominator without remainder.

Let us find a fraction equivalent to $\frac{18}{24}$. Let us divide the numerator and the denominator of $\frac{18}{24}$ by 3 which divides 18 and 24 without remainder.

$$\frac{18}{24} = \frac{18 \div 3}{24 \div 3} = \frac{6}{8}$$

Do the following review exercise to recall the facts you have learnt about fractions.

Review Exercise

(1) Select the unit fractions from the following proper fractions and write them down.

$\frac{2}{3}, \frac{1}{7}, \frac{4}{15}, \frac{1}{3}, \frac{1}{100}$

(2) Fill in the blanks by selecting the suitable value from within the brackets.

(i) $\frac{3}{5}$ is ...... $\frac{1}{5}$ s (1, 2, 3)

(ii) $\frac{2}{7}$ is two ...... s. $\left(\frac{1}{2}, \frac{1}{7}, \frac{1}{5}\right)$

(iii) Five $\frac{1}{6}$ s is equal to .... $\left(\frac{1}{30}, \frac{5}{6}, \frac{1}{5}\right)$

(iv) $\frac{\boxed{\ldots}}{12}$ is equivalent to $\frac{2}{3}$. (2 , 4, 8)

(3) Write down two equivalent fractions for each of the following fractions.

(i) $\frac{2}{3}$ (ii) $\frac{3}{5}$ (iii) $\frac{6}{8}$ (iv) $\frac{36}{48}$

(4) For each of the following fractions, write down the equivalent fraction with the smallest denominator.

$\frac{18}{30}, \frac{16}{24}, \frac{10}{35}$

(5) Write down the fractions $\frac{4}{7}, \frac{1}{7}, \frac{6}{7}, \frac{5}{7}$ in ascending order.

(6) Write down the fractions $\frac{7}{12}, \frac{5}{12}, \frac{2}{3}, \frac{1}{4}$ in descending order.

(7) If Sithmi obtained 21 marks out of a total of 25 marks for a test, express her marks as a fraction of the total marks.

(8) A vendor bought a stock of 50 mangoes of which 8 were spoilt.
   (i) Express the number of spoilt fruits as a fraction of the total number of fruits.
   (ii) Express the number of good fruits as a fraction of the total number of fruits.

10.2 Mixed numbers and improper fractions

1

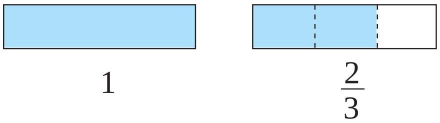
$\frac{1}{2}$

A whole cake and exactly half of an identical cake are shown in the figure. When the whole cake is taken as a unit, it is expressed as 1 and the other part which is exactly half of such a cake, is expressed as $\frac{1}{2}$. Therefore, the total amount of cake in the figure is $1+\frac{1}{2}$ times the whole cake. This is written as $1\frac{1}{2}$, and read as **one and a half**.

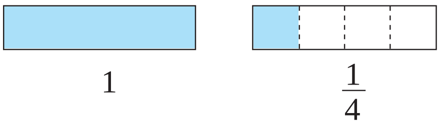
When a number which is the sum of a whole number and a proper fraction is written in this manner, it is defined as a **mixed number**. The whole number in the mixed number is called the **whole number part** and the proper fraction in it is called the **fractional part**.

$1\frac{1}{2}$, $1\frac{7}{8}$, $2\frac{2}{5}$ and $3\frac{1}{3}$ are examples of mixed numbers. The whole number part of the mixed number $2\frac{2}{5}$ is 2, and its fractional part is $\frac{2}{5}$.

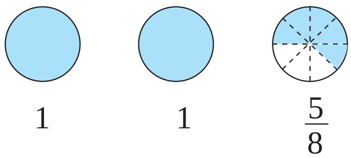
Let us write the mixed numbers represented in the following figures in the above manner.



$1 + \frac{2}{3} = 1\frac{2}{3}$

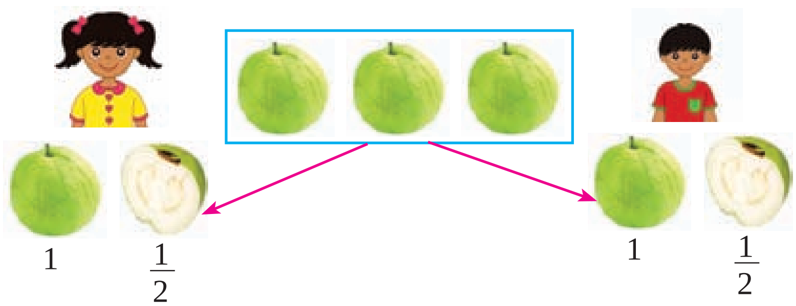


$1 + \frac{1}{4} = 1\frac{1}{4}$



$1 + 1 + \frac{5}{8} = 2 + \frac{5}{8} = 2\frac{5}{8}$

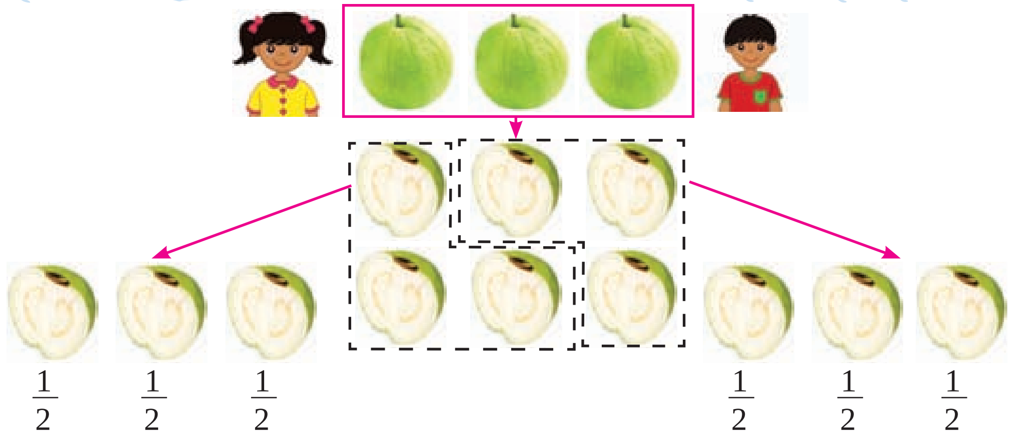
Now let us consider one way of dividing three guavas of the same size equally between two children.



Here, each child receives one whole fruit and half of another fruit.

That is, the total amount that each child receives = $1 + \frac{1}{2}$ fruits. This is written as $1\frac{1}{2}$.

Next, let us consider another way of dividing these three guavas equally between the two children.



3 halves = $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=\frac{3}{2}$.

(Divide each fruit into two equal parts)

3 halves = $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=\frac{3}{2}$.

Accordingly, each child gets 3 halves, which is an amount of $\frac{3}{2}$ guavas.

In both the above cases, each child receives the same amount of guava. Therefore, $\frac{3}{2}=1\frac{1}{2}$.

The numerator of $\frac{3}{2}$ is greater than the denominator.

If the numerator of a fraction is greater or equal to the denominator, it is defined as an **improper fraction**.

We observed above that when three guavas are divided equally between two children, each child receives an amount of $\frac{3}{2}$ guavas. Therefore, $\frac{3}{2}$ represents the same value that is obtained when $3 \div 2$. That is, any proper fraction or improper fraction represents the number that is obtained when its numerator is divided by its denominator.

Example: $\frac{2}{5} = 2 \div 5$ $\quad \frac{11}{3} = 11 \div 3$

$\frac{5}{2}$, $\frac{6}{3}$, $\frac{7}{5}$ and $\frac{11}{4}$ more examples of improper fractions.

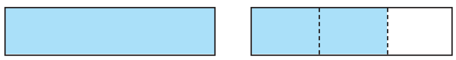
When the whole numbers 1, 2, 3 are expressed as $\frac{2}{2}$, $\frac{6}{3}$, and $\frac{15}{5}$ respectively, they too are considered as improper fractions.

Note that fractions with the numerator and denominator equal to each other are also considered as improper fractions.

10.3 Representing a mixed number as an improper fraction

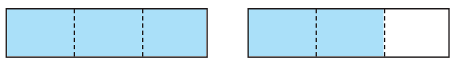
Let us find the shaded region in the figure using two methods.

First method



$1 + \frac{2}{3} = 1\frac{2}{3}$

Second method



The shaded region is five $\frac{1}{3}$s.

1 is equal to three $\frac{1}{3}$ s.

Five $\frac{1}{3}$ s. $= \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{5}{3}$.

According to the above discussion, $1\frac{2}{3} = \frac{5}{3}$.

That is, the mixed number $1\frac{2}{3}$ can also be expressed as the improper fraction $\frac{5}{3}$.

Let us consider the mixed number $1\frac{3}{5}$.

Let us write the mixed number $1\frac{3}{5}$ as an improper fraction.

$$
\begin{aligned}
1\frac{3}{5} &= 1 + \frac{3}{5} \\
&= \frac{5}{5} + \frac{3}{5} \\
&= \frac{8}{5}
\end{aligned}
$$

Example 1

Express $2\frac{3}{4}$ as an improper fraction.

$$2\frac{3}{4} = 1 + 1 + \frac{3}{4}$$
$$= \frac{4}{4} + \frac{4}{4} + \frac{3}{4}$$
$$= \frac{4+4+3}{4}$$
$$= \frac{11}{4}$$

Example 2

Express $3\frac{1}{2}$ as an improper fraction.

$$3\frac{1}{2} = 1 + 1 + 1 + \frac{1}{2}$$
$$= \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{1}{2}$$
$$= \frac{2+2+2+1}{2}$$
$$= \frac{7}{2}$$

Let us now consider an easy method of expressing a mixed number as an improper fraction. Let us consider the mixed number $1\frac{3}{5}$.

$$1 + \frac{3}{5} = \frac{5}{5} + \frac{3}{5}$$
$$= \frac{5+3}{5}$$
$$= \frac{(1 \times 5) + 3}{5} = \frac{8}{5}$$

- Multiply the whole number part in the mixed number, by the denominator of the fractional part and add this to the numerator of the fractional part.
- The value that is obtained is the value of the numerator of the improper fraction which is equal to the given mixed number.
- The denominator of the improper fraction is the same as the denominator of the fractional part of the mixed number.

Let us consider the following examples.

$$2\frac{3}{4} = \frac{(2 \times 4) + 3}{4} = \frac{8 + 3}{4} = \frac{11}{4}$$

$$3\frac{1}{2} = \frac{(3 \times 2) + 1}{2} = \frac{6 + 1}{2} = \frac{7}{2}$$

This process can be done mentally in one step; $7\frac{3}{8} = \frac{59}{8}$.

10.4 Expressing an improper fraction as a mixed number

Let us express $\frac{5}{3}$ as a mixed number .

Method I

$$\frac{5}{3} = \frac{3 + 2}{3}$$

$$= \frac{3}{3} + \frac{2}{3}$$

$$= 1 + \frac{2}{3} = 1\frac{2}{3}$$

Method II

$$\frac{5}{3} = 5 \div 3$$

$$\begin{array}{r|l} & 1 \\ \hline 3 & 5 \\ & 3 \\ \hline & 2 \end{array}$$

The quotient of 5÷3 is 1 and the remainder is 2.

Let us write the above quotient as the whole number part of the solution. The remainder is the numerator of the fractional part. The denominator is the same as that of the improper fraction.

$$\therefore \frac{5}{3} = 1 + \frac{2}{3} = 1\frac{2}{3}$$

Example 1

Express $\frac{17}{10}$ as a mixed number.

Method I

$$\frac{17}{10} = \frac{10+7}{10}$$
$$= \frac{10}{10} + \frac{7}{10}$$
$$= 1\frac{7}{10}$$

Method II

$$\frac{17}{10} = 17 \div 10 = 1 + \frac{7}{10}$$
$$= 1\frac{7}{10}$$

$$\begin{array}{r|l} & 1 \\ 10 & 17 \\ & 10 \\ \hline & 7 \end{array}$$

Example 2

Express $\frac{17}{4}$ as a mixed number.

Method I

$$\frac{17}{4} = \frac{4+4+4+4+1}{4}$$
$$= \frac{4}{4} + \frac{4}{4} + \frac{4}{4} + \frac{4}{4} + \frac{1}{4}$$
$$= 1 + 1 + 1 + 1 + \frac{1}{4}$$
$$\frac{17}{4} = 4\frac{1}{4}$$

Method II

$$\frac{17}{4} = 17 \div 4 = 4 + \frac{1}{4}$$
$$= 4\frac{1}{4}$$

$$\begin{array}{r|l} & 4 \\ 4 & 17 \\ & 16 \\ \hline & 1 \end{array}$$

Exercise 10.1

(1) Of the fractions given below, choose and write down the improper fractions.

(i) $\frac{8}{6}$, $\frac{49}{50}$, $\frac{31}{30}$, $\frac{19}{3}$, $\frac{3}{4}$

(2) Express each of the following mixed numbers as an improper fraction.

(i) $1\frac{1}{4}$ (ii) $2\frac{3}{5}$ (iii) $3\frac{1}{3}$ (iv) $7\frac{5}{8}$

(3) Express each of the following improper fractions as a mixed number.

(i) $\frac{14}{3}$ (ii) $\frac{13}{5}$ (iii) $\frac{26}{3}$ (iv) $\frac{94}{9}$

(4) Write down as a mixed number and as an improper fraction, the amount of guava that each child receives when 23 equal sized guavas are divided equally among 5 children.

10.5 Comparison of fractions

• Comparison of fractions having the same numerator

You have learnt that when two fractions with equal numerators are considered, the fraction with the smaller denominator is greater than the other fraction.

Accordingly, $\frac{4}{5}$ is greater than $\frac{4}{7}$. That is, $\frac{4}{5} > \frac{4}{7}$.

Further, when $\frac{5}{7}, \frac{5}{9}, \frac{5}{8}$ are arranged in ascending order we obtain $\frac{5}{9}, \frac{5}{8}, \frac{5}{7}$. That is, $\frac{5}{9} < \frac{5}{8} < \frac{5}{7}$.

• Comparison of fractions having the same denominator

You have also learnt that when two fractions with equal denominators are considered, the fraction with the larger numerator is greater than the other fraction.

Accordingly, $\frac{3}{5}$ is greater than $\frac{2}{5}$. That is, $\frac{2}{5} < \frac{3}{5}$.

Further, when $\frac{9}{11}, \frac{2}{11}, \frac{15}{11}$ are arranged in ascending order we obtain $\frac{2}{11}, \frac{9}{11}, \frac{15}{11}$.

That is, $\frac{2}{11} < \frac{9}{11} < \frac{15}{11}$.

• Further comparison of fractions

Now let us consider how the greater fraction is identified when two fractions with unequal numerators and unequal denominators are compared by writing equivalent fractions having a common denominator.

Let us compare the fractions $\frac{5}{3}$ and $\frac{7}{6}$.

Let us find the fraction with denominator 6 that is equivalent to $\frac{5}{3}$.

To do this, let us multiply the numerator and denominator of $\frac{5}{3}$ by 2.

$$\frac{5}{3} = \frac{5 \times 2}{3 \times 2} = \frac{10}{6}$$

$$\frac{10}{6} > \frac{7}{6}.$$

Since $\frac{10}{6} = \frac{5}{3}$ we obtain $\frac{5}{3} > \frac{7}{6}$.

Therefore, of the two fractions $\frac{5}{3}$ and $\frac{7}{6}$, the larger fraction is $\frac{5}{3}$.

Let us compare the fraction $\frac{7}{12}$ and $\frac{5}{8}$.

Neither denominator of the fractions $\frac{7}{12}$ and $\frac{5}{8}$ can be written as a multiple of the other. In such situations, the fractions have to be converted into equivalent fractions that have a denominator which is a multiple of the denominators of both fractions. It is convenient to take the least common multiple (LCM) of 12 and 8 in this situation.

$$\begin{array}{r|l} 2 & 12,\ 8 \\ \hline 2 & 6,\ 4 \\ \hline & 3,\ 2 \end{array}$$

The least common multiple of 12 and 18 $\Big\}$ $= 2 \times 2 \times 3 \times 2$

$= 24$

$$\frac{7 \times 2}{12 \times 2} = \frac{14}{24}$$

$$\frac{5 \times 3}{8 \times 3} = \frac{15}{24}$$

$\frac{15}{24} > \frac{14}{24}$ Therefore $\frac{5}{8} > \frac{7}{12}$.

Example 1

Compare the fractions $\frac{17}{12}$ and $\frac{9}{5}$.

There is no number other than 1 which divides both 12 and 5 without remainder.

Therefore, the least common multiple of 12 and 5 $= 12 \times 5 = 60$.

$$\frac{17}{12} = \frac{17 \times 5}{12 \times 5} = \frac{85}{60}$$

$$\frac{9}{5} = \frac{9 \times 12}{5 \times 12} = \frac{108}{60}$$

Since $\frac{108}{60} > \frac{85}{60}$ we obtain, $\frac{9}{5} > \frac{17}{12}$.

A proper fraction is always smaller than an improper fraction with the same denominator.

10.6 Comparison of mixed numbers

- ### Mixed numbers with unequal whole number parts

Let us find the larger number from the mixed numbers $1\frac{1}{2}$ and $3\frac{2}{5}$.

- First, let us examine the whole number parts of the two mixed numbers.
- If the whole number parts are unequal, then the mixed number with the greater whole number part is the larger mixed number.

Accordingly, when the whole number parts of $1\frac{1}{2}$ and $3\frac{2}{5}$ are considered, they are 1 and 3 respectively. Since $3 > 1$, the larger mixed number is $3\frac{2}{5}$.

$3\frac{2}{5} > 1\frac{1}{2}$.

• Mixed numbers with equal whole number parts

Select the larger number from $3\frac{2}{5}$ and $3\frac{1}{2}$.

Method I

- The whole number parts of the above two numbers are equal.
- Therefore, let us compare the fractional parts of these mixed numbers.

Accordingly, let us compare the fractional parts $\frac{2}{5}$ and $\frac{1}{2}$ of the mixed numbers $3\frac{2}{5}$ and $3\frac{1}{2}$.

$$\frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10}$$

$$\frac{1}{2} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10}$$

Since $\frac{5}{10} > \frac{4}{10}$, we obtain $\frac{1}{2} > \frac{2}{5}$.

Therefore, $3\frac{1}{2} > 3\frac{2}{5}$.

Method II

- Express the mixed numbers as improper fractions.
- The larger mixed number can be selected by considering which equivalent improper fraction is larger.

$$3\frac{2}{5} = \frac{17}{5}$$

$$3\frac{1}{2} = \frac{7}{2}$$

Now let us write $\frac{17}{5}$ and $\frac{7}{2}$ as fractions with equal denominators.

$$\frac{17}{5} = \frac{17 \times 2}{5 \times 2} = \frac{34}{10}$$

$$\frac{7}{2} = \frac{7 \times 5}{2 \times 5} = \frac{35}{10}$$

Since $\frac{35}{10} > \frac{34}{10}$, we obtain $\frac{7}{2} > \frac{17}{5}$.

That is, $3\frac{1}{2} > 3\frac{2}{5}$.

Exercise 10.2

(1) For each of the following parts, select and write down the larger/ largest fraction from the given fractions.

(i) $\frac{1}{3}, \frac{1}{5}$ (ii) $\frac{13}{7}, \frac{15}{7}$ (iii) $\frac{5}{11}, \frac{8}{11}, \frac{12}{11}$ (iv) $\frac{11}{3}, \frac{11}{7}, \frac{11}{5}$

(v) $\frac{7}{10}, \frac{4}{5}$ (vi) $\frac{3}{2}, \frac{5}{4}$ (vii) $\frac{3}{4}, \frac{2}{3}$ (viii) $\frac{15}{8}, \frac{7}{3}$

(2) For each of the following parts, select and write down the larger number from the given pair of mixed numbers.

(i) $3\frac{1}{4}, 7\frac{2}{3}$ (ii) $6\frac{2}{5}, 4\frac{1}{2}$ (iii) $5\frac{3}{8}, 5\frac{7}{8}$ (iv) $2\frac{4}{5}, 2\frac{4}{7}$

(v) $6\frac{1}{4}, 6\frac{3}{8}$ (vi) $1\frac{3}{4}, 1\frac{2}{3}$ (vii) $7\frac{5}{6}, 7\frac{4}{5}$ (viii) $6\frac{3}{7}, 6\frac{1}{5}$

(3) Fill in the blanks with the suitable symbol from <, > and =.

(i) $\frac{3}{7}$ .... $\frac{3}{5}$ (ii) $\frac{17}{9}$ ..... $\frac{15}{9}$ (iii) $\frac{25}{8}$ .... $\frac{13}{4}$ (iv) $\frac{4}{5}$ .... $\frac{2}{3}$

(v) $2\frac{1}{6}$ ..... $5\frac{1}{3}$ (vi) $7\frac{1}{2}$ ..... $3\frac{4}{5}$ (vii) $2\frac{1}{5}$ ..... $2\frac{2}{10}$

(viii) $4\frac{2}{3}$ ..... $4\frac{1}{2}$ (ix) $7\frac{3}{8}$ ..... $7\frac{1}{3}$

(4) A person divides a 10 acre land he owns into three equal portions and gives each of his sons a portion. He also divides a 15 acre land he owns into four equal portions and gives each of his daughters a portion. Find out whether the portion a son receives is larger or smaller than the portion a daughter receives. .

(5) The depth of the drain cut by three labourers *A*, *B* and *C* during a day are $1\frac{1}{4}$ m, $2\frac{3}{4}$ m and 2 m respectively. Which labourer has cut the drain with the least depth? Explain your answer.

Summary

- Fractions with the numerator greater or equal to the denominator are defined as improper fractions.
- Numbers which consist of a whole number part and a fractional part are defined as mixed numbers.
- Mixed numbers can be compared by first converting them into equivalent improper fractions.

10 Fractions (Part II)

By studying this lesson you will be able to

- add and subtract fractions.

10.7 Addition of fractions

• Addition of fractions having the same denominator

In Grade 6 you learnt to add proper fractions with equal denominators as well as proper fractions with unequal denominators.

Let us consider the addition of fractions with equal denominators.

$$\frac{2}{8} + \frac{9}{8} = \frac{2+9}{8} = \frac{11}{8}$$

When fractions with equal denominators are added, the denominator of the answer is the same as the denominators of the fractions that are added. The numerator of the answer is the sum of the numerators of the fractions that are added.

The above answer $\frac{11}{8}$ can also be expressed as a mixed number. Then the answer is $1\frac{3}{8}$.

• Addition of fractions with unequal denominators

When fractions with unequal denominators are being added, the given fractions need to be first converted into equivalent fractions with equal denominators, and then added.

Here, it is convenient to convert the given fractions into equivalent fractions that have the least common multiple of the denominators of the given fractions as the denominator.

Find the value of $\frac{7}{10} + \frac{7}{15}$.

The relationship between the denominators of $\frac{7}{10}$ and $\frac{7}{15}$ cannot be identified easily.

In such situations, where the denominators are not related, the given fractions need to be converted into equivalent fractions which have as their denominators a common multiple of the denominators of the given fractions.

Here is it convenient to select the least common multiple of 10 and 15.

$$\begin{array}{r|l} 5 & 10,\ 15 \\ \hline & 2,\ \ 3 \end{array}$$

The least common multiple of 10 and 15 $= 5 \times 2 \times 3$

$= 30$

$$\frac{7}{10} = \frac{7 \times 3}{10 \times 3} = \frac{21}{30}$$

$$\frac{7}{15} = \frac{7 \times 2}{15 \times 2} = \frac{14}{30}$$

$$\frac{7}{10} + \frac{7}{15} = \frac{21}{30} + \frac{14}{30} = \frac{35}{30} = \frac{7}{6} = 1\frac{1}{6}$$

Example 1

Find the value of $\frac{3}{2} + \frac{3}{8}$.

$$\begin{aligned} \frac{3}{2} + \frac{3}{8} &= \frac{3 \times 4}{2 \times 4} + \frac{3}{8} \\ &= \frac{12}{8} + \frac{3}{8} \\ &= \frac{12 + 3}{8} \\ &= \frac{15}{8} \\ &= 1\frac{7}{8} \end{aligned}$$

Example 2

Find the value of $\frac{1}{4} + \frac{2}{5}$.

Here it is convenient to take the least common multiple of 4 and 5 as the denominator each of the equivalent fractions. The least common multiple of 4 and 5 is 20.

$$\frac{1}{4} = \frac{1 \times 5}{4 \times 5} = \frac{5}{20}$$

$$\frac{2}{5} = \frac{2 \times 4}{5 \times 4} = \frac{8}{20}$$

$$\begin{aligned} \frac{1}{4} + \frac{2}{5} &= \frac{5}{20} + \frac{8}{20} \\ &= \frac{13}{20} \end{aligned}$$

Example 3

Find the value of $\frac{17}{12} + \frac{9}{8}$.

The least common multiple of 12 and 8 is 24.

$$\frac{17}{12} + \frac{9}{8} = \frac{34}{24} + \frac{27}{24}$$
$$= \frac{61}{24}$$
$$= 2\frac{13}{24}$$

Example 4

Find the value of $\frac{5}{3} + \frac{3}{8} + \frac{7}{4}$.

The least common multiple of 3, 8 and 4 is 24.

$$\frac{5}{3} + \frac{3}{8} + \frac{7}{4} = \frac{40}{24} + \frac{9}{24} + \frac{42}{24}$$
$$= \frac{91}{24}$$
$$= 3\frac{19}{24}$$

- When adding fractions, some of the steps given in the above examples can be done mentally and the answer can be obtained in a few steps.
- When fractions are simplified, if the answer obtained is a proper fraction, it should be given in its simplest form, and if it is an improper fraction, it should be written as a mixed number.

Exercise 10.3

(1) Evaluate the following.

(i) $\frac{2}{9} + \frac{7}{9} + \frac{5}{9}$ (ii) $\frac{13}{11} + \frac{4}{11}$ (iii) $\frac{7}{6} + \frac{13}{12}$ (iv) $\frac{2}{7} + \frac{3}{5}$

(v) $\frac{12}{5} + \frac{1}{3} + \frac{2}{15}$ (vi) $\frac{13}{4} + \frac{2}{5}$ (vii) $\frac{3}{2} + \frac{5}{4} + \frac{4}{3}$

• Addition of mixed numbers

Let us consider how the two mixed numbers $1\frac{2}{5}$ and $1\frac{1}{5}$ are added together. This is written as $1\frac{2}{5} + 1\frac{1}{5}$.

Method I

The whole number parts and the fractional parts can be added separately.

$$1\frac{2}{5} + 1\frac{1}{5} = 1 + 1 + \frac{2}{5} + \frac{1}{5}$$
$$= 2 + \frac{2+1}{5}$$
$$= 2 + \frac{3}{5}$$
$$= 2\frac{3}{5}$$

Method II

The mixed numbers can be written as improper fractions and added.

$1\frac{2}{5} = \frac{7}{5}$ and $1\frac{1}{5} = \frac{6}{5}$

$$1\frac{2}{5} + 1\frac{1}{5} = \frac{7}{5} + \frac{6}{5}$$
$$= \frac{7+6}{5}$$
$$= \frac{13}{5}$$
$$= 2\frac{3}{5}$$

Method I is more suitable.

Example 1

Find the value of $2\frac{3}{7} + \frac{2}{7}$.

$$2\frac{3}{7} + \frac{2}{7} = 2 + \frac{3}{7} + \frac{2}{7}$$
$$= 2 + \frac{5}{7}$$
$$= 2\frac{5}{7}$$

Example 2

Find the value of $1\frac{1}{3} + 2\frac{5}{12}$.

$$1\frac{1}{3} + 2\frac{5}{12} = (1+2) + \left(\frac{1}{3} + \frac{5}{12}\right)$$
$$= 3 + \left(\frac{1\times 4}{3\times 4} + \frac{5}{12}\right)$$
$$= 3 + \left(\frac{4}{12} + \frac{5}{12}\right)$$
$$= 3 + \frac{9}{12} = 3\frac{9}{12} = 3\frac{3}{4}$$

Example 3

Find the value of $2\frac{2}{3} + \frac{1}{4}$.

$$2\frac{2}{3} + \frac{1}{4} = 2 + \left(\frac{2}{3} + \frac{1}{4}\right)$$
$$= 2 + \left(\frac{8}{12} + \frac{3}{12}\right)$$
$$= 2 + \left(\frac{8+3}{12}\right)$$
$$= 2 + \frac{11}{12}$$
$$= 2\frac{11}{12}$$

Example 4

Find the value of $2\frac{1}{5} + 4\frac{2}{3}$.

$$2\frac{1}{5} + 4\frac{2}{3} = (2+4) + \left(\frac{1}{5} + \frac{2}{3}\right)$$
$$= 6 + \left(\frac{3}{15} + \frac{10}{15}\right)$$
$$= 6 + \left(\frac{3+10}{15}\right)$$
$$= 6 + \frac{13}{15}$$
$$= 6\frac{13}{15}$$

Example 5

Find the value of $1\frac{2}{3} + 2\frac{3}{5} + \frac{5}{6}$.

$$1\frac{2}{3} + 2\frac{3}{5} + \frac{5}{6} = (1+2) + \left(\frac{2}{3} + \frac{3}{5} + \frac{5}{6}\right)$$
$$= 3 + \left(\frac{20}{30} + \frac{18}{30} + \frac{25}{30}\right) = 3 + \frac{63}{30} = 3 + \frac{63 \div 3}{30 \div 3}$$
$$= 3 + \frac{21}{10}$$
$$= 3 + 2\frac{1}{10}$$
$$= 5\frac{1}{10}$$

Exercise 10.4

(1) Evaluate the following.

(i) $3\frac{2}{7} + \frac{3}{7}$ (ii) $2\frac{4}{10} + 3\frac{3}{10}$ (iii) $1\frac{1}{9} + 2\frac{2}{9} + \frac{4}{9}$

(iv) $2\frac{1}{3} + 3\frac{5}{9}$ (v) $\frac{7}{12} + 2\frac{1}{3}$ (vi) $4\frac{3}{5} + 2\frac{1}{10}$

(vii) $2\frac{1}{4} + \frac{2}{3}$ (viii) $5\frac{2}{3} + 3\frac{2}{5}$ (ix) $2\frac{2}{7} + 1\frac{3}{4}$

(x) $4\frac{3}{10} + 3\frac{1}{4}$ (xi) $5\frac{2}{5} + 2\frac{3}{7}$ (xii) $2\frac{7}{12} + 3\frac{5}{8}$

(xiii) $1\frac{2}{3} + 2\frac{1}{3} + 2\frac{5}{6}$ (xiv) $3\frac{1}{4} + 1\frac{1}{2} + \frac{1}{6}$

(xv) $3\frac{5}{6} + 2\frac{3}{4} + 5\frac{1}{3}$

(2) A seamstress states that $1\frac{1}{6}$ m of material is required for a shirt and $2\frac{3}{4}$ m of material is required for a dress. Find the amount of material of a certain type that is required for a shirt and a dress.

(3) A farmer has cultivated paddy in an area of $3\frac{1}{2}$ square kilometres and vegetables in an area of $1\frac{2}{5}$ square kilometres. Find the total cultivated area.

10.8 Subtraction of fractions

Let us now, using examples, describe how to subtract fractions with equal denominators as well as fractions with unequal denominators.

When fractions with unequal denominators are being subtracted, the given fractions are first converted into equivalent fractions with equal denominators and then subtracted.

Example 1

Find the value of $\frac{7}{5} - \frac{1}{5}$.

$$\frac{7}{5} - \frac{1}{5} = \frac{7-1}{5}$$
$$= \frac{6}{5}$$
$$= 1\frac{1}{5}$$

Example 2

Find the value of $\frac{17}{8} - \frac{3}{2}$.

$$\frac{17}{8} - \frac{3}{2} = \frac{17}{8} - \frac{3}{2}$$
$$= \frac{17}{8} - \frac{12}{8}$$
$$= \frac{17-12}{8}$$
$$= \frac{5}{8}$$

Example 3

Find the value of $\frac{1}{2} - \frac{1}{3}$.

$$\frac{1}{2} = \frac{1\times 3}{2\times 3} = \frac{3}{6}, \qquad \frac{1}{3} = \frac{1\times 2}{3\times 2} = \frac{2}{6}$$
$$\frac{1}{2} - \frac{1}{3} = \frac{3}{6} - \frac{2}{6} = \frac{3-2}{6} = \frac{1}{6}$$

Exercise 10.5

(1) Find the value of each of the following.

(i) $\frac{8}{11} - \frac{7}{11}$ (ii) $\frac{13}{12} - \frac{7}{12}$ (iii) $\frac{3}{5} - \frac{1}{5}$ (iv) $\frac{19}{11} - \frac{8}{11}$

(v) $\frac{3}{4} - \frac{1}{2}$ (vi) $\frac{2}{3} - \frac{7}{12}$ (vii) $\frac{15}{7} - \frac{11}{14}$ (viii) $\frac{13}{10} - \frac{1}{2}$

(ix) $\frac{3}{2} - \frac{6}{5}$ (x) $\frac{5}{6} - \frac{3}{4}$ (xi) $\frac{11}{7} - \frac{4}{5}$

(xii) $\frac{9}{8} - \frac{5}{6}$ (xiii) $\frac{7}{8} - \frac{5}{12}$ (xiv) $\frac{8}{9} - \frac{5}{6}$

• Subtracting mixed numbers

Mother had $3\frac{2}{3}$ m of material. She cut $1\frac{1}{3}$ m from this material to sew a dress for her daughter. The amount of material that is remaining can be written as follows.

$$\text{Amount of material remaining} = 3\frac{2}{3} - 1\frac{1}{3}.$$

Method I

In instances such as this, where mixed numbers are being subtracted, the whole number parts and the fractional parts can be simplified separately. Now let us consider how this is done.

$$3\frac{2}{3} - 1\frac{1}{3} = (3-1) + \frac{2}{3} - \frac{1}{3}$$

$$= 2 + \frac{2-1}{3}$$

$$= 2 + \frac{1}{3}$$

$$= 2\frac{1}{3}$$

Method II

This simplification can also be done by converting the mixed numbers into improper fractions. Let us now consider how this is done.

$$3\frac{2}{3} - 1\frac{1}{3} = \frac{11}{3} - \frac{4}{3}$$

$$= \frac{11-4}{3}$$

$$= \frac{7}{3}$$

$$= 2\frac{1}{3}$$

Example 1

Evaluate $2\frac{7}{9} - \frac{2}{9}$.

$$2\frac{7}{9} - \frac{2}{9} = 2 + \left(\frac{7}{9} - \frac{2}{9}\right)$$
$$= 2 + \left(\frac{7-2}{9}\right)$$
$$= 2 + \frac{5}{9}$$
$$= 2\frac{5}{9}$$

Example 2

Evaluate $6\frac{5}{9} - \frac{1}{3}$.

$$6\frac{5}{9} - \frac{1}{3} = 6 + \left(\frac{5}{9} - \frac{1}{3}\right)$$
$$= 6 + \left(\frac{5}{9} - \frac{1 \times 3}{3 \times 3}\right)$$
$$= 6 + \left(\frac{5}{9} - \frac{3}{9}\right)$$
$$= 6 + \frac{2}{9} = 6\frac{2}{9}$$

Example 3

Evaluate $3\frac{4}{5} - 2\frac{1}{5}$.

$$3\frac{4}{5} - 2\frac{1}{5} = (3-2) + \left(\frac{4}{5} - \frac{1}{5}\right)$$
$$= 1 + \left(\frac{4-1}{5}\right)$$
$$= 1\frac{3}{5}$$

Example 4

Evaluate $5\frac{7}{10} - 2\frac{2}{15}$.

$$5\frac{7}{10} - 2\frac{2}{15} = (5-2) + \left(\frac{7}{10} - \frac{2}{15}\right)$$
$$= 3 + \left(\frac{21}{30} - \frac{4}{30}\right)$$
$$= 3 + \frac{17}{30}$$
$$= 3\frac{17}{30}$$

Example 5

Evaluate $7\frac{2}{3} - \frac{1}{4}$.

$$7\frac{2}{3} - \frac{1}{4} = 7 + \left(\frac{2}{3} - \frac{1}{4}\right)$$

The LCM of 3 and 4 is 12.

$$7\frac{2}{3} - \frac{1}{4} = 7 + \left(\frac{2 \times 4}{3 \times 4} - \frac{1 \times 3}{4 \times 3}\right)$$
$$= 7 + \left(\frac{8}{12} - \frac{3}{12}\right)$$
$$= 7 + \frac{5}{12} = 7\frac{5}{12}$$

Example 6

Evaluate $3\frac{1}{5} - 2\frac{1}{10}$.

$$3\frac{1}{5} - 2\frac{1}{10} = (3-2) + \left(\frac{1}{5} - \frac{1}{10}\right)$$
$$= 1 + \left(\frac{1 \times 2}{5 \times 2} - \frac{1}{10}\right)$$
$$= 1 + \left(\frac{2}{10} - \frac{1}{10}\right)$$
$$= 1 + \frac{1}{10}$$
$$= 1\frac{1}{10}$$

Example 7

Evaluate $3\frac{2}{7} - 1\frac{1}{2}$.

Method I

$$3\frac{2}{7} - 1\frac{1}{2} = (3-1) + \left(\frac{2}{7} - \frac{1}{2}\right)$$
$$= 2 + \left(\frac{4}{14} - \frac{7}{14}\right)$$
$$= 2 + \frac{4-7}{14}$$
$$= 1 + 1 + \frac{4-7}{14}$$
$$= 1 + \frac{14}{14} + \frac{4-7}{14}$$
$$= 1 + \frac{14 + 4 - 7}{14} = 1 + \frac{11}{14}$$
$$= 1\frac{11}{14}$$

Method II

$$3\frac{2}{7} - 1\frac{1}{2} = \frac{23}{7} - \frac{3}{2}$$
$$= \frac{46}{14} - \frac{21}{14}$$
$$= \frac{25}{14}$$
$$= 1\frac{11}{14}$$

In such instances, it is easier to first convert the mixed numbers into improper fractions and then simplify them.

Exercise 10.6

(1) Evaluate the following.

(i) $2\frac{3}{5} - 1\frac{1}{5}$ (ii) $4\frac{5}{7} - 1\frac{4}{7}$ (iii) $2\frac{7}{8} - \frac{4}{8}$

(iv) $2 - 1\frac{1}{4}$ (v) $3 - 1\frac{5}{6}$ (vi) $2 - 1\frac{5}{16}$

(vii) $8\frac{7}{10} - 3\frac{2}{5}$ (viii) $2\frac{2}{5} - 1\frac{3}{20}$ (ix) $2\frac{2}{3} - 1\frac{1}{2}$

(x) $3\frac{3}{4} - 1\frac{7}{18}$ (xi) $6\frac{5}{8} - 4\frac{1}{6}$ (xii) $4\frac{3}{10} - 2\frac{4}{15}$

(2) Sachi travelled $3\frac{7}{10}$ kilometres to his brother Gamini's house by going the initial $3\frac{1}{2}$ kilometres by bus and walking the remaining distance. What was the distance that Sachi walked?

(3) A farmer owns a plot of land of area 4 hectares. He has cultivated kurakkan (finger millet) in $2\frac{1}{2}$ hectares. What is the extent of the land in which he has not cultivated kurakkan?

Miscellaneous Exercise

(1) (i) Express $7\frac{3}{5}$ as an improper fraction.

(ii) Express $\frac{50}{11}$ as a mixed number.

(2) (i) Write the fractions $1\frac{1}{4}, \frac{15}{7}, \frac{5}{3}, \frac{1}{2}$ in ascending order.

(ii) Write the fractions $2\frac{5}{3}, 7\frac{1}{3}, \frac{3}{4}, \frac{5}{7}$ in descending order.

(3) Evaluate the following.

(i) $\frac{1}{5} + 1\frac{1}{4} + 3\frac{5}{7}$ (ii) $\frac{3}{5} + 3\frac{5}{7} + 5\frac{1}{4}$ (iii) $7\frac{2}{3} - 4\frac{1}{4}$

(iv) $4\frac{5}{6} - 1\frac{3}{5}$ (v) $4\frac{5}{8} - 2\frac{1}{3}$ (vi) $2\frac{1}{2} - 1\frac{3}{4}$

(4) Malinga walked for three hours at $3\frac{1}{2}$ kilometres per hour. Find the total distance he walked during the three hours as an improper fraction.

Summary

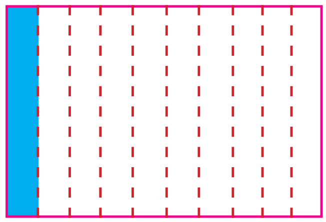
- When fractions are simplified, if the answer obtained is a proper fraction, it should be given in its simplest form, and if it is an improper fraction, it should be written as a mixed number.

11 Decimals

By studying this lesson you will be able to
- represent a fraction with a denominator that can be written as a power of ten, as a decimal number,
- represent a decimal number as a fraction, and
- multiply and divide a decimal number by a whole number.

11.1 Writing a proper fraction with a denominator which is a power of ten, as a decimal number

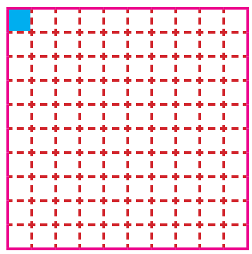
In Grade 6 we learnt how to write a proper fraction with 10 or 100 as the denominator, as a decimal number.



When a unit is divided into 10 equal parts, then one part is equal to $\frac{1}{10}$.

This is denoted as a decimal number, by 0.1.

That is, $0.1 = \frac{1}{10}$.



When a unit is divided into 100 equal parts, then one part is equal to $\frac{1}{100}$.
This is denoted as a decimal number, by 0.01.
That is, $0.01 = \frac{1}{100}$.

We have learnt that when a unit is divided into 1000 equal parts, then one part is equal to $\frac{1}{1000}$.
$\frac{1}{1000}$ is written in decimal form as 0.001. That is, $0.001 = \frac{1}{1000}$.
The number 0.001 is read as zero point zero zero one. The position where 1 is written after the second decimal place in 0.001 is defined as the third decimal place. The place value of the third decimal place is $\frac{1}{1000}$.

Since $\frac{7}{1000}$ is seven $\frac{1}{1000}$ s, we obtain $\frac{7}{1000} = 0.007$ The number 0.007 is read as zero point zero zero seven.

Let us consider $\frac{24}{1000}$.

$$\frac{24}{1000} = \frac{20}{1000} + \frac{4}{1000}$$

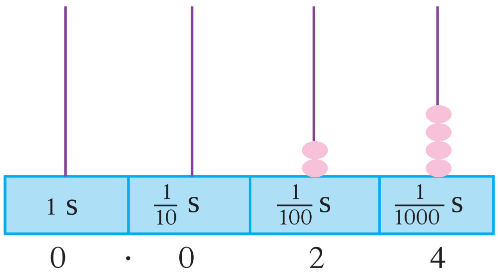
Since, $\frac{20}{1000} = \frac{20 \div 10}{1000 \div 10} = \frac{2}{100}$

$\frac{24}{1000}$ = two $\frac{1}{100}$ s + four $\frac{1}{1000}$ s.

Accordingly, $\frac{24}{1000} = 0.024$.

0.024 is read as zero point zero two four.

Let us represent 0.024 on an abacus.



Example 1

(1) Write each of the following fractions as a decimal number.

(i) $\frac{4}{1000}$ (ii) $\frac{97}{1000}$ (iii) $\frac{751}{1000}$

(i) $\frac{4}{1000} = 0.004$ (ii) $\frac{97}{1000} = 0.097$ (iii) $\frac{751}{1000} = 0.751$

Exercise 11.1

(1) Express each of the following fractions as a decimal number. Represent them on an abacus.

(i) $\frac{9}{10}$ (ii) $\frac{75}{100}$ (iii) $\frac{9}{1000}$ (iv) $\frac{25}{1000}$ (v) $\frac{275}{1000}$

11.2 Writing a proper fraction with a denominator which is not a power of ten, as a decimal number

Let us learn how to express a proper fraction with a denominator which is not a power of 10, as a decimal number.

- The given fraction can easily be written as a decimal number if it can be converted into an equivalent fraction which has a power of 10 as its denominator.

Let us express $\frac{1}{2}$ as a decimal number.

10 can be divided by 2 without remainder. $10 \div 2 = 5$. Therefore, by multiplying the numerator and the denominator of $\frac{1}{2}$ by 5, it can be converted into an equivalent fraction with 10 as the denominator.

$$\frac{1}{2} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10}$$

$$\frac{5}{10} = 0.5$$

Therefore, $\frac{1}{2} = 0.5$.

Let us express $\frac{1}{4}$ as a decimal number.

Although 10 cannot be divided by 4 without remainder, 100 can be divided by 4 without remainder. $100 \div 4 = 25$.

Therefore, by multiplying the numerator and the denominator of $\frac{1}{4}$ by 25, it can be converted into an equivalent fraction with 100 as the denominator.

$$\frac{1}{4} = \frac{1 \times 25}{4 \times 25} = \frac{25}{100}$$

$$\frac{25}{100} = 0.25$$

Therefore, $\frac{1}{4} = 0.25$.

Let us express $\frac{1}{8}$ as a decimal number.

Although 10 and 100 cannot be divided by 8 without remainder, 1000 can be divided by 8 without remainder. $1000 \div 8 = 125$.

Therefore, by multiplying the numerator and the denominator of $\frac{1}{8}$ by 125, it can be converted into an equivalent fraction with 1000 as the denominator.

$$\frac{1}{8} = \frac{1 \times 125}{8 \times 125} = \frac{125}{1000}$$

$$\frac{125}{1000} = 0.125$$

Therefore, $\frac{1}{8} = 0.125$.

According to the above description, the proper fractions that can be converted into equivalent fractions with a power of 10 as the denominator, can easily be expressed as decimal numbers.

That is, if 10, 100, 1000 or any other power of 10 can be divided without remainder by the denominator of a given fraction, then that fraction can be written as a decimal number with one or more decimal places.

Example 1

Express each of the fractions $\frac{1}{5}$, $\frac{13}{25}$ and $\frac{77}{125}$ as a decimal number.

$\frac{1}{5} = \frac{2}{10} = 0.2$

$\frac{13}{25} = \frac{52}{100} = 0.52$

$\frac{77}{125} = \frac{77 \times 8}{125 \times 8} = \frac{616}{1000} = 0.616$

11.3 Writing a mixed number as a decimal number

Now let us consider how a mixed number is expressed as a decimal number.

Let us write $3\frac{5}{20}$ as a decimal number.

$$3\frac{5}{20} = 3 + \frac{5}{20}$$
$$= 3 + \frac{5 \times 5}{20 \times 5} = 3 + \frac{25}{100}$$
$$= 3 + 0.25$$
$$= 3.25$$

Let us write $7\frac{11}{40}$ as a decimal number.

$$7\frac{11}{40} = 7 + \frac{11}{40}$$
$$= 7 + \frac{11 \times 25}{40 \times 25}$$
$$= 7 + \frac{275}{1000}$$
$$= 7.275$$

11.4 Writing an improper fraction as a decimal number

Let us consider how an improper fraction is written as a decimal number.

Let us write $\frac{17}{5}$ as a decimal number.

Method I

$$\frac{17}{5} = 3\frac{2}{5} = 3 + \frac{2}{5}$$
$$= 3 + \frac{4}{10} = 3 + 0.4$$
$$= 3.4$$

Method II

$$\frac{17}{5} = \frac{34}{10} = \frac{30}{10} + \frac{4}{10}$$
$$= 3 + 0.4$$
$$= 3.4$$

Example 1

Express $\frac{9}{8}$ as a decimal number.

Method I

$$\frac{9}{8} = 1 + \frac{1}{8}$$
$$\frac{9}{8} = 1 + \frac{125}{1000}$$
$$= 1 + 0.125$$
$$= 1.125$$

Method II

$$\frac{9}{8} = \frac{9 \times 125}{8 \times 125}$$
$$= \frac{1125}{1000} = \frac{1000}{1000} + \frac{125}{1000}$$
$$= 1 + 0.125$$
$$= 1.125$$

Exercise 11.2

Express the following fractions and mixed numbers as decimal numbers.

(i) $\frac{3}{5}$ (ii) $\frac{3}{4}$ (iii) $\frac{8}{25}$ (iv) $\frac{321}{500}$ (v) $\frac{39}{40}$

(vi) $13\frac{1}{2}$ (vii) $2\frac{7}{50}$ (viii) $2\frac{1}{8}$ (ix) $3\frac{7}{40}$ (x) $5\frac{14}{125}$

(xi) $\frac{13}{10}$ (xii) $\frac{27}{20}$ (xiii) $\frac{7}{5}$ (xiv) $\frac{97}{8}$ (xv) $\frac{251}{250}$

11.5 Writing a decimal number as a fraction

Let us write 0.5 as a fraction.

$0.5 = \frac{5}{10}$

To express $\frac{5}{10}$ in its simplest form, let us divide the numerator and the denominator by 5.

$$0.5 = \frac{5}{10} = \frac{5 \div 5}{10 \div 5} = \frac{1}{2}$$

Let us write 0.375 as a fraction.

There fore, $0.375 = \frac{375}{1000}$

To express $\frac{375}{1000}$ in its simplest form, let us divide the numerator and the denominator by 125.

$$\frac{375}{1000} = \frac{375 \div 125}{1000 \div 125} = \frac{3}{8}$$

$0.375 = \frac{3}{8}$

Let us write 1.75 as a fraction.

$1.75 = 1 + 0.75 = 1 + \frac{75}{100} = 1\frac{75}{100}$

To express $\frac{75}{100}$ in its simplest form, let us divide the numerator and the denominator by 25.

$$\frac{75}{100} = \frac{75 \div 25}{100 \div 25} = \frac{3}{4}$$

Therefore, $1.75 = 1\frac{3}{4}$.

Example 1

Express 1.625 as a fraction in its simplest form.

$$1.625 = 1 + 0.625 = 1 + \frac{625}{1000} = 1 + \frac{625 \div 25}{1000 \div 25} = 1 + \frac{25}{40} = 1 + \frac{25 \div 5}{40 \div 5}$$

$$= 1 + \frac{5}{8}$$

$$= 1\frac{5}{8}$$

Exercise 11.3

Write each of the following decimal numbers as a fraction and express it in the simplest form.

(i) 0.7	(ii) 1.3	(iii) 0.45	(iv) 8.16
(v) 6.75	(vi) 0.025	(vii) 4.225	(viii) 8.625

11.6 Multiplying a decimal number by a whole number

$2 \times 3 = 2 + 2 + 2 = 6$

This illustrates the fact that the product of two whole numbers can be obtained by writing it as a sum.

Now let us find the value of 0.1×3.

$0.1 \times 3 = 0.1 + 0.1 + 0.1$
$= 0.3$

Let us find the value of 0.8×2.

$0.8 \times 2 = 0.8 + 0.8$
$= 1.6$

Let us find the value of 0.35×4.

$0.35 \times 4 = 0.35 + 0.35 + 0.35 + 0.35$
$= 1.40$
$= 1.4$

Let us examine the above answers by considering the following table.

$0.1 \times 3 = 0.3$	$1 \times 3 = 3$
$0.8 \times 2 = 1.6$	$8 \times 2 = 16$
$0.35 \times 4 = 1.40$	$35 \times 4 = 140$

It will be clear to you from observing the above table that, when multiplying a decimal number by a whole number, the answer can be obtained by following the steps given below too.

- Consider the decimal number as a whole number by disregarding the decimal point and multiply it by the given whole number.

- Place the decimal point in the answer that is obtained such that the final answer has the same number of decimal places as the original decimal number.

Now let us find the value of 24.31×6.

First let us multiply the numbers without taking the decimal places into consideration.

$$\begin{array}{r} 2431 \\ \times \quad 6 \\ \hline 14586 \\ \hline\hline \end{array}$$

Since 24.31 has two decimal places, place the decimal point such that the final answer too has two decimal places. Then $24.31 \times 6 = 145.86$

It must be clear to you that, when the whole number by which the decimal number has to be multiplied is large, the method given above is much easier to use than the method of repeatedly adding the decimal number.

Example 1

Find the value of 4.276×12.

$$\begin{array}{r} 4276 \\ \times \quad 12 \\ \hline 8552 \\ 4276 \\ \hline 51312 \\ \hline\hline \end{array}$$

Since 4.276 has three decimal places, the decimal point is placed such that the answer too has three decimal places.

Then, $4.276 \times 12 = 51.312$

Exercise 11.4

Evaluate the following.

(i) 2.45×6	(ii) 0.75×4	(iii) 3.47×15
(iv) 15.28×13	(v) 0.055×3	(vi) 1.357×41

• Multiplying a decimal number by 10, 100 and 1000

Let us consider the following products.

$2.1 \times 10 = 21.0$	$2.1 \times 100 = 210.0$	$2.1 \times 1000 = 2100.0$
$3.75 \times 10 = 37.50$	$3.75 \times 100 = 375.00$	$3.75 \times 1000 = 3750.00$
$23.65 \times 10 = 236.50$	$23.65 \times 100 = 2365.00$	$23.65 \times 1000 = 23650.00$
$43.615 \times 10 = 436.150$	$43.615 \times 100 = 4361.500$	$43.615 \times 1000 = 43615.000$

The following facts are discovered by examining the above products.

- The number that is obtained when a decimal number is multiplied by 10 can be obtained by moving the decimal point in the original decimal number by one place to the right. $37.16 \times 10 = 371.6$
- The number that is obtained when a decimal number is multiplied by 100 can be obtained by moving the decimal point in the original decimal number by two places to the right. $37.16 \times 100 = 3716$
- The number that is obtained when a decimal number is multiplied by 1000 can be obtained by moving the decimal point in the original decimal number by three places to the right. $37.160 \times 1000 = 37\ 160$

Exercise 11.5

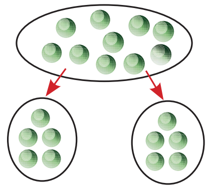
Evaluate the following.

(i) 4.74×10	(ii) 0.503×10	(iii) 0.079×10
(iv) 5.83×100	(v) 5.379×100	(vi) 0.07×100
(vii) 1.2×100	(viii) 0.0056×10	(ix) 0.0307×100
(x) 3.7×1000	(xi) 8.0732×1000	(xii) 6.0051×1000

11.7 Dividing a decimal number by 10, 100 and 1000

$10 = 5 \times 2$ means that there are 2 heaps of five in 10. Therefore, when 10 is divided into two equal heaps there are 5 in each heap.

That is $10 \div 2 = 5$.
You have learnt this in Grade 6.



Now let us find the value of 32.6 ÷ 10.

32.6 ÷ 10 is how many 10s there are in 32.6.
We know that 3.26 × 10 = 32.6.
Therefore, 32.6 ÷ 10 = 3.26

Similarly,
145.56 ÷ 100 is how many 100s there are in 145.56.
Since 1.4556 × 100 = 145.56,
we obtain 145.56 ÷ 100 = 1.4556

6127.3 ÷ 1000 is how many 1000s there are in 6127.3.
Since 6.1273 × 1000 = 6127.3,
we obtain 6127.3 ÷ 1000 = 6.1273.

Let us consider the following divisions.

7871.8 ÷ 10 = 787.18	7871.8 ÷ 100 = 78.718	7871.8 ÷ 1000 = 7.8718
169.51 ÷ 10 = 16.951	169.51 ÷ 100 = 1.6951	169.51 ÷ 1000 = 0.16951
9.51 ÷ 10 = 0.951	9.51 ÷ 100 = 0.0951	9.51 ÷ 1000 = 0.00951

Accordingly,

- The number that is obtained by dividing a decimal number by 10 is equal to the number that is obtained by moving the decimal point in the original decimal number by one decimal place to the left.
  6.0 ÷ 10 = 0.60
- The number that is obtained by dividing a decimal number by 100 is equal to the number that is obtained by moving the decimal point in the original decimal number by two decimal places to the left.
  006.0 ÷ 100 = 0.060 = 0.06
- The number that is obtained by dividing a decimal number by 1000 is equal to the number that is obtained by moving the decimal point in the original decimal number by three decimal places to the left.
  006.0 ÷ 1000 = 0.0060 = 0.006

Exercise 11.6

Evaluate the following.

(i) $27.1 \div 10$ (ii) $1.36 \div 10$ (iii) $0.26 \div 10$ (iv) $0.037 \div 10$

(v) $0.0059 \div 10$ (vi) $58.9 \div 100$ (vii) $3.7 \div 100$ (viii) $97.6 \div 100$

(ix) $0.075 \div 100$ (x) $0.0032 \div 100$ (xi) $4375.8 \div 1000$

(xii) $356.8 \div 1000$

• Dividing a decimal number by a whole number

Let us find the value of $7.5 \div 3$.

Divide the whole number part.

When long division is being performed, place the decimal point in the answer, when the number immediately to the right of the decimal point is being divided. Then continue with the division.

Step 1

$$\begin{array}{r|l} & 2 \\ \cline{2-2} 3 & 7\,.\,5 \\ & 6 \\ \cline{2-2} & 1 \end{array} \qquad \begin{array}{l} 2 \times 3 = 6 \\ 7 - 6 = 1 \end{array}$$

$7 \div 3 = 2$ with a remainder of 1

Since the decimal part of 7.5 occurs after 7, place the decimal point after 2 in the answer.

Step 2

$$\begin{array}{r|l} & 2\,.\downarrow \\ \cline{2-2} 3 & 7\,.\,5 \\ & 6 \\ \cline{2-2} & 1\ \ 5 \end{array}$$

Bring 5 down

Step 3

$$\begin{array}{r|l} & 2\,.\,5 \\ \cline{2-2} 3 & 7\,.\,5 \\ & 6 \\ \cline{2-2} & 1\ \ 5 \\ & 1\ \ 5 \\ \cline{2-2} & \ \ \ \ 0 \end{array} \qquad \begin{array}{l} 5 \times 3 = 15 \\ 15 - 15 = 0 \end{array}$$

Example 1

(i) Find the value of $182.35 \div 7$.

$$\begin{array}{r|l} & 26.05 \\ \hline 7 & 182.35 \\ & 14 \\ \hline & 42 \\ & 42 \\ \hline & 03 \\ & 00 \\ \hline & 35 \\ & 35 \\ \hline & 0 \end{array}$$

Place the decimal point, when the number 3, which is immediately to the right of the decimal place is being divided.

(ii) Find the value of $0.672 \div 12$.

$$\begin{array}{r|l} & 0.056 \\ \hline 12 & 0.672 \\ & 0 \\ \hline & 0\,6 \\ & 0 \\ \hline & 67 \\ & 60 \\ \hline & 72 \\ & 72 \\ \hline & 0 \end{array}$$

$0.672 \div 12 = 0.056$

(iii) Find the value of $2.13 \div 4$.

$$\begin{array}{r|l} & 0.5325 \\ \hline 4 & 2.1300 \\ & 0 \\ \hline & 2\,1 \\ & 2\,0 \\ \hline & 1\,3 \\ & 1\,2 \\ \hline & 10 \\ & 8 \\ \hline & 20 \\ & 20 \\ \hline \end{array}$$

$2.13 \div 4 = 0.5325$

Further knowledge

$$\begin{array}{r|l} & 2\,.\,5 \\ \hline 3 & 7\,.\,5 \\ & 6 \\ \hline & 1\ \ 5 \\ & 1\ \ 5 \\ \hline & 0 \end{array}$$

The digit in the ones place of 7.5 is 7. This denotes 7 ones.
When 7 is divided by 3, we obtain 2 and a remainder of 1.

A remainder of one means 1 ones. That is, ten $\frac{1}{10}$ s.

The digit 5 in 7.5 denotes five $\frac{1}{10}$ s. Therefore, there are fifteen $\frac{1}{10}$ s in the first decimal place. Let us divide this fifteen $\frac{1}{10}$ s by 3. Then we obtain five $\frac{1}{10}$ s with no remainder. That is $7.5 \div 3 = 2.5$

Exercise 11.7

(1) Evaluate the following.

(i) $84.6 \div 2$	(ii) $167.2 \div 4$	(iii) $54.6 \div 3$
(iv) $98.58 \div 6$	(v) $74.5 \div 5$	(vi) $35.86 \div 2$
(vii) $0.684 \div 6$	(viii) $0.735 \div 7$	(ix) $1.08 \div 4$
(x) $7.401 \div 3$	(xi) $8.04 \div 8$	(xii) $11.745 \div 9$

(2) If the height of a child is 145 cm, express this height in metres.

Summary

- When multiplying a decimal number by a whole number, consider the decimal number as a whole number by disregarding the decimal point and multiply the two numbers. Place the decimal point in the answer that is obtained so that it has the same number of decimal places as the original decimal number.

- When a decimal number is multiplied by a power of ten, the number of places the decimal point in the decimal number shifts to the right is equal to the number of zeros in the power of ten by which it is multiplied.

- When a decimal number is divided by a power of ten, the number of places the decimal point in the decimal number shifts to the left is equal to the number of zeros in the power of ten by which it is divided.

12 Algebraic Expressions

By studying this lesson you will be able to

- construct algebraic expressions,
- simplify algebraic expressions, and
- find the value of algebraic expressions by substituting numbers.

12.1 Constructing algebraic expressions

Kavin buys the same amount of milk every day. If this amount is not known, then we cannot represent it by a number although it is a constant value.

As in the above situation, when the numerical value of a constant amount is not known, it is defined as an "**unknown constant**".

The daily income of a certain shop takes different values depending on its daily sales. Since the daily income is not a fixed value, it is a **variable**.

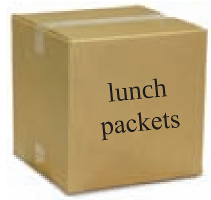
Simple letters of the English alphabet such as $a,b,c,\ldots,x,y,z$ are used to represent unknown constants and variables.

Accordingly, considering the above two examples, the amount of milk bought each day can be denoted by the letter a and the daily income of the shop can be denoted by x.

Let us denote the number of bananas in a bunch in a shop by a. When a comb of 12 bananas is sold, the number of bananas remaining in the bunch can be denoted by $a - 12$.

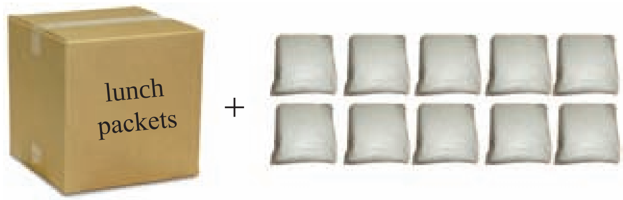
The expression $a - 12$ is an **algebraic expression**. a and 12 are defined as the "**terms**" of this expression.

If the price of a banana is 8 rupees, then $8 \times a$ rupees can be gained by selling all the bananas in the bunch. This is written as $8a$. The coefficient of a in the term $8a$ is 8. There is only one algebraic term in the expression $8a$.



Let us take the number of rice packets sold daily by a vendor as x. If the price of a rice packet is 80 rupees, then the vendor's daily income is $80 \times x$ rupees. We write this as $80x$ rupees.

If the vendor receives a new order to supply 10 more packets daily, then the number of rice packets sold daily will be $x + 10$.



The terms in the expression $x + 10$, are x and 10.

Example 1

The letter m represents a number of unknown value.

(i) Write in terms of m, the number which is three times the given number.

(ii) Write in terms of m, the number which is 15 more than twice the given number.

(i) The number which is three times as large as m is $3m$.

(ii) The number which is twice as large as m is $2m$.

Therefore, the number that is greater than $2m$ by 15 is $2m + 15$.

Exercise 12.1

(1) (i) Construct an algebraic expression for the price of 5 apples by taking the price of one apple as a rupees.

(ii) The price of a pineapple is 10 rupees more than the price of 5 apples. Construct an algebraic expression for the price of a pineapple in terms of a.

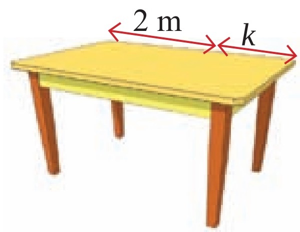
(2) A shop owner buys 12 loaves of bread from a bakery at b rupees per loaf. He then sells these loaves so that each loaf brings him a profit of 3 rupees.

(i) What is the total amount the shop owner pays for the loaves of bread?

(ii) What is the selling price of a loaf of bread?

(iii) A customer buys a loaf of bread and 500g of sugar. The price of 1 kilogram of sugar is 80 rupees. What is the total amount the customer spends?

(3) 1 m = 100 cm.



(i) The length of a table is k centimeters more than 2 meters. Express the length of the table in centimeters in terms of k.

(ii) The width of this table is 50 cm less than its length. Write the width as an expression of k.

12.2 More on constructing algebraic expressions

The algebraic expressions we have constructed so far contain one algebraic symbol, one or more mathematical operations and numbers.

The following table describes algebraic expressions containing one unknown term.

Algebraic Expression	Unknown constant or variable in the algebraic expression	Coefficient of the unknown constant or variable	Terms of the algebraic expression	Mathematical operations in the order they appear in the algebraic expression
$4x$	x	4	$4x$	$\times$
$y+4$	y	1	y , 4	$+$
$p-10$	p	1	p , 10	$-$
$20+3m$	m	3	20 , $3m$	$+$, $\times$
$3a+5$	a	3	$3a$, 5	$\times$, $+$

The mathematical operations of addition, subtraction and multiplication are used in the above expressions. The coefficient of the unknown in each of the expressions is a positive whole number. The operation division is not used in any of these algebraic expressions.

Let us now consider algebraic expressions that have a fraction as the coefficient of the unknown.

There are x number of marbles in a bottle. They are placed in three containers such that each container has the same number of marbles. Then the number of marbles in one container is $x \div 3$. That is, $\frac{x}{3}$.

The width of a hostel room is half its length. If the length is l meters, let us write the width in meters.

The width of the room is $l \div 2$ meters. That is, the width of the room is $\frac{l}{2}$ m.

The length of the adjoining room is one meter more than the width of this room. Let us write the length of the adjoining room as an algebraic expression.

The length of the adjoining room $= \frac{l}{2} + 1$ meters.

Example 1

(1) If more than one meter of cloth is bought, then the price of one meter is p rupees. If less than one meter of cloth is bought, then an additional 10 rupees is charged. Write the price of $\frac{1}{2}$ a meter of cloth as an algebraic expression.

Price of 1m of cloth $= p$ rupees

Since the quantity which is bought is less than 1 m,

the price of $\frac{1}{2}$ m of cloth $= \frac{p}{2} + 10$ rupees.

Example 2

(1) A father sells the 3 plots of land he owns at p rupees per plot. He then divides the money he receives equally among his four children. Write the amount of money received by each child as an algebraic expression.

The money obtained by selling the three plots of land $= 3p$ rupees

The amount of money received by each child $= \frac{3p}{4}$ rupees

Exercise 12.2

(1) Complete the following table.

Algebraic expression	Unknown constant or variable in the algebraic expression	Terms of the algebraic expression
$\frac{a}{2} + 5$	a	$\frac{a}{2}, 5$
$\frac{p}{4} - 8$		
$\frac{x}{5} + 10$		
$25 + \frac{y}{3}$		

(2) Construct an algebraic expression for each of the following situations.

(i) The value of a number is denoted by a. What is the value of the number that is greater by 4 than half the value of the given number?

(ii) In a restaurant, a loaf of bread is sold for p rupees. A person buys $\frac{1}{4}$ of a loaf of bread and a dish of dhal. The dish of dhal costs 30 rupees. Write an algebraic expression for the total amount of money the person has to pay.

(iii) The height of a building is 5 meters less than $\frac{1}{2}$ of its length. If its length is l meters, write the height as an expression of l.

(iv) The price of 1kg of sugar is y rupees. If a 100 rupee note is tendered when $\frac{1}{2}$ kg of sugar is bought, write the balance as an algebraic expression of y.

(3) The price of a box of pencils containing 12 pencils is x rupees.

(i) Write the price of one pencil as an algebraic expression.

(ii) If the price of an eraser is 10 rupees, write the amount of money required to buy 2 pencils and an eraser as an algebraic expression.

(4) Write the expressions given below in words.

The expression $5a - 8$ can be expressed in words as follows.

If a denotes a given value, then $5a - 8$ denotes the value which is 8 less than the value of five times a.

(i) $2a + 8$	(ii) $3x - 15$	(iii) $2(p + 5)$
(iv) $\frac{p}{4} - 4$	(v) $20 - 5p$	(vi) $\frac{x}{2} + 14$
(vii) $\frac{y}{5} - 1$	(viii) $30 + \frac{p}{2}$	(ix) $45 - \frac{y}{3}$

12.3 Constructing algebraic expressions having two unknown terms

The price of a pencil is x rupees and the price of an eraser is y rupees. Let us write the price of 5 pencils and 2 erasers as an algebraic expression.

The price of 5 pencils = $5 \times x$ rupees = $5x$ rupees

The price of 2 erasers = $2 \times y$ rupees = $2y$ rupees

The price of 5 pencils and 2 erasers = $(5x + 2y)$ rupees

The price of 1 kg of sugar is x rupees, the price of 1kg of wheat flour is y rupees and the price of a box of matches is 3 rupees. Let us write the amount of money required to buy 500 g of sugar, 2 kg of wheat flour and 3 boxes of matches as an algebraic expression.

$$\left.\begin{array}{r}\text{The price of 500 g of sugar,}\\ \text{when the price of 1 kg of sugar is } x \text{ rupees}\end{array}\right\} = \frac{x}{2} \text{ rupees}$$

$$\left.\begin{array}{r}\text{The price of 2 kg of wheat flour}\\ \text{when the price of 1 kg of wheat flour is } y \text{ rupees}\end{array}\right\} = 2y \text{ rupees}$$

$$\left.\begin{array}{r}\text{The price of 3 boxes of matches,}\\ \text{when the price of one box of matches is 3 rupees}\end{array}\right\} = 9 \text{ rupees}$$

Therefore, the required amount of money $= (\frac{x}{2} + 2y + 9)$ rupees

Example 1

(i) There are a number of boys and b number of girls in a class. Write the total number of students in the class as an algebraic expression.

The total number of students in the class = $a + b$

(ii) Write the algebraic expression $\frac{x}{2} + \frac{y}{2}$ in words.

"Add one half of the value represented by y to one half of the value represented by x"

Example 2

25 coconuts were bought at a rupees each and all 25 fruits were sold at b rupees each. Assume that b is greater than a. Write an algebraic expression for the profit.

The price of a coconut = a rupees
The amount of money spent on buying 25 coconuts = $25a$ rupees
The amount of money gained by selling 25 coconuts = $25b$ rupees
Profit $= (25b - 25a)$ rupees

Exercise 12.3

(1) Construct algebraic expressions for the following.

(i) A number is represented by a. What is the number that is greater than a by b?

(ii) A number is represented by p. Write the number that is less than p by q.

(iii) The price of a coconut is x rupees.
The price of 1kg of rice is y rupees.
Write an expression in terms of x and y for the price of 4 coconuts and 3 kg of rice.

(iv) The price of 1 kg of sugar is x rupees and the price of a 250 g packet of tea is y rupees. Find the amount of money required to buy 2 kg and 500 g of sugar and 2 packets of tea.

(v) 250 g = $\frac{1}{4}$ kg. 1kg of potatoes is x rupees. A bundle of green leaves is y rupees. Write an algebraic expression for the amount paid if 250 g of potatoes and a bundle of green leaves are bought.

(vi) There are x number of Sinhala books and y number of English books in the school library. $\frac{1}{2}$ the Sinhala books and $\frac{1}{2}$ the English books are Literature books. If the library has issued 23 Sinhala Literature books and 18 English Literature books, then express the number of literature books remaining in the library as an algebraic expression.

(2) Write the following expressions in words.

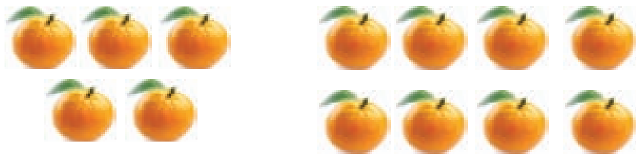
(i) $3x + 5y$ (ii) $2a - 7b$ (iii) $\frac{x}{4} - y + 5$ (iv) $2k + 3p - 8$

12.4 Simplifying the terms of an algebraic expression

Let us consider an algebraic expression similar to one we constructed earlier.

The price of an orange is a rupees. Nimal bought 5 oranges and Deepani bought 8.

Nimal Deepani



Nimal spent $5a$ rupees and Deepani spent $8a$ rupees. So the total amount of money spent by both of them is $5a + 8a$.

Since the number of oranges bought by both of them is 13, the total amount spent is $13 \times a$ rupees. That is $13a$ rupees.

This shows that $5a + 8a = 13a$.

Algebraic terms such as $5a$ and $8a$ which have the same unknown are called "**like terms**". By adding or subtracting several such terms, we can simplify them to one term.

There are no like terms in the algebraic expression $4x + 3y + 5$. Such an expression cannot be simplified further. The terms $4x$, $3y$, 5 of this expression are called "**unlike terms**".

Let us simplify $4x + 3y + x + 2y$.

Let us write the like terms together.

$$\begin{aligned} 4x + 3y + x + 2y &= 4x + 1x + 3y + 2y \\ &= 5x + 5y \end{aligned}$$

Let us simplify $10p + 4k + p - k$.

$$\begin{aligned} 10p + 4k + p - k &= 10p + 1p + 4k - 1k \\ &= 11p + 3k \end{aligned}$$

Example 1

Simplify the following.

(i) $3x + 6k + 5x + 3k + 7$ (ii) $5a + b + 8 + 3a - b - 5$

(i) $3x + 6k + 5x + 3k + 7 = 3x + 5x + 6k + 3k + 7$
$= 8x + 9k + 7$

(ii) $5a + b + 8 + 3a - b - 5 = 5a + 3a + b - b + 8 - 5$
$= 8a + 0 + 3$
$= 8a + 3$

Example 2

There are 25 boys and 15 girls in a Grade 4 class.
There are 28 boys and 11 girls in a Grade 5 class.
The price of a pen is p rupees and the price of an eraser is q rupees. Find the total amount of money needed to give a pen to each boy in Grade 4, an eraser to each girl in Grade 4, an eraser to each boy in Grade 5 and a pen to each girl in Grade 5.

The money needed to give pens and erasers to the students in Grade 4 $= 25p + 15q$

The money needed to give pens and erasers to the students in Grade 5 $= 11p + 28q$

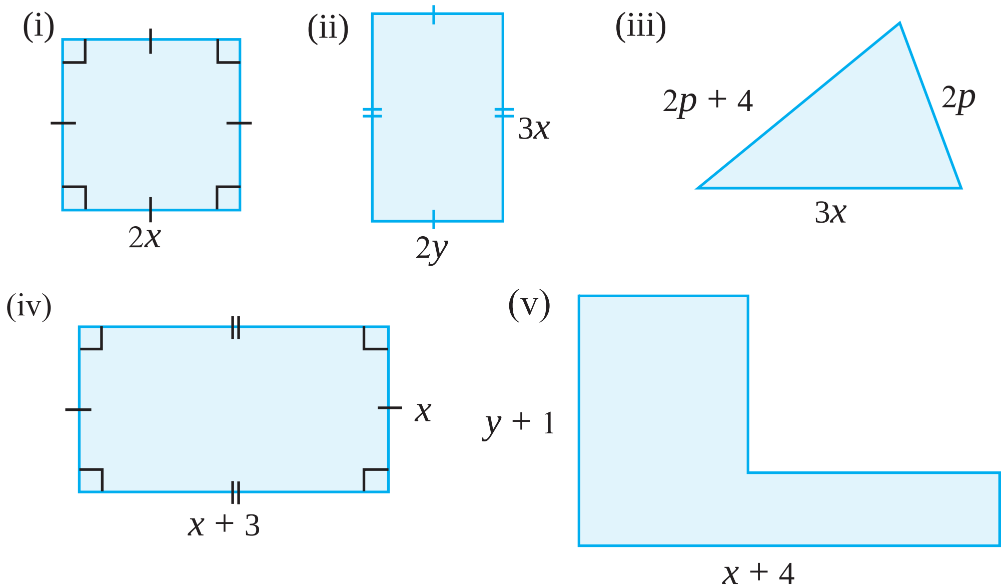
The money needed to give pens and erasers to the students in Grade 4 and Grade 5 $= 25p + 15q + 11p + 28q$
$= 25p + 11p + 15q + 28q$
$= 36p + 43q$

Exercise 12.4

(1) Simplify the following.

(i) $4x + 5y + 3x + 7$ (ii) $3a + 4 + 6b + 3$
(iii) $5p + 4q - 2p + q$ (iv) $10m - 7n + 10n - 4m$
(v) $3k + 5l + 10 + k + 4l - 5$ (vi) $8x - 4y - 11 + x + 7y + 13$

(2) Write an algebraic expression for the perimeter of each of the figures below. Simplify the expression.



12.5 Substituting values for the unknowns in an algebraic expression

When $x = 2$, the expression $x + 3$ takes the value 5. You have learnt in grade 6 that giving a numerical value to the unknown term in an algebraic expression in this manner is called substitution. By substitution, an algebraic expression gets a value.

Let us consider the expression $x + 3$.
When $x = 2$,
$x + 3 = 2 + 3 = 5$.

Let us find the value of $3x - 5$ when $x = 4$.

$$3x - 5 = 3 \times 4 - 5$$
$$= 12 - 5 = 7$$

Let us find the value of $4a - 3$ when $a = 2$.

$$4a - 3 = 4 \times 2 - 3$$
$$= 8 - 3$$
$$= 5$$

Let us now substitute values for the unknowns in an algebraic expression which has two unknown terms and find its value.

Let us find the value of $3x + 4y$ when $x = 4$ and $y = 5$.

$$\begin{aligned} 3x + 4y &= 3 \times 4 + 4 \times 5 \\ &= 12 + 20 \\ &= 32 \end{aligned}$$

Example 1

Find the value of each of the algebraic expressions given below when $x = 4$ and $y = 2$.

(i) $x - y$

$x - y = 4 - 2 = 2$

(ii) $3x - y - 5$

$$\begin{aligned} 3x - y - 5 &= 3 \times 4 - 2 - 5 \\ &= 12 - 2 - 5 \\ &= 10 - 5 \\ &= 5 \end{aligned}$$

Exercise 12.5

(1) Find the value of each of the algebraic expressions given below when $a = 4$.

(i) $3a - 5$ (ii) $5\,(a - 3)$ (iii) $15 - 2a$ (iv) $7a - 5$

(2) For each of the values given to x, find the value of $6x + 4$.

(i) $x = 1$ (ii) $x = 2$ (iii) $x = 5$ (iii) $x = 12$

(3) Find the value of each of the given expressions by substituting the given values.

(i) $4x - 13y + 5$ when $x = 4$ and $y = 1$

(ii) $7a$ - $3b$ $- 8$, when $a = 3$ and $b = 1$

(iii) $2p + k - 5$, when $p = 6$ and $k = 2$

Miscellaneous Exercise

(1) The length of a room is x meters less than twice its width. The width of the room is 3 m. Write an expression in terms of x for the length of the room.

(2) The price of a pen is x rupees and the price of 12 books is y rupees. Nimal buys 2 pens and 3 books. Write an expression for the total amount of money spent by Nimal.

(3) Write each expression given below in words.

(i) $8 + \frac{y}{2}$ (ii) $16 - \frac{a}{3}$

(4) Simplify the following.

(i) $8a + 7b - 3 - 6b - 2a$ (ii) $6x + 5y - 6x - 3y$

(5) Find the value of each of the expressions given below when $x = 7$ and $y = 3$.

(i) $6x - 5y$ (ii) $7x - 3 - 6y$

(6) A father's age was 35 years at the time his son was born.

(i) Write the age of the father, when his son is x years old.

(ii) The mother is 4 years younger to the father. Write the mother's age in terms of x when the son is x years old.

(iii) How many years older is the mother than the son?

Summary

- In an algebraic expression, the number written together with an unknown is called the “coefficient of the unknown”.
- Algebraic terms with the same unknown are called “like terms”.
- Several like terms can be simplified into one term by adding or subtracting them.
- Algebraic terms with different unknowns are called “unlike terms”.
- Two unlike terms cannot be simplified further into one term by adding or subtracting them.

Ponder

(1) A vendor sells 1kg of brinjals for 10 rupees more than twice the price he paid for 1 kg of brinjals. He sells 1kg of papaw for 8 rupees more than three times the price he paid for 1 kg of papaw.

The vendor buys 1kilogram of brinjals and 1kilogram of papaw for x rupees and y rupees respectively.

(i) Write an algebraic expression for the amount the vendor spent to buy 1 kg of brinjals and 1 kg of papaw.

(ii) Write an algebraic expression for the selling price of 1 kg of brinjals.

(iii) Write an algebraic expression for the selling price of 1 kg of papaw.

(iv) Write an algebraic expression for the amount he receives by selling 1 kg of brinjals and 1 kg of papaw.

(v) If the vendor bought 1 kg of brinjals for 35 rupees and 1 kg of papaw for 20 rupees, obtain values for the algebraic expressions in (i), (ii), (iii) and (iv) above.

Glossary

Acute angle	සුළු කෝණය	கூர்ங்கோணம்
Addition	එකතු කිරීම	கூட்டல்ஸ
Algebraic expressions	වීජීය ප්‍රකාශන	அட்சரகணிதக் கோவை
Algebraic symbols	වීජීය සංකේත	அட்சரக் குறியீடு
Algebraic terms	වීජීය පද	அட்சரகணித உறுப்பு
Angle	කෝණය	கோணம்
Axis of symmetry	සමමිති අක්ෂය	சமச்சீர் அச்சு
Bilateral symmetry	ද්වි පාර්ශ්වික සමමිතිය	இருபுடைச் சமச்சீர
Century	ශතකය	நூற்றாண்டு
Coefficient	සංගුණක	வகுத்தல்
Decade	දශකය	தசாப்தம
Decimal numbers	දශම සංඛ්‍යා	முடிவுள்ள தசமம்
Denominator	හරය	பகுதி
Digital root	ඉලක්කම් දර්ශකය	இலக்கச்சுட்டி
Directed number	සදිශ සංඛ්‍යාව	திசைகொண்ட எண்கள்
Division	බෙදීම	வகுத்தல்
Dynamic concept	ගතික සංකල්පය	கோணங்களைப்
Elements	අවයව	மூலகங்கள
Equivalent fraction	තුල්‍ය භාගය	சமவலுப் பின்னங்கள்
Expansion of powers	බල ප්‍රසාරණය	வலுக்களின் விரிவு
Factor	සාධකය	காரணி
Fraction	භාගය	பின்னம்
Greatest common factor	මහා පොදු සාධකය	பொதுக்காரணிகளுள் பெரியது
Improper fraction	විෂම භාගය	முறைமை இல்லாப் பின்னம
Integers	නිඛිල	நிறைவெண்
Leap year	අධික අවුරුද්ද	நெட்டாண்டு
Least common multiple	කුඩා ම පොදු ගුණාකාරය	பொது மடங்குகளுள்
Like terms	සජාතීය පද	நிகர்த்த உறுப்புக்கள
Linear algebraic expressions	ඒකජ වීජීය ප්‍රකාශන	ஒருறுப்பு அட்சரகணிதக்கோவை

Mass	ස්කන්ධය	திணிவு
Mathematical operations	ගණිත කර්ම	கணிதச் செய்கைகள
Millennium	සහස්‍රකය	ஆயிரம் ஆண்டு
Minus	සෘණ	மறை
Mixed number	මිශ්‍ර සංඛ්‍යා	கலப்பு எண
Multiple	ගුණාකාර	மடங்கு
Multiplication	ගුණ කිරීම	பெருக்கல்
Negative integers	සෘණ නිඛිල	மறை நிறைவெண்
Number line	සංඛ්‍යා රේඛාව	எண்கோடு
Numerator	ලවය	தொகுதி
Obtuse angle	මහා කෝණය	பின்வளைகோணம்
Parallel lines	සමාන්තර සරල රේඛා	சமாந்தர நேர்கோடுகள்
Perpendicular distance	ලම්බ දුර	செங்குத்துத் தூரம்
Plus	ධන	நேர்
Positive integers	ධන නිඛිල	நேர் நிறைவெண்
Prime factors	ප්‍රථමක සාධක	முதன்மைக் காரணிகள
Proper fraction	තත්‍ය භාගය (නියම භාගය)	முறைமைப் பின்னம்
Protractor	කෝණමානය	பாகைமானி
Reflex angle	පරාවර්ත කෝණය	நிலைசார் எண்ணக்கரு
Right angle	සෘජු කෝණය	விரிகோணம்சிறியது
Set	කුලකය	தொடை
Set square	විහිත චතුරස්‍රය	மூலை மட்டம்
Side	බාහුව	புயம் (பக்கம்)
Static concept	ස්ථිතික සංකල්පය	இயக்கசார் எண்ணக்கரு
Straight edge	සරල දාරය	நேர்விளிம்பு (வரைகோல்)
Substitution	ආදේශය	பிரதியீடு
Subtraction	අඩු කිරීම	கழித்தல
Symmetrical plane figures	සමමිතික තල රූප	சமச்சீரான தளவுரு
Symmetry	සමමිතිය	சமச்சீர்
Unknowns	අඥාත	தெரியாக் கணியம
Unlike terms	විජාතීය පද	நிகரா உறுப்புக்கள்
Venn diagram	වෙන් රූප සටහන	வென் வரிப்படம
Vertex	ශීර්ෂය	உச்சி
Whole numbers	පූර්ණ සංඛ්‍යා	முழுஎண்கள்

Lesson Sequence

Content	Number of Periods	Competency levels
First Term		
1. Bilateral Symmetry	05	25.1
2. Sets	05	30.1
3. Mathematical Operations on Whole Numbers	04	1.1
4. Factors and Multiples	11	1.3, 1.4
5. Indices	06	6.1
6. Time	05	12.1
7. Parallel Straight Lines	03	27.1
8. Directed Numbers	06	1.2
9. Angles	07	21.1, 21.2
	52	
Second Term		
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11. Decimals	05	3.2
12. Algebraic Expressions	06	14.1, 14.2
13. Mass	06	9.1
14. Rectilinear Plane Figures	06	23.1, 23.2
15. Equations and Formulae	08	17.1, 19.1
16. Length	08	7.1, 7.2
17. Area	06	8.1
18. Circles	04	24.1
19. Volume	05	10.1
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22. Percentages	05	5.1
23. Cartesian Plane	05	20.1
24. Construction of Plane Figures	05	27.2
25. Solids	05	22.1, 22.2
26. Data Representation and Interpretation	08	28.1, 29.1
27. Scale Diagrams	06	13.1
28. Tessellation	05	26.1
29. Likelihood of an Event	06	31.1, 31.2
	50	
Total	170	

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Grade 7

Part - II

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The National Anthem of Sri Lanka

Sri Lanka Matha
Apa Sri Lanka Namo Namo Namo Namo Matha
Sundara siri barinee, surendi athi sobamana Lanka
Dhanya dhanaya neka mal palaturu piri jaya bhoomiya ramya
Apa hata sepa siri setha sadana jeewanaye matha
Piliganu mena apa bhakthi pooja Namo Namo Matha
Apa Sri Lanka Namo Namo Namo Namo Matha
Oba we apa vidya
Obamaya apa sathya
Oba we apa shakthi
Apa hada thula bhakthi
Oba apa aloke
Apage anuprane
Oba apa jeevana we
Apa mukthiya oba we
Nava jeevana demine, nithina apa pubudukaran matha
Gnana veerya vadawamina regena yanu mana jaya bhoomi kara
Eka mavakage daru kela bevina
Yamu yamu vee nopama
Prema vada sema bheda durerada
Namo, Namo Matha
Apa Sri Lanka Namo Namo Namo Namo Matha

අපි වෙමු එක මවකගෙ දරුවෝ
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කිසි කල නොම දිරනා

ආනන්ද සමරකෝන්

ஒரு தாய் மக்கள் நாமாவோம்
ஒன்றே நாம் வாழும் இல்லம்
நன்றே உடலில் ஓடும்
ஒன்றே நம் குருதி நிறம்

அதனால் சகோதரர் நாமாவோம்
ஒன்றாய் வாழும் வளரும் நாம்
நன்றாய் இவ் இல்லினிலே
நலமே வாழ்தல் வேண்டுமன்றோ

யாவரும் அன்பு கருணையுடன்
ஒற்றுமை சிறக்க வாழ்ந்திடுதல்
பொன்னும் மணியும் முத்துமல்ல - அதுவே
யான்று மழியாச் செல்வமன்றோ.

ஆனந்த சமரக்கோன்
கவிதையின் பெயர்ப்பு.

Being innovative, changing with right knowledge
Be a light to the country as well as to the world.

Message from the Hon. Minister of Education

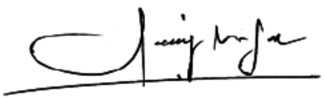
The past two decades have been significant in the world history due to changes that took place in technology. The present students face a lot of new challenges along with the rapid development of Information Technology, communication and other related fields. The manner of career opportunities are liable to change specifically in the near future. In such an environment, with a new technological and intellectual society, thousands of innovative career opportunities would be created. To win those challenges, it is the responsibility of the Sri Lankan Government and myself, as the Minister of Education, to empower you all.

This book is a product of free education. Your aim must be to use this book properly and acquire the necessary knowledge out of it. The government in turn is able to provide free textbooks to you, as a result of the commitment and labour of your parents and elders.

Since we have understood that the education is crucial in deciding the future of a country, the government has taken steps to change curriculum to suit the rapid changes of the technological world. Hence, you have to dedicate yourselves to become productive citizens. I believe that the knowledge this book provides will suffice your aim.

It is your duty to give a proper value to the money spent by the government on your education. Also you should understand that education determines your future. Make sure that you reach the optimum social stratum through education.

I congratulate you to enjoy the benefits of free education and bloom as an honoured citizen who takes the name of Sri Lanka to the world.



Akila Viraj Kariyawasam
Minister of Education

Foreword

The educational objectives of the contemporary world are becoming more complex along with the economic, social, cultural and technological development. The learning and teaching process too is changing in relation to human experiences, technological differences, research and new indices. Therefore, it is required to produce the textbook by including subject related information according to the objectives in the syllabus in order to maintain the teaching process by organizing learning experiences that suit to the learner needs. The textbook is not merely a learning tool for the learner. It is a blessing that contributes to obtain a higher education along with a development of conduct and attitudes, to develop values and to obtain learning experiences.

The government in its realization of the concept of free education has offered you all the textbooks from grades 1-11. I would like to remind you that you should make the maximum use of these textbooks and protect them well. I sincerely hope that this textbook would assist you to obtain the expertise to become a virtuous citizen with a complete personality who would be a valuable asset to the country.

I would like to bestow my sincere thanks on the members of the editorial and writer boards as well as on the staff of the Educational Publications Department who have strived to offer this textbook to you.

W. M. Jayantha Wickramanayaka,
Commissioner General of Educational Publications,
Educational Publications Department,
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2019.04.10

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Message of the Boards of Writers and Editors

This textbook has been compiled in accordance with the new syllabus which is to be implemented from 2016 for the use of grade seven students.

We made an effort to develop the attitude "We can master the subject of Mathematics well" in students.

In compiling this textbook, the necessity of developing the basic foundation of studying mathematical concepts in a formal manner was specially considered. This textbook is not just a learning tool which targets the tests held at school. It was compiled granting due consideration to it as a medium that develops logical thinking, correct vision and creativity in the child.

Furthermore, most of the activities, examples and exercises that are incorporated here are related to day to day experiences in order to establish mathematical concepts in the child. This will convince the child about the importance of mathematics in his or her daily life. Teachers who guide the children to utilize this textbook can prepare learning tools that suit the learning style and the level of the child based on the information provided here.

Learning outcomes are presented at the beginning of each lesson. A summary is provided at the end of each lesson to enable the child to revise the important facts relevant to the lesson. Furthermore, at the end of the set of lessons related to each term, a revision exercise has been provided to revise the tasks completed during that term.

Every child does not have the same capability in understanding mathematical concepts. Thus, it is necessary to direct the child from the known to the unknown according to his / her capabilities. We strongly believe that it can be carried out precisely by a professional teacher.

In the learning process, the child should be given ample time to think and practice problems on his or her own. Furthermore, opportunities should be given to practice Mathematics without restricting the child to just the theoretical knowledge provided by mathematics.

Our firm wish is that our children act as intelligent citizens who think logically by studying Mathematics with dedication.

Boards of Writers and Editors

Contents

13

Mass

By studying this lesson you will be able to
- identify milligramme as a unit used to measure masses,
- identify the relationship between the units gramme and milligramme,
- add and subtract masses expressed in grammes and milligrammes, and
- multiply and divide masses expressed in milligrammes, grammes, and kilogrammes by a whole number.

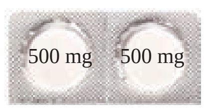
13.1 Units used to measure mass

You have learnt before that gramme and kilogramme are units used to measure masses. Now let us identify another unit which is used to measure masses.

The masses of the nutrients included in a 100 grammes packet of food for children, “Thriposha”, are indicated as below.

Protein 20.0 g	Carbohydrate 61.9 g
Fat 7.8 g	Iron 18 mg

The mass of paracetamol in each of the paracetamol tablets shown in the figure is indicated as 500 mg.



Based on the above information, you will notice that in addition to kilogramme (kg) and gramme (g), the unit milligramme which is smaller than the other two units is also used to measure a mass more precisely. “Milligramme” is denoted by mg.

1 gramme is 1000 milligrammes. That is, 1 g = 1000 mg

13.2 The relationship between grammes and milligrammes

• Expressing a mass given in grammes in milligrammes

Now let us consider how a mass given in grammes is expressed in milligrammes.

$$\text{Since } 1 \text{ g} = 1000 \text{ mg},$$
$$2 \text{ g} = 2 \times 1000 \text{ mg} = 2000 \text{ mg}$$
$$3 \text{ g} = 3 \times 1000 \text{ mg} = 3000 \text{ mg}$$

Therefore, to express a mass given in grammes, in terms of milligrammes, the given number of grammes should be multiplied by 1000.

Example 1

Express 7.656 g in milligrammes.

$$7.656 \text{ g} = 7.656 \times 1000 \text{ mg}$$
$$= 7656 \text{ mg}$$

Example 2

Express 2 g 650 mg in milligrammes.

$$2 \text{ g } 650 \text{ mg} = 2 \times 1000 \text{ mg} + 650 \text{ mg}$$
$$= 2000 \text{ mg} + 650 \text{ mg}$$
$$= 2650 \text{ mg}$$

Example 3

Express 7.656 g in grammes and milligrammes.

$$7.656 \text{ g} = 7 \text{ g} + 0.656 \text{ g}$$
$$= 7 \text{ g} + 0.656 \times 1000 \text{ mg}$$
$$= 7 \text{ g} + 656 \text{ mg}$$
$$= 7 \text{ g } 656 \text{ mg}$$

Example 4

Express $3\frac{1}{2}$ g in milligrammes.

$$3\frac{1}{2} \text{ g} = 3 \text{ g} + \frac{1}{2} \text{ g}$$
$$= 3 \times 1000 \text{ mg} + 500 \text{ mg}$$
$$= 3000 \text{ mg} + 500 \text{ mg}$$
$$= 3500 \text{ mg}$$

• Expressing a mass given in milligrammes in terms of grammes

Next let us consider how a mass given in milligrammes can be expressed in grammes.

$$\text{Since } 1000 \text{ mg} = 1 \text{ g},$$
$$2000 \text{ mg} = \frac{2000}{1000} \text{ g} = 2 \text{ g}$$
$$3000 \text{ mg} = \frac{3000}{1000} \text{ g} = 3 \text{ g}$$

Therefore, to express a mass given in milligrammes in terms of grammes, the given number of milligrammes should be divided by 1000.

Example 1

Express 2758 mg in grammes.

$2758 \text{ mg} = \frac{2758}{1000} \text{ g}$

$= 2.758 \text{ g}$

Example 2

Express 2225 mg in grammes and milligrammes.

$2225 \text{ mg} = 2000 \text{ mg} + 225 \text{ mg}$

$= \frac{2000}{1000} \text{ g} + 225 \text{ mg}$

$= 2 \text{ g} + 225 \text{ mg}$

$= 2 \text{ g } 225 \text{ mg}$

Accordingly, when an amount of 1000 mg or more, is expressed in terms of grammes and milligrammes, care should be taken to ensure that the amount of milligrammes written is less than 1000.

Example 3

Express 3 g 675 mg in grammes.

$3 \text{ g } 675 \text{ mg} = 3 \text{ g} + 675 \text{ mg}$

$= 3 \text{ g} + \frac{675}{1000} \text{ g}$

$= 3 \text{ g} + 0.675 \text{ g}$

$= 3.675 \text{ g}$

Exercise 13.1

(1) Fill in the blanks.

(i) 8 g 42 mg = 8 g + ... mg
= ...... mg + ... mg
= ...... mg

(ii) $3750 \text{ mg} = \frac{3750}{1000} \text{ g}$
= .... g

(iii) 1.275 g = 1 g + ... mg
= .... mg + ... mg
= .... mg

(iv) 1.275 g = 1.275 × .... mg
= .... mg

(2) Express the following masses in grammes.

(i) 1245 mg (ii) 1475 mg (iii) 2 g 875 mg (iv) 12 g 8 mg

(3) Express the following masses in milligrammes.

(i) 8 g (ii) 15 g (iii) 3 g 750 mg (iv) 2 g 75 mg
(v) 2.5 g (vi) 3.005 g (vii) 3.61 g (viii) $1\frac{3}{4}$ g

(4) Express each of the following masses in terms of grammes and milligrammes.

(i) 2350 mg (ii) 3.75 g (iii) 12.05 g (iv) 1.005 g

(5) Complete the following table.

g	g mg	mg
1.4 g	1 g 400 mg	1400 mg
3.65 g	....	....
5.005 g	....	....
....	1 g 975 mg	....
....	5 g 5 mg	....
....	....	6007 mg
....	....	12 535 mg

13.3 Addition of masses expressed in grammes and milligrammes

The mass of the chocolates in a box of mass 15 g 350 mg, is 750 g 800 mg. Let us find the total mass of the box of chocolates.
To do this, let us add the mass of the box and the mass of the chocolates.

Method I

	g	mg
	15	350
+	750	800
	766	150

Let us add the quantities in the milligrammes column.
350 mg + 800 mg = 1150 mg
1150 mg = 1000 mg + 150 mg
= 1 g + 150 mg
Let us write 150 mg in the milligrammes column.

Let us carry the 1 g to the grammes column and add the amounts in the grammes column.
1 g + 15 g + 750 g = 766 g
Let us write 766 g, in the grammes column.

Total mass of the box of chocolates is 766 g 150 mg.

Method II

Let us express each of the masses in grammes, and then simplify.

15 g 350 mg = 15.350 g
750 g 800 mg = 750.800 g
766.150 g = 766 g + 150 mg

	g
	15 . 350
+	750 . 800
	766 . 150

Exercise 13.2

(1) Simplify the following.

(i)
$$\begin{array}{rrr} & \text{g} & \text{mg} \\ & 250 & 170 \\ + & 35 & 630 \\ \hline \hline \end{array}$$

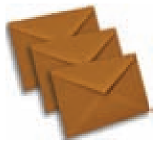
(ii)
$$\begin{array}{rrr} & \text{g} & \text{mg} \\ & 15 & 150 \\ & 20 & 675 \\ + & 30 & 265 \\ \hline \hline \end{array}$$

(iii) 10 g 255 mg + 5 g 805 mg

(iv) 150 g 750 mg + 50 g 360 mg

(2) The mass of the sweetmeats in a box of mass 19 g 750 mg, is 480 g 250 mg. Find the total mass of the box of sweetmeats.

(3) The masses of three letters received by a post office are 10 g 150 mg, 5 g 975 mg and 8 g 900 mg respectively. Show that the total mass of all three letters exceeds 25 g.



13.4 Subtraction of masses expressed in grammes and milligrammes

The total mass of a box of sweetmeats is 500 g 250 mg. The mass of the empty box is 100 g 750 mg. Accordingly, let us find the mass of the sweetmeats in the box.

To find the mass of the sweetmeats, the mass of the empty box needs to be subtracted from the total mass.

Method I

$$\begin{array}{rrr} & \text{g} & \text{mg} \\ & 500 & 250 \\ - & 100 & 750 \\ \hline & 399 & 500 \\ \hline \hline \end{array}$$

Since 750 mg cannot be subtracted from 250 mg, let us carry 1 g, that is 1000 mg, from the 500 g in the grammes column to the milligrammes column and add it to the 250 mg in the milligrammes column.
Then, 1000 mg + 250 mg = 1250 mg.
1250 mg – 750 mg = 500 mg
Let us write the 500 mg in the milligrammes column.

Let us subtract 100 g from the 499 g remaining in the grammes column.
Then, 499 g – 100 g = 399 g
Let us write the 399 g, in the grammes column.

The mass of the sweetmeats in the box is 399 g 500 mg.

Method II

Let us express each of the masses in grammes, and then simplify.

500 g 250 mg = 500.250 g

100 g 750 mg = 100.750 g

399.500 g = 399 g 500 mg

$$\begin{array}{r} \text{g} \\ 500\,.\,250 \\ -\,100\,.\,750 \\ \hline 399\,.\,500 \\ \hline\hline \end{array}$$

The mass of the sweetmeats in the box is 399 g 500 mg.

Exercise 13.3

(1) Simplify the following.

(i)
$$\begin{array}{rrr} & \text{g} & \text{mg} \\ & 50 & 750 \\ - & 20 & 250 \\ \hline \\ \hline\hline \end{array}$$

(ii)
$$\begin{array}{rrr} & \text{g} & \text{mg} \\ & 150 & 200 \\ - & 75 & 300 \\ \hline \\ \hline\hline \end{array}$$

(iii) 250 g 550 mg – 150 g 105 mg

(iv) 60 g – 25 g 150 mg

(2) The total mass of a biscuit packet with biscuits is 210 g 150 mg. The mass of the empty packet is 2 g 300 mg. What is the mass of the biscuits in the biscuit packet?

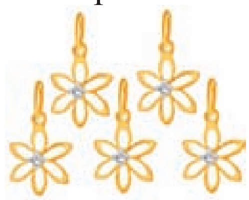
(3) When a certain amount was used from a quantity of margarine of mass 150 g, the remaining mass was 105 g 350 mg. Find the mass of the margarine that was used.

(4) A mass of 160 g 450 mg of gold was left over after making jewellery from a block of gold of mass 205 g 375 mg. Find the mass of the gold that was used to make the jewellery.

13.5 Multiplication of a mass by a whole number

➢ The mass of gold used to produce a particular pendent is 6 g 500 mg. Let us find the total mass of gold required to produce 5 such pendants.

To produce 5 pendants, 5 portions of gold of mass 6 g 500 mg each are required. Therefore, to find the total mass of gold that is required, 6 g 500 mg should be multiplied by 5.



Method I

Let us express 6 g 500 mg in milligrammes and then multiple by 5.

6 g 500 mg = 6500 mg

6500 mg × 5 = 32 500 mg

$$\begin{array}{r} \text{mg} \\ 6500 \\ \times 5 \\ \hline 32\,500 \\ \hline\hline \end{array}$$

32 500 mg = 32 g 500 mg

That is, the total mass required to produce 5 pendants is 32 g 500 mg.

Method II

$$\begin{array}{ll} \text{g} & \text{mg} \\ 6 & 500 \\ & \times\ 5 \\ \hline 32 & 500 \\ \hline\hline \end{array}$$

First, let us multiply 500 mg by 5.

500 × 5 mg = 2500 mg

2500 mg = 2000 mg + 500 mg = 2 g + 500 mg

Let us write 500 mg in the milligrammes column.

Let us multiply 6 g by 5. 6 g × 5 = 30 g

Now let us add the 2 g obtained from the multiplication done in the milligrammes column, to 30 g.

30 g + 2 g = 32 g

Let us write 32 g in the grammes column.

- Let us simplify 5 kg 120 g × 12.

Method I

$$\begin{array}{rr} \text{kg} & \text{g} \\ 5 & 120 \\ & \times 12 \\ \hline 61 & 440 \\ \hline\hline \end{array}$$

First multiply 120 g by 12.

120 g × 12 = 1440 g = 1 kg 440 g

Let us multiply 5 kg, by 12.

5 kg × 12 = 60 kg

5 kg 120 g × 12 = 60 kg + 1 kg 440 g

= 60 kg +1 kg + 440 g

= 61 kg 440 g

5 kg 120 g × 12 = 61 kg 440 g

Method II

Let us express 5 kg 120 g in grammes and then multiply by 12.

5 kg 120 g = 5120 g

Let us multiply 5120 g by 12.

61 440 g = 61 kg 440 g

$$
\begin{array}{r}
\text{g} \\
5120 \\
\times\ 12 \\
\hline
10240 \\
5120 \\
\hline
61440 \\
\hline\hline
\end{array}
$$

Example 1

The mass of a lorry which transports goods is 2250 kg. It is loaded with 60 cement bags of mass 50 kg each. When entering an old bridge, the driver sees a notice which indicates that a mass greater than 5300 kg cannot be transported across the bridge. The mass of the driver and his assistant is 140 kg. Is this vehicle allowed to cross the bridge?

Mass of vehicle = 2250 kg

Mass of cement = 50 kg × 60 = 3000 kg

Mass of two passengers = 140 kg

Therefore the total mass of the vehicle = 2250 kg + 3000 kg + 140 kg

= 5390 kg

Since the total mass of the vehicle is more than 5300 kg, it is not allowed to cross the bridge.

Exercise 13.4

(1) Simplify the following.

(i)
$$
\begin{array}{rr}
\text{g} & \text{mg} \\
150 & 100 \\
 & \times\ 5 \\
\hline
 & \\
\hline\hline
\end{array}
$$

(ii)
$$
\begin{array}{rr}
\text{g} & \text{mg} \\
175 & 375 \\
 & \times\ 4 \\
\hline
 & \\
\hline\hline
\end{array}
$$

(iii)
$$
\begin{array}{lrr}
 & \text{kg} & \text{g} \\
 & 12 & 100 \\
\times & & 8 \\
\hline
 & & \\
\hline\hline
\end{array}
$$

(iv)
$$
\begin{array}{lrr}
 & \text{kg} & \text{g} \\
 & 5 & 250 \\
\times & & 4 \\
\hline
 & & \\
\hline\hline
\end{array}
$$

(v) 12 g 150 mg × 12

(vi) 16 g 650 mg × 13

(vii) 10 kg 375 g × 15

(viii) 5 kg 650 g × 25

(2) Find the quantity of rice that needs to be purchased for a week for a household that requires 1 kg 750 g of rice daily.

(3) The mass of a certain type of biscuit is 3 g 750 mg. Packets containing 25 of these biscuits each are issued to the market. Find the total mass of the biscuits in one packet.

(4) Four gunny bags of mass 760 g each are filled with 40 kg of sugar per bag. Find the total mass of the 4 gunny bags filled with sugar.

(5) 20 incense sticks of mass 650 mg per stick are in a packet of mass 2 g.

(i) Find the mass of the incense sticks in one packet.
(ii) Find the total mass of one packet of incense sticks.
(iii) Find the total mass of 12 such packets.

13.6 Division of a mass by a whole number

➢ The mass of 5 tablets is 1 g 750 mg. Let us find the mass of one tablet. To do this, 1 g 750 mg should be divided by 5.

Method I

$$
\begin{array}{r|ll}
 & \text{g} & \text{mg} \\
 & 0 & 350 \\ \hline
5 & 1 & 750 \\
 & 0 & \\ \hline
 & 1 \rightarrow & 1000 \\ \cline{3-3}
 & & 1750 \\
 & & \underline{1750} \\
 & & 0000
\end{array}
$$

Let us divide the gramme quantity first.
Since there are no 5s in 1, let us write '0' in the place where the answer is to be written in the grammes column and carry the remaining 1 g as 1000 mg to the milligrammes column.

Then let us find the amount of milligrammes in the milligrammes column.
1000 mg + 750 mg = 1750 mg
Let us divide 1750 mg by 5. 1750 mg ÷ 5 = 350 mg

The mass of one tablet is 350 mg.

Method II

Express 1 g 750 mg in milligrammes and then divide by 5.

$$
\begin{array}{r|l}
 & \text{mg} \\
 & 350 \\ \hline
5 & 1750 \\
 & 15 \\ \hline
 & 25 \\
 & \underline{25} \\
 & 00 \\
 & \underline{00} \\
 & 00
\end{array}
$$

1 g 750 mg = 1750 mg
1750 mg ÷ 5 = 350 mg

The mass of one tablet is 350 mg.

- ➢ A mass of 16 kg 200 g of sugar is stored in three bags in equal quantities. Let us find the mass of sugar in one of these bags.

To do this, 16 kg 200 g should be divided by 3.

Method I

	kg	g
	5	400
3	16	200
	15	
	1 →	1000
		1200
		1200
		0000

Let us divide 16 kg in the kilogrammes column by 3.
Let us carry the remaining 1 kg to the grammes column as 1000 g
Next let us find the amount of grammes in the grammes column
1000 g + 200 g = 1200 g
Let us divide 1200 g by 3.
1200 g ÷ 3 = 400 g

The mass of sugar in one bag is 5 kg 400 g.

Method II

Let us express 16 kg 200 g in grammes and divide by 3.

16 kg 200 g = 16 kg + 200 g
= 16 000 g + 200 g
= 16 200 g
16 200 g ÷ 3 = 5400 g

	g
	5400
3	16200
	15
	12
	12
	00
	00
	00
	00

The mass of sugar in one bag is 5 kg 400 g.

Example 1

A quantity of 19.2 kg of a particular type of sweetmeat is purchased and stored in equal quantities in 6 boxes. Find the mass of the sweetmeats contained in one box.

Mass of sweetmeats in 6 boxes = 19.2 kg
Mass of sweetmeats in one box = 19.2 kg ÷ 6
= 3.2 kg

	kg
	3.2
6	19.2
	18
	12
	12
	0

Exercise 13.5

(1) Simplify the following.

(i) 8 g 160 mg ÷ 8 (ii) 1 g 575 mg ÷ 3 (iii) 6 g 125 mg ÷ 5

(iv) 7 g 140 mg ÷ 3 (v) 10 g 400 mg ÷ 4

(2) Simplify the following.

(i) 4 kg 800 g ÷ 4 (ii) 4 kg 230 g ÷ 3 (iii) 8 kg 350 g ÷ 5

(iv) 12 kg 600 g ÷ 7

(3) A quantity of 1.6 kg of fertilizer from a quantity of 4 kg is used on a coconut plant. If the remaining amount is used on 8 orange plants in equal quantities, find the amount of fertilizer used on one orange plant in grammes.

(4) The mass of the biscuits in a biscuit packet is indicated as 75 g. If the packet contains 12 biscuits, find the mass of one biscuit.

(5) The total mass of 306 biscuits of the same type is 3 kg 978 g.

(i) Find the mass of one biscuit.

(ii) If these biscuits are put into packets such that each packet contains 34 biscuits, find the mass of the biscuits in one packet.

(iii) Find the total mass of the biscuits in 5 such packets.

13.7 Mass Estimation

The mass of an olive obtained from a stack of olive fruits is about 5 g. Estimate the total mass of 100 olives.

The total mass of 100 olives is approximately 5×100 g; that is, 500 g.

Exercise 13.6

(1) The mass of 10 nelli fruits obtained from a stack of fruits is 27 g 225 mg. Estimate the total mass of 100 nelli fruits.

(2) A household having only 4 adults and no children eat rice for all 3 meals in a day. An adult usually consumes 125 g of rice for breakfast, 100 g for lunch and 75 g for dinner.

(i) Estimate the amount of rice that is required for one adult of this household for one day.

(ii) Estimate the number of kilogrammes of rice that is required for this household for a week.

(iii) Estimate the amount of rice that is required for all 4 adults for a month.

(3) Information on the quantities of the nutrients included in a 100 g packet of "Thriposha" which is given to children with malnutrition is given below.

Protein 20.0 g   Carbohydrate 61.9 g

Fat 7.8 g   Iron 18 mg

If a child is given 50 g of 'Thriposha' per day, estimate the mass of each of the nutrients that can be expected to be consumed by a child in a month.

(i) Protein   (ii) Fat

(iii) Iron   (iv) Carbohydrate

Miscellaneous Exercise

(1) The amount of paracetamol in a paracetamol tablet is 375 mg. If the amount of paracetamol taken by an adult should be less than 2 g per day, what is the maximum number of tablets that an adult can take in a day?

(2) A mass of 100 g of cheese is issued to the market in a box of mass 2 g 500 mg. Find the mass of 100 such boxes of cheese.

(3) If 60 equal sized sesame balls are made from a mixture containing 500 g of sesame seeds and 250 g of jaggery, find the mass of one sesame ball in grammes and milligrammes.

(4) The total mass of a box containing 80 tea bags is 276 g. The mass of the empty box is 26 g. Find the mass of one tea bag and express it in grammes and milligrammes.

(5) When passengers flying oversees travel in a group, if the average mass of their bags does not exceed 30 kg, there are no extra charges for overweight bags. However, if the average mass of the bags exceeds 30 kg, then those with bags that exceed 30 kg have to pay overweight charges. The following are the masses of the bags of 5 passengers who are travelling in a group.

Hasintha - 20 kg 250 g   Mangala - 29 kg 750 g   Sithumini - 32 kg

Dileepa - 32 kg 150 g   Sashika - 28 kg 70 g

Based on the above information, show with reasons whether Dileepa and Sithumini have to pay overweight charges.

$$\text{Average mass of the bags} = \frac{\text{total mass of the bags of all the group members}}{\text{number of group members}}$$

Summary

- Milligramme (mg), gramme (g) and kilogramme (kg) are a few units used to measure mass.
  1 kg = 1000 g   1 g = 1000 mg
- To express a mass given in grammes, in terms of milligrammes, the given number of grammes should be multiplied by 1000.
- To express a mass given in milligrammes in terms of grammes, the given number of milligrammes should be divided by 1000.

14

Rectilinear plane figures

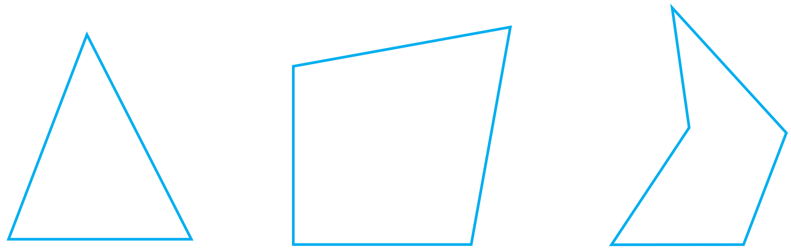
(Part I)

By studying this lesson you will be able to

- identify what a polygon is and
- identify convex polygons, concave polygons and regular polygons.

14.1 Polygons

Consider each of the following plane figures.

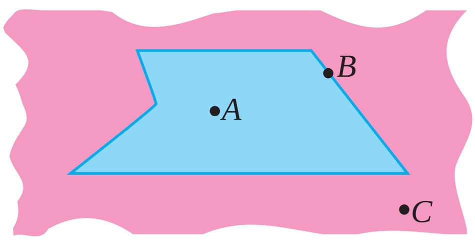


The above figures are all bounded by straight line segments. Furthermore, the straight line segments do not intersect each other in these plane figures, and only two straight line segments meet at each vertex point. Such plane figures are called **polygons**.

A closed plane figure bounded by three or more straight line segments is called a **polygon**.

Each of the line segments by which a given polygon is bounded is called a **side** of the polygon and each of the points at which two of these sides meet is called a **vertex** of the polygon.

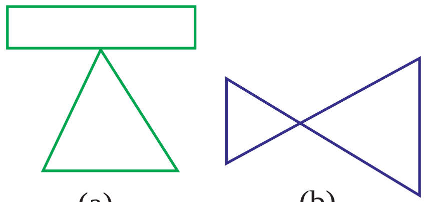
The region bounded by the straight line segments of a polygon (shaded blue) is called the interior region of the polygon, and the region outside (shaded pink) is called the exterior region of the polygon.



A is a point in the interior region of the polygon,
B is a point on the polygon and
C is a point in the exterior region of the polygon.

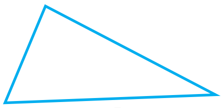
An angle in the interior region of a given polygon, between two sides which meet at a vertex is called an **angle** of the polygon.

Figure (a) shown here has three straight lines which meet at a particular point. Figure (b) has two straight lines which intersect at a point. Therefore, these plane figures are **not polygons**.

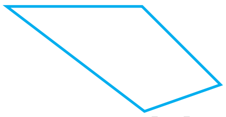


(a) (b)

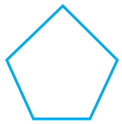
A polygon should have at least three sides. Polygons with three sides are **triangles**. Polygons with 4 sides are **quadrilaterals**, polygons with 5 sides are **pentagons** and polygons with 6 sides are **hexagons**.



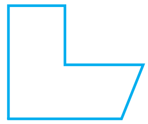
Triangle



Quadrilateral

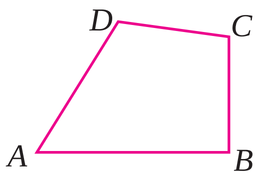


Pentagon



Hexagon

The vertices of a polygon are named using capital letters of the English alphabet. Then the polygon itself, the sides and the angles of the polygon can be named by using these letters.



- In the above given quadrilateral, the vertices have been named A, B, C and D. The quadrilateral is called $ABCD$.
- The sides of quadrilateral $ABCD$ are AB, BC, CD and DA. In the same way, the sides can also be named BA, CB, DC and AD.
- The angles of quadrilateral $ABCD$ are $A\hat{B}C$, $B\hat{C}D$, $C\hat{D}A$ and $D\hat{A}B$. In the same way, the angles can also be named $C\hat{B}A$, $D\hat{C}B$, $A\hat{D}C$ and $B\hat{A}D$. In a polygon, the number of sides and the number of angles are both equal to the number of vertices.

Exercise 14.1

(1) The way a polygon is named, based on the number of sides it has, is given in the following table.

Number of sides	Name of polygon	Number of angles	Number of vertices
3	Triangle		
4	Quadrilateral		
5	Pentagon		
6	Hexagon		
7	Heptagon		
8	Octagon		
9	Nonagon		
10	Decagon		

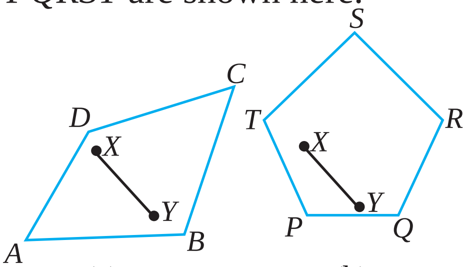
(i) Copy the table and complete the columns named "number of angles" and "number of vertices".

(ii) Draw a sketch of each type of polygon named in the above table. Name the vertices of each polygon you drew using capital letters of the English alphabet. Name the sides and the angles of each polygon.

(2) Cut 4 strips of paper of breadth around 5 cm. By folding each paper appropriately, make a triangle, a quadrilateral, a pentagon and a hexagon and cut each shape out. Paste these shapes in your book.

14.2 Convex polygons and Concave polygons

A quadrilateral *ABCD* and a pentagon *PQRST* are shown here.



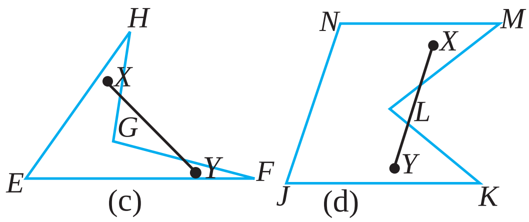
(a) (b)

- When joining any two points marked inside a polygon with a straight line, as shown in the figure, if the straight line, lies entirely inside the polygon, that is, it never goes outside the polygon, then that polygon is known as a **convex polygon**.

  That is, the straight line joining any two points inside a convex polygon does not intersect the sides of the polygon.

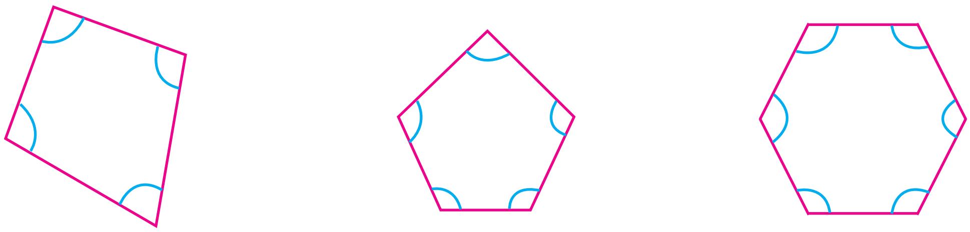
A quadrilateral *EFGH* and a pentagon *JKLMN* are shown here.

- If there are two points in the interior of a polygon such that the straight line joining these two points does not lie entirely inside the polygon, then that polygon is known as a **concave polygon**.

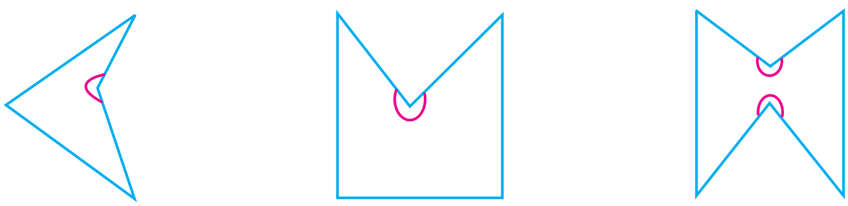


  That is, in a concave polygon, there are two points inside the polygon such that the straight line which joins the two points intersects certain sides of the polygon.

No angle of a convex polygon is a reflex angle.



At least one angle of a concave polygon is a reflex angle.



- If no interior angle of a polygon is a reflex angle, then such a polygon is a convex polygon.
- If at least one interior angle of a polygon is a reflex angle, then such a polygon is a concave polygon.

Exercise 14.2

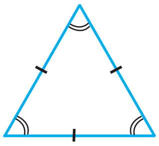
(1) Draw a concave polygon with 1 reflex angle, with 2 reflex angles and with 3 reflex angles. Name each polygon based on the number of sides.

(2) State two facts that distinguish a triangle from the other polygons.

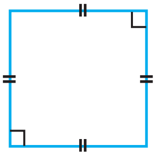
14.3 Regular polygons

A polygon with all sides equal in length and all angles equal in magnitude is called a **regular polygon.**

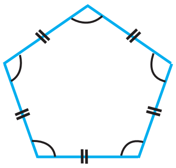
- A triangle with all three sides equal in length and all three angles equal in magnitude is a **regular triangle** or an **equilateral triangle**.



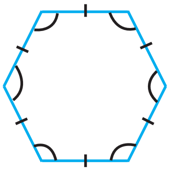
- A quadrilateral with all four sides equal in length and all four angles equal in magnitude is a **regular quadrilateral** or a **square**.



- A pentagon with all five sides equal in length and all five angles equal in magnitude is a **regular pentagon**.

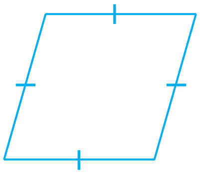


- A hexagon with all six sides equal in length and all six angles equal in magnitude is a **regular hexagon**.

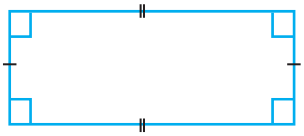


There are polygons with all sides equal in length, which are not regular polygons.

For example, in a rhombus, all four sides are equal in length, but all four angles are not equal in magnitude. Therefore a **rhombus is not a regular polygon.**



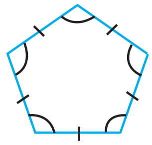
There are polygons with all angles equal in magnitude, which are not regular polygons.



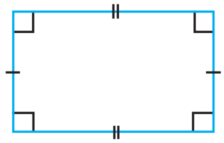
For example, in a rectangle, all four angles are equal in magnitude but all four sides need not be equal in length. Therefore, a **rectangle is not a regular polygon**.

Exercise 14.3

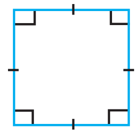
(1) Use the data in the below given polygons to complete the table.



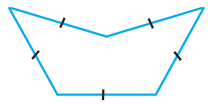
(a)



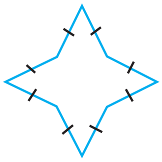
(b)



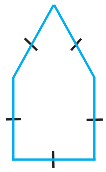
(c)



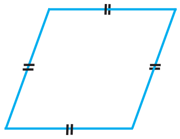
(d)



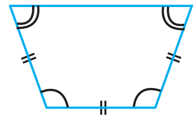
(e)



(f)



(g)



(h)

Figure	**Convex / Concave**	**Is it regular?**	**If it is not regular, the reason**
a			
b			
c			
d			
e			
f			
g			
h			

(2) Create various polygons by folding a piece of paper of length 50 cm and breadth 5 cm. Using a pen, draw straight lines along the folds. Name the polygons you obtain.

Summary

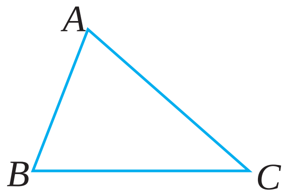
- A closed rectilinear plane figure consisting of three or more straight line segments is a polygon.
- No interior angle of a convex polygon is a reflex angle.
- At least one interior angle of a concave polygon is a reflex angle.
- A polygon with all sides equal in length and all angles equal in magnitude is called a regular polygon.

14 Rectilinear plane figures (Part II)

By studying this lesson you will be able to
- identify acute angled triangles, right angled triangles and obtuse angled triangles, and
- identify equilateral triangles, isosceles triangles and scalene triangles.

14.4 Triangles

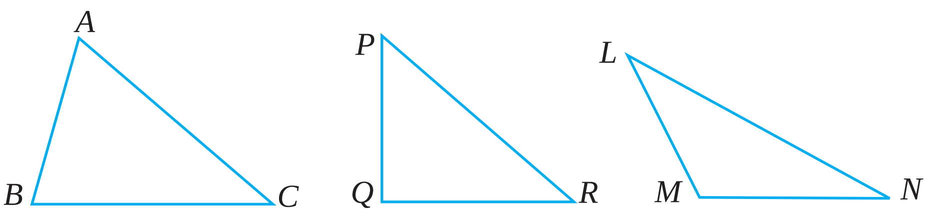
You have learnt that a polygon consisting of three straight line segments is a triangle. There are three angles and three sides in a triangle. These are called the elements of the triangle.



AB, BC and CA are the three sides of the triangle ABC. Furthermore, $A\hat{B}C$, $B\hat{C}A$ and $C\hat{A}B$ are the three angles of the triangle ABC.

Activity 1

Step 1 - Complete the table given below by naming the sides and the angles of each of the given triangles.

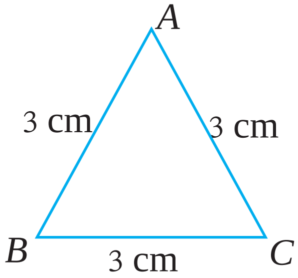


Triangle	**Sides**	**Angles**
ABC	AB, AC, BC,	$A\hat{B}C$, $B\hat{A}C$, $B\hat{C}A$,
PQR		
LMN		

14.5 Classification of triangles according to the length of the sides

• Equilateral triangles

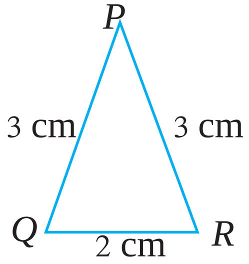
Each side of triangle ABC is of length 3 cm.
That is, $AB = BC = CA = 3$ cm.
All sides of triangle ABC are equal in length.



A triangle of which all three sides are equal in length is known as an **equilateral triangle**.

• Isosceles triangles

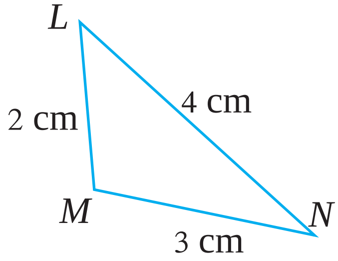
In triangle PQR, $PQ = PR = 3$ cm.
The other side which is QR is 2 cm in length. That is, PQ and PR are equal in length in triangle PQR.



A triangle of which two sides are equal in length is known as an **isosceles triangle.**

• Scalene triangles

In triangle LMN, $LM = 2$ cm,
$MN = 3$ cm and $NL = 4$ cm.
That is, all sides of triangle LMN are of different lengths.

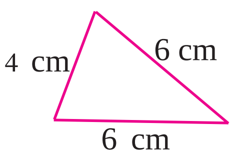


A triangle of which all three sides are unequal in length is known as a **scalene triangle.**

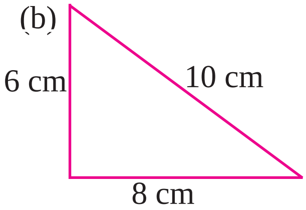
Exercise 14.4

(1) Examine the below given triangles and state whether each triangle is an equilateral triangle, an isosceles triangle or a scalene triangle.

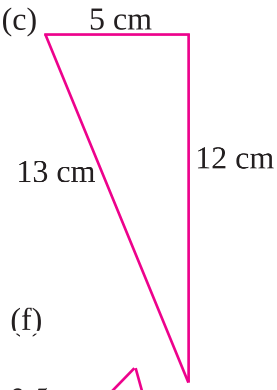
(a)



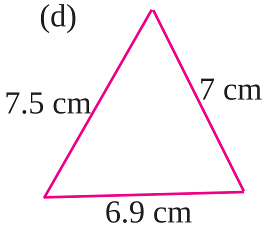
(b)



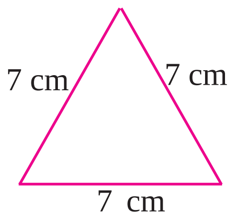
(c)



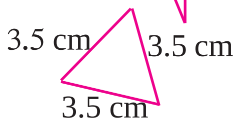
(d)



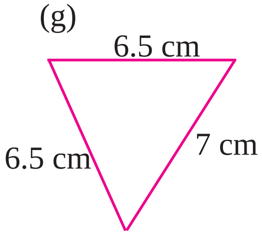
(e)



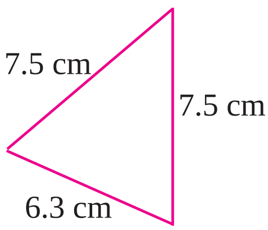
(f)



(g)



(h)

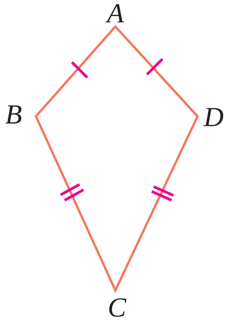


(2) Complete the table.

Length of each side of the triangle			Type of triangle based on the lengths of the sides
(cm)	(cm)	(cm)	
6	3	6	
4	4	4	
3	6	5	
5	6	8	

(3) "All equilateral triangles are isosceles triangles". Do you agree with this statement? Give reasons.

(4) A quadrilateral is shown in the figure. Name according to the lengths of the sides, the triangles that are obtained by



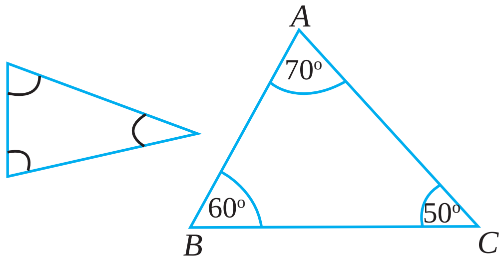
(i) joining only AC
(ii) joining only BD

(5) By folding a rectangular shaped paper, create an equilateral triangle and an isosceles triangle, cut these triangles out, and paste them in your book.

14.6 Classification of triangles according to the angles

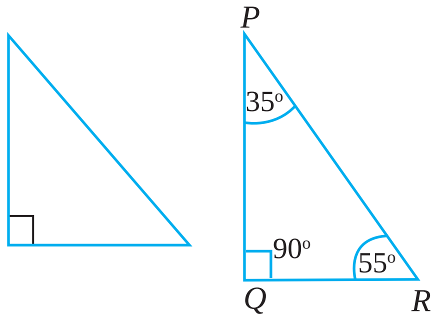
- ### Acute angled triangle

If all three angles of a triangle are acute angles, then the triangle is called an **acute angled triangle.**



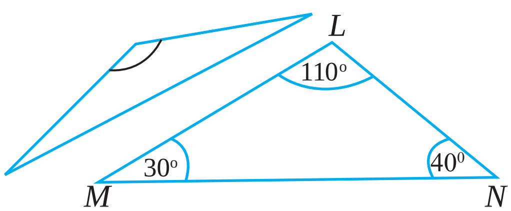
- ### Right angled triangle

If one angle of a triangle is a right angle, then the triangle is called a **right angled triangle**. The other two angles of a right angled triangle are acute angles.



- ### Obtuse angled triangle

If one angle of a triangle is an obtuse angle, then the triangle is called an **obtuse angled triangle.** The other two angles of an obtuse angled triangle are acute angles.

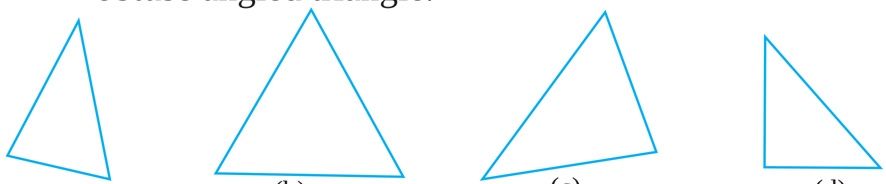


Activity 2

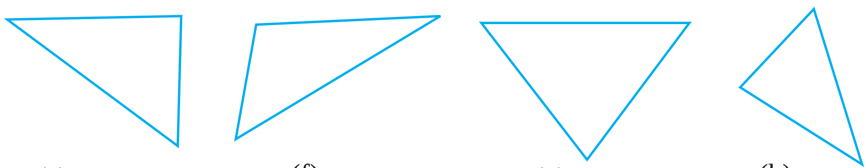
Step 1 - Obtain a right angled corner by folding a piece of paper.

Step 2 - Using the right angled corner, compare the angles of the below given triangles.

Step 3 - Accordingly, write down for each of the triangles whether it is an acute angled triangle, a right angled triangle or an obtuse angled triangle.



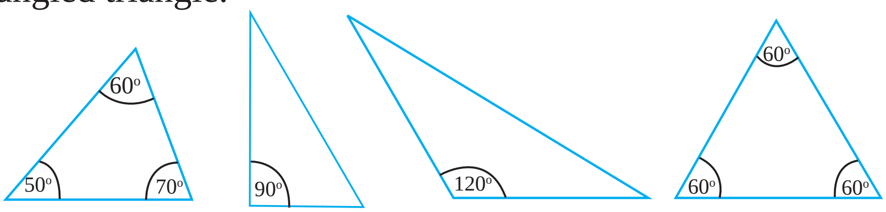
(a) (b) (c) (d)



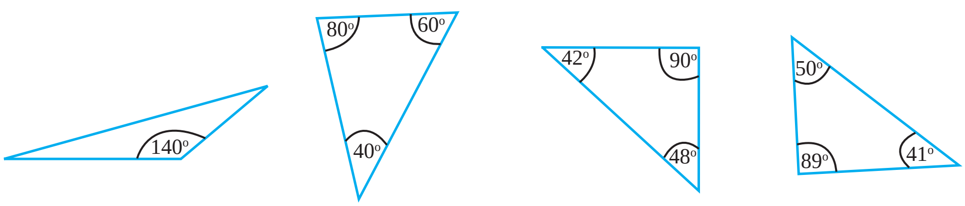
(e) (f) (g) (h)

Exercise 14.5

(1) By considering the data in the below given triangles, classify each of them as an acute angled triangle, right angled triangle or an obtuse angled triangle.

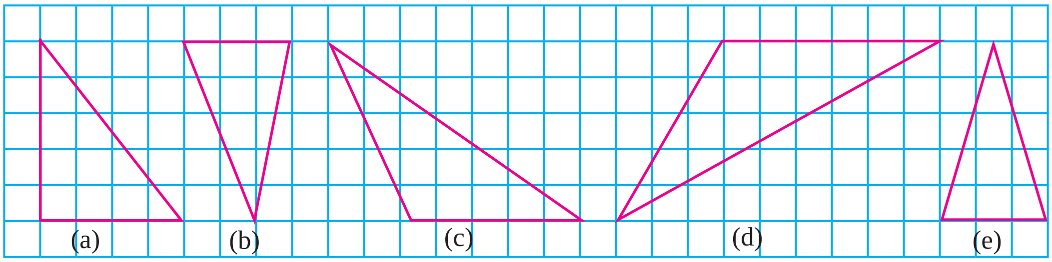


(a) (b) (c) (d)



(e) (f) (g) (h)

(2) Classify each of the triangles given below, according to its angles.



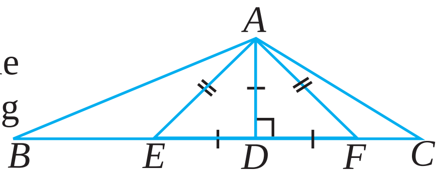
(3) Do the following by considering the given figure.

(i) Name 3 isosceles triangles.

(ii) Name 2 right angled triangles.

(iii) Name an obtuse angled triangle and a right angled triangle having AB as a side.

(iv) Name a scalene triangle.



(4) Do the following using the data in the figure.

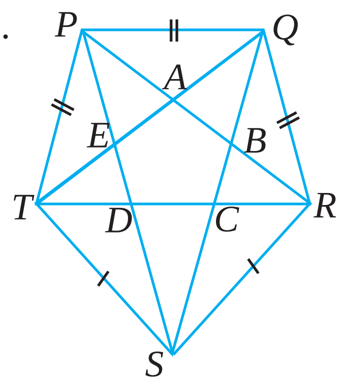
(i) Name 3 isosceles triangles.

(ii) Name 2 scalene triangles.

(iii) Name 2 convex pentagons.

(iv) Name 2 concave pentagons.

(v) Name a hexagon.



Summary

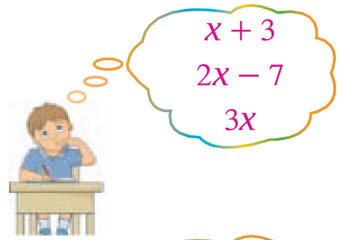
- If all three sides of a triangle are equal in length, then it is called an equilateral triangle.
- If any two sides of a triangle are equal in length, then it is called an isosceles triangle.
- If all three sides of a triangle are different in length, then it is called a scalene triangle.
- In a triangle, if all three angles are acute, then it is called an acute angled triangle.
- In a triangle, if one angle is a right angle, then it is called a right angled triangle.
- In a triangle if one angle is an obtuse angle, then it is called an obtuse angled triangle.

15

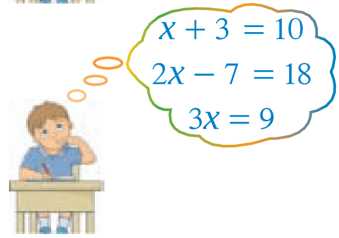
Equations and Formulae

By studying this lesson you will be able to
- construct a simple equation in one unknown,
- solve simple equations,
- construct simple formulae, and
- find the value of any variable of a formula by substituting positive whole numbers for the other variables.

15.1 Constructing simple equations



You have learnt earlier to construct algebraic expressions by using algebraic symbols for unknown values, numbers for known values and operations.



When the value represented by an algebraic expression is equal to the value of a given number, it can be expressed as "algebraic expression = number".

When the value represented by an algebraic expression is equal to the value represented by another algebraic expression, it can be expressed as
"first algebraic expression = second algebraic expression".

Relationships of the above forms are called **equations**.

Consider the equations $x + 3 = 10$, $2x - 7 = 18$ and $3x = 9$. There is only one unknown in each of these equations. The index of the unknown in each equation is also 1.

An equation containing only one unknown of index one is called a **simple equation**.

In the equation $x + 5 = 8$, the value of the algebraic expression $x + 5$ on the left-hand side has been equated to 8 on the right-hand side.

An equation always contains the symbol "=". Apart from this, it also contains unknown terms, numbers and operations.

- A vendor had x mangoes. He bought another 24 mangoes. He now has a total of 114 mangoes. Let us express this information by an equation.

The number of mangoes the vendor had initially $= x$

The number of mangoes the vendor bought $= 24$

The total number of mangoes the vendor has now $= x + 24$

Moreover, since the total number of mangoes the vendor has now is 114,

$$x + 24 = 114$$

- "The price of a loaf of bread reduced by 4 rupees. The new price of a loaf of bread is 50 rupees." Let us express this by an equation.

Let us take the initial price of a loaf of bread as b rupees.

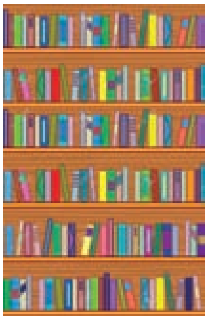
Since the price of a loaf of bread reduced by 4 rupees,

the new price of a loaf of bread $= b - 4$

Moreover, since the new price of a loaf of bread is 50 rupees,

$$b - 4 = 50$$

- Books have been placed on the shelves of a bookcase in a library such that each shelf contains x books. There are 6 shelves in the bookcase. After 10 of these books were issued to students, the number of books remaining in the bookcase reduced to 104. Let us express this information by an equation.



The total number of books that were on the 6 shelves $= 6x$

Number of books issued to students $= 10$

$\therefore$ the number of books remaining in the bookcase $= 6x - 10$

Moreover, since the number of books remaining is 104,

$$6x - 10 = 104$$

Example 1

When 13 is added to twice the value of a given number, the value that is obtained is 85. Represent this information by an equation.

Let us take the number as a.

Twice this number $= 2 \times a = 2a$

The number that is added $= 13$

The number that is obtained $= 2a + 13$

Moreover, since the number that is obtained is 85,

$$2a + 13 = 85$$

Example 2

In a certain year, a father was three times the age of his daughter on the day of her wedding. The girl's mother is 4 years younger to her father. That year, the mother's age was 62. By taking the age of the daughter on the day of her wedding as x years, construct an equation to represent this information.

Three times the age of the girl on the day of her wedding $= 3x$

$\therefore$ the father's age that year $= 3x$

The age of the mother who is four years younger to the father $= 3x - 4$

Moreover, since the mother's age that year was 62,

$$3x - 4 = 62$$

Exercise 15.1

(1) Each part, construct an equation to represent the information given in the statements.

(i) When 7 is added to the number represented by x, the value obtained is 20.

(ii) Nimal's present age is x years. In another 5 years he will be 18 years old.

(iii) When 12 is subtracted from the number represented by y, the value obtained is 27.

(iv) Saman received a salary of x rupees in January. The amount remaining from his salary after he had sent Rs 5000 to his mother was Rs 8000.

(v) When a number x is multiplied by 2, the value that is obtained is 34.

(vi) The amount spent on three pencils of the same type which cost p rupees each was 54 rupees.

(vii) The price of 1 kg of rice is r rupees. When 80 rupees is added to the price of 4 kg of rice, the value is 500 rupees.

(viii) The age of a father was three times the age of his son on the day of the son's wedding. The mother's age on this day was 60 years. The mother is 6 years younger to the father. Take the son's age to be x years.

(ix) Due to the price of a newspaper increasing by 10 rupees, its price is now 30 rupees.

(x) When a piece of cloth of length 70 cm is cut out, a piece of length 40 cm is left over.

(xi) 200 rupees was needed to purchase 5 mangosteens and one pineapple priced at 100 rupees.

(xii) When 12 is subtracted from five times a certain number, the value obtained is 98.

(xiii) When 4 is added to three times a certain number, the value obtained is 73.

(xiv) Sameera needed to buy a book worth 500 rupees. He saved an equal amount of money each day for 7 days. To purchase the book, he needed to add another 129 rupees to the amount he had saved.

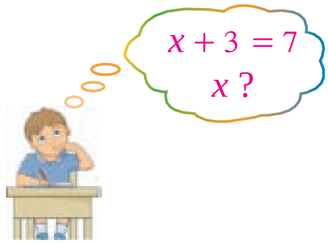
15.2 Solving simple equations

The equality symbol "=" in an equation expresses the fact that the value represented on the left-hand side of the symbol is equal to the value represented on the right-hand side.

What we mean by solving a simple equation is finding the value of the unknown term which satisfies the equation (that is, the value for which the equation holds true). This value is called the solution of the equation. A simple equation has only one solution.

For example, when 4 is substituted for x in the equation $x + 3 = 7$, the left-hand side of the equation is equal to the right-hand side.

Therefore, the solution to the equation $x + 3 = 7$ is $x = 4$.



• Solving simple equations using algebraic methods

You have learnt that the equality symbol "=" in an equation expresses the fact that the value on the left-hand side of this symbol is equal to the value on the right-hand side.

When solving simple equations, the value that the unknown should take for the left-hand side to be equal to the right-hand side of the equation can be found as follows.

➢ Let us find the value of the unknown which satisfies the equation $a + 8 = 10$.

When the same number is subtracted from the two sides of an equation, the new values that are obtained on the two sides are equal.

Let us subtract 8 from both sides of the equation $a + 8 = 10$.

$$a + 8 - 8 = 10 - 8 \qquad (8 - 8 = 0)$$
$$\therefore a = 2$$

➢ Let us find the value of the unknown which satisfies the equation $x - 7 = 10$.

In this equation, the value of $x - 7$ is equal to 10.

When the same number is added to the two sides of an equation, the new values that are obtained on the two sides are equal.

When 7 is added to the two sides of the equation $x - 7 = 10$, the left-hand side is equal to x and the right-hand side is equal to 17.

$$x - 7 + 7 = 10 + 7 \qquad (-7 + 7 = 0)$$
$$\therefore x = 17$$

➢ Let us solve the equation $5x = 10$.

When the two sides of an equation are divided by the same non-zero number, the new values that are obtained on the two sides are equal.

Let us divide both sides of the equation $5x = 10$ by 5.

$$\frac{5x}{5} = \frac{10}{5} \qquad \left(\frac{5}{5} = 1\right)$$
$$\therefore \; x = 2$$

When the value that is obtained is substituted for the unknown in the equation and simplified, if the same number is obtained on the two sides of the equation, then the accuracy of your answer is established.

Let us establish this, through the following examples.

Example 1

Solve $3y - 2 = 10$.

$$3y - 2 = 10$$
$$3y - 2 + 2 = 10 + 2 \quad \text{(let us add 2 to both sides)} \; (-2 + 2 = 0)$$
$$3y = 12$$
$$\frac{3y}{3} = \frac{12}{3} \quad \text{(let us divide both sides by 3)} \; \left(\frac{3}{3} = 1\right)$$
$$\therefore \; y = 4$$

Let us examine whether the solution $y = 4$ that you obtained is correct.

When $y = 4$,

Left-hand side $= 3y - 2$
$= 3 \times 4 - 2$
$= 12 - 2$
$= 10$

Right-hand side $= 10$

Therefore, left-hand side = right-hand side

Therefore, the solution $y = 4$ is correct.

Example 2

It costs 96 rupees to buy four books of the same price and 3 pencils priced at 8 rupees each. Find the price of a book.

Let us take the price of a book as x rupees.

Then the price of four books $= 4x$ rupees

The price of 3 pencils, each priced at 8 rupees $= 3 \times 8$ rupees $= 24$ rupees

Therefore, $4x + 24 = 96$

$$4x + 24 - 24 = 96 - 24$$
$$4x = 72$$
$$\frac{4x}{4} = \frac{72}{4}$$
$$x = 18$$

$\therefore$ the price of a book is 18 rupees.

Let us examine whether the solution $x = 18$ is correct.

When $x = 18$,

Left-hand side $= 4x + 24$

$= 4 \times 18 + 24 = 72 + 24 = 96$

Right-hand side $= 96$

That is, left-hand side = right-hand side

$\therefore$ the solution $x = 18$ is correct.

• Another method of solving simple equations

The inverse operations of the mathematical operations addition, subtraction, multiplication and division which we use in equations are respectively subtraction, addition, division and multiplication.

Another method of solving a simple equation of the above form is performing the inverse operations of the operations on the left-hand side, on the value on the right-hand side.

Let us solve the equation $3x + 7 = 10$.

The left-hand side of this equation is $3x + 7$.

The right-hand side is 10.

$x \longrightarrow \boxed{\times 3} \longrightarrow 3x \longrightarrow \boxed{+7} \longrightarrow 3x+7$ (left-hand side)

$x \longleftarrow \boxed{\div 3} \longleftarrow 3x \longleftarrow \boxed{-7} \longleftarrow 3x+7$

$1 \longleftarrow \boxed{\div 3} \longleftarrow 3 \longleftarrow \boxed{-7} \longleftarrow 10$ (right-hand side)

$\therefore x = 1$

Example 1

Solve $x - 7 = 10$.

$x \longrightarrow \boxed{-7} \longrightarrow x-7$ (left-hand side)

$\underset{17}{x} \longleftarrow \boxed{+7} \longleftarrow \underset{10}{x-7}$ (right-hand side)

$\therefore x = 17$

Example 2

Solve $5x = 30$.

$x \longrightarrow \boxed{\times 5} \longrightarrow 5x$ (left-hand side)

$\underset{6}{x} \longleftarrow \boxed{\div 5} \longleftarrow \underset{30}{5x}$ (right-hand side)

$\therefore x = 6$

Example 3

Solve $3y - 2 = 10$.

$y \longrightarrow \boxed{\times 3} \longrightarrow 3y \longrightarrow \boxed{-2} \longrightarrow 3y-2$ (left - hand side)

$\underset{4}{y} \longleftarrow \boxed{\div 3} \longleftarrow \underset{12}{3y} \longleftarrow \boxed{+2} \longleftarrow \underset{10}{3y-2}$ (right-hand side)

$\therefore y = 4$

Exercise 15.2

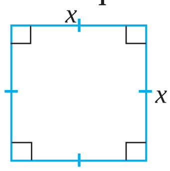
(1) Solve each of the following equations.

(i) $x + 6 = 7$ (ii) $x + 4 = 20$ (iii) $x - 5 = 14$ (iv) $x - 3 = 27$
(v) $6x = 48$ (vi) $7b = 56$ (vii) $2x + 5 = 9$ (viii) $8x + 7 = 79$
(ix) $7x - 5 = 51$ (x) $9x - 7 = 101$ (xi) $11x + 1 = 12$

(2) The price of a banana in a comb with 18 fruits is y rupees. If it costs 170 rupees to buy this comb of bananas and a pineapple priced at 80 rupees, find the value of y.

15.3 Formulae

Let us develop the relationship between the length of a side of a square and the perimeter of the square.



Let the length of a side of a square be x cm and the perimeter of the square be p cm.

Since the perimeter of a square is the sum of the lengths of the four sides of the square,

$p = x + x + x + x \;\; = 4\,x$

Equations such as the above are called **formulae**.

Here p is called the subject of the formula.

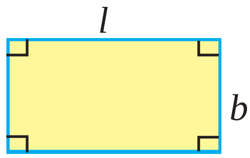
Accordingly, the relationship between the perimeter and the length of a square of length x units and perimeter p units is $p = 4x$.

This formula can be used to find the perimeter of any square of which the side length is known.

Since the units of the values on the two sides of a formula are the same, it is not necessary to state the units.

Formula for the perimeter of a rectangular lamina can also be developed as above.

Let the length of this rectangular lamina be l units, the breadth be b units and the perimeter be P in the same units.



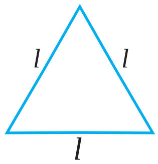
Then $P = l + b + l + b$

This can be written either as $P = 2l + 2b$ or $P = 2\,(l + b)$.

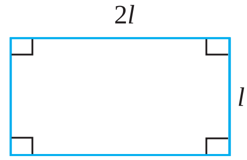
This formula can be used to find the perimeter of any rectangle of which the length and breadth are known.

Exercise 15.3

(1) The perimeter of an equilateral triangle of side length l units is P units. Construct a formula expressing the relationship between P and l.



(2) The breadth and the length of the given rectangle are l units and $2l$ units respectively. Construct a formula for its perimeter P in terms of l.



(3) The breadth of the rectangle in the figure is x cm. If the length is 10 cm more than the breadth and the perimeter is P in the same units, construct a formula expressing the relationship between P and x.

(4) There is a fixed charge of 100 rupees in the monthly electricity bill of a certain city. Apart from this, households that consume less than 100 units of electricity per month have to pay an additional amount of 8 rupees per unit consumed. If the monthly electricity bill of a consumer who uses n units (where $n < 100$) during a month is p rupees, construct a formula for p in terms of n.

(5) A machine produces N milk packets during the first hour. Every hour thereafter, it produces n packets. If T packets are produced in t hours, construct a formula for T in terms of n, N and t.

15.4 Substituting numerical values for the variables in a formula

If the length, breadth and perimeter of a rectangle are l, b and P respectively, then $P = 2l + 2b$.

The length and the breadth of a certain rectangle are 13 cm and 7 cm respectively. Let us calculate its perimeter by using the above formula.

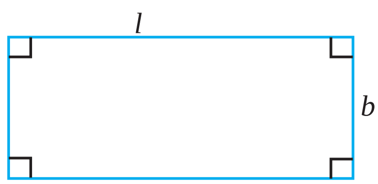
$$P = 2l + 2b$$

$l = 13$ cm and $b = 7$ cm

Therefore, $P = 2 \times 13 + 2 \times 7$ cm

$= 26 + 14$ cm

$= 40$ cm



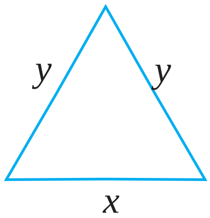
Exercise 15.4

(1) Find the value of N when $Q = 13$ and $D = 20$ in the formula $N = 18 + QD$.

(2) If the area of a square lamina of side length x units is A square units, a formula for A, in terms of x is $A = x^2$. Find the value of A when $x = 8$.

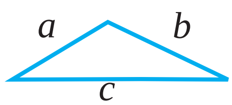
(3) (i) If the perimeter of the given triangle is P, develop a formula for P.

(ii) Find the value of P when $x = 16$ cm and $y = 12$ cm.



(4) (i) If the perimeter of the given triangle is P, construct a formula for P.

(ii) Find the value of P when $a = 4$ cm, $b = 5$ cm and $c = 6$ cm.



(5) If the area of a rectangular lamina of length l units and breadth b units is A square units, the formula for A, in terms of l and b is $A = lb$. Find the value of A when $l = 6$ cm and $b = 3$ cm.

Summary

- The relationship that is obtained when two mathematical expressions, of which at least one is algebraic, are equated to each other is an equation.
- The solution of an equation is the value of the unknown for which the equation holds true.
- A relationship between several variables can be expressed as a formula.
- The value of the any variable of a formula can be found by substituting positive whole numbers for the other variables.

16 Length

By studying this lesson you will be able to

- add and subtract length measurements,
- multiply and divide length measurements by a whole number, and
- find the perimeter of a rectilinear plane figure.

16.1 Units of length

All the words, height, depth, width and thickness describe a certain length. You have already learnt that the units millimetre (mm), centimetre (cm), metre (m) and kilometre (km) are used to measure lengths. The relationships between these units are given below.

1 centimetre	= 10 millimetres
1metre	= 100 centimetres
1 kilometre	= 1000 metres

1 cm	= 10 mm
1 m	= 100 cm
1 km	= 1000 m

You have also learnt to convert a length given in a certain unit to another unit using the above relationships. Do the following review exercise to revise what you have learnt.

Review Exercise

(1) Fill in the blanks.

(i) 13 mm = 10 mm + ... mm
= ...... cm + ... mm
= 1.3 cm

(ii) 45 mm = ... cm ...... mm
= .... cm

(iii) 728 cm = ... m ...... cm
= .... m

(iv) 7075 m = ... km .... m
= .... km

(v) 305 mm = ... cm

(vi) 150 cm = ... m

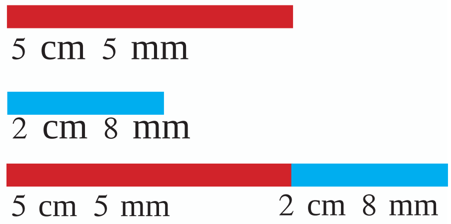
(vii) 1540 m = ... km

(viii) 5.3 cm = ... mm

(ix) 3.25m = ... cm

(x) 2.375 km = ... m

16.2 Addition of length measurements



The figure shows two ribbons, one red and the other blue. The red ribbon is of length 5 cm 5 mm. The blue ribbon is of length 2 cm 8 mm. The figure also shows the two ribbons pasted on a piece of paper such that one end of the blue ribbon touches one end of the red ribbon. Let us find the length of the pasted ribbon.

To do this we need to add the lengths of the two ribbons.

Method I

$$\begin{array}{rcc} & \text{cm} & \text{mm} \\ & 5 & 5 \\ + & 2 & 8 \\ \hline & 8 & 3 \\ \hline\hline \end{array}$$

Let us add the quantities in the millimetres column.
$5\ \text{mm} + 8\ \text{mm} = 13\ \text{mm}$
$13\ \text{mm} = 1\ \text{cm} + 3\ \text{mm}$
Let us write 3 mm in the millimetres column and carry the 1 cm to the centimetres column.
Then, $1\ \text{cm} + 5\ \text{cm} + 2\ \text{cm} = 8\ \text{cm}$
Let us write 8 cm in the centimetres column.

So, the total length is 8 cm and 3 mm.

Method II

Let us express each of the length measurements in centimetres, and then simplify.

$5\ \text{cm}\ 5\ \text{mm} = 5.5\ \text{cm}$

$2\ \text{cm}\ 8\ \text{mm} = 2.8\ \text{cm}$

$8.3\ \text{cm} = 8\ \text{cm}\ 3\ \text{mm}$

$$\begin{array}{rc} & \text{cm} \\ & 5\,.\,5 \\ + & 2\,.\,8 \\ \hline & 8\,.\,3 \\ \hline\hline \end{array}$$

- Let us simplify $5\ \text{m}\ 65\ \text{cm} + 15\ \text{m}\ 70\ \text{cm}$.

Method I

$$\begin{array}{rcc} & \text{m} & \text{cm} \\ & 5 & 65 \\ + & 15 & 70 \\ \hline & 21 & 35 \\ \hline\hline \end{array}$$

Let us add the quantities in the centimetres column.
$65\ \text{cm} + 70\ \text{cm} = 135\ \text{cm}$
$135\ \text{cm} = 1\ \text{m} + 35\ \text{cm}$
Let us write the 35 cm in the centimetres column and carry the 1 m to the metres column.
Then, $1\ \text{m} + 5\ \text{m} + 15\ \text{m} = 21\ \text{m}$
Let us write 21 m in the metres column.

Method II

Let us express each of the length measurements in metres, and then simplify.

$5 \text{ m } 65 \text{ cm} = 5.65 \text{ m}$

$15 \text{ m } 70 \text{ cm} = 15.70 \text{ m}$

$21.35 \text{ m} = 21 \text{ m } 35 \text{ cm}$

$$\begin{array}{rr} & \text{m} \\ & 5\,.\,65 \\ + & \underline{15\,.\,70} \\ & \underline{\underline{21\,.\,35}} \end{array}$$

- Let us simplify $3 \text{ km } 30 \text{ m} + 980 \text{ m}$.

Method I

$$\begin{array}{rrr} & \text{km} & \text{m} \\ & 3 & 30 \\ + & & \underline{980} \\ & \underline{\underline{4}} & \underline{\underline{10}} \end{array}$$

Let us add the quantities in the metres column.
$30 \text{ m} + 980 \text{ m} = 1010 \text{ m}$
$1010 \text{ m} = 1 \text{ km} + 10 \text{ m}$
Let us write the 10 m in the metres column and carry the 1 km to the kilometres column.
$3 \text{ km} + 1 \text{ km} = 4 \text{ km}$
Let us write the 4 km in the kilometres column.

Method II

Let us express each of the length measurements in kilometres, and then simplify.

$3 \text{ km } 30 \text{ m} = 3.030 \text{ km}$

$980 \text{ m} = 0.980 \text{ km}$

$4.010 \text{ km} = 4 \text{ km } 10 \text{ m}$

$$\begin{array}{rr} & \text{km} \\ & 3\,.\,030 \\ + & \underline{0\,.\,980} \\ & \underline{\underline{4\,.\,010}} \end{array}$$

Example 1

Simplify $12 \text{ m } 70 \text{ cm} + 8 \text{ m } 5 \text{ cm} + 15 \text{ m } 80 \text{ cm}$.

Method I

$$\begin{array}{rrr} & \text{m} & \text{cm} \\ & 12 & 70 \\ & 8 & 05 \\ + & \underline{15} & \underline{80} \\ & \underline{\underline{36}} & \underline{\underline{55}} \end{array}$$

Method II

$12 \text{ m } 70 \text{ cm} = 12.70 \text{ m}$

$8 \text{ m } 5 \text{ cm} = 8.05 \text{ m}$

$15 \text{ m } 80 \text{ cm} = 15.80 \text{ m}$

$$\begin{array}{rr} & \text{m} \\ & 12\,.\,70 \\ + & 8\,.\,05 \\ & \underline{15\,.\,80} \\ & \underline{\underline{36\,.\,55}} \end{array}$$

$36.55 \text{ m} = 36 \text{ m } 55 \text{ cm}$

Exercise 16.1

(1) Simplify the following.

(i) $\begin{array}{rrr} & \text{cm} & \text{mm} \\ & 5 & 6 \\ + & \underline{12} & \underline{3} \\ & \underline{\underline{}} & \underline{\underline{}} \end{array}$ (ii) $\begin{array}{rrr} & \text{cm} & \text{mm} \\ & 13 & 6 \\ + & \underline{17} & \underline{8} \\ & \underline{\underline{}} & \underline{\underline{}} \end{array}$ (iii) $\begin{array}{rrr} & \text{m} & \text{cm} \\ & 4 & 35 \\ + & \underline{7} & \underline{80} \\ & \underline{\underline{}} & \underline{\underline{}} \end{array}$ (iv) $\begin{array}{rrr} & \text{km} & \text{m} \\ & 3 & 70 \\ + & \underline{1} & \underline{5} \\ & \underline{\underline{}} & \underline{\underline{}} \end{array}$

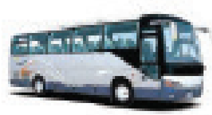
(v) 2 km 780 m + 1 km 530 m

(vi) 57 cm 8 mm + 8 cm 7 mm + 12 cm 7 mm

(vii) 8 m 53 cm + 7 m 75 cm + 4 m 2 cm

(viii) 1 km 730 m + 4 km 20 m + 950 m

(2) Nipuna travels 1 km and 370 m from his house to the bus halt by bicycle. From there he travels 5 km and 680 m to school by bus. Find the total distance Nipuna travels when going to school from his house.



(3) A ribbon is cut into 3 pieces in order to make a wall hanging.

First piece	-	12 cm 8 mm
Second piece	-	8 cm 4 mm
Third piece	-	4 cm

In order to cut all the above pieces, what is the minimum length of the ribbon that is required?

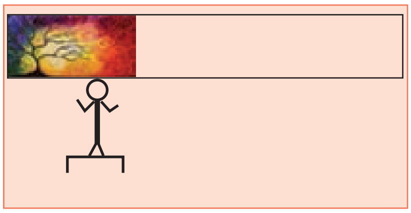
(4) There are three iron rods of the same type of length 1 m 23 cm, 2 m 9 cm and 1 m 73 cm respectively. A new rod can be made by selecting two of these rods and soldering them together without altering their original lengths.

(i) Find the length of the longest rod that can be made in this manner.

(ii) Find the length of the shortest rod that can be made in this manner.

16.3 Subtraction of lengths

- The length of a classroom wall is 5 m 50 cm. It has been decided to draw a picture along the top edge of the wall. On a particular day, the picture is drawn to a length of 1 m 80 cm. Let us find the remaining length of the wall on which the picture needs to be drawn.



To do this we need to subtract the length of the picture drawn from the length of the entire wall.

Method I

$$5 \text{ m } 50 \text{ cm} = 5.50 \text{ m}$$
$$1 \text{ m } 80 \text{ cm} = 1.80 \text{ m}$$
$$5 \text{ m } 50 \text{ cm} - 1 \text{ m } 80 \text{ cm} = 3.70 \text{ m}$$
$$= 3 \text{ m } 70 \text{ cm}$$

$$\begin{array}{r} \text{m} \\ 5\,.\,50 \\ -\,1\,.\,80 \\ \hline 3\,.\,70 \\ \hline \end{array}$$

So the length of the wall remaining on which the picture has to be drawn is 3 m 70 cm.

Method II

$$\begin{array}{rrr} & \text{m} & \text{cm} \\ & 5 & 50 \\ - & 1 & 80 \\ \hline & 3 & 70 \\ \hline \end{array}$$

50 is less than 80. So let us carry over 1 m from the 5 m in the metres column to the centimetres column.
Then 4 m will remain in the metres column.
$100 \text{ cm} + 50 \text{ cm} = 150 \text{ cm}$
$150 \text{ cm} - 80 \text{ cm} = 70 \text{ cm}$
Let us write 70 cm in the centimetres column.
Now let us reduce 1 m from the 4 m in the metres column.
$4 \text{ m} - 1 \text{ m} = 3 \text{ m}$
Let us write 3 m in the metres column.

So the length of the wall remaining on which the picture has to be drawn is 3 m 70 cm.

Example 1

A piece of length 7 cm 5 mm is cut from a ribbon of length 32 cm 3 mm. What is the length of the remaining piece of ribbon?

Let us simplify 32cm 3mm – 7 cm 5 mm.

Method I

$$\begin{array}{rrr} & \text{cm} & \text{mm} \\ & 32 & 3 \\ - & 7 & 5 \\ \hline & 24 & 8 \\ \hline \end{array}$$

3 is less than 5. Let us carry over 1 cm from the 32 cm in the centimetres column to the millimetres column. Then there will be 31 cm remaining in the centimetres column.
$10 \text{ mm} + 3 \text{ mm} = 13 \text{ mm}$
$13 \text{ mm} - 5 \text{ mm} = 8 \text{ mm}$
Let us write 8 mm in the millimetres column.
From the remaining 31 cm in the centimetres column, let us subtract 7 cm.
$31 \text{ cm} - 7 \text{ cm} = 24 \text{ cm}$

Method II

Let us express each of the length measurements in centimetres, and then simplify.

$32\ \text{cm}\ 3\ \text{mm} = 32.3\ \text{cm}$

$7\ \text{cm}\ 5\ \text{mm} = 7.5\ \text{cm}$

Length of the remaining piece of ribbon is $24.8\ \text{cm} = 24\ \text{cm}\ 8\ \text{mm}$

$$\begin{array}{r} \text{cm} \\ 32\,.\,3 \\ -\ 7\,.\,5 \\ \hline 24\,.\,8 \\ \hline\hline \end{array}$$

Example 2

Simplify $6\ \text{km}\ 50\ \text{m} - 2\ \text{km}\ 700\ \text{m}$.

Method I

$$\begin{array}{rrr} & \text{km} & \text{m} \\ & 6 & 50 \\ - & 2 & 700 \\ \hline & 3 & 350 \\ \hline\hline \end{array}$$

50 is less than 700. Let us carry over 1 km from the 6 km in the kilometres column to the metres column.

$1000\ \text{m} + 50\ \text{m} = 1050\ \text{m}$

$1050\ \text{m} - 700\ \text{m} = 350\ \text{m}$

Let us write 350 m in the metres column. From the remaining 5 km, in the kilometres column, let us subtract 2 km.

$5\ \text{km} - 2\ \text{km} = 3\ \text{km}$

Let us write 3 km in the kilometres column.

Method II

$6\ \text{km}\ 50\ \text{m} = 6.050\ \text{km}$

$2\ \text{km}\ 700\ \text{m} = 2.700\ \text{km}$

$3.350\ \text{km} = 3\ \text{km}\ 350\ \text{m}$

$$\begin{array}{rr} & \text{km} \\ & 6\,.\,050 \\ - & 2\,.\,700 \\ \hline & 3\,.\,350 \\ \hline\hline \end{array}$$

Exercise 16.2

(1) Simplify.

(i) 10 cm 8 mm – 2 cm 5 mm (ii) 15 cm 5 mm – 9 mm

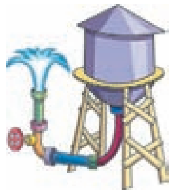
(iii) 7 m 85 cm – 4 m 75 cm (iv) 75 m 5 cm – 57 m 85 cm

(v) 12 km 300 m – 8 km 500 m (vi) 24 km 75 m – 15 km 350 m

(2) Ruvini is 1 m 35 cm tall. Gayani is 1 m 48 cm tall. By how many centimetres is Gayani taller than Ruvini?

(3) From a piece of cloth of length 35 m in a shop, a length of 20 m 80 cm was sold. Find the length of the remaining cloth.

(4) A water tank is 1 m 30 cm deep. Water is filled to a height of 80 cm in this tank. We want to fill the tank completely. To do this, what is the height the water that must be added now?



(5) A worker is assigned to dig a trench of length 15 m. On a particular day he digs a length of 3 m 40 cm. Find the length of the trench remaining for him to dig.

(6) During an inter house sports meet of a school, it was required to run a distance of 10 km for the marathon. Nisham participated in this event. After running a distance of 8 km 850 m, he was injured and could not complete the race. Find the remaining distance that Nisham should have run to complete the race.

16.4 Multiplication and division of measurements of length

- **Multiplication of a measurement of length by a whole number**

➢ A ribbon of length 1 m 80 cm is required to decorate a present. Let us find the length of ribbon required to decorate 8 presents.

To do this, we need a ribbon which is of length eight times that of the piece of ribbon that is required to decorate one present. So 1 m 80 cm must be multiplied by 8.

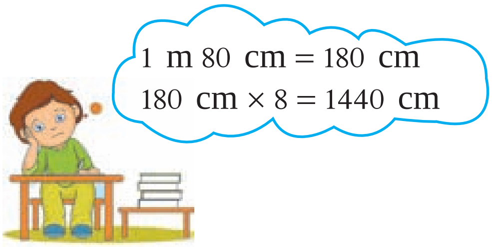
Method I

	m	cm
	1	80
×		8
	14	40

80 cm × 8 = 640 cm
Since 640 cm = 6 m 40 cm,
let us write the 40 cm in the centimetres column and carry the 6 m to the metres column.
1 m × 8 = 8 m.
Let us add the 8 m to the 6 m.
8 m + 6 m = 14 m
Let us write the 14 m, in the metres column.

Method II

Let us express 1 m 80 cm, in centimetres and then multiply by 8.



$$\begin{array}{rr} & \text{cm} \\ & 180 \\ \times & 8 \\ \hline & 1440 \\ \hline \hline \end{array}$$

Therefore the total length = 1440 cm = 14 m 40 cm

➢ Let us simplify 3 cm 7 mm × 5.

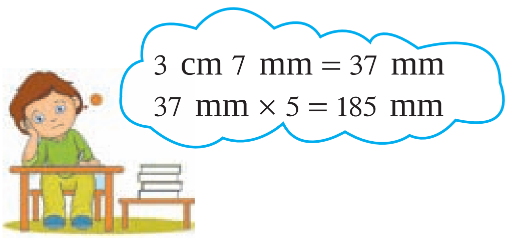
Method I

	cm	mm
	3	7
×		5
	18	5

7 mm × 5 = 35 mm
35 mm = 3 cm 5 mm
Let us write 5 mm in the millimetres column.
3 cm × 5 = 15 cm
Let us add the 3 cm to the 15 cm.
3 cm + 15 cm = 18 cm
Let us write 18 cm in the centimetres column.

Method II

Let us express 3 cm 7 mm, in millimetres and then multiply by 5.



$$\begin{array}{rr} & \text{mm} \\ & 37 \\ \times & 5 \\ \hline & 185 \\ \hline \hline \end{array}$$

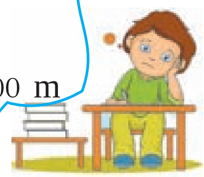
185 mm = 18 cm 5 mm
3 cm 7 mm × 5 = 18 cm 5 mm

➢ Let us simplify 3 km 175 m × 12.

Method I

km	m
3	175
×	12
38	100

Let us first multiply 175 m by 12.
175 m × 12 = 2100 m
= 2 km 100 m
Now let us multiply 3 km by 12.
3 km × 12 = 36 km
3 km 175 m × 12 = 36 km + 2 km 100 m
= 38 km 100 m



Method II

Let us express 3 km 175 m, in metres and then multiply by 12.

$$\begin{array}{r} 3175 \\ \times \quad 12 \\ \hline 6350 \\ 3175 \\ \hline 38100 \\ \hline \end{array}$$

3 km 175 m = 3175 m
3175 m × 12 = 38 100 m
38 100 m = 38 km 100 m
∴ 3 km 175 m × 12 = 38 km 100 m

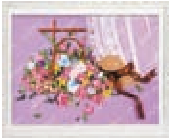
Exercise 16.3

(1) Simplify.

(i) 5 cm 2 mm × 5 (ii) 12 cm 7 mm × 5 (iii) 5 m 25 cm × 7
(iv) 2 m 50 cm × 15 (v) 35 km 7 m × 6 (vi) 2 km 450 m × 16

(2) Cloth of length 1 m 35 cm is required to sew a child's dress. Find the length of cloth required to sew 8 such dresses.

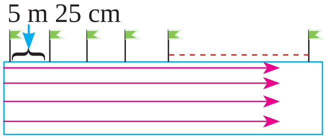
(3) Seven pieces of ribbon, each of length 12 cm 5 mm, are required to make a wall hanging. What is the minimum length of the ribbon needed to cut these seven pieces?



(4) In a play ground, the running tracks are straight as shown in the figure. Along the edge of the running track, flags are placed 5 m 25 cm apart as shown in the figure. There are 21 such flags.

(i) How many such 5 m 25 cm gaps are there along the row of flags?

(ii) Find the distance between the first and the 21^{st} flag.



(5) Twelve tiles are stacked one on top of another. Each tile is of thickness 2 cm 4 mm. Find the height of the stack of tiles.

(6) To get to the second floor of a two storey house, it is necessary to climb 35 steps, each of height 15.75 cm.
   (i) Find how many centimetres above the first floor, the second floor is located.
   (ii) Express this height in metres.

• Division of a measurement of length by a whole number

Let us now study how to divide measurements of length by a whole number.

➢ Suppose we are given a wire of length 5 m 46 cm and cut it into 2 equal pieces. Let us find the length of one piece.

Here we need to divide the length of the wire by 2.

Method I

Let us express 5 m 46 cm in centimetres and then divide by 2.

$$\begin{array}{r|l} & \underline{273\text{ cm}} \\ 2 & 546\text{ cm} \\ & \underline{4} \\ & 14 \\ & \underline{14} \\ & \quad 6 \\ & \quad \underline{6} \\ & \quad \underline{\underline{0}} \end{array}$$

5 m 46 cm = 546 cm

546 cm ÷ 2 = 273 cm

273 cm = 2 m 73 cm

So the length of one piece = 2 m 73 cm

Method II

$$\begin{array}{r|l} & \underline{2\text{ m} \quad 73\text{ cm}} \\ 2 & 5\text{ m} \quad 46\text{ cm} \\ & \underline{4} \\ & 1\text{ m} \rightarrow \underline{100\text{ cm}} \\ & \qquad 146\text{ cm} \\ & \qquad \underline{146\text{ cm}} \\ & \qquad \quad 00 \end{array}$$

Let us divide the 5 m in the metres column by 2.
Let us carry the remainder which is 1 m to the centimetres column.
Then the number in the centimetres column is
100 cm + 46 cm = 146 cm.
146 cm ÷ 2 = 73 cm

So the length of one piece = 2 m 73 cm.

Example 1

Simplify $65\text{ cm } 7\text{ mm} \div 9$.

$$\begin{array}{r|l} & 73\text{ mm} \\ \hline 9 & 657\text{ mm} \\ & 63 \\ \hline & \ \ 27\text{ mm} \\ & \ \ 27\text{ mm} \\ \hline & \ \ \ 00 \end{array}$$

Method I

Let us express 65 cm 7 mm, in millimetres and then divide by 9.

$65\text{ cm } 7\text{ mm} = 657\text{ mm}$

$65\text{ cm } 7\text{ mm} \div 9 = 73\text{ mm}$

$= 7\text{ cm } 3\text{ mm}$

Method II

$$\begin{array}{r|l} & 7\text{ cm } 3\text{ mm} \\ \hline 9 & 65\text{ cm } 7\text{ mm} \\ & 63 \\ \hline & \ \ 2 \rightarrow 20\text{ mm} \\ & \qquad 27\text{ mm} \\ & \qquad 27\text{ mm} \\ \hline & \qquad 00 \end{array}$$

Let us divide the 65 cm in the centimetres column by 9. Let us take the remaining 2 cm, to the millimetres column as 20 mm and find the amount in the millimetres column.

$20\text{ mm} + 7\text{ mm} = 27\text{ mm}$

$27\text{ mm} \div 9 = 3\text{ mm}$

$65\text{ cm } 7\text{ mm} \div 9 = 7\text{ cm } 3\text{ mm}$

Example 2

Simplify $8\text{ km } 740\text{ m} \div 5$.

$$\begin{array}{r|l} & 1748\text{ m} \\ \hline 5 & 8740\text{ m} \\ & 5 \\ \hline & 37 \\ & 35 \\ \hline & \ \ 24 \\ & \ \ 20 \\ \hline & \ \ \ 40 \\ & \ \ \ 40 \\ \hline & \ \ \ 00 \end{array}$$

Method I

Let us express 8 km 740 m in metres and then divide by 5.

$8\text{ km } 740\text{ m} = 8740\text{ m}$

$8740\text{ m} \div 5 = 1748\text{ m}$

$8\text{ km } 740\text{ m} \div 5 = 1748\text{ m}$

$= 1\text{ km } 748\text{ m}$

Method II

$$\begin{array}{r|ll} & 1\text{ km} & 748\text{ m} \\ \hline 5 & 8\text{ km} & 740\text{ m} \\ & 5 & \\ \hline & 3 \rightarrow & 3000\text{ m} \\ & & 3740 \\ & & 35 \\ \hline & & \ \ 24 \\ & & \ \ 20 \\ \hline & & \ \ \ 40 \\ & & \ \ \ 40 \\ \hline & & \ \ \ 00 \end{array}$$

Let us divide the 8 km in the kilometres column by 5.

Let us take the remaining 3 km, to the metres column as 3000 m. Then the amount in the metres column is

$3000\text{ m} + 740\text{ m} = 3740\text{ m}$.

$3740\text{ m} \div 5 = 748\text{ m}$

$8\text{ km } 740\text{ m} \div 5 = 1\text{ km } 748\text{ m}$

Exercise 16.4

(1) Fill in the blanks.

(i)

$$\begin{array}{r|l} & \text{.... cm .... mm} \\ 12 & 43 \text{ cm } 2 \text{ mm} \\ & \underline{36} \\ & \text{....} \rightarrow \text{....} \\ & \quad 72 \text{ mm} \\ & \quad \underline{\text{....}} \\ & \quad \text{....} \end{array}$$

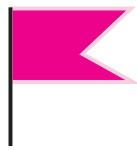
(ii) 43 cm 2 mm = ... mm
43 cm 2 mm ÷ 12 = ... mm ÷ 12
= .... mm
= .... cm .... mm

(2) Simplify the following.

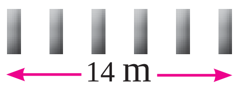
(i) 15 cm 6 mm ÷ 3 (ii) 96 cm 6 mm ÷ 7 (iii) 12 m 48 cm ÷ 8
(iv) 205 m 70 cm ÷ 10 (v) 8 km 40 m ÷ 3 (vi) 2 km 750 m ÷ 5

(3) If a wire of length 8 m is cut into 20 equal parts, find the length of one part.

(4) A piece of cloth of length 35 m was used to sew 25 flags of equal size for a festival. If the entire piece of cloth was used to sew the flags, then find the length of the material used to sew one flag.



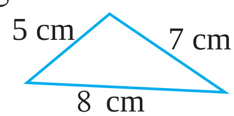
(5) A plot of land is of length 14 m. The figure shows how 6 concrete poles are placed along one side of the border of this plot of land. The gap between any two nearby poles is the same. Find the gap between two nearby poles.



(6) A quantity of 57.6 m of material was bought for costumes and distributed equally among 24 members of a band. Find the quantity of material one person received.

16.5 Perimeter

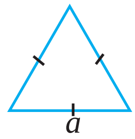
In grade 6 you learnt that the length around a closed plane figure is called its perimeter.
Let us find the perimeter of the triangle shown in the figure.



The sum of the lengths of all three sides of the triangle } = 8 cm + 7 cm + 5 cm
= 20 cm

Therefore the perimeter of the triangle = 20 cm

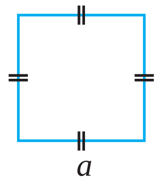
- Perimeter of an equilateral triangle



If the side length of an equilateral triangle is a units and the perimeter is p units, then

$$p = a + a + a$$
$$p = 3a$$

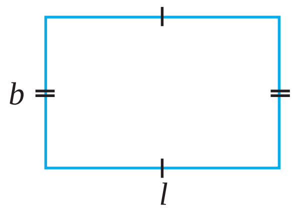
- Perimeter of a square



If the side length of a square is a units and the perimeter is p units, then

$$p = a + a + a + a$$
$$p = 4a$$

- Perimeter of a rectangle

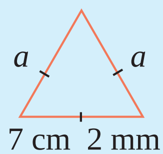


If in a rectangle, the length is l units, the width is b units and the perimeter is p units, then

$$p = l + b + l + b$$
$$p = 2l + 2b$$
or
$$p = 2(l + b)$$

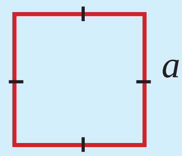
Example 1

The length of an equilateral triangle is 7 cm 2 mm. Find its perimeter.



Perimeter of the triangle $= 3a$
$= 3 \times (7 \text{ cm } 2 \text{ mm})$
$= 21 \text{ cm } 6 \text{ mm}$

Example 2



The perimeter of a square is 25 cm 6 mm. Find the length of a side.

If the length of a side is a units, then

the perimeter of the square $= 4a = 25 \text{ cm } 6 \text{ mm}$

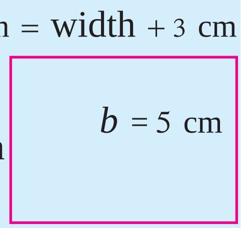
$\therefore$ the length of a side $= a = 25 \text{ cm } 6 \text{ mm} \div 4$

The length of a side is 6 cm 4 mm.

```
     6 cm  4 mm
4 | 25 cm  6 mm
    24
     1 → 10 mm
         16 mm
         16 mm
         00
```

Example 3

The length of a rectangle is 3 cm greater than its width. If the width is 5 cm, then find the perimeter.



The length of the rectangle = width + 3 cm

= 5 cm + 3 cm = 8 cm

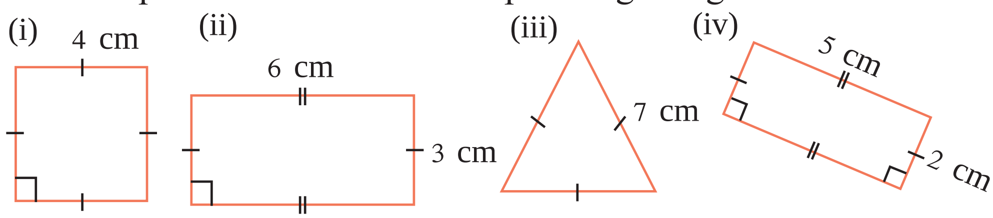
The perimeter of the rectangle = $2l + 2b = 2 \times 8 + 2 \times 5$ cm

= 16 + 10 cm

= 26 cm

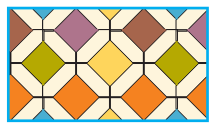
Exercise 16.5

(1) Find the perimeter of each of the plane figures given below.

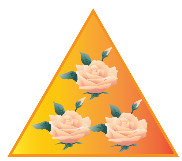


(2) (i) The figure denotes a square shaped stamp of side length 2.4 cm. Find the perimeter of the stamp.

(ii) Find the perimeter of the rectangular shaped tile of length 24 cm and breadth 5 cm shown in the figure.

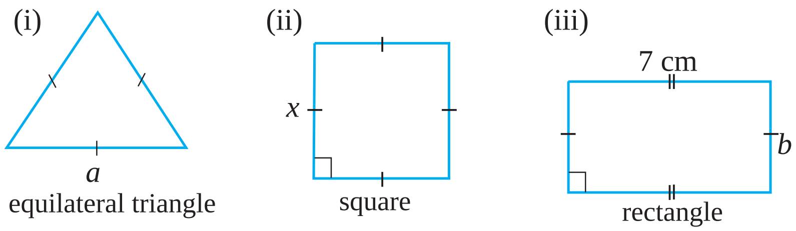


(iii) The perimeter of an equilateral triangle shaped wall hanging is 48 cm 6 mm. Find the length of a side.



(iv) The perimeter of a square shaped handkerchief is 40 cm. Find the length of a side.

(3) The perimeter of each figure below is 24 cm. Find the values of a, x and b.



(4) (i) Find the perimeter of a square shaped plot of land of side length 50 m.

(ii) Find the total length of five strands of wire needed to build a fence around the above plot of land.

(5) Three rectangular shaped laminas are shown in the following figure. Each of them are of length 4 cm and width 3 cm. One half of each of these laminas is shaded.

Figure a

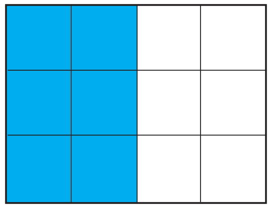


Figure b

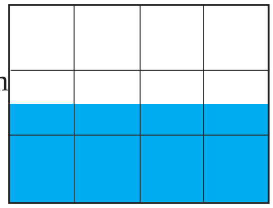
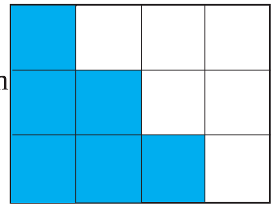


Figure c



(i) Find the perimeter of a rectangle of length 4 cm and width 3 cm

(ii) Find the perimeter of the shaded region of Figure a.

(iii) Find the perimeter of the shaded region of Figure b.

(iv) Find the perimeter of the shaded region of Figure c.

(v) If a rectangular sheet of paper is divided into two equal parts, will the perimeter of one of these parts be equal to half the perimeter of the rectangle?

Summary

- 10 mm = 1 cm   100 cm = 1 m   1000 m = 1 km
- If a side of an equilateral triangle is a, then its perimeter is $3a$.
- If a side of a square is a, then its perimeter is $4a$.
- If the length a rectangle is l and its width is b, then its perimeter is $2l + 2b$. That is $2\,(l + b)$.

Ponder

There are four iron rods of length 85 cm, 1m 23 cm, 2 m 9 cm and 1 m 73 cm respectively. Find the length of the longest and the shortest rod that can be made by soldering three of these rods together. Assume that the lengths of the rods do not change when they are soldered together.

17 Area

By studying this lesson you will be able to

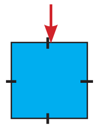
- identify the units used to measure areas,
- find the areas of squares and rectangles using formulae,
- find areas of composite plane figures, and
- solve problems related to area

17.1 Area

You have learnt in grade 6 that the extent of a surface is called the area of that surface.

The area of a square lamina of side length 1 cm is used as the standard unit to measure areas. This is defined as one square centimetre and is denoted by 1 cm^2.

1 cm × 1cm square lamina



Area = 1 cm^2

Two birthday cards are shown in the figure. The extent of the surface of each card is called the area of each picture.

(a)

(b)

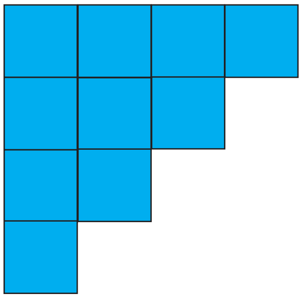
You can identify that the area of (b) is greater than the area of (a).

Do the following review exercise to recall the above facts which were learnt in Grade 6.

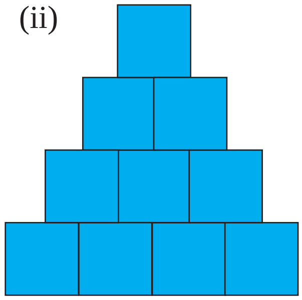
Review Exercise

(1) Considering the area of a small square to be 1 cm^2, find the area of each of the figures given below.

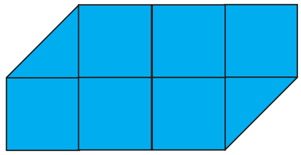
(i)



(ii)



(iii)

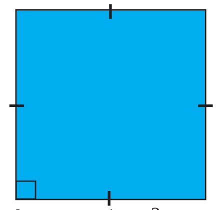


17.2 More on units used to measure areas

The unit 1 cm^2 is not sufficient to measure the areas of surfaces such as walls, parapet walls, the floor of a classroom and flower beds. Even the length measurements of such surfaces are obtained using metres and not centimetres.

Consider a square shaped portion of a floor of side length 1 m. It is too large to be drawn in a book. A reduced shape of such a surface is shown in the figure.

1 m ×1 m
square lamina



Area = 1 m^2

The area of a square lamina of side length 1 m is one square metre. This is denoted by 1 m^2.

The area of the square shaped portion of the floor shown in the figure is 1 m^2. Do the following activity to gain an understanding of the extent of a 1 m^2 surface area.

Activity 1

Step 1 - Get a few newspapers, a pair of scissors, a meter ruler or a measuring tape and some glue.

Step 2 - Paste the newspapers together appropriately and cut out a square lamina of side length 1 m.

Step 3 - Cut out another square lamina of side length 1 cm.

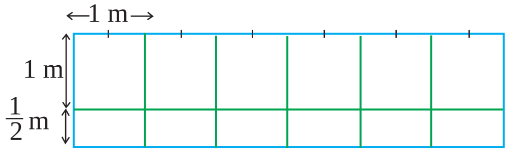
Step 4 - What is the area of each square lamina you cut out?

Step 5 - Can you easily identify how many times the area of the small square the area of the large square is?

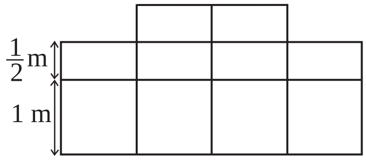
By doing the above activity you would have realized that a surface area of 1 m^2 is very large compared to a surface area of 1 cm^2.

Exercise 17.1

(1) The figure shows how the wall of a school has been divided into square shaped and rectangular shaped sections for paintings to be done on them. What is the total surface area allocated for paintings in square metres?

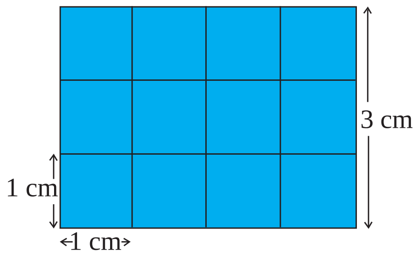


(2) What is the area of the figure shown here which is made out of equal sized squares and equal sized rectangles?



17.3 Formulae for the area of a square and the area of a rectangle

The rectangular lamina shown in the figure which is of length 4 cm and breadth 3 cm is divided into square laminas of side length 1 cm.



Since there are 12 small squares, the area of this rectangle is 12 cm^2. The length of this rectangle is 4 cm.

Number of squares in a row $= 4$

Number of rows $= 3$

$\therefore$ Total number of squares $= 4 \times 3$

$= 12$

$\therefore$ The area of the figure $= 12\ \text{cm}^2$

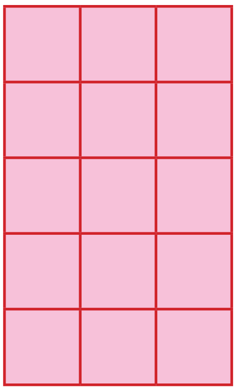
As the length of the rectangle is 4 cm and the breadth is 3 cm,

the area of the figure = (length $\times$ breadth) cm^2

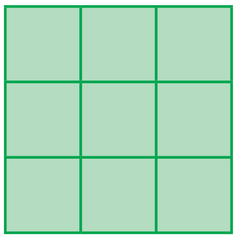
Based on the above explanation it is clear that the area of a rectangle can be found using its length and its breadth, without counting the squares of area 1cm^2. Do the following activity to establish this further.

Activity 2

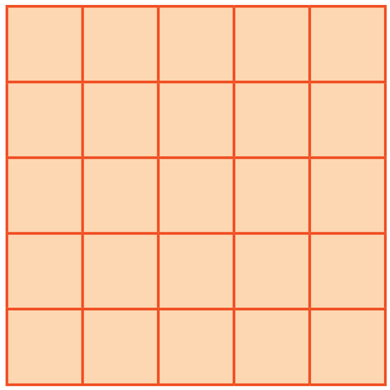
Consider the length of each side of a small square into which each of the shapes given below are divided, to be 1 cm. Copy and complete the table based on these shapes.



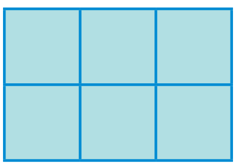
(a)



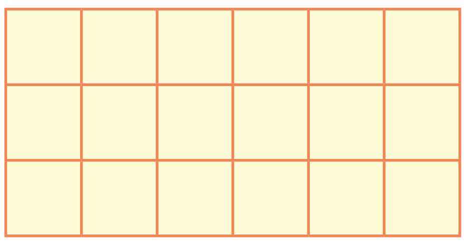
(b)



(c)



(d)



(e)

Figure	Number of squares in a row	Number of rows	Specific name of the figure	Total number of squares	Area	Area of the rectangle = length × breadth
a	3	5	Rectangle	$3 \times 5 = 15$	$15\ cm^2$	$3\ cm \times 5\ cm = 15\ cm^2$
b	....	....	....	....	....	....
c	....	....	....	....	....	....
d	....	....	....	....	....	....
e	....	....	....	....	....	....

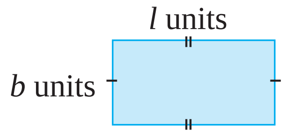
- **Formula for the area of a rectangle**

It is clear from this activity that the area of the rectangle obtained by counting the small squares can also be found using the length and the breadth of the rectangle.

Now let us obtain a formula for the area of a rectangle of length l units and breadth b units

The area of the rectangle = length × breadth

$\therefore$ The area of the rectangle = $l \times b$ square units



If the area of a rectangle of length l units and breadth b units is A square units, then $A = lb$.

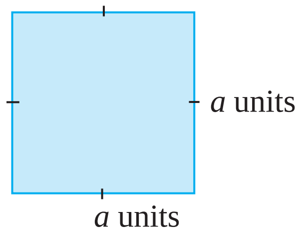
- **Formula for the area of a square**

Let us similarly, obtain a formula for the area of a square.

The area of the square = length × breadth

$= a \times a = a^2$

$\therefore$ The area of the square = a^2 square units



If the area of a square of side length a units is A square units, then $A = a^2$

Example 1

Find the area of the rectangular wall hanging of length 12 cm and breadth 5 cm.

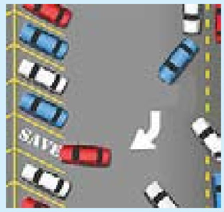
$\left.\begin{array}{r}\text{The area of a rectangle of length } l \text{ and}\\ \text{breadth } b\end{array}\right\} = lb$

$\therefore$ The area of the wall hanging $= 12 \times 5\ \text{cm}^2$

$= 60\ \text{cm}^2$

Example 2

The length of a side of a square shaped car park is 30 m. Find the area of the park.



The area of a square of side length $a = a^2$

$\left.\begin{array}{r}\therefore \text{The area of the car park of side length}\\ 30\ \text{m}\end{array}\right\} = 30 \times 30\ \text{m}^2$

$= 900\ \text{m}^2$

Example 3

The breadth of a rectangular plot of land of area equal to the area of another rectangular plot of land of length 12 m and breadth 3 m, is 4 m. Find the length of this plot of land.

The area of a rectangle of length l and breadth $b = lb$

$\left.\begin{array}{r}\text{The area of the rectangular plot of land of length}\\ 12\ \text{m and breadth } 3\ \text{m}\end{array}\right\} = 12 \times 3\ \text{m}^2$

$= 36\ \text{m}^2$

The length of the plot of land of breadth 4 m $= 36 \div 4$ m

$= 9$ m

Let us take the length of the rectangular plot of land as l.

$A = lb$

$36 = l \times 4$

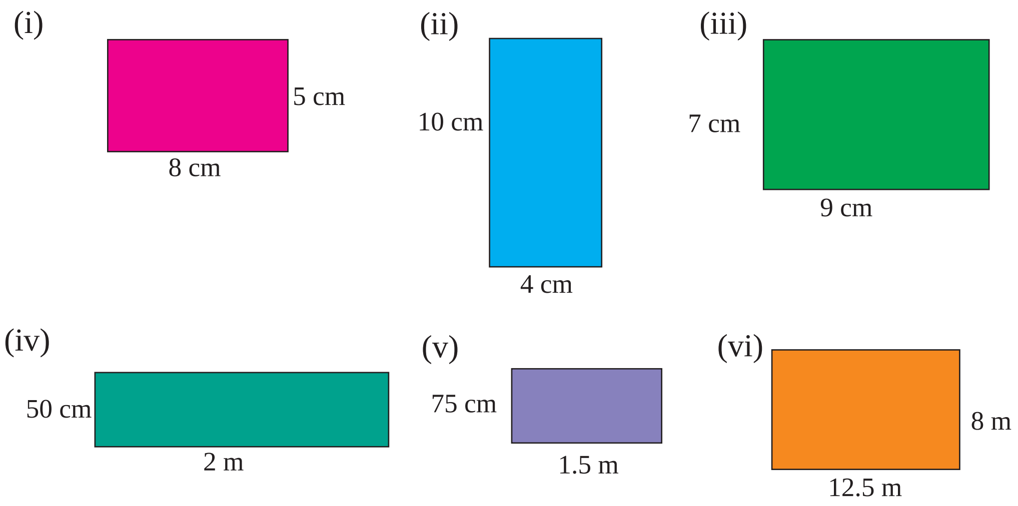
$4l = 36$

$l = \frac{36}{4}\ \text{m} = 9\ \text{m}$

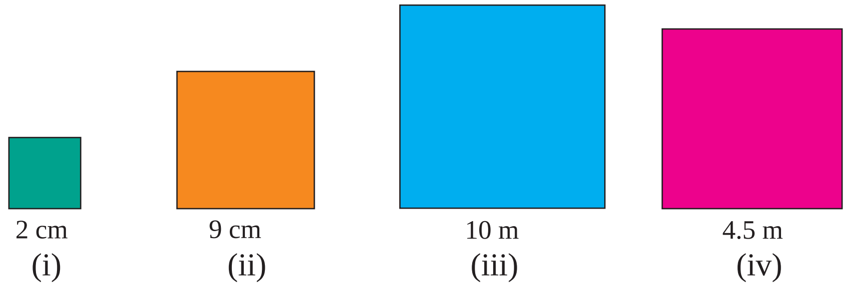
$\therefore$ the length of the plot of land is 9 m.

Exercise 17.2

(1) Find the area of each of the rectangular laminas given below.



(2) Find the area of each of the square laminas given below.



(3) The length of a rectangular plot of land is 9 m and its breadth is 4 m.

(i) Find the area of this plot of land.

(ii) Draw figures of two other plane shapes of the same area and mark the dimensions on the figure.

(4) The floor of a classroom takes the shape of a square of side length 10 m.

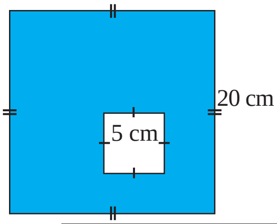
(i) Find the area of the floor of the classroom.

(ii) Another classroom which has the same area as the above classroom has a rectangular floor. If the breadth of the floor of this classroom is 5 m, find the length of the floor.

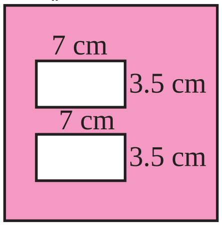
(5) The area of a flower bed is 36 m². An incomplete table containing the dimensions of several flower beds of the same area is given below. Copy and complete the table.

Length (m)	Breadth (m)	Area (m²)	Shape of the flower bed	Perimeter of the flower bed
9	....	36	Rectangle	....
18	....	36	....	....
12	....	36	....	....
6	....	36	....	....

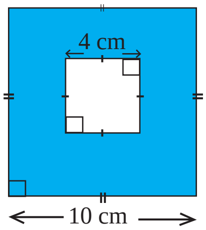
(6) The figure shows a shaded square shaped lamina of side length 20 cm within which a square shaped lamina of side length 5 cm has been shaded white. Find the area of the region shaded in blue in the figure.

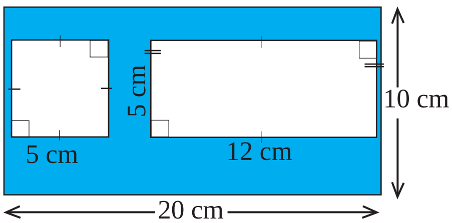


(7) Two rectangular shaped parts of length 7 cm and breadth 3.5 cm have been shaded in white in the square shaped piece of paper in the figure of area 616 cm². Find the area of the region shaded in pink.



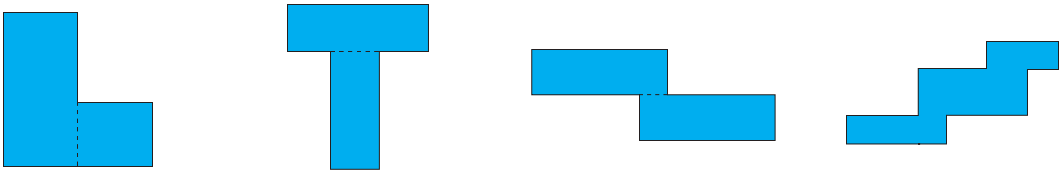
(8) Find the area of the shaded region in each of the following figures.





17.4 Areas of composite plane figures

Composite plane figures that can be divided into several rectangles are shown here.



Activity 3

Step 1 - From coloured paper, cut out the following shapes with the given dimensions.
- 3 rectangles of length 5 cm and breadth 4 cm
- 3 rectangles of length 6 cm and breadth 3 cm
- 3 rectangles of length 4 cm and breadth 1 cm
- 3 squares of side length 2 cm
- 3 squares of side length 3 cm

Step 2 - Find the area of each of the above plane figures and write it on the lamina.

Step 3 - Prepare 3 composite figures using 2 different laminas at a time and paste them in your exercise book.

Step 4 - Prepare another 3 composite figures using 3 different shapes at a time and paste them in your exercise book as well.

Step 5 - Find the areas of the pasted composite plane figures by considering the areas of the rectangles and squares prepared at the beginning of the activity, and write them next to the relevant composite figure.

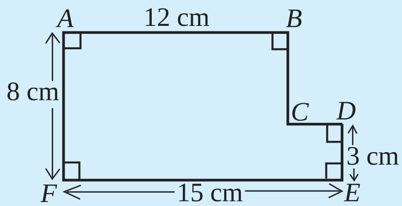
Step 6 - Write down the procedure of finding the area of a composite figure.

Based on the above activity, the procedure of finding the area of a composite plane figure can be expressed in 3 steps.

- Divide the composite figure into sections which are squares and of rectangles of which the area can be found.
- Find the area of each divided section.
- Find the sum of the areas.

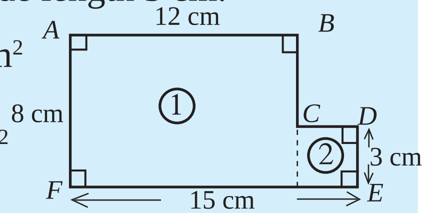
Example 1

Find the area of the figure *ABCDEF* based on the given measurements.



Method I

This figure can be divided into two sections as a rectangle of length 12 cm and breadth 8 cm and a square of side length 3 cm.



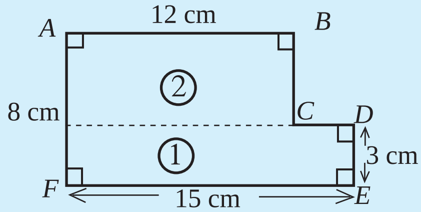
The area of rectangle (1) $= 12 \times 8 \text{ cm}^2$
$= 96 \text{ cm}^2$

The area of square (2) $= 3 \times 3 \text{ cm}^2$
$= 9 \text{ cm}^2$

$\therefore$ The area of the whole figure $= (96 + 9) \text{ cm}^2$
$= 105 \text{ cm}^2$

Method II

The area of the above figure can also be found by dividing it into two rectangles, where one is of length 15 cm and breadth 3 cm and the other is of length 12 cm and breadth 5 cm.



The area of rectangle (1) $= 15 \times 3 \text{ cm}^2$
$= 45 \text{ cm}^2$

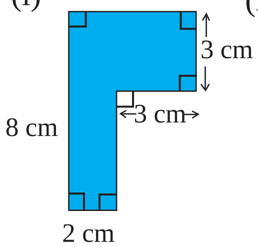
The area of rectangle (2) $= 12 \times 5 \text{ cm}^2$
$= 60 \text{ cm}^2$

$\therefore$ The area of the whole figure $= 45 + 60 \text{ cm}^2$
$= 105 \text{ cm}^2$

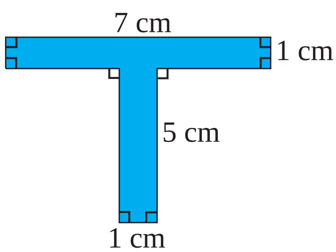
Exercise 17.3

(1) Several composite figures that can be separated into rectangles are shown here. Copy the given figures in your exercise book and find the area of each figure.

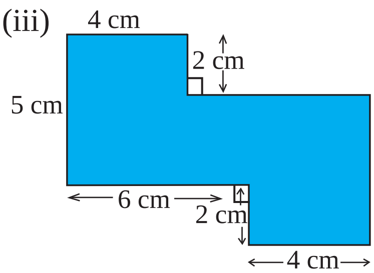
(i)



(ii)



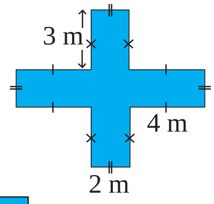
(iii)



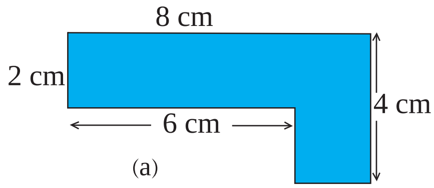
(2) Find the

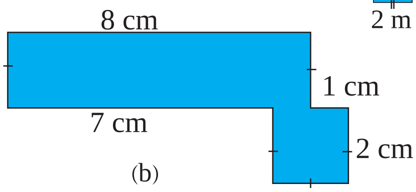
(i) area and

(ii) perimeter of the given figure.



(3)





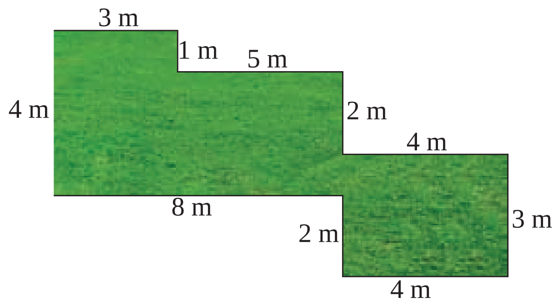
(i) Find the area of figure (a) and figure (b) separately.

(ii) Is the area of figure (a) equal to the area of figure (b)?

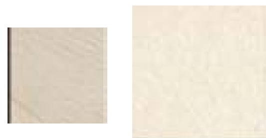
(iii) Find the perimeter of figure (a) and figure (b) separately.

(iv) Is the perimeter of figure (a) equal to the perimeter of figure (b)?

(4) Find the area of the plot of land given in the figure.



(5) It is proposed to lay tiles on a rectangular floor of length 6 m and breadth $4\frac{1}{2}$ m. It is required to select a suitable tile from a square tile of side length 30 cm and a square tile of side length 40 cm. The tiles are to be laid such that the edges of the tiles are parallel to the walls.

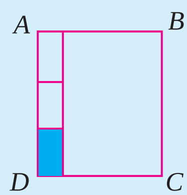


(i) Write down which tile you will select to avoid any wastage. Explain the reason for your selection.

(ii) Find the number of tiles that are required based on your selection.

17.5 Estimation of the areas of plane figures

Example 1



The area of the shaded region in the figure is 6 cm^2. Estimate the area of the rectangle *ABCD*.

Area of a thin strip = 6×3 cm^2 = 18 cm^2

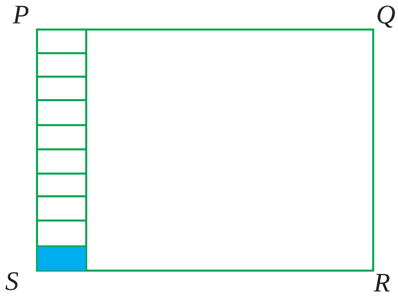
There are about 5 thin strips.

$$\text{The area of 5 strips} = 18 \times 5 \text{ cm}^2$$
$$= 90 \text{ cm}^2$$

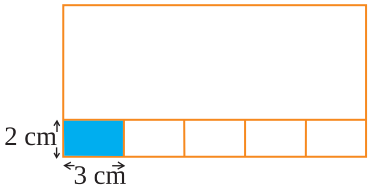
$\therefore$ The area of the rectangle $ABCD$ is approximately = 90 cm^2

Exercise 17.4

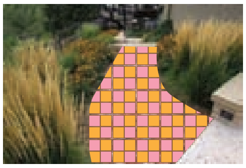
(1) $PQRS$ is a rectangle. The area of the shaded region in the figure is 120 cm^2. Find an approximate value for the area of the rectangular lamina $PQRS$.



(2) Based on the information given in the figure,
 (i) find the area of the shaded region.
 (ii) estimate the area of the whole figure.



(3) It is required to lay concrete bricks along a straight road of breadth 4 m to a distance of 100 m. The top surface of a concrete brick is square shaped of side length 40 cm. Estimate the minimum number of concrete bricks required to pave the whole road.



Summary

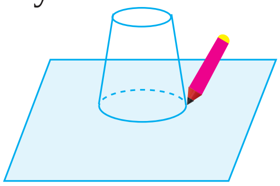
- Square centimetre (cm^2) and square metre (m^2) are two units used to measure areas.
- The area of a rectangle of length l units and breadth b units is lb square units.
- The area of a square of side length a units is a^2 square units.

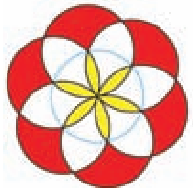
18 Circles

By studying this lesson you will be able to
- draw circles by handling a pair of compasses accurately,
- identify what the centre, radius and diameter of a circle are, and
- create circle designs using a pair of compasses.

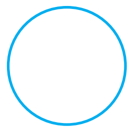
18.1 Drawing Circles

You are already capable of drawing circles and creating circle designs using different objects with circular shapes. Observe the figures given below to recall what you have learnt earlier on this subject.

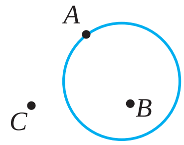




A figure drawn using a tumbler is shown here. You have learnt that the entire curved line of the figure is called a circle.



In this figure, A is a point on the circle, B is a point inside the circle and C is a point outside the circle.



When circles are drawn using various objects, the size of each circle drawn depends on the size of the circular shape of the object used to draw the circle. Therefore the above method is not suitable to draw a circle of the size of your choice. Let us investigate other methods of drawing circles without using objects with circular shapes. Let us do Activity 1.

Activity 1

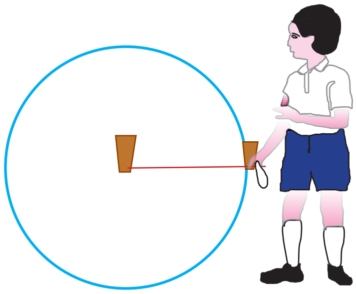
Get two sticks and a thread.

Step 1 - Fix one stick in the middle of a flat sandy land and tie a thread of a particular length to the stick as shown in the figure.

Step 2 - Tie the other end of the thread to another stick.

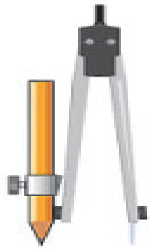
Step 3 - Mark a curved line on the sand by keeping one end of the second stick in contact with the sandy land while moving right around the stick fixed in the middle of the flat land, ensuring that the thread is tightly stretched at all times.

Step 4 - Repeat the above activity several times using threads of different lengths.



You will realize that the size of the circle depends on the length of the thread that is used.

There is a tool called "**a pair of compasses**" in the mathematical instruments box which can be used instead of the two sticks and the piece of thread, to do the above activity. Different lengths can be obtained using a pair of compasses, as was achieved above by using pieces of thread of different lengths.



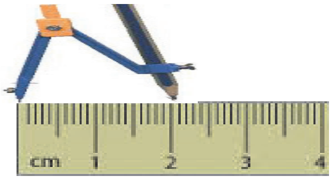
Now let us do the above activity using the pair of compasses. When you are preparing the pair of compasses, it is convenient to use a short pencil. The pencil should be fixed such that the point of the pencil and the point of the pair of compasses are at the same level when the pair of compasses is fully compressed.

Activity 2

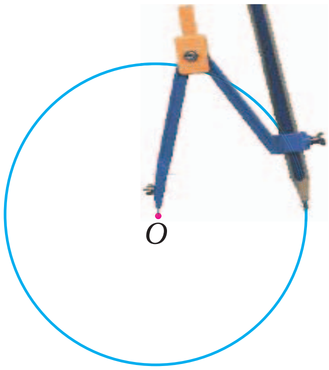
Get a pair of compasses with a correctly fixed pencil, a ruler and a white paper.

Step 1 - Mark a point called O towards the centre of the white paper.

Step 2 - Adjust the pair of compasses such that the distance between the pencil point and the point of the pair of compasses is 2 cm.



Step 3 - Keep the point of the pair of compasses fixed at O and draw a curved line on the piece of paper by moving the pencil point one whole round about the point O, while ensuring that the distance between the pencil point and the point of the pair of compasses remains unchanged. You will see that a circle is drawn around point O.



Step 4 - Construct several other circles by changing the distance between the point of the pair of compasses and the pencil point.

Exercise 18.1

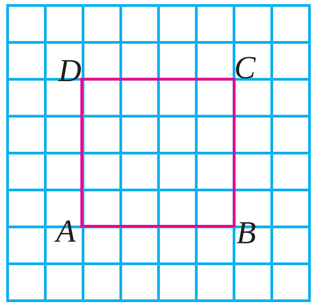
(1) Draw a circle by keeping a distance of 4 cm between the point of the pair of compasses and the pencil point.

(2) Mark a point O on a clean sheet of paper. Keep the point of the pair of compasses on point O and draw 3 circles by changing the distance between the point of the pair of compasses and the pencil point.

(3) (i) Draw a straight line segment AB of length 3 cm.

(ii) Keep the point of the pair of compasses at point A and extend it until the pencil point reaches point B. Now draw the circle which goes around point A.

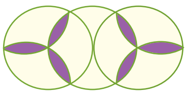
(iii) Keep the point of the pair of compasses on point B and extend it until the pencil point reaches point A. Now draw the circle which goes around point B.

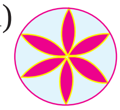


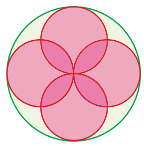
(4) (i) Draw a square $ABCD$ in your square ruled exercise book by taking the length of 4 squares of your exercise book as the length of a side of the square.

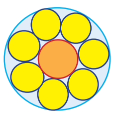
(ii) Take the distance between the point of the pair of compasses and the pencil point to be the length of 2 squares of your exercise book and draw 4 circles around the points A, B, C and D.

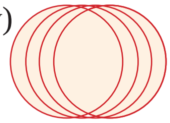
(5) Several circle designs created by using a pair of compasses and a pencil are shown below. Create these designs or some other circle designs using a pair of compasses and a pencil.

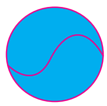
(i) 

(ii) 

(iii) 

(iv) 

(v) 

(vi) 

(6) Create a design suitable for a wall hanging by drawing circles using a pair of compasses and a pencil.

18.2 Centre, radius and diameter of a circle

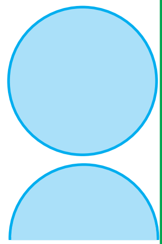
• The centre of a circle

Activity 3

Step 1 - Draw a circle on a piece of paper using a pair of compasses and a pencil.

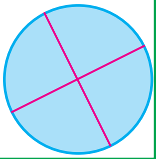
Step 2 - Cut along the circle and separate out the circular lamina.

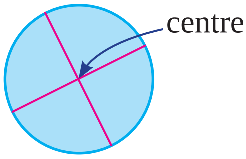
Step 3 - Fold the circular lamina into two equal parts.



Step 4 - Open out the folded lamina and fold it again into two equal parts along another line.

Step 5 - Open out the folded lamina again and mark the fold lines in a dark colour using a ruler.





Observe how the two fold lines intersect each other. You will notice that the point of intersection of the fold lines and the point at which the point of the pair of compasses was kept while drawing the circle are the same. This point is called the **centre** of the circle.

• The radius of a circle

Activity 4

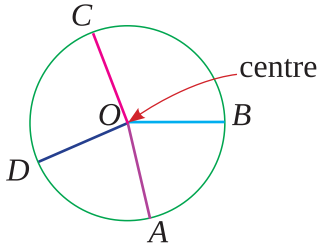
Step 1 - Draw a circle on a piece of paper using a pair of compasses and a pencil.

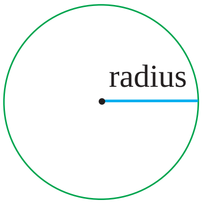
Step 2 - Mark the centre of the circle as O.

Step 3 - Mark several points on the circle and name them as A, B, C and D.

Step 4 - Join each of these points to the centre.

Step 5 - Using a ruler, measure the lengths of the straight line segments that were obtained in step 4.





You will notice that the lengths of these straight line segments are all equal to each other and identical to the distance between the points of the pair of compasses and the pencil that was used to draw the circle. Accordingly, the distance from the center of a circle to any point on the circle is the same constant value.

The straight line segment joining the center of a circle to a point on the circle is called **a radius** of the circle. The term 'radius' is also used for the length of the radius.

- **The diameter of a circle**

Activity 5

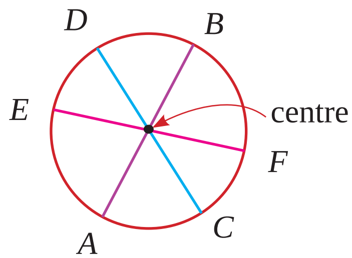
Step 1 - Draw a circle on a piece of paper using a pair of compasses and a pencil.

Step 2 - Mark the centre of the circle as O.

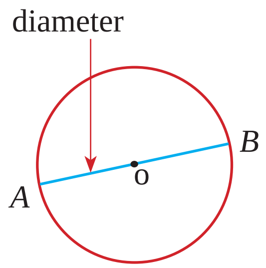
Step 3 - Draw a straight line segment through the point O using the ruler, and name the intersecting points of the straight line segment and the circle as A and B.

Step 4 - Measure the length of the straight line segment AB using the ruler.

Step 5 - Draw several such straight line segments by changing the position of the ruler. Observe that the lengths of all these straight line segments are equal.



A straight line segment joining two points on a circle, which passes through the centre of the circle, is called **a diameter** of the circle. The term 'diameter' is also used for the length of a diameter.



According to this figure, AB is a diameter and OA and OB are radii of the circle.

$AB = OA + OB$

Further, $OA = OB$ (radii of the circle)

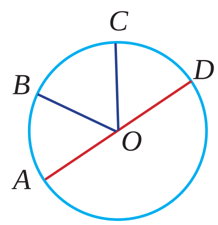
$AB = OA + OA$

That is, $AB = 2\,OA$

The diameter of a circle is twice its radius.

Exercise 18.2

(1) Do the following for the circle shown in the figure.



(i) Name the centre of the circle.

(ii) Name the radii.

(iii) Name a diameter.

(2) (i) Draw a circle of radius 4 cm.

(ii) Name the centre of the circle as O and a point on the circle as X.

(iii) Produce XO until it meets the circle again at Y.

(iv) Write down the name used to define XY. Measure the length of XY and write it down.

(3) Draw a straight line segment AB such that AB = 3 cm. Draw two circles of radius 3 cm each, taking the points A and B as the two centres.

(i) Name the points of intersection of the two circles as P and Q.

(ii) Measure the lengths of AP and BP.

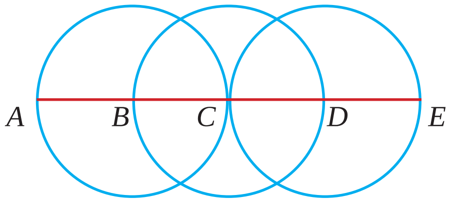
(iii) Join PA and produce it until it meets the circle with centre A again at R.

(iv) What name is used to define the straight line segment PR?

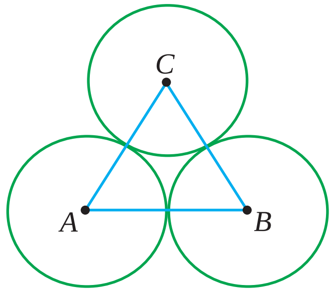
(4) B, C and D are the centres of the circles shown in the figure. The radii of all three circles are equal. Here AE = 10 cm.

(i) Find the length of AC.

(ii) Find the radius of each circle.



(5) ABC is an equilateral triangle. The perimeter of the triangle ABC is 12 cm. Three circles of equal radii and centres A, B and C respectively have been drawn as shown in the figure.



(i) Calculate the length of the side AC.

(ii) Calculate the radius of the circle with centre A.

(iii) Calculate the diameter of the circle with centre B.

(6) (i) Draw a circle of radius 3 cm. Name its centre O.

(ii) Mark a point on the circle and name it A.

(iii) Draw a circle of radius 3 cm with A as its centre. Mark one of the intersecting points of this circle and the first circle as B.

(iv) Draw a circle of radius 3 cm with B as the centre.

(v) Similarly, draw another 4 circles such that the centres of these circles all lie on the first circle and the radius of each circle is 3 cm.

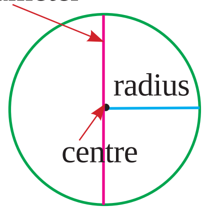
(vi) Do all the circles with centres that lie on the first circle pass through O?

(7) (i) Draw a straight line segment AB of length 4 cm. Draw a circle such that AB is a diameter.

(ii) Draw two circles having A and B as their centres and AB the radius of each circle.

Summary

- A straight line segment joining the centre of a circle to a point on the circle is called a radius of the circle.
- A straight line segment joining two points on a circle, which passes through the centre of the circle, is called a diameter of the circle.
- The diameter of a circle is twice its radius.



Ponder

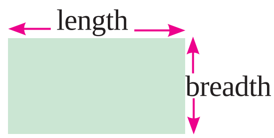
(1) Get a rectangular piece of paper and draw the largest possible circle that can be drawn on that piece of paper using a pair of compasses.

19 Volume

By studying this lesson you will be able to

- identify what volume means,
- identify the different units used to measure volume, and
- find the volume of a cube and a cuboid.

19.1 Identifying what volume means



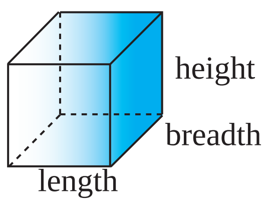
You have learnt that area is the extent of a plane surface.
Now let us see what volume is.

Let us consider the following objects.

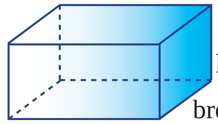
Each of the above objects occupies a certain amount of space. This space is called the **volume** of the object.

Now let us consider a cube and a cuboid.

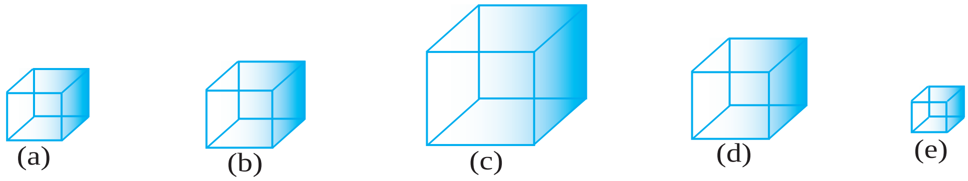
A cube consists of 6 equal square shaped faces. It has 12 edges of equal length. As shown in the figure, the length, breadth and height of a cube are equal.



A cuboid consists of three pairs of rectangular plane surfaces; each pair being equal. It has three sets of 4 edges of equal length; totalling 12 edges. As shown in the figure, the length, breadth and height can be different to each other.



The following figure depicts five cubes.



When these cubes are arranged in ascending order of their volumes we obtain *e*, *a*, *b*, *d*, *c*.

Activity 1

Step 1 - Collect at least 4 solid cube or cuboid shaped objects.

Step 2 - See whether you can arrange them in increasing order of their volumes.

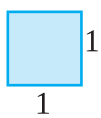
Step 3 - Inquire from your teacher whether the order in which you arranged the objects is correct.

19.2 Measuring the volume of solid objects using arbitrary units

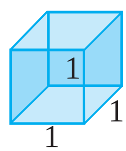
By comparing the amount of space occupied by a die with the amount of space occupied by a brick, we can easily say that the volume of the brick is greater than the volume of the die.

However, it is difficult to compare the volumes of objects such as statues and logs which are of different shapes by just observing them. Therefore let us consider the units that are used to measure volumes.

A straight line segment of length 1 unit



A square of area 1 square unit

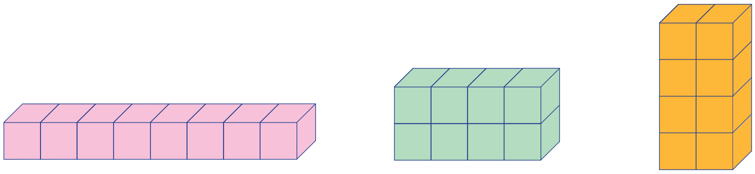


A cube of volume 1 cubic unit

The unit to measure area is 1 square unit which is the area of a square of side length 1 unit.

The unit to measure volume is 1 cubic unit which is the volume of a cube of side length 1 unit.

The following figure depicts a few cuboids that have been created using 8 identical cubes. Now let us find the volume of each of these cuboids.



(a) (b) (c)

Let us take the volume of a small cube to be 1 cubic unit. Then,

since there are 8 cubes in figure (a), the volume of the cuboid in figure (a) is 8 cubic units,

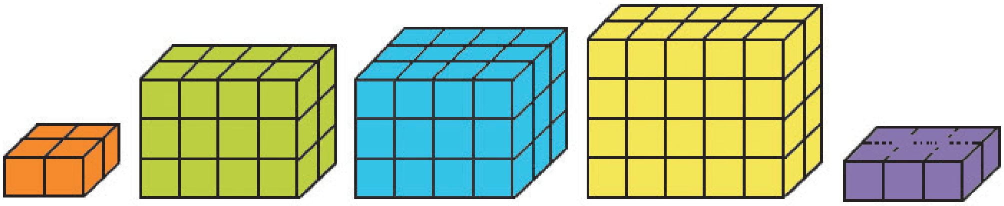
since there are 8 cubes in figure (b), the volume of the cuboid in figure (b) is 8 cubic units, and

since there are 8 cubes in figure (c), the volume of the cuboid in figure (c) is 8 cubic units.

Although the length, breadth and height of these cuboids take different values, their volumes are all equal.

Exercise 19.1

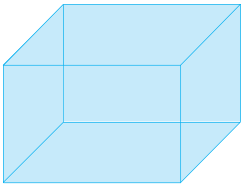
(1) Find the volume of each of the solid objects in the given figure by counting the number of small cubes each object contains. Consider the volume of a small cube to be 1 cubic unit.

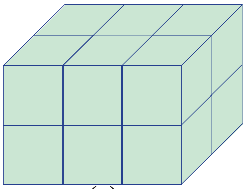


(a) (b) (c) (d) (e)

- ## More on measuring the volume of solid objects using arbitrary units

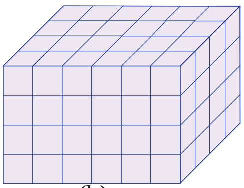
Consider how the volume of the cuboid shown below has been found.





(a)

Here the cuboid has been divided into 12 smaller cubes of side length 1 unit. Let us take the volume of one small cube to be 1 cubic unit. Then the volume of the cuboid is 12 cubic units.



(b)

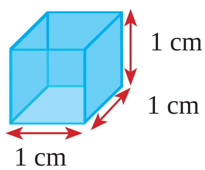
Here the cuboid has been divided into 96 small cubes of side length 1 unit. Let us take the volume of one small cube to be 1 cubic unit. Then the volume of this cuboid is 96 cubic units.

Understand that the volume of the small cube that we used as our unit to measure volume is different in the above two cases. Accordingly, two different numerical values were obtained for the volume of the cuboid.

As indicated above, an arbitrary unit can be used to measure the volume of a solid object. It is important to mention the unit that was used when writing the volume of an object, as the numerical value depends on the unit used, as seen above.

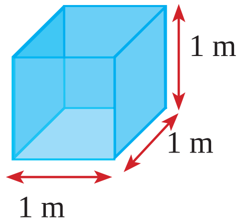
19.3 Standard units used to measure volume

We obtained different numerical values for the volume of a solid object, which depended on the unit that was used. To avoid this variance, standard units are used to measure volumes.



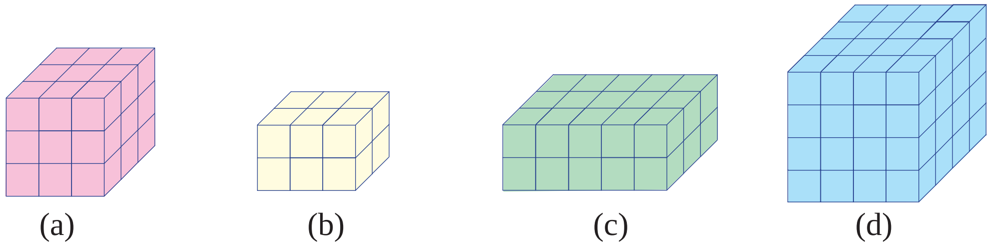
The volume of a cube of side length 1 cm is used as the standard unit of volume. It is defined as 1 cubic centimetre and written as 1 cm^3.

The volume of a cube of side length 1 metre is used as the unit to measure larger volumes. Its volume is 1 cubic metre. One cubic metre is written as 1 m^3.



Exercise 19.2

(1) Find the volume of each of the following solid objects in cubic centimetres. Consider the volume of a small cube to be 1 cm^3.



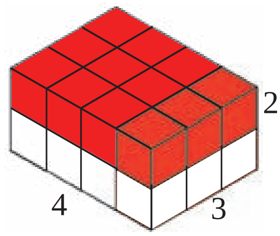
19.4 Another method of finding the volume of a cube or a cuboid

Let us consider an easier method of finding the volume of a cube and a cuboid.

• The volume of a cuboid

A cuboid of length 4 units, breadth 3 units and height 2 units is shown here.

The portion highlighted in red consists of 12 cubes of volume 1 cubic unit each.

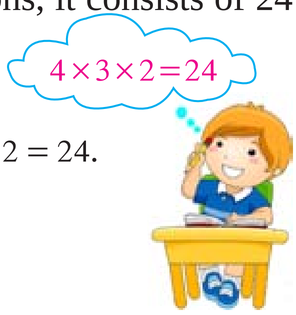


$4 \times 3 = 12$

Since the whole cuboid consists of two such portions, it consists of 24 cubes of volume 1 cubic unit each.

$12 \times 2 = 24$

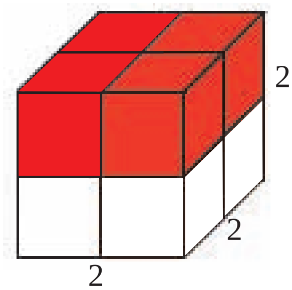
Therefore, the volume of the whole cuboid $= 4 \times 3 \times 2 = 24$.



Volume of a cuboid $=$ length $\times$ breadth $\times$ height

• The volume of a cube

A cube of side length 2 units is shown here.



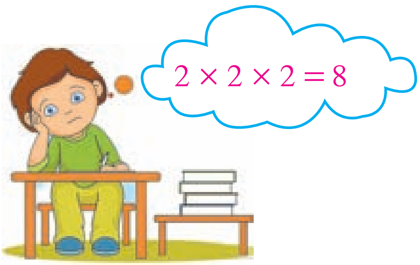
The portion highlighted in red consists of 4 cubes of volume 1 cubic unit each.

$2 \times 2 = 4$

Since the whole cube consists of two such portions, it consists of 8 cubes of volume 1 cubic unit each.

$4 \times 2 = 8$

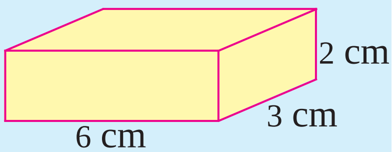
Therefore,
the volume of the whole cube of side length 2 units = $2 \times 2 \times 2 = 8$



Volume of the cube = length × breadth × height
= side length × side length × side length
= $(\text{side length})^3$

Example 1

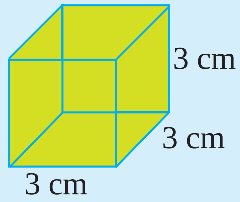
Find the volume of the cuboid in the figure.



Length of the cuboid = 6 cm
Breadth of the cuboid = 3 cm
Height of the cuboid = 2 cm
Volume of the cuboid = length × breadth × height
= 6 cm × 3 cm × 2 cm
= 36 cm^3

Example 2

Find the volume of the cube in the figure.



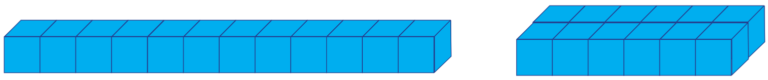
Volume of the cube $=$ length $\times$ breadth $\times$ height

$= 3 \text{ cm} \times 3 \text{ cm} \times 3 \text{ cm}$

$= 27 \text{ cm}^3$

Exercise 19.3

(1) The following figure depicts two cuboids that have been formed using 12 cubes of volume 1 cm^3 each.

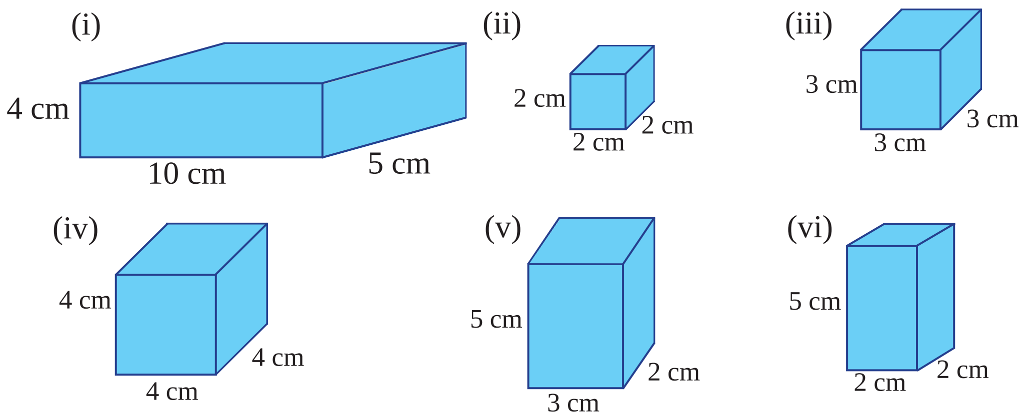


(i) Find the volume of each cuboid.

(ii) Find the length, breadth and height of each cuboid.

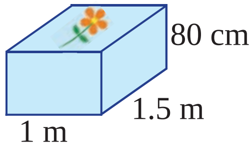
(iii) Write the length, breadth and height of another cuboid of volume 12 cm^3.

(2) Calculate the volume of each of the following solids.



(3) The volume of a cuboid shaped box is 60 cm^3. The length and breadth of the box are respectively 6 cm and 2 cm. Calculate its height.

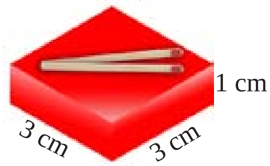
(4) The length, breadth and height of a cuboid shaped container are 1.5 m, 1 m and 80 cm respectively.



(i) Find the height of the container in centimetres.

(ii) Find the volume of the container in cubic centimetres.

(5) The figure shows a matchbox of length, breadth and height equal to 3 cm, 3 cm and 1 cm respectively.



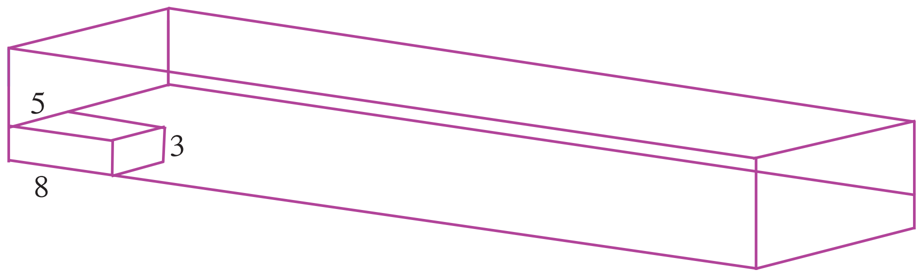
(i) Find the volume of this matchbox.

(ii) There are three layers, each consisting of 4 matchboxes in a package containing 12 of these matchboxes. Find the length, breadth and height of this package.

(iii) Show that the volume of this package is 108 cm^3.

19.5 Estimation of Volume

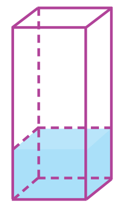
The length, breadth and height of a cake of soap are 8 cm, 5 cm and 3 cm respectively. The maximum number of cakes of soap that can be packed in the given box is 92. Estimate the volume of the box.



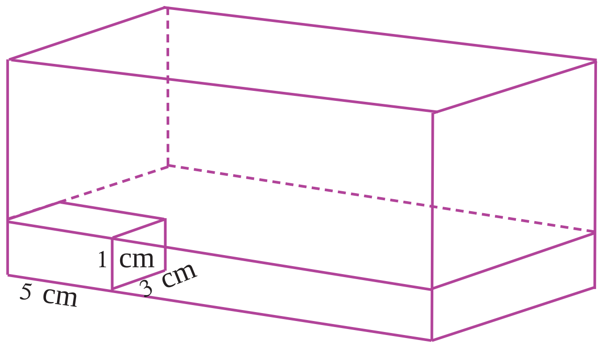
The volume of a cake of soap is approximately $8 \times 5 \times 3$ cm^3; that is, 120 cm^3. Therefore, the volume of the box is approximately 120×92 cm^3, that is, 11 040 cm^3.

Exercise 19.4

(1) The volume of the shaded cuboid portion in the figure is 16 cm^3. Estimate the volume of the whole cuboid.



(2) The length, breadth and height of a matchbox are 5 cm, 3 cm and 1 cm respectively. Matchboxes are packed in the box as shown in the figure. Estimate the volume of the box.



Summary

- The volume of a solid is the amount of space it occupies.
- Arbitrary units can be used to measure volumes. When stating the volume, the units used should also be mentioned.
- A cube of side length 1 cm is used as the standard unit of volume.
- Cubic centimetre (cm^3) and cubic metre (m^3) are two units that are used to measure volumes.
- The volume of a cuboid of length, breadth and height equal to a, b and c units respectively is $a \times b \times c$ cubic units.
- The volume of a cube of side length a units = a^3 cubic units.

20

Liquid Measurements

By studying this lesson you will be able to

- multiply liquid quantities expressed in millilitres and litres by a whole number and
- divide liquid quantities expressed in millilitres and litres by a whole number.

20.1 Units used to measure liquid quantities

There are occasions when you have to purchase liquid types such as milk, coconut oil and syrups. You have already learnt in grade 6, that millilitre and litre are two units that are used to measure liquid quantities. A quantity of one litre of a liquid is equal to a quantity of 1000 millilitres of that liquid.

$$1\ l = 1000\ \text{ml}$$

To express a liquid quantity in millilitres, which is given in litres, the quantity given in litres should be multiplied by 1000.

To express a liquid quantity in litres, which is given in millilitres, the quantity given in millilitres should be divided by 1000.

Do the following exercise to revise your grade 6 knowledge.

Review Exercise

(1) (i) Express 6 l in millilitres.

(ii) Express 7 l 300 ml in millilitres.

(iii) Express 3758 ml in litres and millilitres.

(iv) Express 10 065 ml in litres and millilitres.

(2) Simplify.

(i)
$$\begin{array}{rr} l & \text{ml} \\ 7 & 250 \\ +\,4 & 350 \\ \hline \\ \hline\hline \end{array}$$

(ii)
$$\begin{array}{rr} l & \text{ml} \\ 3 & 50 \\ +\,7 & 975 \\ \hline \\ \hline\hline \end{array}$$

(iii)
$$\begin{array}{rr} l & \text{ml} \\ 6 & 50 \\ -\,3 & 875 \\ \hline \\ \hline\hline \end{array}$$

(iv)
$$\begin{array}{rr} l & \text{ml} \\ 3 & 45 \\ -\,2 & 165 \\ \hline \\ \hline\hline \end{array}$$

(3) Find out how much of drink can be prepared by adding 1 *l* 250 ml of fruit juice to 2 *l* 650 ml of water, and express this amount in litres and millilitres.

(4) There was 10 *l* 750 ml of water in a bucket. Geetha watered plants with a quantity of 5 *l* 850 ml of water from the bucket. How much of water is remaining in the bucket now?

20.2 Multiplication of liquid quantities expressed in millilitres and litres by a whole number

➢ Binuli drinks a 200 ml glass of *kola kenda* daily. Let us find out how much *kola kenda* she drinks in 4 days.

Amount of *kola kenda* consumed per day = 200 ml
Amount of *kola kenda* consumed in 4 days = 200 ml × 4
= 800 ml

➢ 1 *l* 750 ml of fuel is required to operate a generator for one hour. Let us find the amount of fuel required to operate the generator for 3 hours.

Method I

1 *l* 750 ml = 1750 ml
1750 ml × 3 = 5250 ml
5250 ml = 5 *l* 250 ml

$$\begin{array}{r} \text{ml} \\ 1750 \\ \times \quad 3 \\ \hline 5250 \\ \hline\hline \end{array}$$

Method II

1 *l* 750 ml = 1.750 *l*
1.75 *l* × 3 = 5.25 *l*
5.25 *l* = 5 *l* 250 ml

$$\begin{array}{r} 1.75 \\ \times \quad 3 \\ \hline 5.25 \\ \hline\hline \end{array}$$

Method III

l	ml
1	750
×	3
5	250

Let us multiply the 750 ml in the millilitres column by 3.

750 ml × 3 = 2250 ml

2250 ml = 2000 ml + 250 ml = 2 l 250 ml

Let us write 250 ml in the millilitres column and carry the 2 l to the litres column.

Let us multiply the amount of litres in the litres column by 3.

1 l × 3 = 3 l. Now let us add the 2 l we carried from the millilitres column.

3 l + 2 l = 5 l

Finally let us write 5 l in the litres column.

Exercise 20.1

(1) Multiply.

(i)

l	ml
4	25
×	5

(ii)

l	ml
2	350
×	4

(iii) 5 l 750 ml × 13

(iv) 8 l 575 ml × 15

(2) Multiply the liquid quantities given below by the given number and express the answer in litres and millilitres.

(i) 250 ml × 5 (ii) 515 ml × 7 (iii) 750 ml × 16

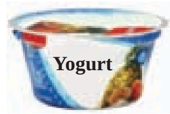
(3) A bottle contains 375 ml of drink. Express the total amount of drink in 6 such bottles in litres and millilitres.

(4) A cordial bottle contains 1 l 750 ml of cordial. How much of cordial is there in 6 such bottles?

(5) A house without electricity requires 1 l 650 ml of kerosene oil per day. Find the amount of kerosene oil required by that house for a week.

(6) 2 l 225 ml of diesel is required to operate a generator for one hour. Find the amount of diesel required to operate the generator for 8 hours.

(7) 50 ml of milk is used to produce one cup of yoghurt. Find the total amount of milk required to produce 150 such cups of yoghurt.



(8) A bucket used for bathing can be filled completely with 5 *l* 650 ml of water. If a person pours water from this bucket (completely filled) 60 times whenever he bathes, find how much of water he uses on each occasion that he bathes.

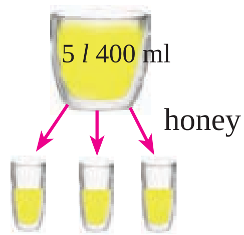
(9) A 540 *l* water tank is filled with water. Due to a crack in a pipe, water leaks out at a speed of 6 *l* 750 ml per minute.

(i) What is the total amount of water that leaks during 8 minutes?

(ii) Show that the tank will be empty if the leakage continues for 80 minutes.

20.3 Division of liquid quantities expressed in millilitres and litres by a whole number

➢ The total amount of honey collected from a honeycomb is 5 *l* 400 ml. If this amount is divided equally among 3 people, how much of honey will one person receive?



Amount of honey received by one person = 5 *l* 400 ml ÷ 3

Method I

5 *l* 400 ml = 5400 ml
5400 ml ÷ 3 = 1800 ml
5 *l* 400 ml ÷ 3 = 1800 ml
= 1 *l* 800 ml

```
     1800 ml
3 | 5400 ml
    3
    24
    24
      00
      00
      00
```

Method II

$$
\begin{array}{r|ll}
 & l & \text{ml} \\
 & 1 & 800 \\
\hline
3 & 5 & 400 \\
 & \underline{3} & \\
 & 2 \rightarrow & \underline{2000} \\
 & & 2400 \\
 & & \underline{2400} \\
 & & \underline{\underline{0000}}
\end{array}
$$

When 5 l is divided by 3, the remainder is 2 l.
When this remainder of 2 l is taken to the millilitres column, it is 2000 ml.
2 l = 2000 ml
When 2000 ml is added to 400 ml we get 2400 ml.
2400 ml ÷ 3 = 800 ml

One person receives 1 l 800 ml of honey.

Exercise 20.2

(1) Evaluate the following.

(i) 750 ml ÷ 3 (ii) 9 l 750 ml ÷ 3 (iii) 2 l 200 ml ÷ 5
(iv) 4 l 50 ml ÷ 3 (v) 18 l 900 ml ÷ 6 (vi) 13 l 50 ml ÷ 3

(2) The 45 000 litres of fuel in a bowser is issued in equal amounts to six filling stations. Find the amount issued to one filling station.

(3) 10 l 728 ml of milk is poured in equal quantities into 12 pots for curdling. Find how much of milk is poured into one pot.

(4) A motor vehicle requires 1 l 560 ml of fuel to travel a distance of 24 km. Find how much fuel it requires to travel a distance of 1 km.

(5) If 4 l 50 ml of a drink is poured equally into 9 glasses, how many milliliters of drink will one glass contain?

(6) A quantity of 1 l 950 ml of a perfume is put into 30 small bottles in equal amounts and issued to the market. What quantity of perfume is there in one bottle in millilitres?

(7) When the drink prepared by mixing 1.54 l of fruit juice with 1.7 l of water is poured in equal amounts into 12 tumblers, how many litres of drink will one tumbler contain?

(8) A soft drink manufacturing company produces 800 bottles of drink of regular size in a day. If the total amount of drink produced in a day is 300 *l*, how much drink is included in one bottle?

Miscellaneous Exercise

(1) An Ayurvedic syrup is issued to the market in 80 bottles containing 750 ml of syrup in a day.

(i) Find the total volume of syrup issued in a day.

(ii) A customer uses a bottle of syrup he purchased for 30 days. He drinks an equal amount twice a day.

(a) Find the amount of syrup he consumes in a day.

(b) Find the amount of syrup he consumes on each occasion.

(iii) If the daily production of syrup is increased to 86 *l* 250 ml, how many bottles can be issued in a day?

(2) A motor vehicle can be driven 16 km on one litre of fuel. A person spends 1.5 *l* of fuel daily to go to office and return home.

(i) Find the total distance travelled by the vehicle in a day.

(ii) Find the amount of fuel required by him for 22 working days.

(iii) If the total distance he travelled during a certain month is 480 km, find the total number of litres of fuel that was consumed during that month.

Summary

- 1 *l* = 1000 ml
- To express a liquid quantity in litres, which is given in millilitres, the quantity given in millilitres should be divided by 1000.
- To express a liquid quantity in millilitres, which is given in litres, the quantity given in litres should be multiplied by 1000.

Revision Exercise - 2

(1) (i) Find the value of 6.785×1000.

(ii) Simplify $3\frac{1}{3} - 1\frac{1}{4}$.

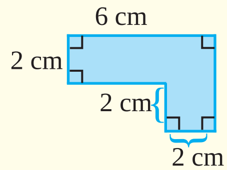
(iii) Find the value of $2a + 5$, if $a = 4$.

(iv) Express 5.075 g, in grammes and milligrammes.

(v) Solve $2x + 5 = 7$.

(vi) Simplify 96 cm 6 mm $\div 7$.

(vii) Find the area of the given figure.

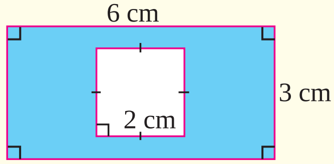


(viii) Find the volume of a cube of side length 5 cm.

(ix) Write $1\frac{5}{7}$ as an improper fraction.

(x) Write $\frac{17}{5}$ as a mixed number.

(xi) Find the area of the shaded region.



(xii) Find the side length of a square land of perimeter 22 m.

(xiii) Find the breadth of a rectangular land of area $24\ \text{m}^2$ and length 8 m.

(2) (a) Fill in the blanks using $<$ or $>$ appropriately.

(i) $\frac{3}{4}$ ..... $\frac{1}{4}$ (ii) $\frac{1}{4}$ ..... $\frac{5}{12}$ (iii) $3\frac{5}{8}$ ..... $3\frac{1}{3}$

(b) Simplify the following.

(i) $3\frac{5}{12} + \frac{7}{12}$ (ii) $2\frac{2}{7} + \frac{9}{14}$ (iii) $2\frac{5}{8} - 1\frac{1}{8}$ (iv) $3\frac{7}{8} - 2\frac{2}{3}$

(c) Dileepa and Sithumina sat for a multiple choice question paper. From the total number of questions, Dileepa answered $\frac{5}{8}$ correctly and Sithumina answered $\frac{3}{4}$ correctly. Who answered more questions correctly? Give reasons for your answer.

(d) In a test, Rahuman received 0.36 of the total marks and Rahuldev received $\frac{9}{25}$ of the total marks. Show that Rahuman and Rahuldev received the same amount of marks.

(3) (a) Convert the following fractions and mixed numbers into decimals.

(i) $\frac{648}{1000}$ (ii) $\frac{6}{20}$ (iii) $\frac{7}{8}$ (iv) $2\frac{1}{4}$

(b) Simplify.

(i) 0.875×100 (ii) 3.25×6 (iii) 0.005×22

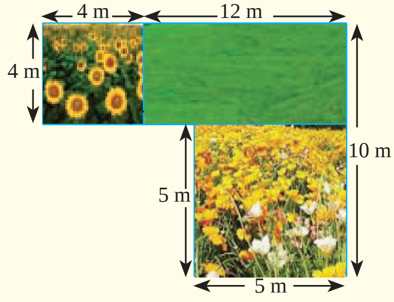
(iv) $127.5 \div 10$ (v) 24.68×8 (vi) $13.75 \div 1000$

(4) The given figure shows a home garden.

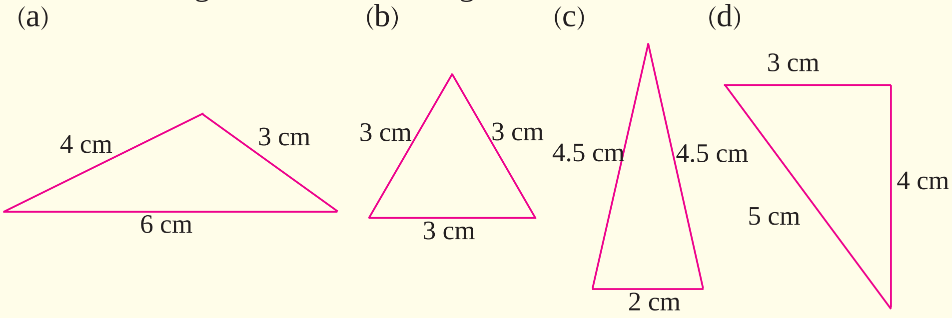
(i) Find the perimeter of the garden.

(ii) Find the area of the garden where flowers are grown.

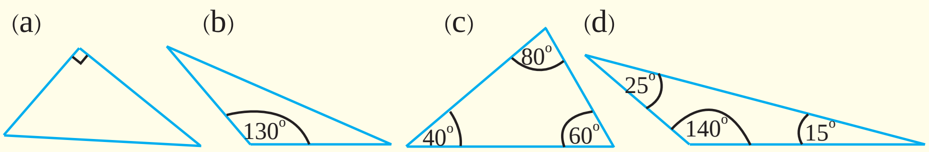
(iii) Find the total area of the garden.



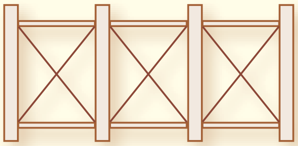
(5) (i) Each of the following triangles state whether it is an equilateral triangle, an isosceles triangle or a scalene triangle.



(ii) Each of the following triangles state whether it is an acute angled triangle, a right angled triangle or an obtuse angled triangle.



(6) The gate shown in the figure has 4 vertical posts. Each is of height 1.75 m.

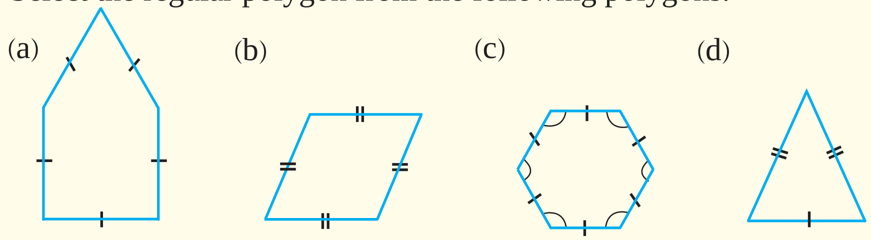


(i) If the posts are made from a metal pipe, find the total length of the pipe.

(ii) The total length of the metal bar used to cut the 6 horizontal bars was 8.4 m. Find the length of one horizontal bar.

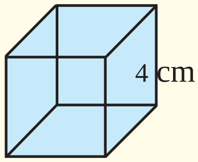
(7) (a) (i) Draw a concave polygon with 1 reflex angle and 6 sides.

(ii) Select the regular polygon from the following polygons.



(8) (a) (i) Find the volume of the given cube.

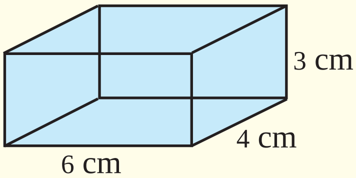
(ii) Calculate the volume of a cube of length twice the length of the above given cube.



(b) A cuboid is shown in the figure.

(i) Find the volume of this cuboid.

(ii) What is the height of a cuboid of volume 96 cm^3, if its length and breadth are the same as those of the cuboid shown in the figure?



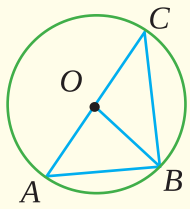
(9) *O* is the centre of the circle in the figure. *AC* is a straight line.

(i) What is the special name given to *AC* ?

(ii) What is the special name given to the length *OB* ?

(iii) Name two isosceles triangles in the figure.

(iv) If *AB* = 6 cm, *BC* = 8 cm and the radius of the circle is 5 cm, find the perimeter of each of the triangles *OBC*, *AOB* and *ACB*.



(10) Information on the quantities of milk bought by three households during a week from a milkman is given below.

(i) Household *A* buys 1*l* 500 ml of milk per day on all seven days of the week. Find the total quantity of milk that household *A* buys during a week.

(ii) Household *B* buys the same amount of milk on each of the seven days of a week. The total amount of milk household *B* buys during a week is 12 *l* 250 ml. Find the amount of milk household *B* buys per day.

(iii) Find the total quantity of milk bought during a week by household *C*, if 7 *l* 500 ml of milk in total is bought during the five week days and 2 *l* 750 ml of milk in total is bought on Saturday and Sunday.

(iv) During the school holidays, the milkman is asked to deliver 250 ml more milk per week than the normal amount he delivers. If an equal amount of milk is delivered each day, find the amount he delivers to household *C* per day during the holidays.

(11) A certain brand of biscuits is introduced to the market in packets.

(i) The mass of a biscuit is 8 g 250 mg. If a packet contains 25 biscuits, find the total mass of the biscuits in a packet.

(ii) The mass of the empty packet is 760 mg. Find the total mass of a packet of biscuits.

(iii) 12 such packets of biscuits are packed in a box of mass 40 g, and such boxes containing packets of biscuits are distributed to wholesale dealers. Find the total mass of one such box that is bought by a wholesale dealer.

(12) (a) (i) Solve $9x + 7 = 97$.

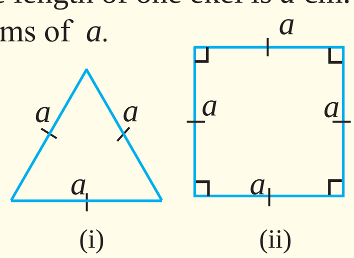
(ii) When Nimal gave Rs. 200 to buy 8 books, he received a balance of Rs. 40. Construct an equation using this information, by taking the price of a book to be Rs. x. Find the price of a book.

(b) The figure shows two frames in the shape of rectilinear plane figures, which have been made using ekels of equal length. The length of one ekel is a cm.

(i) Find the perimeter of the first figure in terms of a.

(ii) Find the perimeter of the second figure in terms of a.

(iii) If the total length of the ekels used to make these two frames is 42 cm, construct an equation in terms of a. Solve it and find the value of a.



(13) The cost of printing the cover of a certain book is Rs. y while the cost of printing a page of the book is Rs. p.

(i) If the book has 45 pages, and it costs Rs. c to print one copy of it, construct a formula for c in terms of p and y.

(ii) If the cost of printing the book is Rs. 115, and the cost of printing the cover is Rs. 25, find in rupees, the cost p of printing a page of this book.

(14) Two athletes train on two days of the week. The distances run on the two days are given below.

Day	Shanuka	Kavindu
Monday	2 km 800 m	1 km 200 m
Tuesday	4 km 400 m	3 km 800 m

(i) Who runs a longer distance during the training period of two days?

(ii) How much further does Shanuka run on Tuesday than on Monday?

(iii) On Tuesday, how much further does Shanuka run than Kavindu?

(iv) What is the total distance run by Shanuka during 4 weeks of such training?

21 Ratios

By studying this lesson you will be able to

- divide a quantity in a given ratio,
- find the total value or the values of the other terms when the value of a term of a ratio is given, and
- apply knowledge on ratios in practical situations.

21.1 Ratios and Equivalent Ratios

You have learnt in grade six that a ratio is a numerical relationship between two or more quantities expressed in similar units.

Let us focus on a few instances in daily life where ratios are applied.

A label pasted on a bottle of fruit juice recommends that, two parts juice be mixed with three parts water.

Therefore, to make a consumable drink from the bottled juice, one can mix 2 litres of juice with 3 litres of water.

We say that the fruit drink is made using juice and water in the ratio 2:3.

The mixed quantities (in litres) of juice and water are expressed by the ratio 2:3. This is read as 'two-to-three' or 'two-is-to-three'. The numbers 2 and 3 are called the **terms of the ratio**.

When we write a ratio, it is essential to write the terms in the correct order – that is, the order in which we mention the quantities. In the above example, we wrote juice first and water second. The same order is followed for the terms – we write 2 as the first term of the ratio and 3 as the second term.

When the terms of a given ratio are multiplied by the same positive whole number, we get an **equivalent ratio**.

That is, $1 : 3 = 2 : 6 = 3 : 9 = 4 : 12 = 5 : 15.$

Now let us consider an example where three items are mixed.
A concrete mixture is made by mixing together cement, sand and granite.

The ratio in which cement, sand and granite are mixed to prepare this concrete mixture is written as 1 : 3 : 4. It is read '1 to 3 to 4' or '1 is to 3 is to 4'. Here, 1, 3 and 4 are the terms of the ratio.

Let us multiply each term of the ratio 1 : 3 : 4 by 2.
Then we get the ratio 2 : 6 : 8. The ratio 2 : 6 : 8 is equivalent to the ratio 1 : 3 : 4.

A ratio should be written such that its terms are whole numbers that cannot be simplified further.

If the terms of a given ratio are whole numbers, and if the highest common factor of the terms is 1, then the ratio is said to be in the simplest form.

When the terms of a ratio are whole numbers, to write the ratio in its simplest form,

- check whether the terms have common factors.
- if the terms have common factors, then divide each term of the ratio by the highest common factor of the terms.

Example 1

Write a ratio equivalent to $4 : 1 : 6$.
Multiplying the terms in the ratio by 3 we obtain,

$$4 : 1 : 6 = 4 \times 3 : 1 \times 3 : 6 \times 3$$
$$= 12 : 3 : 18$$

Example 2

Express the ratio $8 : 4 : 12$ in its simplest form.
The HCF of the terms 8, 4 and 12 is 4. Dividing the terms of the ratio by 4 we obtain,

$$8 : 4 : 12 = 8 \div 4 : 4 \div 4 : 12 \div 4$$
$$= 2 : 1 : 3$$

Example 3

The sides of a triangle are 8 cm, 6 cm 5 mm and 50 mm. Find the ratio of the lengths of the sides of the triangle and express it in the simplest form.
Let us express the lengths in similar units.
8 cm = 80 mm, 6 cm 5 mm = 65 mm, 50 mm
The ratio of the lengths of the sides = 80 : 65 : 50
The ratio of the lengths of the sides in the simplest form } = 16 : 13 : 10

Exercise 21.1

(1) Write down the ratio for each of the following examples, and express it in the simplest form.
   (i) The number of boys in a class is 20 and the number of girls is 25.
   (ii) The price of a pen is Rs. 15, the price of a pencil is Rs. 10 and the price of an eraser is Rs. 5.
   (iii) The ingredients for a cake are 1 kg flour, 500 g sugar and 500 g margarine.
   (iv) The price of a mandarin is Rs. p, the price of an orange is Rs. q and the price of an apple is Rs r.

(2) For each of the following ratios, write down two equivalent ratios.

(i) 2 : 3 (ii) 6 : 5 : 7 (iii) 1 : 4 : 5

(3) Express each of the following ratios in its simplest form.

(i) 12 : 18 (ii) 28 : 70 : 42 (iii) 25 : 100 : 125

(4) The sides of a triangle are 7 cm, 50 mm and 6 cm 5 mm. Find the ratio of the lengths of the sides and express it in the simplest form.

21.2 Dividing in a ratio

• Dividing a given quantity in a ratio

There are instances in day-to-day life when people need to divide items among themselves. On some occasions, this is done in equal amounts, while on other occasions it is done in unequal amounts.
At the beginning of this lesson, we discussed about mixing juice and water in the ratio 2:3.

Five units of fruit drink are made by mixing 2 parts juice with 3 parts water.

The ingredients of this drink are juice and water.

Since the **number of parts of juice is 2** and the **number of parts of water is 3**, the number of parts of drink is 5.

Let us find the amount of each ingredient in the drink, if 10 litres of the drink were made.

Ratio of juice to water $= 2 : 3$

Total number of parts $= 2 + 3$

$= 5$

Volume of five parts $= 10\ l$

Volume of one part $= \frac{10}{5}\ l$

$= 2\ l$

Parts of juice $= 2$

Volume of juice $= 2\ l \times 2$

$= 4\ l$

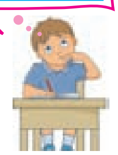
Parts of water $= 3$

Volume of water $= 2\ l \times 3$

$= 6\ l$

Juice Water
2 : 3

Number of parts	Volume
5	10
2	?
3	?



Note
When using this method, problem solving is facilitated by writing the given ratio in its simplest form and then finding the total number of parts relevant to it.

Example 1

Cement, sand and granite in a concrete mixture are in the ratio $1:3:4$. Find the quantities of cement, sand and granite in 16 cubic metres of concrete.

$$
\begin{aligned}
\text{Ratio of cement to sand to granite} &= 1:3:4\\
\text{Total number of parts} &= 1+3+4=8\\
\text{Size of 8 parts} &= 16\text{ m}^3\\
\text{Size of a single part} &= \frac{16}{8}\text{ m}^3 = 2\text{ m}^3\\
\text{Number of parts of cement} &= 1\\
\text{Quantity of cement} &= 1\times 2\text{ m}^3 = 2\text{ m}^3\\
\text{Number of parts of sand} &= 3\\
\text{Quantity of sand} &= 3\times 2\text{ m}^3 = 6\text{ m}^3\\
\text{Number of parts of granite} &= 4\\
\text{Quantity of granite} &= 4\times 2\text{ m}^3 = 8\text{ m}^3
\end{aligned}
$$

Example 2

The ingredients to make 3 kg of cake are butter, sugar and flour, mixed in the ratio 1 : 2 : 3. Find the mass of each ingredient in the cake.

$$
\begin{aligned}
\text{Ratio of butter to sugar to flour} &= 1:2:3\\
\text{Total number of parts} &= 1+2+3=6\\
\text{Total mass of the 6 parts of cake mixture} &= 3\text{ kg}\\
\text{Mass of a single part} &= \frac{3}{6}\text{kg}\\
&= \frac{3000}{6}\text{ g} = 500\text{ g}\\
\text{Parts of butter} &= 1\\
\text{Mass of butter} &= 1\times 500\text{ g} = 500\text{ g}\\
\text{Parts of flour} &= 3\\
\text{Mass of flour} &= 3\times 500\text{ g} = 1500\text{ g}\\
&= 1\text{ kg } 500\text{ g}\\
\text{Parts of sugar} &= 2\\
\text{Mass of sugar} &= 2\times 500\text{ g}\\
&= 1000\text{ g} = 1\text{ kg}
\end{aligned}
$$

Example 3

Nadaraja and Mohommad made a profit of Rs. 7000 from their small business. They decide to divide the profit in the ratio 3 : 4, which is the ratio in which they invested in the business. Find how much of the profit each person receives.

The ratio in which the profit is divided between Nadaraja and Mohommad $= 3 : 4$

Total number of parts $= 3 + 4 = 7$

Total profit $=$ Rs. 7000

Value of a single part $= \text{Rs. } \frac{7000}{7}$

$= \text{Rs. } 1000$

Number of parts Nadaraja receives $= 3$

Value of profit Nadaraja receives $= \text{Rs. } 1000 \times 3$

$= \text{Rs. } 3000$

Number of parts Mohommad receives $= 4$

Value of profit Mohommad receives $= \text{Rs. } 1000 \times 4$

$= \text{Rs. } 4000$

Exercise 21.2

(1) Rs. 1500 was divided between Sumudu and Kumudu in the ratio 2 : 3. Find the amount each received.

(2) Copper is added to gold in gold jewellery, such that the ratio of copper to gold is 1 : 11. Find the mass of gold and copper needed to make a necklace of mass 60 grammes.

(3) The ratio of boys to girls in a school is 5 : 4. If the total number of students in the school is 1800, find the number of boys and the number of girls there are in the school.

(4) A land owner divides his land of 1800 m^2 between his son and his daughter in the ratio 5:3. How much of the land does the son receive?

(5) Rice flour, sugar and coconut are mixed in the ratio 4 : 3: 1 to prepare a certain sweetmeat mixture. Find the mass of each ingredient in 2 kg of the sweetmeat.

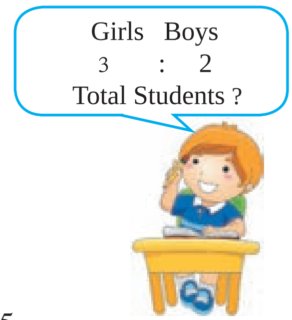
(6) A high nutrient food item is made of green gram, soya and rice mixed in the ratio 2 : 1 : 3. Compute the amount of rice in a 840 g packet of this food item.

(7) The ratio of the high-school students enrolled in the science, technology and arts streams in a school is 3 : 5 : 7. If the total number of high-school students in the school is 600, how many students are enrolled in the arts stream?

(8) The ratio of length to breadth of a rectangular playground is 3 : 2. If its perimeter is 600 m, find its length and its breadth.

• Calculating the total amount, when the amount of one item in a ratio is given

The ratio of girls to boys in a class is 3 : 2. If the number of girls in the class is 24, let us find how many students there are in total in the class.



$$\begin{aligned} \text{Ratio of girls to boys} &= 3 : 2 \\ \text{Number of parts of girls in the class} &= 3 \\ \text{Number of girls} &= 24 \end{aligned}$$

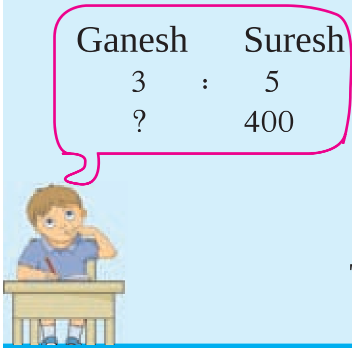
Since the number of students in 3 parts is 24,

$$\begin{aligned} \text{the number of students in one part} &= \frac{24}{3} = 8 \\ \text{Total number of parts} &= 3 + 2 = 5 \\ \text{Total number of students} &= 8 \times 5 \\ &= 40 \end{aligned}$$

Example 1

A sum of money was divided between Ganesh and Suresh in the ratio 3 : 5. Suresh received Rs. 400. What was the total amount that was divided between the two of them?

The ratio in which the money was divided between Ganesh and Suresh	= 3 : 5
Parts for Suresh	= 5
Amount received by Suresh	= Rs. 400
Therefore, the value of 5 parts	= Rs. 400
Value of one part	= Rs. 400 ÷ 5
	= Rs. 80
Number of parts	= 3 + 5 = 8
Total amount that was divided	= Rs. 80 × 8
	= Rs. 640



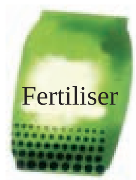
Exercise 21.3

(1) Sugar and flour were mixed in the ratio 3 : 5 to prepare a mixture for a sweetmeat. If the mass of sugar used was 750 g, find the total mass of the sweetmeat mixture.

(2) Sirimal rides a bike from his house to the bus halt and then takes a bus when he travels to school. The ratio of the distance he travels by bike to the distance he travels by bus is 2 : 7. If the distance he travels by bus is 14 km, what is the distance to the school from his house?

(3) A fruit drink is made using water and orange juice in the ratio 5 : 7. If the quantity of orange juice used is 350 ml, what is the total volume of the fruit drink that is made?

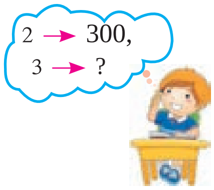
(4) The ratio of Nitrogen to Phosphorus to Potassium in a fertiliser is 5 : 2 : 1. If the mass of Phosphorus in a bag of this fertiliser is 250 g, what is the total mass of the bag?



(5) When making a mixture of plaster, the ratio in which cement, lime and sand are mixed is 2:3:5. If the quantity of lime in a mixture of plaster is 6 pans, what is the total quantity of the mixture, measured in pans?

- **When the amount related to one term in a ratio is given, determining the other amounts**

The ratio in which a sum of money was divided between Siyam and Kandan is 2 : 3. If Siyam received Rs. 300, let us find how much money Kandan received.



Ratio in which the money was divided between Siyam and Kandan $\}$ = 2 : 3
Parts Siyam received = 2
Money Siyam received = Rs.300
Since two parts is worth Rs. 300,
the value of one part = Rs. $300 \div 2$
= Rs. 150
Parts Kandan received = 3
Money Kandan received = Rs. 150×3
= Rs. 450

Example 1

The ratio in which cement, sand and granite are mixed in order to prepare a concrete mixture is 2 : 3 : 4. Let us find the quantities of cement and granite that should be mixed with 9 pans of sand and the total amount of concrete mixture that is made.

Parts of sand = 3
Pans of sand = 9
Quantity of sand in one part $= \frac{9}{3}$ pans = 3 pans
Parts of cement = 2
$\therefore$ quantity of cement $= 3 \times 2$ pans = 6 pans
Parts of granite = 4
$\therefore$ quantity of granite $= 3 \times 4$ pans = 12 pans
Total number of parts $= 2 + 3 + 4 = 9$
Quantity of concrete mixture $= 3 \times 9$ pans = 27 pans

Exercise 21.4

(1) Sesame balls are made by mixing sesame and jaggery in the ratio 5 : 4. How much jaggery is required to make sesame balls, if 500 g of sesame is used?

(2) The ratio of female workers to male workers in an office is 3 : 2. If there are 18 female workers, find the number of male workers.

(3) Tea and milk are mixed in the ratio 2 : 5 when making milk tea. How many millilitres of tea should be used to make milk tea, if 100 ml of milk is used?

(4) Mr. Perera's savings to expenditure ratio is 3:7. If his savings during a certain month was Rs. 6000, how much money did he spend that month?

(5) An alloy is made by mixing masses of zinc and copper in the ratio 2:5.

   (i) If the mass of zinc in a sample of this alloy is 6 kg, what is the mass of copper?

   (ii) If the mass of copper in a sample of this alloy is 10 kg, what is the mass of zinc?

   (iii) Find the mass of copper in 28 kg of the alloy.

   (iv) If the mass of zinc in a sample of this alloy is 2 kg, what is the mass of the sample?

Miscellaneous Exercise

(1) Silver and copper were mixed in the ratio 2 : 3 to make a statue. If the mass of the statue is 1425 g, find the mass of silver in the statue.

(2) Kamalini, Nimal and Tharaka divided several veralu (olives) among themselves in the ratio 1 : 3 : 5. If Tharaka received 15 veralu, how many veralu did Kamalini receive? Find also the number of veralu that Nimal received.

(3) The ratio of Sinhalese to Tamils to Muslims in a certain city is 5 : 4 : 3. If the total population in the city is 7200, find how many Sinhalese there are in the city.

Summary

- Multiplying (or dividing) the terms of a given ratio by a fixed number gives an equivalent ratio.
- If all the terms of a ratio are whole numbers and their HCF is one, the ratio is said to be in its simplest form.
- The total number of parts of a ratio is the sum of the terms of the ratio. The number of parts of each item is the term in the ratio associated with that item.

  For example, a concrete mixture in which cement, sand and granite are mixed in the ratio 3 : 6 : 8 has 3 parts of cement, 6 parts of sand and 8 parts of granite. Thus, the total number of parts is 17.
- When ingredients are mixed in a given ratio, if the amount of one ingredient or the total amount is known, it is possible to find the amount of a single part, by dividing the known amount by the relevant number of parts. Thereby the individual amounts of the other ingredients and the total amount can be determined.

22

Percentages

By studying this lesson you will be able to,

- identify a percentage,
- use the symbol %, to indicate an amount as a fraction of 100, and
- write a fraction with denominator equal to a factor of 100, as a percentage.

22.1 Introduction to the concept of percentage

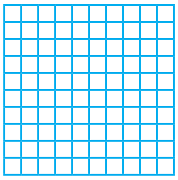
Some advertisements taken from a newspaper and a leaflet are shown below.

In all these advertisements the symbol 1 % appears after a number. % is known as the **percentage sign**. The percentage sign is used in various instances.

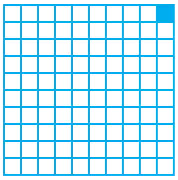
5% of the eggs in the basket are rotten. This means that 5 eggs out of 100 eggs are rotten. The ratio of the number of rotten eggs to the number of eggs in the basket is 5:100.

The yield from paddy seeds is 3500%. Accordingly, when you plant 100 paddy seeds you will get a yield of 3500. Therefore the ratio of the yield to the amount of seeds planted is 3500 : 100.

Let us study percentages using a 10×10 square grid.

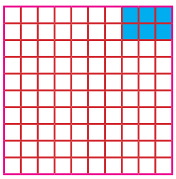
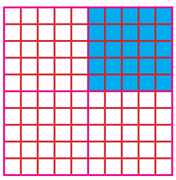
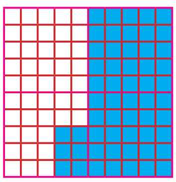
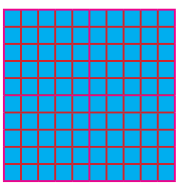


The region of the 10×10 square grid is taken as 1 unit.



Considering it as one unit, the grid is divided into 100 small squares. Of these squares, exactly one is coloured. That is, $\frac{1}{100}$ of the entire grid is coloured. As a percentage, this is 1%. This is read as "**one percent**". This indicates a portion of a unit as a percentage.

The below given table is prepared by taking the initial number of squares as 100.

Figure	Coloured part	As a fraction	As a decimal number	As a percentage
	6 of the 100 squares	$\frac{6}{100}$	0.06	6%
	25 of the 100 squares	$\frac{25}{100}$	0.25	25%
	56 of the 100 squares	$\frac{56}{100}$	0.56	56%
	100 of the 100 squares	$\frac{100}{100}$	1.00	100%

Exercise 22.1

(1) Write the percentages given in words using the percentage sign.

(i) Two percent (ii) Twenty percent
(iii) Hundred percent (iv) Hundred and seventy five percent
(v) Twelve and a half percent (vi) Thirty point five percent

(2) Write down how each of the percentages given below is read.

(i) 25 % (ii) 180 % (iii) 7.5 %

(3) Write the percentage corresponding to each of the following fractions of a unit.

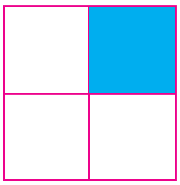
(i) $\frac{9}{100}$ (ii) $\frac{30}{100}$ (iii) $\frac{100}{100}$ (iv) $\frac{105}{100}$

(4) Write the fraction corresponding to each of the percentages given below.

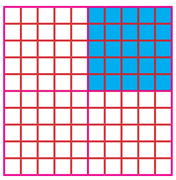
(i) 33 % (ii) 100 % (iii) 85 % (iv) 1 %

22.2 More on representing fractions as percentages

Let us now consider a fraction which does not have 100 as the denominator. Let us learn to write it as a percentage.



Observe this figure. We see that $\frac{1}{4}$ of the whole figure is coloured.



This figure has been divided into 100 equal sized squares. Here $\frac{25}{100}$ of the whole figure is coloured. That is 25% of the whole figure is coloured.

See that the coloured parts of both figures are the same. So $\frac{1}{4} = \frac{25}{100}$.
That is $\frac{1}{4} = 25\%$.

Thus, a given fraction can be written as an equivalent fraction with 100 as the denominator. Then we can represent the given fraction as a percentage.

Example 1

Write $\frac{3}{10}$ as a percentage.

As $100 \div 10 = 10$, let us multiply the denominator and the numerator by 10.

$$\frac{3}{10} = \frac{3 \times 10}{10 \times 10} = \frac{30}{100} = 30\%$$

Example 2

Write $\frac{5}{4}$ as a percentage.

As $100 \div 4 = 25$, let us multiply the denominator and the numerator of $\frac{5}{4}$ by 25.

$$\frac{5}{4} = \frac{5 \times 25}{4 \times 25} = \frac{125}{100} = 125\%$$

Example 3

Write 3 as a percentage.

$$3 = \frac{3}{1} = \frac{3 \times 100}{1 \times 100} = \frac{300}{100} = 300\%$$

Example 4

Write $2\frac{1}{2}$ as a percentage.

$$2\frac{1}{2} = \frac{5}{2} = \frac{5 \times 50}{2 \times 50} = \frac{250}{100} = 250\%$$

Example 5

Of the 25 students in a class, 13 are girls. Represent the number of girls, as a percentage of all the students in the class.

The number of girls, as a fraction of all the students in the class is $\frac{13}{25}$.

$$\frac{13}{25} = \frac{13 \times 4}{25 \times 4} = \frac{52}{100} = 52\%$$

$\therefore$ the number of girls, as a percentage of all the students in the class is 52%.

Exercise 22.2

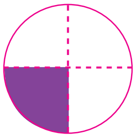
(1) Write each of the fractions given below as a percentage.

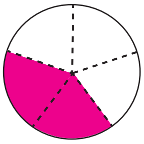
(i) $\frac{3}{4}$ (ii) $\frac{1}{10}$ (iii) $\frac{15}{20}$ (iv) $\frac{3}{2}$ (v) $\frac{13}{10}$ (vi) $1\frac{2}{5}$ (vii) $1\frac{7}{20}$

(2) For each of the figures given below, write the shaded part as a fraction of the whole figure. Indicate this as a percentage.

(i) 

(ii) 

(iii) 

(iv) 

(3) The total marks given for an assignment was 25. Prathapa got 21 for this assignment.

(i) Write her marks as a fraction of the total marks.

(ii) Write her marks as a percentage of the total marks.

(4) A children's society has 20 members. Only 17 members attended a meeting on a certain day.

(i) Write the number that attended the meeting that day as a fraction of the total number of members.

(ii) Write the above fraction as a percentage.

(5) The same Mathematics test paper was given to both Class A and Class B of grade 7. Malinda who was in Class A got 22 marks out of 25 for the test, while Suresh who was in Class B, got 18 marks out of 20.

(i) Express the marks Malinda got, as a percentage of the total marks.

(ii) Express the marks Suresh got, as a percentage of the total marks.

(iii) Of the two, who has shown more mathematical ability at the test?

(6) A vendor bought 50 mangoes, of which 8 were spoilt.

(i) Express the number of spoilt mangoes as a percentage of the total number of mangoes.

(ii) Express the number of good mangoes as a percentage of the total number of mangoes.

(7) 20 students attended an eye clinic. Of them, 5 had problems with their eye sight. Of all the students who came to the clinic find the percentage of students who didn’t have problems with their eye sight.

(8) Last year, Mr. Perera's salary was 50 000 rupees per month. This year his salary has increased to 65 000 rupees per month. Find the increment as a percentage of last year’s monthly salary.

(9) You can harvest 5 kg of ginger from 1 kg of ginger. Express the harvest as a percentage of the ginger that is planted.

(10) For every 100 bean seeds that are planted from a packet, 85 germinate. Write the percentage of germinating seeds.

22.3 Representing decimal numbers as percentages

We have already learnt how to represent a decimal number as a fraction. Recalling what was learnt earlier, let us consider how a decimal number is represented as a percentage.

Activity 1

Copy the table given below in your exercise book and fill in the blanks.

Decimal number	The number as a fraction	The number as a fraction having 100 as the denominator	The number as a percentage of the original amount
0.5	$\frac{5}{10}$	$\frac{5 \times 10}{10 \times 10} = \frac{50}{100}$	50%
2.3	$\frac{23}{10}$	....	....
0.25	$\frac{25}{100}$	....	25%
1.75	....	....	....

A given decimal number with one or two decimal places can be represented as a percentage, by first representing it as a fraction having 100 as the denominator.

➢ **This can also be done by multiplying the given decimal number or fraction by 100 and placing the % symbol in the answer.**

- Let us represent 0.5 as a percentage.
  Let us multiply 0.5 by 100 and then place the % symbol in the answer.
  $0.5 \times 100 = 50$
  $\therefore$ 50% is 0.5 represented as a percentage.

- Let us represent 0.25 as a percentage.
  0.25 represented as a percentage is 0.25×100 %; that is, 25%.

Example 1

Let us represent 1.08 as a percentage.
1.08 represented as a percentage is 1.08×100 % ; that is, 108%.

Exercise 22.3

(1) Write each of the given decimal numbers as a fraction. Then write it as a percentage.

(i) 0.3 (ii) 0.5 (iii) 0.1 (iv) 0.33
(v) 0.45 (vi) 0.03 (vii) 0.08 (viii) 0.01

(2) Multiply each of the given decimal numbers and fractions by 100, and represent it as a percentage of the original amount.

(i) 0.7 (ii) $\frac{2}{5}$ (iii) 0.65 (iv) $\frac{3}{4}$
(v) 0.08 (vi) 0.05 (vii) 1.5 (viii) 1.25

(3) A person spends $\frac{2}{5}$ of his monthly income on his children's education and 0.25 of his monthly income on food items.

(i) Express the amount he spends on his children's education as a percentage of his income.

(ii) Express the amount he spends on food items as a percentage of his monthly income.

(iii) For which of the above two needs does he spend the greater portion of his monthly income?

(4) Kamal had to pay a certain amount of money to an institution. He pays $\frac{1}{4}$ in January, 23% in February and 0.52 of the amount in March.

(i) Express the amount of money he pays in January and March as a percentage of the total amount he had to pay.

(ii) Now compare your answers and decide in which month he has paid the most.

Summary

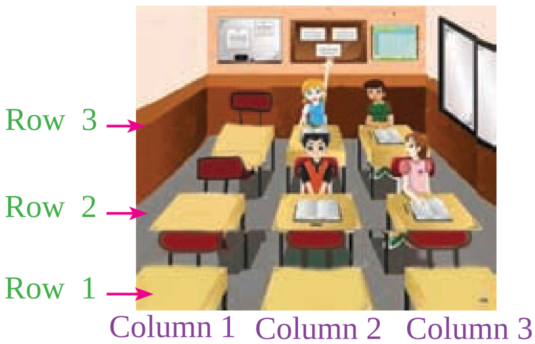
- When amounts which are parts of 100 are written with the percentage symbol %, we say that they are written as percentages.
- A given fraction or decimal number can be written as a percentage, by first writing it as a fraction having 100 as the denominator.
- A given decimal number can be represented as a percentage by multiplying it by 100 and placing the % symbol in the answer.

23 Cartesian Plane

By studying this lesson you will be able to
- identify what a Cartesian plane is,
- identify a point on a Cartesian plane by its coordinates, and
- plot a point on a Cartesian plane when its coordinates are given.

23.1 Identifying a location

The locations of several students seated in a classroom are shown in the figure. Let us describe the location of each of the students.



Location of several students

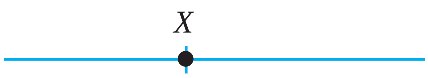
Location		Name of the student
Column	Row	
3	3	Nimal
2	2	Sesath
3	2	Mala
2	3	Mayuri

Mayuri's location is in the 3rd row of the 2nd column.
You may observe, as indicated in the table, that the location of each of the students in the classroom can be indicated exactly, in a similar manner.

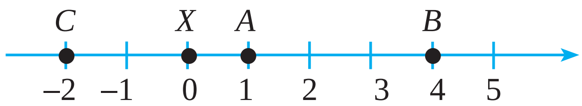
Now let us see how the location of a point can be determined with respect to a fixed point.

• Location of a point with respect to a fixed point

A fixed point on a straight line is indicated as X.



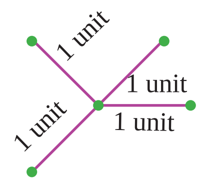
Considering point X as 0 (zero), number the straight line as a number line. Now, with respect to the point X, we can represent any point on the line by a number.



Accordingly, with respect to the point X, the positions of the points A, B and C can be represented by the numbers 1, 4 and -2 r espectively.

Points A and B are located to the right of point X, at a distance of 1 unit and 4 units respectively from X. Point C is located to the left of point X, at a distance of 2 units from X.

There are many points on a plane which are at a distance of 1 unit from a fixed point on the plane. Therefore, it is not possible to exactly determine the location of a point at a distance of 1 unit from a particular point on a plane, using only one number line.

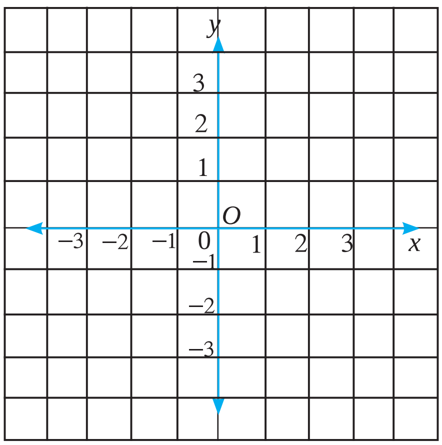


In 1637, Rene Descartes (1596 AD – 1650 AD) of French origin presented a method of representing the exact location of a point on a plane using a grid. Such a grid is called a **Cartesian plane.**


Rene Descartes

23.2 Cartesian plane

A Cartesian plane is shown in the figure.

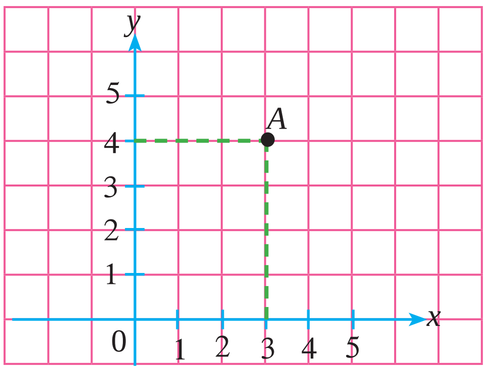


- O is a fixed point on this Cartesian plane.
- Here, two number lines intersect perpendicularly at point O.
- The number zero of each number line is positioned at point O. It is called the **origin**.
- As indicated in the figure, one number line is called the ***x*-axis** and the other number line is called the ***y*-axis**.
- Any point on the plane can be exactly identified by two numbers based on point O.
- These two numbers are called the **coordinates** of that point.

23.3 Identifying a point on a Cartesian plane by its coordinates

A is a point on the given Cartesian plane.

Let us see how the point A on the Cartesian plane can be exactly identified by two numbers.



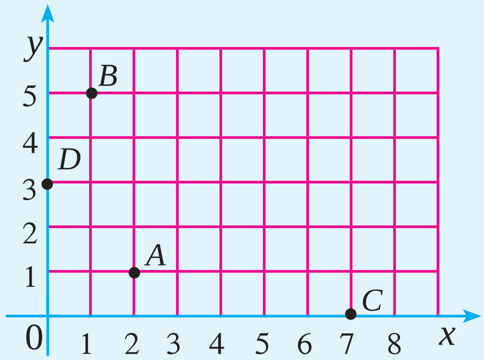
The line drawn from point A which is perpendicular to the x - axis, meets the x - axis at 3. The line drawn from point A which is perpendicular to the y - axis, meets the y-axis at 4.

Accordingly, the x - coordinate of the point A is defined as 3 and the y - coordinate of A is defined as 4. The coordinates of A are written as (3, 4) by writing the x - coordinate first and the y-coordinate second, within brackets. This is written in short as $A(3,4)$.

Accordingly, the coordinates of the origin O are (0, 0).

Example 1

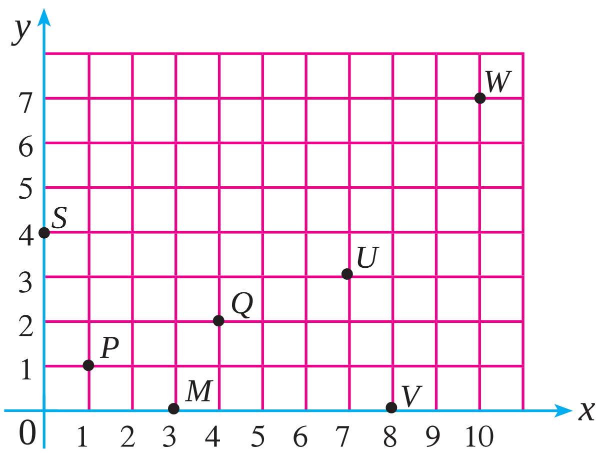
Write down the coordinates of the points on the given cartesian plane as ordered pairs.



Point	x - coordinate	y - coordinate	Coordinates
A	2	1	(2,1)
B	1	5	(1,5)
C	7	0	(7,0)
D	0	3	(0,3)

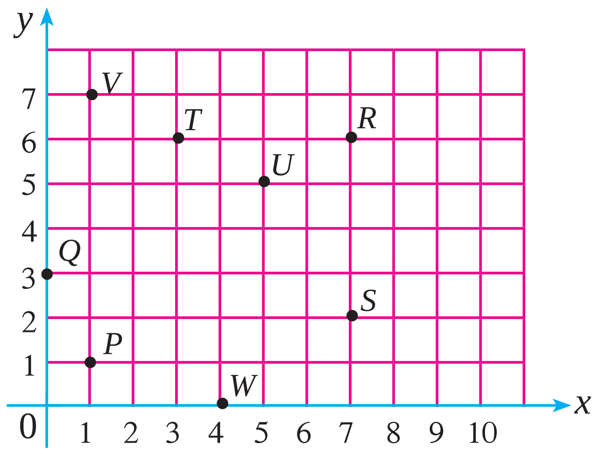
Exercise 23.1

(1) Copy the given table in your book and complete it based on the coordinates of the points represented on the Cartesian plane.

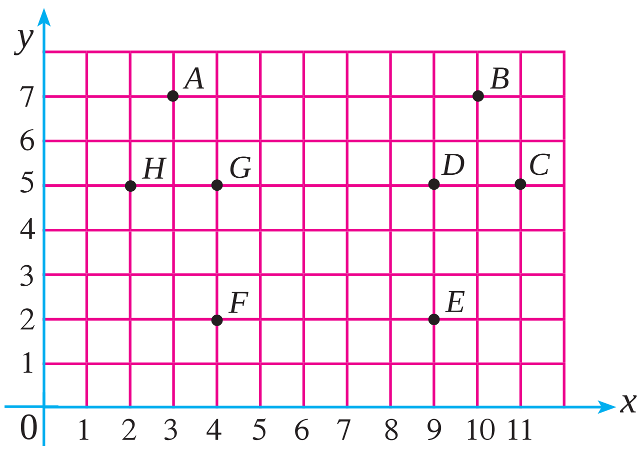


Point	x - coordinate	y - coordinate	Coordinates	Name of the point with its coordinates
P	1	1	(1,1)	P (1,1)
Q				
S				
V				
U				
W				
M				

(2) Write down the coordinates of the points on the given Cartesian plane.



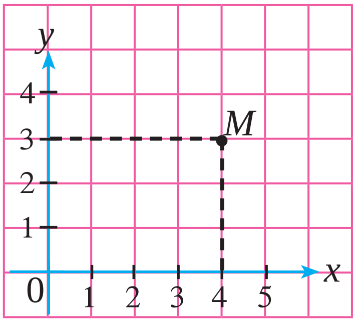
(3) Write down the coordinates of the points on the given Cartesian plane.



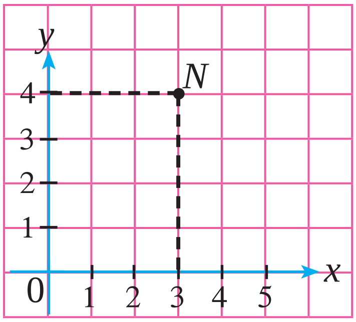
23.4 Plotting points on a Cartesian plane

Let us see how the point M (4, 3) is plotted on a cartesian plane. From the origin O, move 4 units to the right along the x - axis, and from there, move 3 units upwards parallel to the y - axis and then mark M.

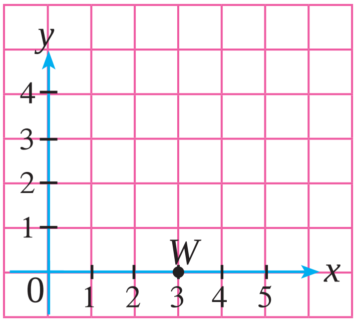
(i) Plotting the point M (4,3)



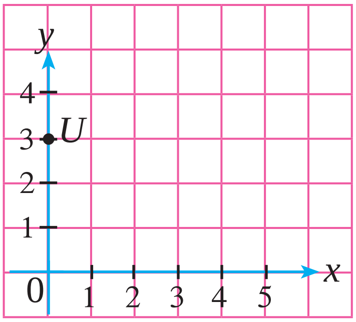
(ii) Plotting the point N (3,4)



(iii) Plotting the point W (3,0)



(iv) Plotting the point U (0,3)



- The coordinates of a point with y - coordinate zero, i.e., a point on the x - axis, is of the form (x, 0).
- The coordinates of a point with x - coordinate zero, i.e., a point on the y - axis, is of the form (0, y).
- The coordinates of the point with the x and y - coordinates both equal to zero is (0, 0). This point is the origin.

Exercise 23.2

(1) Draw a suitable Cartesian plane and plot the following points.

A (2, 5), B (4, 3), C (2, 1), D (0, 6), E (3, 6), F (7, 0)

(2) Plot the following points on a Cartesian plane and join them with straight line segments in the order of the letters and return to the starting point.

(i) A (1, 7), B (2, 1), C (5, 5), D (8, 1), E (9, 7)

(ii) A (5, 1), B (5, 3), C (0, 5), D (0, 6), E (5, 4), F (5, 5), G (10, 5), H (10, 1)

(iii) A (1, 4), B (0, 4), C (0, 7), D (1, 7), E (1, 6), F (7, 6), G (7, 7), H (10, 7), I (10, 4), J (7, 4), K (7, 5), L (1, 5)

(3) Shanuka says that "the vertices of a square are positioned at P (2, 2), Q (2, 7), R (7, 7), S (7, 2)". Plot these points on a cartesian plane and verify the validity of the above statement.

(4) Draw a Cartesian plane and plot four points such that the x - coordinate and y - coordinate values of each point are equal to each other. Write down the coordinates of the four points.

(5) (i) Plot the points given below on a Cartesian plane and join them in the order of the letters with straight line segments.

A (4, 1), B (4, 2), C (4, 3), D (4, 4)

(ii) Extend the line that is obtained.

(iii) Write the coordinates of two other points on this line.

(6) (i) Plot the points given below on a Cartesian plane and join them in the order of the letters with straight line segments.

P (2, 3), Q (4, 3), R (6, 3), S (7, 3)

(ii) Extend the line that is obtained.

(iii) Write the coordinates of two other points on this line.

Summary

- Any point on a Cartesian plane can be denoted by an ordered pair (x, y).
- The number denoted by x is called the x - coordinate and the number denoted by y is called the y - coordinate of the point (x, y).

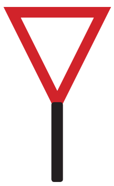
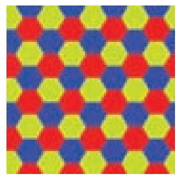
24 Construction of Rectilinear Plane Figures

By studying this lesson you will be able to

- construct a straight line segment of given length,
- construct an equilateral triangle of given side length, and
- construct a hexagon by means of an equilateral triangle or a circle.

24.1 Constructions

The following figure presents examples of equilateral triangles and regular hexagons that can be observed around us.

Equilateral triangles and regular hexagons are two types of convex polygons that are important in geometry.

In geometry, it is necessary to draw as well as construct plane figures. When a plane figure is being drawn, it is done according to the given data, without paying much attention to the measurements. However, when a plane figure is being constructed, attention needs to be paid to the measurements, and a figure of the correct size should be constructed according to the given data.

Geometrical constructions are done with a pair of compasses and a straight edge.

When it is necessary to measure lengths and angles, then the appropriate measuring instruments need to be used.

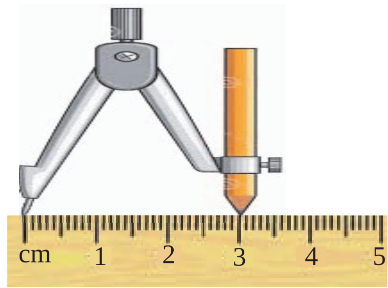
24.2 Construction of a straight line segment

You have learnt earlier that a straight line segment is a portion of a straight line.

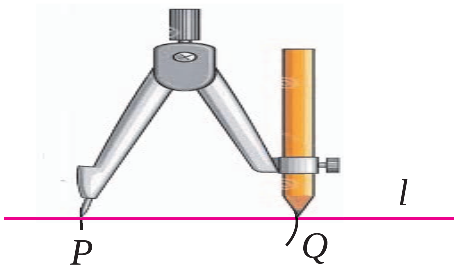
Now let us construct the straight line segment PQ of length 3 cm.

Step 1 - Draw a straight line using a ruler. Name it l. Mark a point on the straight line l and name it P.

Step 2 - Place the pair of compasses on the ruler and set it so that the point of the pair of compasses and the pencil point are at a distance of 3 cm apart.



Step 3 - Place the point of the pair of compasses on the point P and mark a point on the line which is 3 cm from P and name it Q.



Step 4 - Write "3 cm" between the two points P and Q.

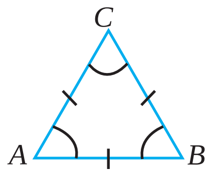
Now you have constructed a straight line segment PQ of length 3 cm. To indicate that the length of this straight line segment is 3 cm, we write $PQ = 3$ cm.

➤ Construct straight line segments of the lengths given below.

(i) $AB = 7$ cm (ii) $XY = 7.8$ cm

24.3 Construction of an equilateral triangle

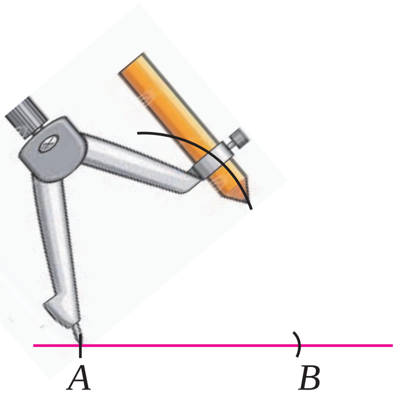
You have learnt earlier that an equilateral triangle is a triangle with sides which are equal in length and angles which are equal in magnitude.



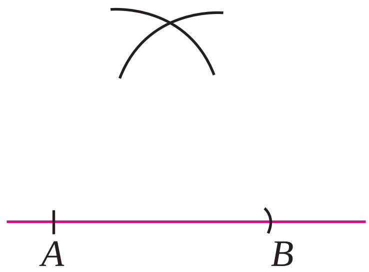
Let us construct an equilateral triangle of side length 3 cm.

Step 1 - Construct a straight line segment *AB* of length 3 cm using a pair of compasses and a ruler.

Step 2 - Set the pair of compasses so that the point of the pair of compasses and the pencil point are at a distance of 3 cm apart. Place the point of the pair of compasses on the point *A* and construct an arc as shown in the figure.

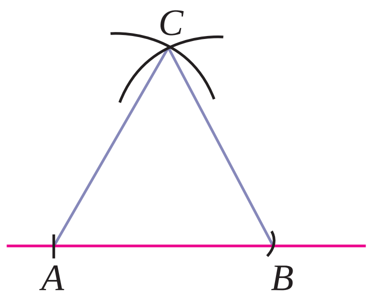


Step 3 - Place the point of the pair of compasses at the point *B* and construct another arc such that it intersects the first arc. If the arcs do not intersect, place the point of the pair of compasses at *A* and lengthen the initial arc.



Name the point of intersection of the two arcs as *C*.

Step 4 - Join *AC* and *BC*.



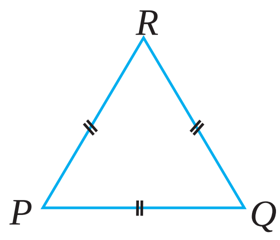
Then you will obtain the equilateral triangle *ABC* of side length 3 cm.

➤ (i) Construct two equilateral triangles of side length 4 cm and 5.7 cm.
(ii) Measure the angles of the above two triangles that you constructed.

Exercise 24.1

(1) Construct the straight line segment *LM* of length 6 cm using a straight edge and a pair of compasses.

(2) Draw the straight line *l* and construct the straight line segment *PQ* of length 7.5 cm on it.



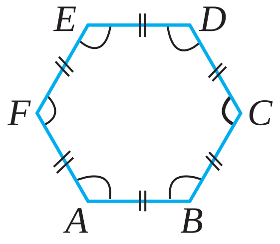
(3) (i) Construct the equilateral triangle *PQR* in the figure. Measure and write down the magnitude of the angle *PQR*.

(ii) Mark the mid points of the sides of the triangle *PQR* and name them *X*, *Y* and *Z*. Draw the triangle *XYZ*.

(4) (i) Cut out 6 equilateral triangles, each of side length 3 cm, from different coloured paper.

(ii) Mark a point *O* on a piece of paper and paste the triangles on this paper such that one vertex of each triangle coincides with *O* and adjacent triangles have a side which touches one side of each triangle next to it. What is the shape of the figure you obtain by doing this?

24.4 Constructing a regular hexagon

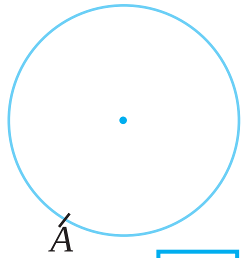


In the figure is a regular hexagon *ABCDEF*. A hexagon is a closed convex polygon bounded by 6 straight line segments. In a regular hexagon,

- the sides are of equal length
- the angles are of equal magnitude

Now let us see how a regular hexagon is constructed.

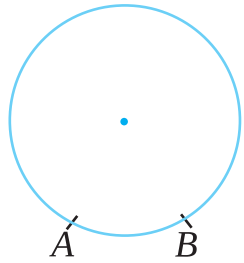
• Constructing a regular hexagon by means of a circle



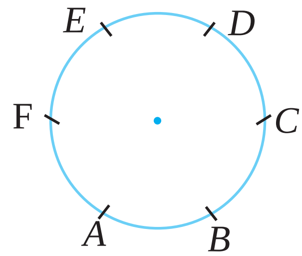
Step 1 - Construct a circle of radius 1.5 cm using a pair of compasses.

Step 2 - Mark a point *A* on this circle.

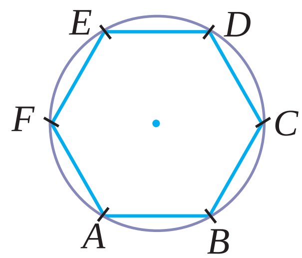
Step 3 - Set the pair of compasses so that the point of the pair of compasses and the pencil point are at a distance of 1.5 cm apart. Place the point of the pair of compasses on point *A*, draw an arc which intersects the circle and name this point *B*.



Step 4 - Similarly, place the point of the pair of compasses on the point *B* and mark the point *C*. Now, place the point on *C* and mark the point *D*, place the point on *D* and mark the point *E* and finally place the point on *E* and mark the point *F*.



Step 5 - Join the points *A*, *B*, *C*, *D*, *E* and *F* respectively.



You have now constructed the regular hexagon *ABCDEF* of side length 1.5 cm. By measuring the angles of the regular hexagon establish the fact that they are of equal magnitude.

- Construct a regular hexagon of side length 3.5 cm by following the above steps.

• Constructing a regular hexagon using an equilateral triangle

Step 1 - Construct the equilateral triangle *ABC* of side length 4 cm.
Step 2 - Construct the equilateral triangle *BCD* by taking *BC* as a side.
Step 3 - Construct the equilateral triangle *CDE* by taking *CD* as a side.
Step 4 - Construct the equilateral triangle *CEF* by taking *CE* as a side.
Step 5 - Construct the equilateral triangle *CFG* by taking *CF* as a side.
Step 6 - Join *A* and *G*.

Then you will obtain the regular hexagon *ABDEFG* of side length 4 cm. A regular hexagon of any side length can be constructed in the above manner.

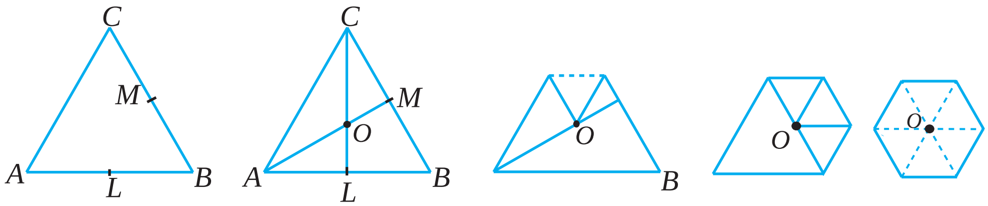
- Construct a regular hexagon of side length 3 cm.

Activity 1

Step 1 - Construct the equilateral triangle *ABC* of side length 3 cm on a piece of paper.

Step 2 - Mark the midpoint of *AB* as *L* and the midpoint of *BC* as *M*.

Step 3 - Join *MA* and *LC*. Name the point of intersection of *LC* and *MA* as *O*. Cut out the triangular lamina *ABC*.



Step 4 - Fold the triangle such that each vertex coincides with *O*. The figure that is obtained by doing this is a regular hexagon.

Step 5 - Measure the length of a side of the regular hexagon.

- The length of a side of the regular hexagon is 1 cm.
- That is, the length of a side of the original equilateral triangle is 3 times the length of a side of the regular hexagon.

- Construct a regular hexagon of side length 3 cm by following the above steps.

Exercise 24.2

(1) (i) Construct the circle of radius 5 cm and centre *O*.

(ii) Construct the regular hexagon *ABCDEF* of side length 5 cm with its vertices on the above circle.

(iii) Join *OA*, *OB*, *OC*, *OD*, *OE* and *OF*. How many triangles do you get? Are they all equilateral triangles?

(2) Construct a regular hexagon of side length 6 cm.

(3) (i) Construct a straight line segment *AB* of length 5 cm.
(ii) Create two equilateral triangles, each of which has *AB* as a side.

(4) (i) Construct a circle of radius 4 cm.
(ii) Construct a regular hexagon with its vertices on the above circle.
(iii) By producing three sides of the hexagon obtain an equilateral triangle.

(5) (i) Construct a circle of radius 5 cm.
(ii) Construct a regular hexagon with its vertices on the above circle.
(iii) Construct three equilateral triangles on three sides of the hexagon, leaving out a side of the hexagon between two triangles.
(iv) What is the shape of the total figure?

Summary

- The construction of an equilateral triangle can be done in four steps.
  - Construct a straight line segment.
  - Taking the same length as the straight line segment onto the pair of compasses, construct an arc placing the point of the pair of compasses at one end of the line segment.
  - Construct an arc from the other end point, using the same length as above, such that it intersects the earlier arc.
  - Join the intersection point to the end points of the straight line segment.
- A regular hexagon can be constructed by performing the following steps.
  - Construct a circle.
  - Divide the circle into 6 equal parts by intersecting the circle with arcs of the same length as the radius of the circle.
  - Join the points of intersection.

25

Solids

By studying this lesson you will be able to
- prepare models of a square pyramid and a triangular prism,
- draw the net of a square pyramid and a triangular prism on a square ruled paper, and
- know Euler's relationship for the above solids by considering the number of edges, vertices and faces of these solids.

25.1 Introduction of Solids

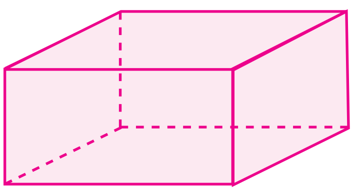

A die

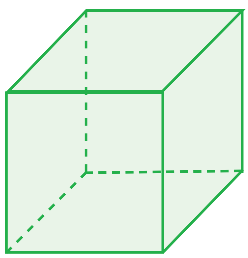

A brick

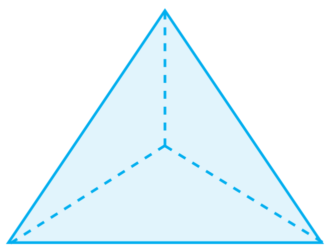

A putt


A concrete post

You have learnt that an object such as a die, an iron ball or a concrete pillar, which has a specific shape and which occupies a certain amount of space is called a solid object. You have also learnt in grade 6 that the surfaces of solid objects can be plane surfaces or curved surfaces.


A cuboid


A cube


A regular tetrahedron

Do the review exercise to recall the facts you have learnt about solids.

Review Exercise

(1) (i) Write down the number of faces, edges and vertices of a cuboid.
(ii) Draw a net that can be used to construct a cuboid.

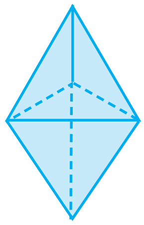
(2) (i) What is the shape of a face of a cube?
(ii) Draw a net that can be used to construct a cube.

(3) Write down the number of faces, edges and vertices of a regular tetrahedron.

(4) (i) Draw the shape of a face of a regular tetrahedron.
(ii) Draw a net that can be used to make a regular tetrahedron.

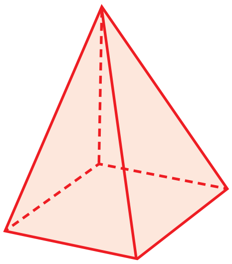
(5) Below is the figure of a solid object constructed by pasting two faces of two identical regular tetrahedrons, one on the other.

(i) How many faces are there in this solid?
(ii) How many edges are there in this solid?
(iii) How many vertices are there in this solid?



25.2 Square pyramid

Tombs of the Pharaohs who ruled in Egypt were built in this shape. They are called pyramids.

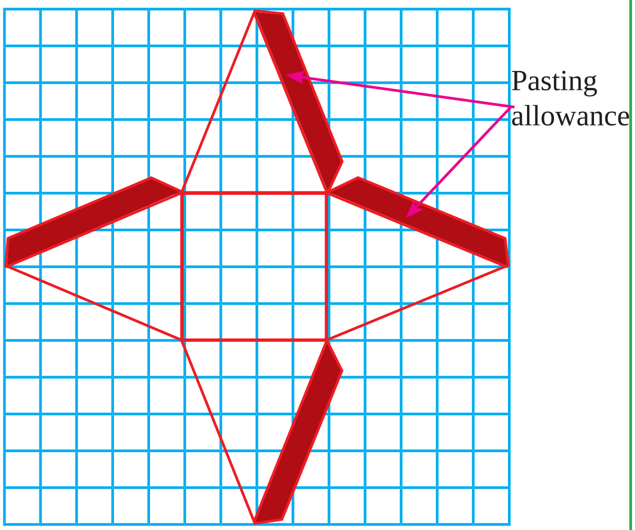


A solid object with a square base and four equal triangular faces is called a **square pyramid**. The figure illustrates a square pyramid.

Let us identify the characteristics of a square pyramid by engaging in the following activity.

Activity 1

Step 1- Draw the given figure on a square ruled paper. Cut out the figure that you drew and either copy it or paste it on a thick piece of paper such as a Bristol board.

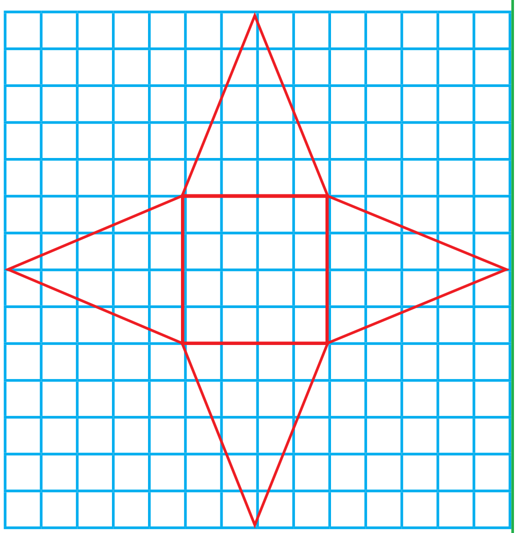


Step 2 - Cut out the figure drawn or pasted on the Bristol board and prepare a model of a square pyramid by folding along the edges and pasting along the pasting allowances.

Step 3 - Based on the model you prepared, find the number of faces, edges and vertices of a square pyramid. Examine the specific features of the model.

Step 4 - Write down the specific features you identified in your exercise book.

Step 5 - Measure and write down the lengths of the edges of the model.



The figure you obtain by removing the pasting allowances of the above figure which was used to prepare a model of a square pyramid, is called the "**net of the square pyramid**".

The object you prepared during the above activity is a model of a square pyramid.

Features you can identify in a square pyramid

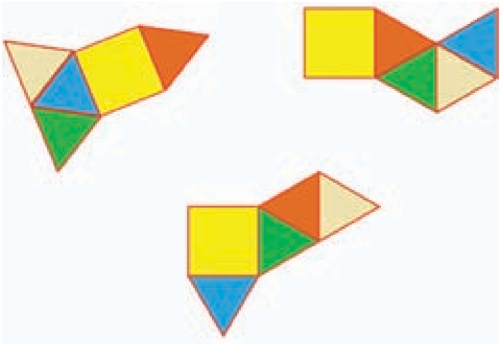
- There are 5 faces in a square pyramid.
- One face has a square shape.
- The other 4 faces take the shape of equal triangles.
- There are 5 vertices in a square pyramid.
- There are 8 edges in a square pyramid. All are straight edges.

Activity 2

(1) Draw each shape given in the figure on a square ruled paper.

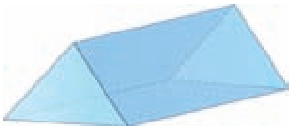
(2) Cut out each shape, fold along the edges and paste them using sellotape.

(3) What is the name of each of the solids you get?



25.3 Triangular Prism

A figure of a kaleidoscope which is an object through which a pattern of multiple images can be observed is given here. It is made out of 3 rectangular plane mirrors.

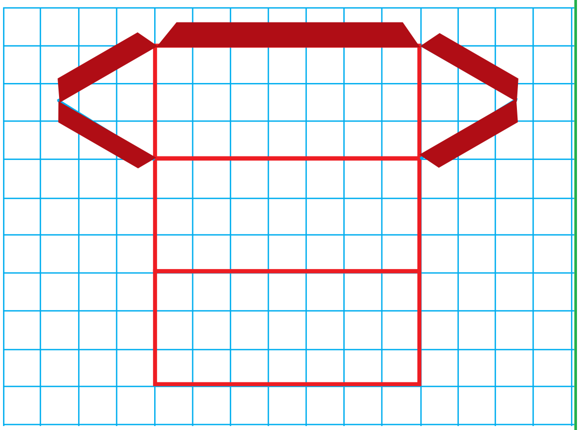


A solid object which has 3 rectangular plane faces and two triangular faces is called a "**triangular prism**".

Let us identify the characteristics of a triangular prism by engaging in the following activity.

Activity 3

Step 1 - Draw the given figure on a square ruled paper. Cut out the figure that you drew and either copy it or paste it on a thick piece of paper such as a Bristol board.

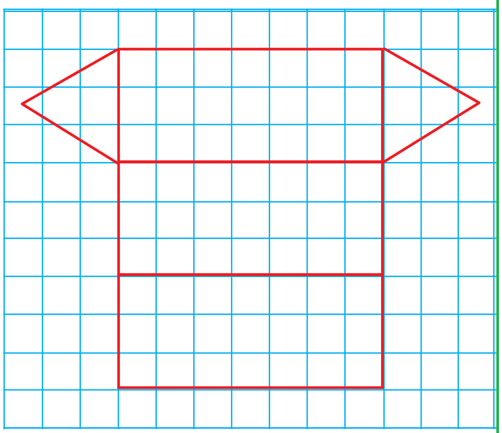


Step 2 - Cut out the figure drawn or pasted on the Bristol board and prepare a model of a triangular prism by folding along the edges and pasting along the pasting allowances.

Step 3 - Based on the model you prepared, find the number of faces, edges and vertices of a triangular prism. Examine other specific features of the model.

Step 4 - Write down the specific features you identified in your exercise book.

The figure you obtain by removing the pasting allowances of the above figure which was used to prepare a model of a triangular prism, is called the "**net of the triangular prism**".



Features you can identify in a triangular prism

- There are 5 faces in a triangular prism.
- There are 2 triangular shaped faces in a triangular prism. They are equal in size and shape.
- The other 3 faces of a triangular prism are of rectangular shape.
- There are 9 edges in a triangular prism. All are straight edges.
- There are 6 vertices in a triangular prism.

Exercise 25.1

(1) Write down the number of faces, edges and vertices of a square pyramid.

(2) Construct two square pyramids of equal measurements using Bristol board.

(i) Paste together the square faces of the two pyramids you constructed.

(ii) Write down the number of faces, edges and vertices of the solid object you obtained in the above step.

(3) Draw the figure of another net which can be used to prepare a square pyramid.

(4) Write down the number of faces, edges and vertices of a triangular prism.

(5) Write down the number of faces, edges and vertices of the solid you obtain by overlapping and pasting together two equal rectangular faces of two identical triangular prisms.

(6) Draw different nets that can be used to construct a triangular prism.

25.4 Euler's Relationship

Fill in the blanks in the table given below based on the solids you studied in grade 6 and by observing the solids you constructed in activities 1 and 3.

Solid	Number of vertices (V)	Number of faces (F)	Sum of the number of vertices and the number of faces $(V+F)$	Number of edges (E)
Cube	8	6	$8+6=14$	12
Cuboid	....	....	....	....
Regular tetrahedron	....	....	....	....
Square pyramid	....	....	....	....
Triangular prism	....	....	....	....

After completing the table, turn your attention to the $(V+F)$ column and the "E" column. With regard to the above solids, notice that the values in the $(V+F)$ column are always greater by 2 than the values in the "E" column.

Accordingly, for the above solids, the sum of the number of faces and the number of vertices is equal to the value obtained by adding 2 to the number of edges.

Number of Vertices	+	Number of Faces	=	Number of Edges + 2
V	+	F	=	$E + 2$

The above relationship which is true for solids with plane faces only, was first presented in the 18th century by a Swiss mathematician called "Leonhard Euler" who lived in Switzerland. Therefore this relationship was later called Euler's formula.

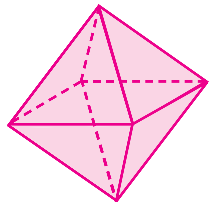
Exercise 25.2

(1) A certain solid has 6 faces and 8 vertices. Find the number of edges of the solid using Euler's relationship.

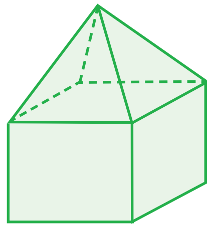
(2) If a certain solid has 8 edges and 5 faces, find the number of vertices of the solid.

(3) Verify Euler's relationship for a triangular prism, by considering the number of faces, vertices and edges it has.

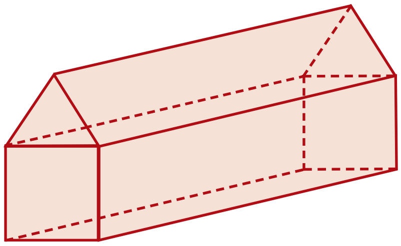
(4) A solid constructed by coinciding and pasting the square faces of two identical square pyramids is shown in the figure.



(i) Find the number of edges, faces and vertices of this solid.
(ii) Show that the above values satisfy Euler's relationship.



(5) A solid constructed by combining a cube and a square pyramid is shown in the figure. Find the number of edges, faces and vertices of the solid and check whether they satisfy Euler's relationship.



(6) The solid shown in the figure has been constructed using a cuboid and a triangular prism. Validate Euler's relationship for this solid.

(7) Construct a cube and 6 square pyramids with bases that are equal to a face of the cube. Construct a composite solid by pasting the square faces of the 6 pyramids on the six faces of the cube.
(i) How many edges, faces and vertices are there in the composite solid?
(ii) Do these values agree with Euler's relationship?

Summary

- A solid that consists of a square base and 4 identical triangular faces having a common vertex is called a square pyramid.
- A square pyramid consists of 8 edges, 5 faces and 5 vertices.
- A solid with 3 rectangular faces and 2 parallel triangular faces is called a triangular prism.
- A triangular prism consists of 9 edges, 5 faces and 6 vertices.
- If a solid has E number of edges, F number of faces and V number of vertices, $V + F = E + 2$ denotes Euler's relationship.

26 Data Representation and Interpretation

By studying this lesson you will be able to

- represent data in column/bar graphs and multiple column graphs and
- interpret data represented in column graphs and multiple column graphs

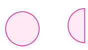
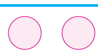
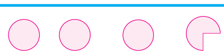
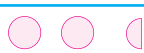
26.1 Column/Bar Graphs

Let us consider briefly what you learnt in Grade 6 about representing data in tables and pie-charts.

The given table provides information on how 39 employees of a certain office travel to work. The employees have been divided into four groups based on their mode of transport. Each group is called a category.

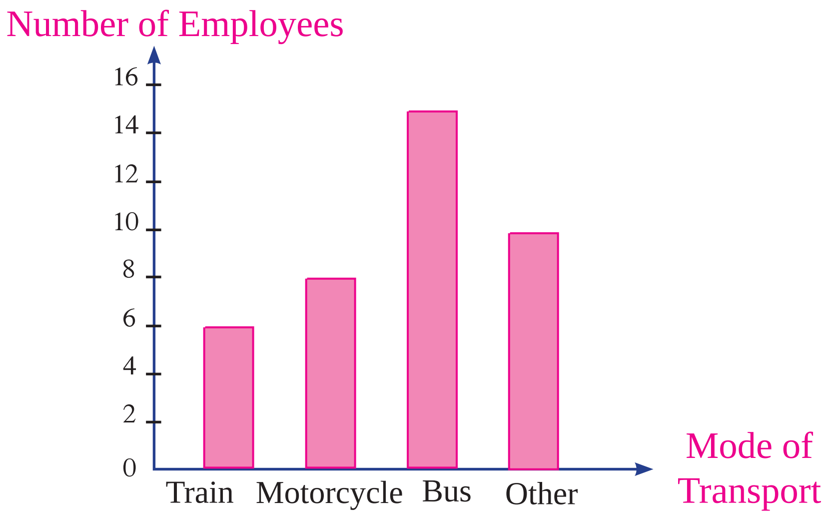
Mode of Transport	Number of Employees
Train	6
Motorcycle	8
Bus	15
Other	10

Let us represent this information in a picture graph. Let us represent 4 employees by  . Accordingly, two employees are represented by half a circular shape  , three employees are represented by three quarter of a circular shape  and one employee is represented by a quarter of a circular shape  .

Mode of Transport	Number of Employees
Train	
Motorcycle	
Bus	
Other	

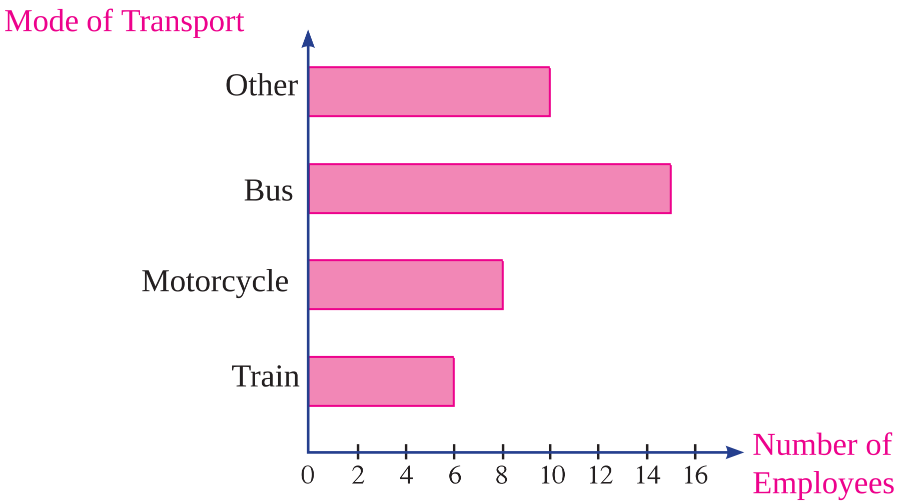
 Represents four employees

Let us represent this information using columns of equal width instead of pictures. Then a graph of the following form is obtained.



Graphs of the above form are called **column graphs**. These columns are of equal width and the gaps between the columns too are equal in size. The height of each column is equal to the number (amount) corresponding to the given category.

This information can also be represented by horizontal bars. Then the graph of the following form is obtained.

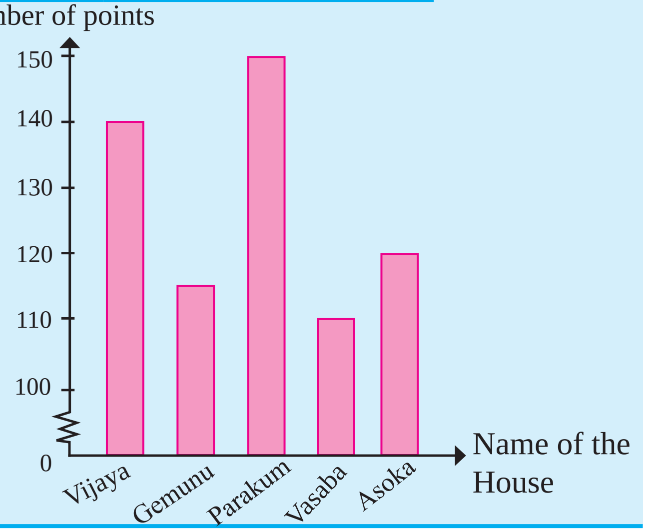


Example 1

The total points gained by each of the houses at the 2015 annual sports meet of a school with more than 5000 students are given in the following table. Represent this information in a column graph.

Name of the House	Total number of points
Vijaya	140
Gemunu	115
Parakum	150
Vasaba	110
Asoka	120

To indicate that the distance between 0 and 100 along the vertical axis in the column graph, which represents the total number of points, is less than what is should be, the symbol ≷ is used.

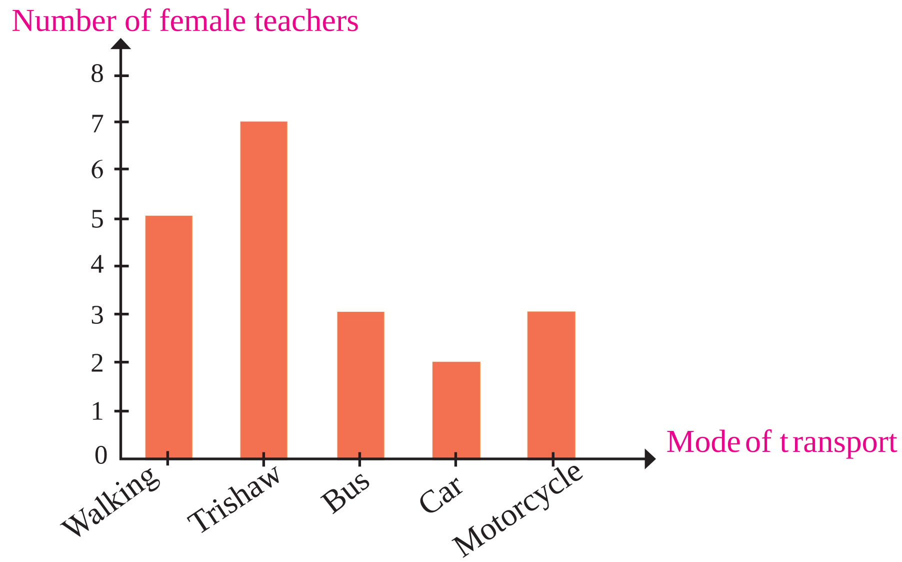


26.2 Multiple Column Graphs

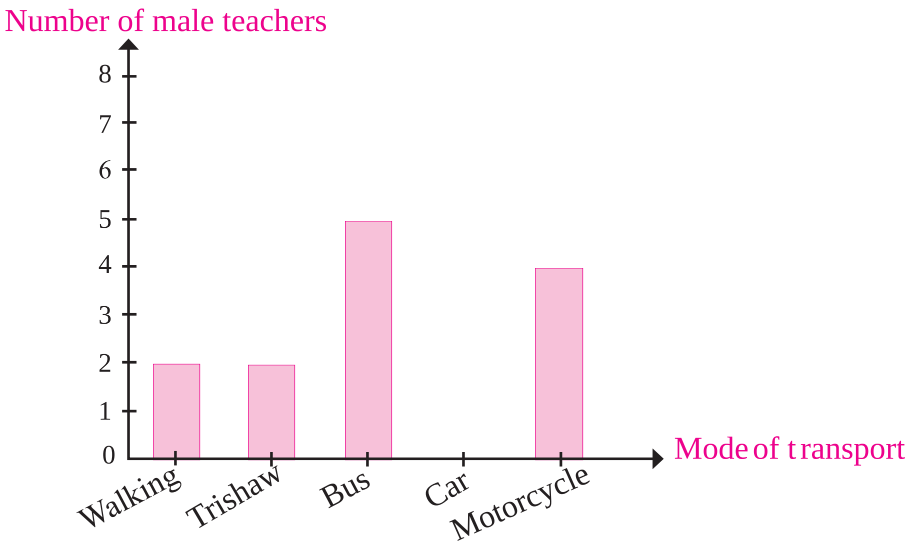
The following table provides information on how the teachers of a certain Maha Vidyalaya in a rural area travel to school. The teachers have been separated into five categories, and each category has been further divided into two subcategories named Male and Female.

Mode of Transport	Teacher	
	Female	Male
Walking	5	2
Trishaw	7	2
Bus	3	5
Car	2	0
Motorcycle	3	4

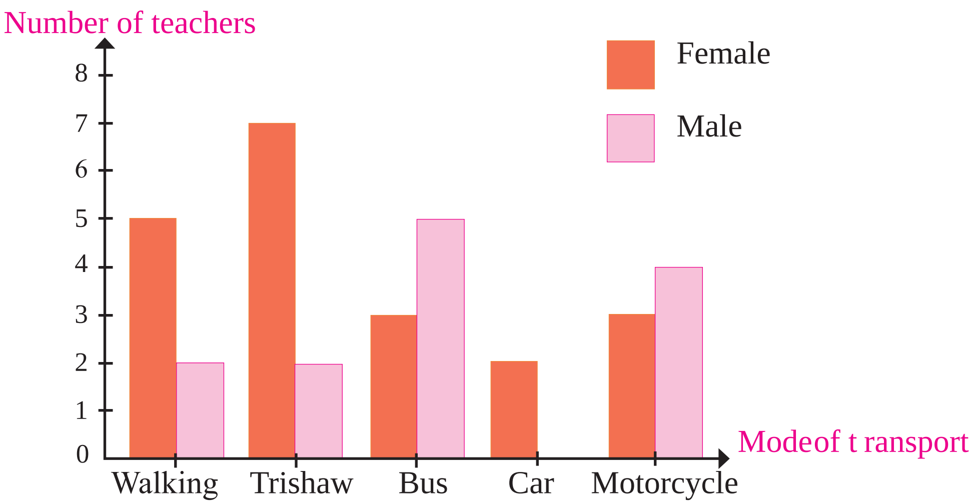
Information on how the female teachers travel to school is represented in the following column graph.



Information on how the male teachers travel to school is represented in the following column graph.



Information on how all the teachers travel to school is represented in the following multiple column graph.



In this graph too the columns are of equal width. The columns corresponding to the subcategories of each category have been drawn such that they border each other. Such graphs are called **multiple column graphs**.

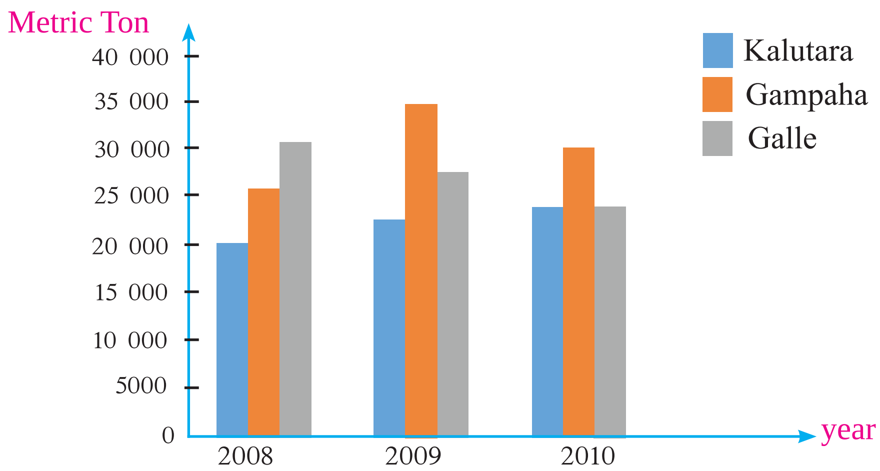
In the above example, the first two graphs have been drawn to provide an explanation. When drawing such graphs, represent all the information in one graph as in the third graph.

By representing the information by a multiple column graph, the information can be compared more easily.

26.3 Interpretation of Data

Now let us extract information from column graphs and multiple column graphs.

The following graph provides information on the paddy production during the Yala season in the districts of Gampaha, Kalutara and Galle.



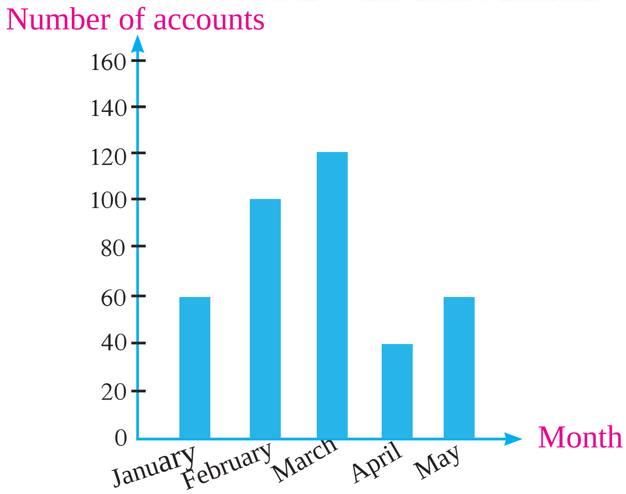
Let us carefully observe the above graph.

- It is a multiple column graph.
- During the Yala season, in the Gampaha district, the paddy production was highest in the year 2009 and lowest in the year 2008.
- The paddy production in the Kalutara district gradually increased during the period 2008 – 2010.
- The paddy production in the Galle district gradually decreased during the period 2008 – 2010.
- The paddy production in the year 2008 in all three districts was 75 000 metric tons.

Several conclusions that were drawn from the graph have been given above.

Exercise 26.1

(1) The following graph represents information on the savings accounts that were opened during the first five months of a year at a branch of a certain bank.



(i) In which month has the most number of savings accounts been opened?

(ii) In which month has the least number of savings accounts been opened?

(iii) In which two months have an equal number of savings accounts been opened?

(iv) How many savings account holders opened their accounts in the month of January?

(v) How many savings account holders in total opened accounts during the period from January to March?

(vi) How many more people had opened accounts in March than in April?

(2) A table with information on the number of coconuts that were plucked from a certain estate during the year 2014 is given below.

Month	Coconut Yield (To the nearest 10 fruits)
January	200
March	280
May	200
July	400
September	250
November	150

Represent this information by a column graph and answer the following questions based on the graph.

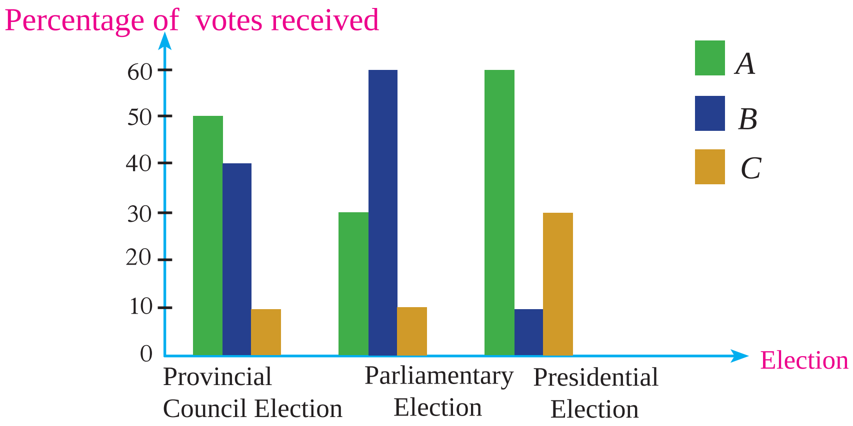
(i) Name the month with the highest yield.

(ii) In which month was the yield the lowest?

(iii) Write down the two months in which the yields were the same.

(iv) Is it easier to extract information from the table or from the column graph?

(3) The following graph represents information on the percentage of votes received by three political parties from the total votes cast during the three most recently held elections in a certain electoral district.



(a) Answer the following questions based on the above multiple column graph.

(i) Which party has received the most number of votes in the Provincial Council Election?

(ii) Which party has succeeded in increasing their percentage of votes from the Provincial Council Election to the Parliamentary Election?

(iii) In which election has the political party A received the highest percentage of votes?

(iv) Which party has received a lesser percentage of votes in the Presidential Election than in the Parliamentary Election?

(v) Which party received the highest percentage of votes in the Parliamentary Election?

(b) Draw another multiple column graph to represent the above information with the horizontal axis denoting the percentage of votes received by the three parties *A*, *B* and *C* and the vertical axis representing the three elections.

(4) A table prepared by the sports teacher of a certain school on the types of sports that students in grades 6 – 11 participate in is given below. Each grade has 100 students. (Assume that students who participate in one type of sport do not participate in the other type).

Grade	Number of Students	
	Indoor sports	Outdoor sports
6	10	90
7	35	65
8	15	85
9	15	85
10	40	60
11	45	55

Represent the information in the above table by a suitable multiple column graph and answer the following questions.

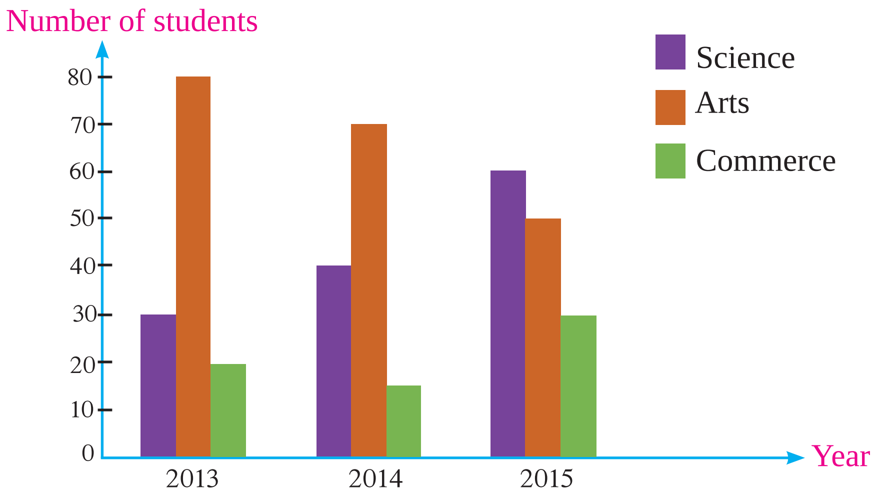
(i) Students of which grade participate in outdoor sports the most?

(ii) Which grade has the most number of students participating in indoor sports?

(iii) Which grade has the least number of students participating in outdoor sports?

(iv) Which grade has the greatest difference between the number of students who participate in outdoor sports and the number of students who participate in indoor sports?

(5) The following multiple column graph provides information on the number of students who entered the different A'level subject streams at a certain school during three successive years.



(i) Which stream shows a gradual increase in the number of students entering the stream?

(ii) Which stream shows a gradual decrease in the number of students entering the stream?

(iii) In which year has the greatest number of students joined the A'level classes of this school?

(iv) If all the students who joined the A'level class in 2013 sat the examination in 2015, how many students in total faced the A'level examination in 2015 from this school?

Summary

- When data has been represented in a column graph or a multiple column graph, it can be interpreted, and information can be compared by considering the heights of the columns of the graph.

27 Scale Diagrams

By studying this lesson you will be able to

- identify scale diagrams, and
- draw scale diagrams and calculate actual measurements using the scale.

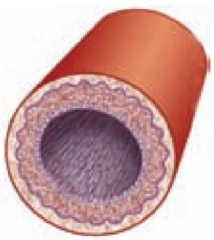
27.1 Scale Diagrams

When the shapes of various objects in the environment are being drawn, it is most often difficult to draw them to the actual measurements of the shape. In such situations, the shape is drawn by decreasing or increasing the measurements by a common ratio depending on the size of the shape.

Since the figure is drawn by increasing or decreasing all the measurements by a common ratio, the shape of the figure will be exactly the same as the original shape and only the size will be different. Figures drawn in this manner are called **scale diagrams**. A few such scale diagrams are shown below.

The plan of the floor area of a house; size has been decreased

The map of Sri Lanka; size has been decreased



The cross section of a blood vessel; size has been increased

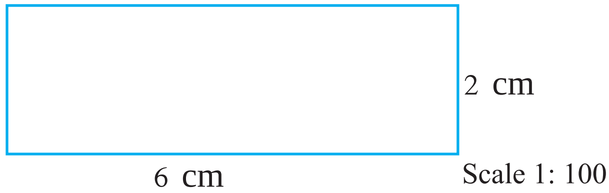
27.2 The Scale of a Scale Diagram

Suppose you want to draw a scale diagram of a flower bed of length 6 m and breadth 2 m in your book. You need to first select a suitable scale.

Suppose 1 cm in the scale diagram represents a length of 1 m of the flower bed.

Since 1 m equals 100 cm, a length of 1 cm in the scale diagram represents 100 cm of actual length. As the same unit has been used, this can be expressed as a ratio as 1:100. This ratio is considered as **the scale** of the scale diagram.

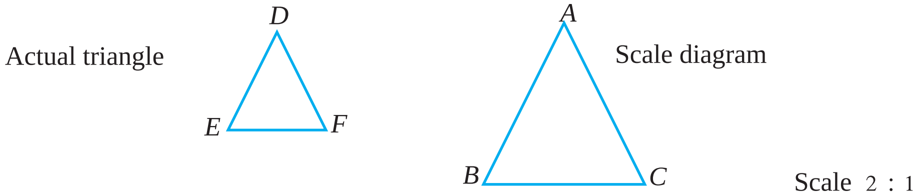
Based on the selected scale, a scale diagram of the flower bed can be drawn, with the actual length of the flower bed which is 6 m represented by 6 cm and the actual breadth which is 2 m represented by 2 cm in the scale diagram.



The scale written as 1:100 in the figure expresses the fact that an actual length of 100 cm is represented by 1 cm in the scale diagram.

Observe carefully how the scales of various scale diagrams are indicated.

The scale diagram of the given triangle has been drawn to the scale 2:1.



Example 1

Express as a ratio, the scale of a scale diagram where 200 cm is represented by 1cm.

Since the same unit has been used, the scale can be expressed as a ratio as 1:200.

Example 2

Express as a ratio, the scale of a scale diagram where 9 m is represented by 2 cm.

Length represented by 2 cm $= 9$ m
Length represented by 2 cm $= 900$ cm
Length represented by 1 cm $= 900 \div 2$ cm
$= 450$ cm

The scale is $1 : 450$

Example 3

Express as a ratio, the scale of a scale diagram where 2 mm is represented by 1 cm.

Length represented by 1cm $= 2$ mm
Length represented by 10 mm $= 2$ mm

The scale is $10 : 2$ or $5 : 1$
This scale is used to magnify a small object.

Exercise 27.1

(1) Express the scale as a ratio in each of the following cases.

(i) Representing 20 cm by 1 cm
(ii) Representing 8 m by 2 cm
(iii) Representing 1 m by 4 cm
(iv) Representing 1 mm by 5 cm
(v) Representing 6 mm by 3 cm

27.3 Drawing scale diagrams

Let us gain an understanding of scale diagrams by considering the following examples.

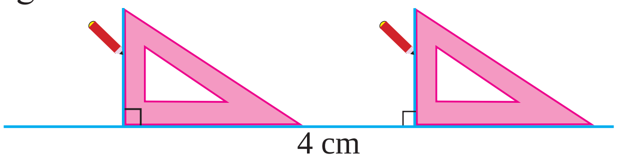
Let us draw a scale diagram of the blackboard in the classroom.

- The blackboard is rectangular in shape.
- Its length is 4 m and its breadth is 1 m.
- Let us consider that 1 m is represented by 1 cm as the scale. That means the scale is 1:100.
- So the scale diagram should be a rectangle of length 4 cm and breadth 1 cm.
- Let us mark the measurements in a sketch.

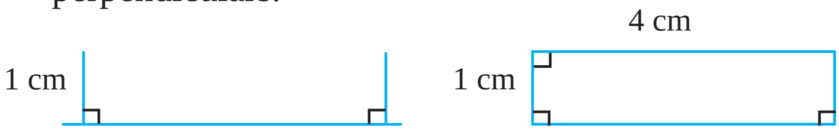
Follow the given steps to draw the scale diagram with this length and breadth.

Step 1 - Draw a straight line segment of length 4 cm using the ruler and the pencil.

Step 2 - Draw two perpendiculars of length 1 cm each at the two ends of the straight line segment using the set square as shown in the figure.



Step 3 - Complete the rectangle by joining the end points of the two perpendiculars.



Exercise 27.2

(1) The length of a hall in a particular school is 20 m and the width (breadth) is 8 m.

  (i) Select a suitable scale to draw the floor plan of the hall and write it as a ratio.

  (ii) Draw a scale diagram of the floor plan of the hall.

(2) The side length of a square shaped land is 24 m. Draw a scale diagram of the land using the scale 1:600.

(3) The length of a rectangular building is 30 m and the width is 18 m.

  (i) Select a suitable scale to draw the scale diagram of the floor of the building.

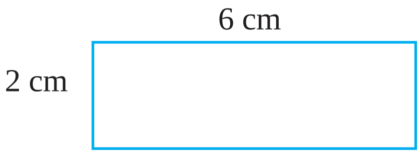
  (ii) Draw the scale diagram of the floor of the building using the selected scale.

27.4 Obtaining actual measurements from scale diagrams

Let us see how the actual measurements can be obtained from a given scale diagram by considering a few examples.

A scale diagram of a land drawn to the scale 1:500 is shown in the figure. Let us find;

(i) the actual length of the land,
(ii) the actual width of the land,
(iii) the actual area of the land.



The scale 1:500 indicates that 500 cm or 5m of the actual length of the land is represented by 1 cm in the scale diagram.

Therefore;

(i) the actual length of the land $= 6 \times 5 \text{ m} = 30 \text{ m}$
(ii) the actual width of the land $= 2 \times 5 \text{ m} = 10 \text{ m}$
(iii) the actual area of the land $= \text{length} \times \text{width} = 30 \times 10 \text{ m}^2$
$= 300 \text{ m}^2$

Example 1

A square shaped land is drawn to the scale 1: 400. The side length of the scale drawing is 2.5 cm. Calculate the side length of the land.

1 : 400 means that 400 cm or 4 m is represented by 1 cm in the scale diagram.

Therefore, side length of the land $= 2.5 \times 4 \text{ m}$
$= 10 \text{ m}$

Example 2

What length in a scale diagram drawn to the scale 1: 10 000 represents an actual length of 1 km?

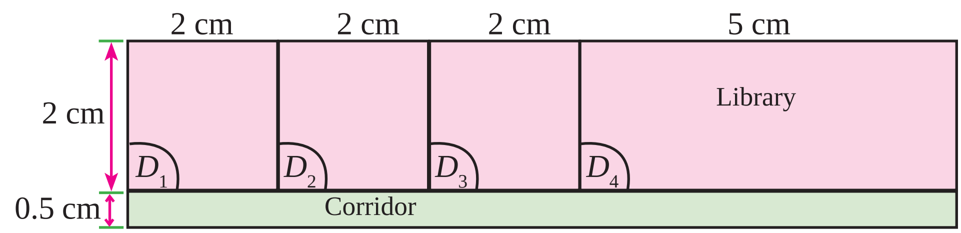
1:10 000 means that 10 000 cm is represented by 1 cm in the scale diagram.
$10\ 000 \text{ cm} = 100 \text{ m} = 0.1 \text{ km}$
That is, 0.1 km is represented by 1 cm in the scale diagram.
$\therefore$ 1 km is represented by 10 cm in the scale diagram.

Exercise 27.3

(1) In a map drawn to the scale 1:200,
   (i) find the actual length represented by 3 cm.
   (ii) find the actual length represented by 5 cm.
   (iii) what length in the map represents an actual length of 8 m?

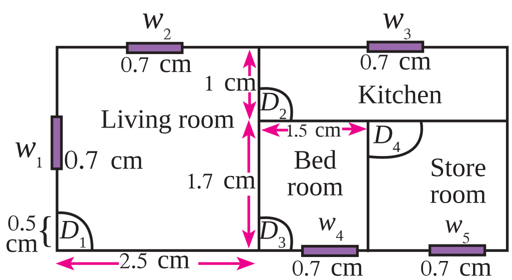
(2) In a map drawn to the scale 1:200 000,
   (i) what is the actual distance between two cities indicated by a distance of 7 cm?
   (ii) what length in the map represents a distance of 1 km?
   (iii) If the distance from Colombo to Balangoda along the A4 road is 142 km, what is the distance between the two cities in the map?

(3) A scale diagram of the ground floor of a multi-storey building in a school is shown below. The floor plan consists of 3 classrooms, a library and a corridor. The scale is 1:200.



   (i) Find the actual length and width of a classroom in metres.
   (ii) Find the actual area of a classroom.
   (iii) Find the actual area of the library.
   (iv) Find the actual area of the corridor.

(4) The floor plan of a house is shown in the figure. The scale is 1:200.



   (i) Find the actual width of the door D_1.
   (ii) Find the actual length of the window w_1.
   (iii) Find the actual length and width of the bedroom and hence find the area of the bedroom.

(iv) Find the area of the living room.

(v) It is proposed to lay tiles in the living room. Estimate the number of square tiles of side length 50 cm required for this purpose.

Summary

- When a scale diagram of a shape is being drawn, it has to be done by decreasing or increasing the measurements by a common ratio, depending on the size of the shape.
- The ratio of a unit length to the actual length represented by a unit length in a scale diagram is considered as the scale of the scale diagram.

28

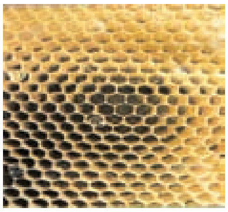
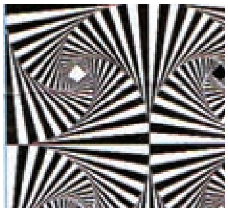
Tessellation

By studying this lesson you will be able to
- understand what tessellation is,
- identify pure tessellations and semi pure tessellations, and
- create tessellations.

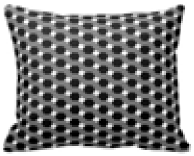
28.1 Introducing Tessellation

Figures of surfaces which are attractive due to a certain shape occurring repeatedly in an organized manner are given below. Each of these creations enhances the beauty of the environment.

The fact that the shape that recurs is of one size and the shapes are organized in a pattern without any gaps in between them reveals the wonder of nature. Let us consider such creations further.

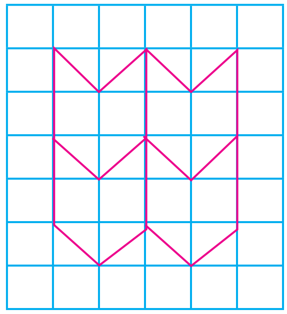
We have seen how bricks and tiles have been laid in attractive designs on the floors, roofs and courtyards of places of worship. Moreover, most bed spreads and clothes have beautiful designs on them. Several such designs are shown below. See whether you can identify the shapes in them.



Tessellation is the process of creating a design consisting of the repeated use of one or more shapes, closely fitted together without gaps or overlaps, on a plane surface.

Based on this description, we can identify the creations in the above figures as tessellations.

Activity 1



Step 1 - Create a design by repeatedly drawing the shape in this figure on a page in your square ruled exercise book.

Step 2 - Colour your design appropriately and make it an attractive work of art.

On completing the above activity you would have ended up with a very attractive tessellation.

28.2 Pure Tessellation

Activity 2

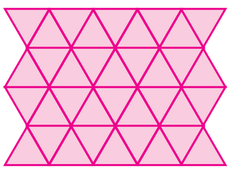


Figure 1

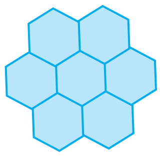


Figure 2

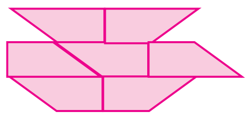


Figure 3

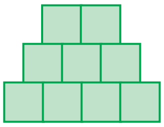


Figure 4

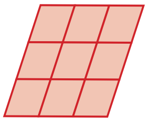


Figure 5

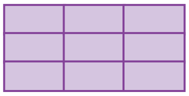


Figure 6

Figures of several tessellations that have been created using various shapes are given above. Copy the table given below and complete it after carefully observing the above tessellations.

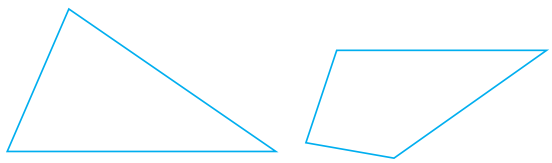
Figure	Sketch of the shape
1	
2	
3	
4	
5	
6	

From the above activity it is clear that tessellation can be done using various shapes.

Tessellation that is done using just one shape is called pure tessellation.

According to this, all the tessellations considered in the above activity are pure tessellations.

Activity 3



Step 1 - Copy the triangle in the figure and cut out 10 triangular laminas of the same size using coloured paper.

Step 2 - By using the cut out laminas, create a pure tessellation and paste it on a page of your exercise book.

Step 3 - Copy the given quadrilateral and create a pure tessellation as above, and paste it on a page of your exercise book.

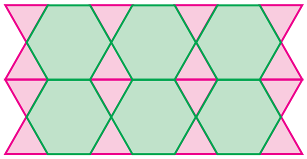
Exercise 28.1

(1) Write down two facts that need to be considered when creating a tessellation.

(2) What is a pure tessellation?

(3) Create a pure tessellation by using any shape you like and paste it on your exercise book.

28.3 Semi pure tessellation

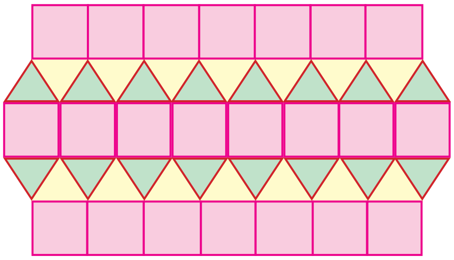


The above figure shows two tessellations that have been created using different shapes. Examine and see whether you can identify the shapes in the two figures.

Tessellation that is done using two or more different shapes is called semi pure tessellation.

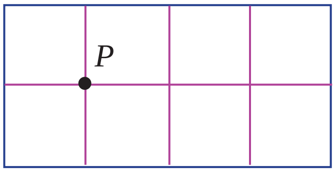
Activity 4

The figure shows a tessellation that has been created using triangles and quadrilaterals.



Create another tessellation using triangles and quadrilaterals and paste it in your exercise book.

The figure shows a tessellation that has been created using squares. A point at which vertices of several of these squares meet has been marked as P. As depicted in the figure, the angles of four squares are around the point P. Let us consider the sum of the angles around the point P.



The magnitude of an interior angle of the square $= 90^{o}$

$\therefore$ the sum of the angles around the point $P = 90^{o} \times 4 = 360^{o}$

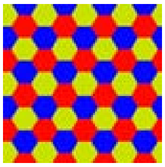
We can similarly show that the sum of the angles around any point is equal to 360^{o}.

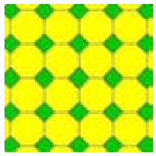
The sum of the angles around a vertex point of a tessellation created using rectilinear plane figures is 360^{o}.

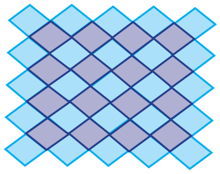
Accordingly, the shapes that are selected to create a tessellation should be such that the angle of 360° around a point can be covered by them without gaps and overlaps.

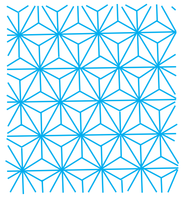
Exercise 28.2

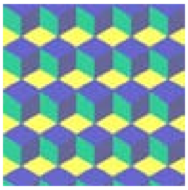
(1) For each of the following tessellations, write down with reasons whether it is a pure tessellation or a semi pure tessellation.

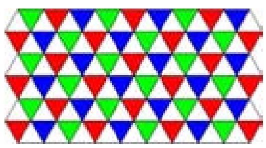

(i)

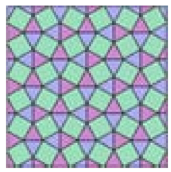

(ii)


(iii)

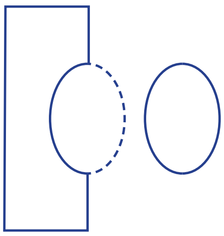

(iv)

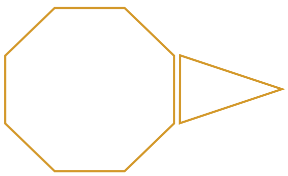

(v)

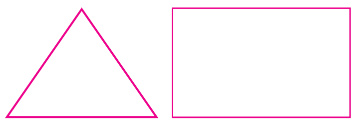

(vi)


(vii)

(2) From the following, select the pairs of shapes that can be used to create semi pure tessellations.


(a)

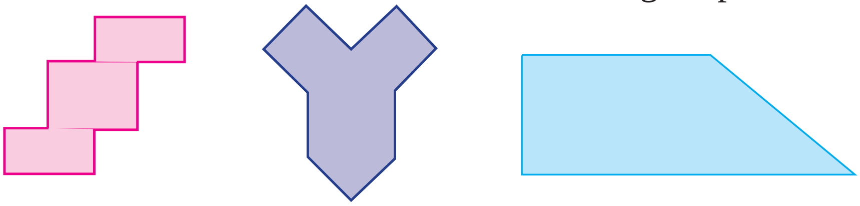

(b)


(c)

Activity 5

(1) Create a semi pure tessellation using two or more shapes that you like, and paste it in your exercise book.

(2) Create tessellations with each of the following shapes.



28. 4 Creating tessellation designs

Activity 6

Step 1 - Cut out a rectangular shaped lamina.

Step 2 - On the lamina that you cut out, draw any shape that you like as shown in Figure 1. Now cut and separate out the shape you drew.

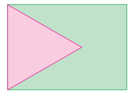


Figure 1

Step 3 - Paste the two parts that you obtained in Step 2 on a piece of cardboard as shown in Figure 2.

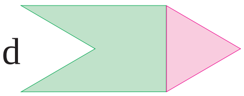
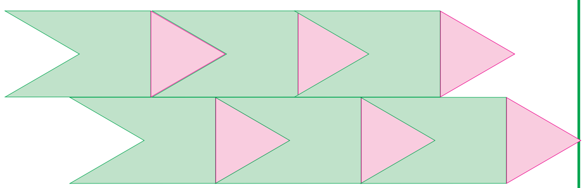


Figure 2

Step 4 - Using the net that you prepared in Step 3, cut out laminas using coloured paper and create a tessellation design.



- Following the steps of activity 6 above, create various attractive tessellation designs using different nets and display them.

Summary

- Tessellation is the process of creating a design consisting of the repeated use of one or more shapes, closely fitted together without gaps or overlaps, on a plane surface.
- Tessellation done using just one shape is called pure tessellation.
- Tessellation done using two or more shapes is called semi pure tessellation.

29 Likelihood of an Event Occurring

By studying this lesson you will be able to
- identify the events that definitely occur, the events that definitely do not occur and the event that occur randomly and
- describe the outcomes of an experiment.

29.1 Events

Let us consider the following events.

1. A stone which is lifted and released, falling downwards
2. The sun rising from the West
3. A tossed coin landing heads up
4. The next page turned in a Mathematics book being a whole numbered page
5. The cricketer getting out in the next ball
6. Appearance of the moon on a new-moon day
7. The sun rising tomorrow
8. Rain occurring this afternoon
9. A heavy stone floating on water
10. A train leaving at the scheduled time

Now let us divide these events into the following three types; the events that definitely occur, the events that definitely do not occur and the events which we cannot be certain will occur or not.

We know that the events 1, 4 and 7 will definitely occur. We also know that the events 2, 6 and 9 will definitely not occur. If we consider event 3, we cannot say for certain that a tossed coin will land heads up. Likewise it is not certain whether the events 5, 8 and 10 will occur or not.

So, the events we come across are of three types. They are, events that definitely occur, events that definitely do not occur and events that we cannot be certain will occur or not. The events which we cannot be certain will occur or not are called **random events**.

Activity 1

Write 2 examples each of random events, events that definitely occur and events that definitely do not occur.

Discuss your answers with the others in the class.

Exercise 29.1

(1) Write down whether each of the events given below, is an event which definitely occurs, an event which definitely does not occur or a random event.

(a) Of the two competing football teams A and B, team A winning the game

(b) When a red regular cube is tossed, the side that lands up being red

(c) A ball taken out of a bag which contains only 5 white balls, being a black ball

(d) The next passenger getting down from a bus being a woman

(e) When a regular cube with each of its six sides marked with one of 1, 2, 3, 4, 5 and 6 is rolled, the side with 5 turning up

(f) A stone thrown at a Mango tree with fruits, hitting a Mango

(g) A piece of wood placed on water, floating

(h) The youngest participant in a 100 m race for those under thirteen winning the race

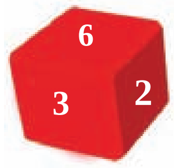
(i) Chathuri being a student who obtains more than 75 marks for Mathematics in the grade 7 year end examination

(2) In a school having 700 students, a prefect is chosen by the votes of all the students. Aravinda and Suranga have been proposed for this position.

(i) Find the least number of votes that Aravinda must get if he is to become the prefect.

(ii) Will a prefect always be elected through this method?

(3) The faces of a die are numbered 1, 2, 3, 4, 5 and 6. The die is rolled once. Write down whether each of the events given below is an event which will definitely occur, an event which will definitely not occur or a random event.



(i) Obtaining 8
(ii) Obtaining an even number
(iii) Obtaining 4
(iv) Obtaining a number which is less than 7

29.2 Experiments and Outcomes

The event, "the first commuter getting down from the bus is a woman" is a random event. This is because the first commuter getting down from the bus can be either a woman or a man. Before someone gets down from the bus, we cannot be certain which of these two events will occur. The experiment here is "observing whether the first commuter getting down from the bus is a woman or a man". The outcome will be either "the commuter is a woman" or "the commuter is a man".

For the event of "a stone which is lifted and released falling downwards" the relevant experiment is "observing a stone which is lifted and released". The outcome is "the stone falling downwards".

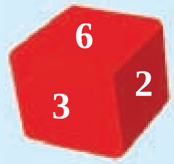
Furthermore, in experiments such as observing whether the sun rises from the East and observing whether a stone lifted and released falls downwards, the outcomes are definitely known before the experiment is conducted.

Let us consider the event of "a tossed coin landing tails up". In this case we cannot be certain whether heads or tails will land up. So the event is a random event. Here, the experiment is "observing the side that lands up when a coin is tossed". The outcome will be either "heads landing up" or "tails landing up".

Let us consider the event of “rain occurring this afternoon”. This is a random event. The experiment is “observing whether it rains this afternoon”. The outcome will be either “raining this afternoon” or “not raining this afternoon”.

Example 1

The faces of a die are numbered 1, 2, 3, 4, 5 and 6. The die is rolled once and the number on the face that turns upward is observed. Write the set of outcomes of this experiment.



getting 1, getting 2, getting 3, getting 4, getting 5 and getting 6.

Exercise 29.2

(1) Write down the experiments and corresponding outcomes for each of the events in a, b, c, d and e under Exercise 29.1 (1).

29.3 The likelihood of obtaining each of the possible outcomes of an experiment

Let us examine the nature of each of the experiments given below.

- The faces of a regular die are numbered 1, 2, 3, 4, 5 and 6. The die is rolled once and the number on the face that turns upward is observed.

  The outcomes of this experiment are getting 1, getting 2, getting 3, getting 4, getting 5 and getting 6. If each of these outcomes is equally likely to occur, then the die used in this experiment is called a “fair die” or an “unbiased die”.

- A coin is tossed once and the side that lands up is observed.

The outcomes of this experiment are getting head and getting tail. If either one of these outcomes is equally likely to occur, then the coin used in this experiment is called “a fair coin “or an “unbiased coin”

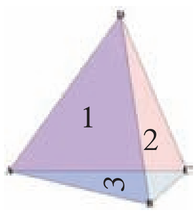
- A coin with one side made of aluminum and the other side of copper, where the quantities of aluminum and copper used are equal, is tossed once and the side that lands up is observed.

The outcomes of this experiment are “the aluminum side lands up” and “the copper side lands up”. Since the density of copper is more than the density of aluminum, the likelihood of the aluminum side landing up is greater than the likelihood of the copper side landing up. Therefore this is not a fair coin.

- A coconut shell similar to the one in the figure is tossed once and the side that lands up is observed.

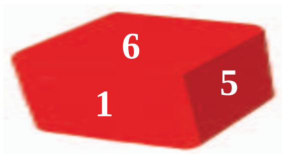
The outcomes of this experiment are “the shell lands up and “the shell lands down. Although there are only two outcomes, the likelihood of landing with the shell up is greater than the likelihood of landing with the shell down. Therefore, the coconut shell is not an unbiased object.

- The faces of a regular tetrahedron are numbered 1, 2, 3 and 4. The tetrahedron is rolled once and the number on the face that turns downward is observed.



The outcomes of this experiment are getting 1, getting 2, getting 3 and getting 4. If each of these is equally likely to occur, then this regular tetrahedron is a fair one.

- The faces of a cuboid are numbered 1, 2, 3, 4, 5 and 6. The cuboid is rolled once and the number on the face that turns upward is observed.



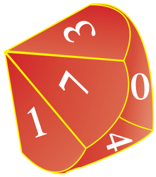
The outcomes of this experiment are getting 1, getting 2, getting 3, getting 4, getting 5 and getting 6. The areas of the faces of the cuboid are not the same. It is more likely for the cuboid to land with a face having the greatest area downwards. Therefore, of the six outcomes, certain events are more likely to occur than the others. So the cuboid is not a "fair object".

If each of the outcomes of an experiment is equally likely to occur, then the object used in the experiment is called a fair or an unbiased object.

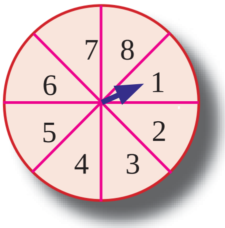
Exercise 29.3

(1) For each experiment below, write the set of outcomes and write down whether the experiment is carried out using a biased object or an unbiased object.

(i) The top in the figure, with its faces marked from 0 to 9 is spun and the face that touches the ground when it stops spinning is observed.



(ii) The figure shows a circular disc divided into 8 equal parts. The parts are numbered 1 to 8. One end of the indicator is fixed to the center and the other end is rotated. The number of the portion where the indicator stops is observed.



(2) Consider the figure below. Each circular disc is made to rotate around its centre. When the discs stop rotating, the colour indicated by the arrow head is observed. Explain whether each of the discs used in this experiment is fair or not.

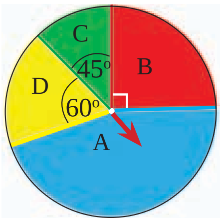


Figure 1

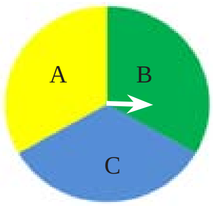


Figure 2

(3) Write two examples of experiments which are done using fair objects.

Summary

- The events which occur in our environment fall into one of the following three types. The events which definitely occur, the events which definitely do not occur and random events.
- The events that can occur in an experiment are called the outcomes of the experiment.
- If an experiment is carried out using an object, the object is considered to be an unbiased object (fair object) if all the outcomes of the experiment are equally likely and it is considered to be a biased object if the outcomes are not equally likely.

Revision Exercise 3

(1) (i) Write down a ratio equivalent to $2:8:5$.

(ii) Write down the number of faces, edges and vertices of a square pyramid.

(iii) Write down $1\frac{2}{5}$ as a decimal number.

(iv) Find the value of $64 - 125 \div 5$.

(v) Solve $2x + 8 = 16$.

(vi) Write down the ratio $14:49:35$ in its simplest form.

(vii) Find the highest common factor and least common multiple of 63 and 42.

(viii) Construct the straight line segment AB of length 6 cm.

(ix) Construct a circle of radius 4 cm.

(x) Write down the number of faces, edges and vertices in a triangular prism.

(xi) Write down all possible outcomes of the experiment of rolling an unbiased cubic die which has its six sides marked 1, 2, 3, 4, 5 and 6.

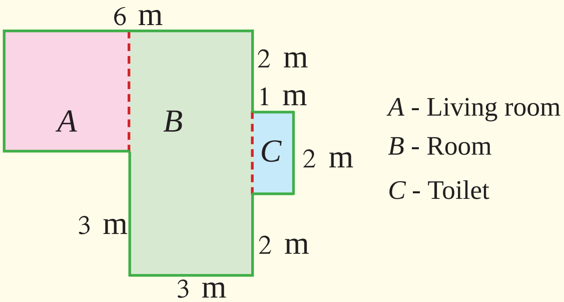
(xii) The length and width of a rectangular land drawn to the scale $1:200$ are 7 cm and 2.5 cm respectively. Find the actual length and width of the land.

(xiii) In a nutritious instant food packet, green gram, soya and rice are mixed in the ratio $1:1:3$. Find the amount of rice that is included in one such 100 g food packet.

(xiv) Write down Euler's relationship.

(xv) Construct an equilateral triangle of side length 8 cm. Name it ABC.

(2) The floor plan of a restroom in a tourist inn is shown below.



(i) The living room is square shaped. What is the length of a side of this room?

(ii) Find the area of the living room.

(iii) Find the area of the room.

(iv) Find the area of the toilet.

(v) Find the total perimeter of the restroom.

(vi) It is required to tile the floor of the room with 50 cm $\times$ 50 cm square tiles. Find the number of tiles that can be laid in a widthwise row and the number of tiles that can be laid in a lengthwise row. Thereby, obtain the total number of tiles that is required for this purpose.

(vii) Draw a scale diagram of this floor plan using the scale 1 : 100.

(viii) What is the ratio of the length of the room to that of the toilet?

(3) (a) It has been decided to recruit male and female workers to a newly opened garment factory in the ratio 4 : 9.

(i) If the total number of workers that are to be recruited is 260, find separately, the number of male and female workers that are to be recruited.

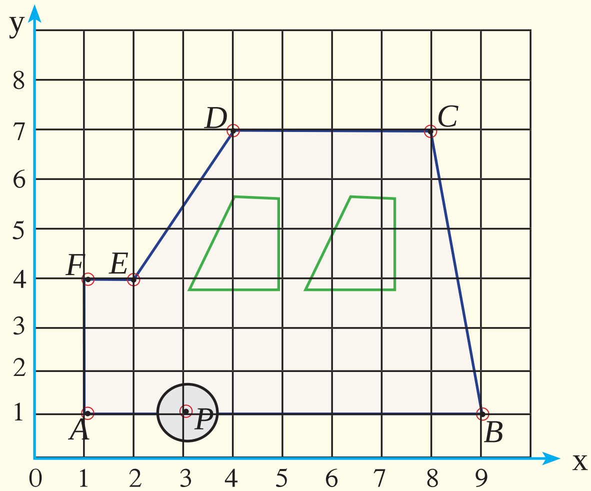
(ii) The ratio of the monthly salary of a male worker to that of a female worker is 5 : 4. If the monthly salary of a female worker is Rs 24 000, find the monthly salary of a male worker.

(4) 25 contestants participated in the 1st round of a poetry recitation competition. 12 contestants qualified for the 2nd round.

(i) Express the number of contestants who qualified for the 2nd round as a fraction of the total number of contestants.

(ii) Express the number of contestants who qualified for the 2nd round as a percentage of the total number of contestants.

(5) An incomplete figure of a motor car which has been drawn in a Cartesian plane is shown here.



(i) Draw this diagram in a Cartesian plane.

(ii) Which point is represented by the ordered pair (4,7)?

(iii) Write down the coordinates of the points *A*, *P*, *B*, *C*, D, *E* and *F* as ordered pairs.

(iv) If the coordinates of the centre of the back wheel is (7, 1), mark this centre and draw the wheel.

(6) (i) Construct a circle of radius 6 cm.

(ii) Construct a regular hexagon with its vertices on this circle.

(iii) Construct an equilateral triangle on each side of the hexagon, external to it.

(iv) Find the perimeter of one of the two largest triangles that you get when you complete the above step.

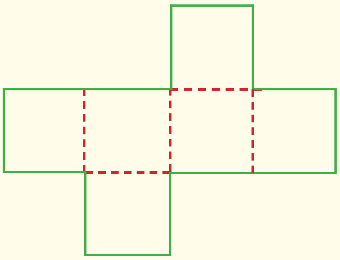
(v) What is the shape you get when you connect the vertices of the 6 equilateral triangles that do not lie on the original hexagon?

(7) (i) 5 m is represented by 1 cm in a scale diagram. Express this scale as a ratio.

(ii) Find the actual length of a house which is represented by 8 cm in a scale diagram drawn to the scale 1: 200.

(iii) The length of a school building is 20 m and its width is 6 m. Draw a scale diagram of this building using the scale 1 : 100.

(8) A net of a solid object is shown here. There are 6 equal squares of side length 6 cm.



(i) Write down the name of the solid that can be constructed by folding along the dotted lines.

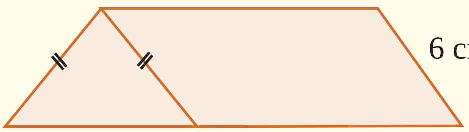
(ii) Considering the number of vertices, edges and faces of this solid object, show that Euler's relationship is satisfied by these values.

(iii) Obtain the total surface area of the solid by finding the area of each face.

(iv) Find the length of an edge of a solid of the same shape whose total surface area is 384 cm^2.

(v) Show that the volume of that solid is 512 cm^3.

(9) A prism is shown in the figure. The triangular faces are isosceles.

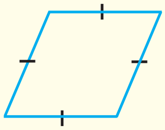
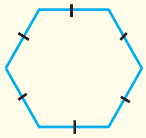
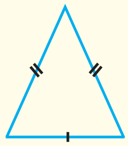
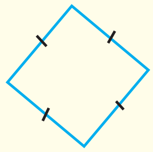


(i) Draw the 3 rectangular faces of the prism separately and mark their dimensions.

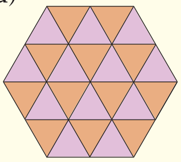
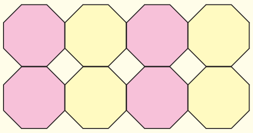
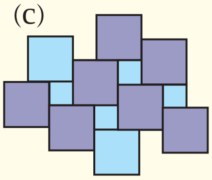
(ii) Find the area of each of these faces separately.

(iii) There are 10 edges and 6 vertices in a solid with plane faces. Find the number of faces that the solid has using Euler's relationship.

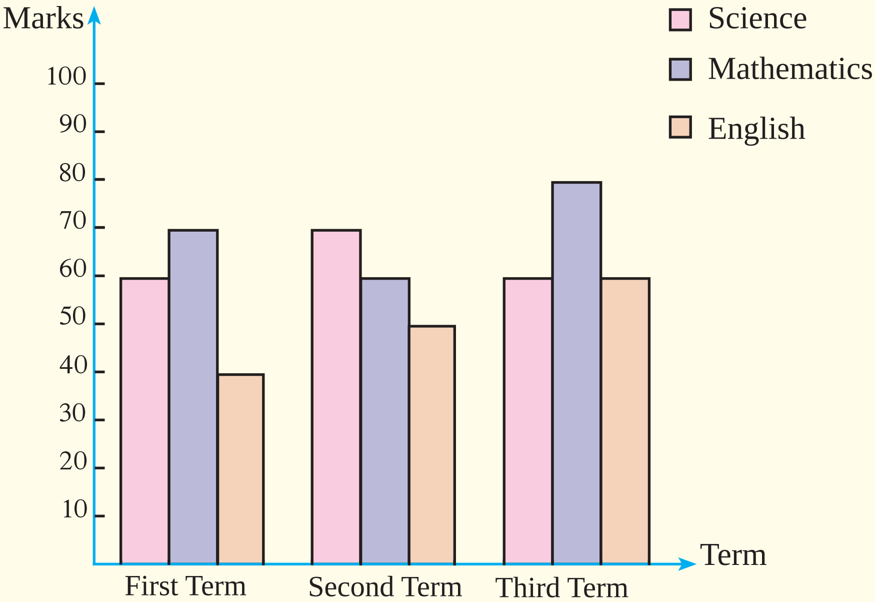
(10) (i) From the following plane shapes, select the ones that can be used for pure tessellation.

(a)

(b)

(c)

(d)


(ii) Select and separately write down the pure tessellations and the semi pure tessellation.

(a)

(b)

(c)


(11) The marks obtained by a student during 3 terms for Mathematics, Science and English are shown in the multiple column graph.



(i) Which subject shows a continuous increase in the marks?

(ii) For which subject has the student obtained identical marks in two terms?

(iii) By how many marks has the total marks obtained in the 3rd term for all 3 subjects increased when compared with the total marks obtained in the 1st term for all 3 subjects?

(12) If each employee is provided with 7.5 metres of material to sew uniforms, calculate the number of metres of material that is required for 12 employees.

(13) If the thickness of a DVD is 2.3 mm, find the thickness of a package consisting of 5 such DVDs.

Lesson Sequence

Content	Number of Periods	Competency levels
First Term		
1. Bilateral Symmetry	05	25.1
2. Sets	05	30.1
3. Mathematical Operations on Whole Numbers	04	1.1
4. Factors and Multiples	11	1.3, 1.4
5. Indices	06	6.1
6. Time	05	12.1
7. Parallel Straight Lines	03	27.1
8. Directed Numbers	06	1.2
9. Angles	07	21.1, 21.2
	52	
Second Term		
10. Fractions	10	3.1
11. Decimals	05	3.2
12. Algebraic Expressions	06	14.1, 14.2
13. Mass	06	9.1
14. Rectilinear Plane Figures	06	23.1, 23.2
15. Equations and Formulae	08	17.1, 19.1
16. Length	08	7.1, 7.2
17. Area	06	8.1
18. Circles	04	24.1
19. Volume	05	10.1
20. Liquid Measurements	04	11.1
	68	
Third Term		
21. Ratios	05	4.1
22. Percentages	05	5.1
23. Cartesian Plane	05	20.1
24. Construction of Plane Figures	05	27.2
25. Solids	05	22.1, 22.2
26. Data Representation and Interpretation	08	28.1, 29.1
27. Scale Diagrams	06	13.1
28. Tessellation	05	26.1
29. Likelihood of an Event Occurring	06	31.1, 31.2
	50	
Total	170	

Glossary

Acute - angled triangle	සුළු කෝණී ත්‍රිකෝණය	கூர்ங்கோண முக்கோணி
Area	වර්ගඵලය	பரப்பளவு
Biased	නැඹුරු	சமநேர்தகவற்ற
Category	ප්‍රවර්ග	வகைகுறி
Centre	කේන්ද්‍රය	மையம
Circle	වෘත්තය	வட்டம்
Closed plane figures	සංවෘත තලරූප	மூடிய தளவுரு
Column graph/ bar graph	තීර ප්‍රස්තාර	சலாகை வரைபு
Compound plane figures	සංයුක්ත තලරූප	கூட்டுத் தளவுருக்கள
Concave polygon	අවතල බහු-අස්‍රය	குழிவுப் பல்கோணி
Construction	නිර්මාණය	அமைப்பு
Convex polygon	උත්තල බහු-අස්‍රය	குவிவுப் பல்கோணி
Cartesian plane	කාටීසීය තලය	தெக்காட்டின் தளம்
Coordinates of a point	ලක්ෂ්‍යයක ඛණ්ඩාංක	புள்ளியொன்றின் ஆள்கூறுகள
Cube	ඝනකය	சதுரமுகி
Cuboid	ඝනකාභය	கனவுரு
Data	දත්ත	தரவுகள்
Desired units	අභිමත ඒකක	எதேச்சை அலகுகள்
Diameter	විෂ්කම්භය	விட்டம
Edge	දාරය	விளிம்பு
Equilateral triangle	සමපාද ත්‍රිකෝණය	சமபக்க முக்கோணி
Equilateral triangle	සමපාද ත්‍රිකෝණය	சமபக்க முக்கோணி
Euler's relationship	ඔයිලර්ගේ සම්බන්ධතාව	ஒயிலரின் தொடர்பு
Event	සිද්ධි	நிகழ்ச்சி
Experiment	පරීක්ෂණ	பரிசோதனை
Face	මුහුණත	முகம்
Formula	සූත්‍රය	சூத்திரம்
Information	තොරතුරු	தகவல்கள
Isosceles triangle	සමද්විපාද ත්‍රිකෝණය	இருசமபக்க முக்கோணி
Length	දිග	நீளம்
Line segment	සරල රේඛා ඛණ්ඩය	நேர்கோட்டுத் துண்டம்
Liquid measurements	ද්‍රව මිනුම්	திரவ அளவீடுகள்
Multiple-column graph	බහු තීර ප්‍රස්තාර	கூட்டுச் சலாகை வரைபு
Obtuse - angled triangle	මහා කෝණී ත්‍රිකෝණය	விரிகோண முக்கோணி
Occurrence	සිදු වීම	நிகழ்வு ∴ நேர்கை
Origin	මූල ලක්ෂ්‍යය	உற்பத்தி

Pair of compasses	කවකටුව	கவராயம்
Percentage	ප්‍රතිශතය	சதவீதம்
Perimeter	පරිමිතිය	சுற்றளவு
Polygon	බහු අස්‍රය	பல்கோணி
Prism	ප්‍රිස්මය	அரியம்
Probability	සම්භාවිතාව	நிகழ்தகவு
Pure tessellation	ශුද්ධ ටෙසලාකරණය	தூய தெசலாக்கம
Pyramid	පිරමීඩය	கூம்பகம்
Radius	අරය	ஆரை
Random event	අහඹු සිදුවීම්	எழுமாற்று நிகழ்வு
Ratio	අනුපාතය	விகிதம்
Rectangle	සෘජුකෝණාස්‍රය	செவ்வகம்
Regular hexagon	සවිධි ෂඩස්‍රය	ஒழுங்கான அறுகோணி
Regular polygon	සවිධි බහු-අස්‍රය	ஒழுங்கான பல்கோணி
Right angled triangle	සෘජු කෝණී ත්‍රිකෝණය	செங்கோண முக்கோணி
Scale	පරිමාණය	அளவிடை
Scale diagram	පරිමාණ රූප	அளவிடை ப்படம்
Scalene triangle	විෂම ත්‍රිකෝණය	சமனில்பக்க முக்கோணி
Semi - pure tessellation	අර්ධ ශුද්ධ ටෙසලාකරණය	அரைத் தூய தெசலாக்கம
Shapes	හැඩතල	வடிவங்கள்
Solids	ඝන වස්තු	திண்மங்கள்
Square	සමචතුරස්‍රය	சதுரம்
Square pyramid	සමචතුරස්‍ර පිරමීඩය	சதுரக் கூம்பகம்
Standard units	සම්මත ඒකක	நியம அலகுகள்
Straight edge	සරල දාරය	நேர் விளிம்பு
Tessellation	ටෙසලාකරණය	தெசலாக்கம்
Triangle	ත්‍රිකෝණය	முக்கோணி
Triangular prism	ත්‍රිකෝණ ප්‍රිස්මය	முக்கோண அரியம்
Unbiased	නොනැඹුරු	சமநேர்தகவுடைய
Units	ඒකක	அலகுகள்
Vertex	ශීර්ෂය	உச்சி
Volume	පරිමාව	கனவளவு
x - axis	x අක්ෂය	ஒ அச்சு
y - axis	y අක්ෂය	ல அச்சு
x - coordinate	x - ඛණ්ඩාංකය	ஒ ஆள்கூறு
y - coordinate	y - ඛණ්ඩාංකය	ல ஆள்கூறு