MATHEMATICS

Grade 8

Part - I

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The National Anthem of Sri Lanka

Sri Lanka Matha
Apa Sri Lanka Namo Namo Namo Namo Matha
Sundara siri barinee, surendi athi sobamana Lanka
Dhanya dhanaya neka mal palaturu piri jaya bhoomiya ramya
Apa hata sepa siri setha sadana jeewanaye matha
Piliganu mena apa bhakthi pooja Namo Namo Matha
Apa Sri Lanka Namo Namo Namo Namo Matha
Oba we apa vidya
Obamaya apa sathya
Oba we apa shakthi
Apa hada thula bhakthi
Oba apa aloke
Apage anuprane
Oba apa jeevana we
Apa mukthiya oba we
Nava jeevana demine, nithina apa pubudukaran matha
Gnana veerya vadawamina regena yanu mana jaya bhoomi kara
Eka mavakage daru kela bevina
Yamu yamu vee nopama
Prema vada sema bheda durerada
Namo, Namo Matha
Apa Sri Lanka Namo Namo Namo Namo Matha

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ආනන්ද සමරකෝන්

ஒரு தாய் மக்கள் நாமாவோம்
ஒன்றே நாம் வாழும் இல்லம்
நன்றே உடலில் ஓடும்
ஒன்றே நம் குருதி நிறம்

அதனால் சகோதரர் நாமாவோம்
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நன்றாய் இவ் இல்லினிலே
நலமே வாழ்தல் வேண்டுமன்றோ

யாவரும் அன்பு கருணையுடன்
ஒற்றுமை சிறக்க வாழ்ந்திடுதல்
பொன்னும் மணியும் முத்துமல்ல - அதுவே
யான்று மழியாச் செல்வமன்றோ.

ஆனந்த சமரக்கோன்
கவிதையின் பெயர்ப்பு.

Being innovative, changing with right knowledge
Be a light to the country as well as to the world.

Message from the Hon. Minister of Education

The past two decades have been significant in the world history due to changes that took place in technology. The present students face a lot of new challenges along with the rapid development of Information Technology, communication and other related fields. The manner of career opportunities are liable to change specifically in the near future. In such an environment, with a new technological and intellectual society, thousands of innovative career opportunities would be created. To win those challenges, it is the responsibility of Sri Lankan Government and myself, as the Minister of Education, to empower you all.

This book is a product of free education. Your aim must be to use this book properly and acquire the necessary knowledge out of it. The government in turn is able to provide free textbooks to you, as a result of the commitment and labour of your parents and elders.

Since we have understood that the education is crucial in deciding the future of a country, the government has taken steps to change curriculum to suit the rapid changes of the technological world. Hence, you have to dedicate yourselves to become productive citizens. I believe that the knowledge this book provides will suffice your aim.

It is your duty to give a proper value to the money spent by the government on your education. Also you should understand that education determines your future. Make sure that you reach the optimum social stratum through education.

I congratulate you to enjoy the benefits of free education and bloom as an honoured citizen who takes the name of Sri Lanka to the world.

Akila Viraj Kariyawasam
Minister of Education

Foreword

The educational objectives of the contemporary world are becoming more complex along with the economic, social, cultural and technological development. The learning and teaching process too is changing in relation to human experiences, technological differences, research and new indices. Therefore, it is required to produce the textbook by including subject related information according to the objectives in the syllabus in order to maintain the teaching process by organizing learning experiences that suit to the learner needs. The textbook is not merely a learning tool for the learner. It is a blessing that contributes to obtain a higher education along with a development of conduct and attitudes, to develop values and to obtain learning experiences.

The government in its realization of the concept of free education has offered you all the textbooks from grades 1-11. I would like to remind you that you should make the maximum use of these textbooks and protect them well. I sincerely hope that this textbook would assist you to obtain the expertise to become a virtuous citizen with a complete personality who would be a valuable asset to the country.

I would like to bestow my sincere thanks on the members of the editorial and writer boards as well as on the staff of the Educational Publications Department who have strived to offer this textbook to you.

W. M. Jayantha Wickramanayaka
Commissioner General of Educational Publications,
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2019.04.10

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Message of the Boards of Writers and Editors

This textbook has been compiled in accordance with the new syllabus which is to be implemented from 2017 for the use of grade eight students.

We made an effort to develop the attitude "We can master the subject of Mathematics well" in students.

In compiling this textbook, the necessity of developing the basic foundation of studying mathematical concepts in a formal manner was specially considered. This textbook is not just a learning tool which targets the tests held at school. It was compiled granting due consideration to it as a medium that develops logical thinking, correct vision and creativity in the child.

Furthermore, most of the activities, examples and exercises that are incorporated here are related to day to day experiences in order to establish mathematical concepts in the child. This will convince the child about the importance of mathematics in his or her daily life. Teachers who guide the children to utilize this textbook can prepare learning tools that suit the learning style and the level of the child based on the information provided here.

Learning outcomes are presented at the beginning of each lesson. A summary is provided at the end of each lesson to enable the child to revise the important facts relevant to the lesson. Furthermore, at the end of the set of lessons related to each term, a revision exercise has been provided to revise the tasks completed during that term.

Every child does not have the same capability in understanding mathematical concepts. Thus, it is necessary to direct the child from the known to the unknown according to his / her capabilities. We strongly believe that it can be carried out precisely by a professional teacher.

In the learning process, the child should be given ample time to think and practice problems on his or her own. Furthermore, opportunities should be given to practice mathematics without restricting the child to just the theoretical knowledge provided by mathematics.

We would like to bestow our sincere thanks on Dr. Jayampathi Rathnayake, Department of Mathematics, University of Colombo, Dr. Anuradha Mahasinghe, Department of Mathematics, University of Colombo and W. K. A. Samanmalee, WP/ JP/ President College, Maharagama.

Our firm wish is that our children act as intelligent citizens who think logically by studying mathematics with dedication.

Boards of Writers and Editors

Contents

1 Number Patterns

By studying this lesson you will be able to,

- identify the *n*th term of a given number pattern, and
- find any term of a number pattern when the *n*th term is given.

1.1 Number patterns and terms of a number pattern

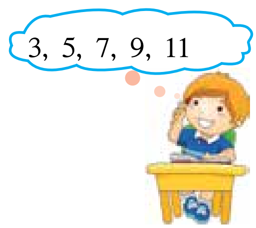
Let us write the odd numbers from 3 to 11 in ascending order.

3, 5, 7, 9, 11

This is the number pattern of the odd numbers from 3 to 11 written in ascending order.

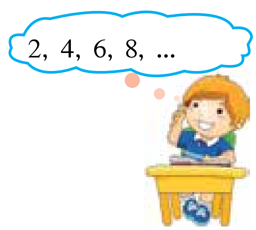
- When numbers are written in a certain order according to a specific method or rule, starting from a certain number, it is called a **number pattern**.
- Every number in a given number pattern is called a **term of the number pattern**.
- The first number of a number pattern is called the first term and the following numbers in order are called the second term, third term, fourth term, etc.
- Commas are used to separate the terms of a number pattern.

Let us consider again the number pattern 3, 5, 7, 9, 11 of the odd numbers from 3 to 11 written in ascending order.



The first term of the above pattern is 3 and the fourth term is 9. The last or the 5th term is 11. There are only five terms in this number pattern. Therefore the number of terms is finite.

Such number patterns, where the number of terms is finite, are called **finite number patterns**.



Let us write the even numbers starting from 2 in ascending order.

2, 4, 6, 8, …

You learnt in Grade 6 that this is the number pattern of the even numbers starting from 2 written in ascending order.

Since the exact number of terms in this number pattern cannot be specified, that is, since it is infinite, we cannot write down all its terms. Therefore, the first few terms are written such that the pattern can be identified, and as above, three dots are used to denote the rest of the terms.

Such number patterns where the number of terms is not finite, are called **infinite number patterns**.

Example 1

Write the terms of each of the following number patterns.

(i) The number pattern of the prime numbers between 1 and 17, written in ascending order.

(ii) The number pattern of the odd numbers starting from 1, written in ascending order.

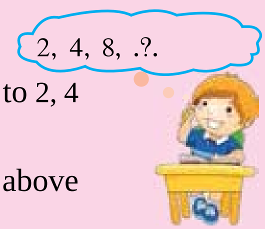
(iii) The number pattern starting from 1 and followed by the terms 2 and 1 written alternatively.

(i) 2, 3, 5, 7, 11, 13 (ii) 1, 3, 5, 7, 9, …

(iii) 1, 2, 1, 2, 1, 2, …

Note

Consider the number pattern 2, 4, 8,…



A number pattern with the first, second and third terms equal to 2, 4 and 8 respectively is given above.

We can easily write two different number patterns with the above first three terms.

(i) 2, 4, 8, 16, 32, 64, …
Here a term is multiplied by 2 to get the next term.

(ii) 2, 4, 8, 10, 20, 22, 44, …
Here, the second term is obtained by adding two to the first term, the third term is obtained by multiplying the second term by two, the fourth term is obtained by adding two to the third term, etc.

An important fact that can be learnt from this is that there can be more than one number pattern having the same first few terms.

Exercise 1.1

(1) Fill in the blanks.

(i) In the number pattern 1, 3, 5, 7, 9, …

the first term = ...
the second term = ...
the fourth term = ...

(ii) In the number pattern 4, 8, 12, 16, 20, …
the first term = ...
the second term = ...
the third term = ...

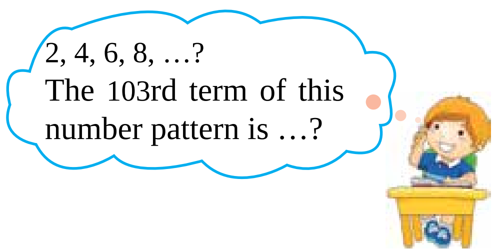
(2) Write the terms of each of the following number patterns.

(i) The number pattern of the even numbers between 1 and 9 written in ascending order.
(ii) The number pattern of the multiples of 6 from 6 to 36 written in ascending order.
(iii) The number pattern of the even numbers greater than 7 written in ascending order.
(iv) The number pattern of the prime numbers starting from 2 written in ascending order.

(3) Copy the below given statements in your exercise book and mark the correct statements with a $\checkmark$ and the incorrect statements with a $\times$.
(i) The terms of a number pattern have to be in ascending order.
(ii) The terms of a number pattern have to be different from each other.
(iii) If the 10th terms of two number patterns are different, then the two number patterns are different to each other.

1.2 The general term of a number pattern

Let us consider how we can easily find any term of a number pattern.



When the nth term of a number pattern is written as an algebraic expression in n, it is called the **general term of the number pattern**.

Using the general term, the value of any term in the number pattern can be found.

The general term of the number pattern of the multiples of a number

➤ Let us consider the number pattern of the multiples of 2, starting from 2 and written in ascending order.

This number pattern is 2, 4, 6, 8, …

Although the terms from the fifth term onwards are not written, we know that the fifth term is 10, the sixth term is 12 and the seventh term is 14.

Let us find the *n*th term of this number pattern.
The table given below, shows how the value of each term is obtained.

Term	Value of the term	How the value of the term is obtained
First term	2	2×1
Second term	4	2×2
Third term	6	2×3
Fourth term	8	2×4
⋮	⋮	⋮
Tenth term	$\boxed{?}$	2×10
⋮	⋮	⋮
*n*th term	$\boxed{?}$	$2 \times n$
⋮	⋮	⋮

According to the 3rd column of the table, the *n*th term of the above number pattern is $2 \times n$; that is $2n$.

The *n*th term of this number pattern is $2n$. This is called the **general term** of this number pattern. By substituting suitable values for n in $2n$, we can obtain the values of the relevant terms.

The value of n in the general term of a number pattern should always be a positive integer.

The above number pattern is the same as **the even numbers starting from 2 and written in ascending order**.

- The general term of the number pattern of the even numbers starting from 2 and written in ascending order is $2n$.
- The general term of the number pattern of the multiples of 2 starting from 2 and written in ascending order is $2n$.

➤ Let us consider the number pattern of the multiples of 3 starting from 3 and written in ascending order.

This number pattern is 3, 6, 9, 12, …

How the values of the terms of this number pattern are obtained is shown in the following table.

Term	**Value of the term**	**How the value of the term is obtained**
First term	3	3×1
Second term	6	3×2
Third term	9	3×3
Fourth term	12	3×4
⋮	⋮	⋮
Tenth term	$\boxed{?}$	3×10
⋮	⋮	⋮
nth term	$\boxed{?}$	$3 \times n$
⋮	⋮	⋮

According to the third column of this table, the nth term in this number pattern is $3 \times n$; that is $3n$.

The general term of the number pattern of the multiples of 3 starting from 3 and written in ascending order is $3n$.

Accordingly,

- the general term of the number pattern of the multiples of 4 starting from 4 and written in ascending order is $4n$.
- the general term of the number pattern of the multiples of 7 starting from 7 and written in ascending order is $7n$.

Example 2

The general term of the number pattern of the multiples of 3 starting from 3 and written in ascending order is $3n$.

(i) Find the 13th term of this number pattern.
(ii) Find which term is 87 of this number pattern.

(i) The general term of the number pattern of the multiples of 3 starting from 3 and written in increasing order is $3n$

The 13th term of this number pattern $= 3 \times 13 = 39$

(ii) $3n = 87$

Let us find the value of n that satisfies this equation.

$$\frac{3n}{3} = \frac{87}{3}$$
$$n = 29$$

$\therefore$ 87 is the 29th term of this number pattern.

Example 3

In the number pattern of the multiples of 4 starting from 4 and written in ascending order, with general term $4n$,

(i) what is the 10th term?
(ii) what is the 11th term?
(iii) which term is 100?
(iv) is 43 a term of this number pattern? What are the reasons for your answer?

(i) The general term of the number pattern of the multiples of $4 = 4n$

$$\text{10th term} = 4 \times 10 = 40$$

(ii) The general term of the number pattern of the multiples of $4 = 4n$

$$\text{11th term} = 4 \times 11 = 44$$

(iii) Since the general term of the number pattern of the multiples of 4 is $4n$,

$$4n = 100$$
$$\frac{4n}{4} = \frac{100}{4}$$
$$n = 25$$

$\therefore$ 100 is the 25th term.

(iv) When $4n = 43$

$$\frac{4n}{4} = \frac{43}{4}$$
$$n = 10\frac{3}{4} \text{ (This is not a positive integer)}$$

$\therefore$ 43 cannot be a term of this number pattern.

43 is not a multiple of 4. Therefore, it can be said that 43 is not a term of this number pattern.

Exercise 1.2

(1) Copy the table given below and complete it.

Number pattern	First term	General term
5, 10, 15, 20, ...		
10, 20, 30, 40, ...		
8, 16, 24, 32, ...		
7, 14, 21, 28, ...		
12, 24, 36, 48, ...		
1, 2, 3, 4, ...		

(2) Write the number pattern of the multiples of 5 between 3 and 33 written in ascending order.

(3) In the number pattern 11, 22, 33, 44, … of the multiples of 11 starting from 11 and written in ascending order,
(i) what is the general term?
(ii) what is the 9th term?
(iii) which term is 121?

(4) In the number pattern 9, 18, 27, 36, … of the multiples of 9 starting from 9 and written in ascending order,
(i) what is the general term ?
(ii) what is the 11th term?
(iii) which term is 270?

(5) In the number pattern with general term $100n$,
(i) what is the 11th term?
(ii) which term is 500?

(6) What is the smallest multiple of 3 larger than 100? Which term is it in the number pattern of the multiples of 3 starting from 3?

(7) What is the nth term (general term) of the pattern of the even numbers greater than 1 but less than 200 written in ascending order? The smallest value of n is 1. What is its largest value?

(8) It has been estimated that in a country having a population of 2 million people, the population will increase by 2 million people every 25 years. Estimate the population of the country in 200 years.

• The general term of the pattern of the odd numbers

You have learnt earlier that odd numbers are numbers which have a remainder of 1 when divided by 2.

1, 3, 5, 7, ... is the pattern of the odd numbers starting from 1 written in ascending order.

Since we obtain a remainder of 1 when an odd number is divided by 2, we should obtain an odd number when 1 is subtracted from any multiple of 2.

Accordingly, let us identify how the pattern of the odd numbers is developed by considering the following table.

Term	Multiples of 2	Multiples of 2 - 1	Odd number
First term	$2 = 2 \times 1$	$(2 \times 1) - 1$	$2 - 1 = 1$
Second term	$4 = 2 \times 2$	$(2 \times 2) - 1$	$4 - 1 = 3$
Third term	$6 = 2 \times 3$	$(2 \times 3) - 1$	$6 - 1 = 5$
⋮	⋮	⋮	⋮
Tenth term	$20 = 2 \times 10$	$(2 \times 10) - 1$	$20 - 1 = 19$
⋮	⋮	⋮	⋮
nth term	$2n = 2 \times n$	$(2 \times n) - 1$	$2n - 1$
⋮	⋮	⋮	⋮

The general term of the pattern of the odd numbers starting from 1 and written in ascending order can be expressed in terms of the general term of the pattern of even numbers starting from 2, written in ascending order.

The general term of the pattern of the odd numbers starting from 1 and written in ascending order is $2n - 1$.

Example 4

In the pattern of the odd numbers 1, 3, 5, 7, ... starting from 1,

(i) what is the general term?
(ii) what is the 72nd term?
(iii) which term is 51?

(i) Since this is the pattern of the odd numbers starting from 1, the general term is $2n - 1$.

(ii) When $n = 72$, the seventy second term
$$= 2 \times 72 - 1$$
$$= 144 - 1$$
$$= 143$$

(iii) Let us take that 51 is the nth term of this number pattern.

Then, $2n - 1 = 51$
$$2n - 1 + 1 = 51 + 1$$
$$2n = 52$$
$$\frac{2n}{2} = \frac{52}{2}$$
$$n = 26$$

51 is the 26th term of this number pattern.

Exercise 1.3

(1) In the pattern of the odd numbers starting from 1 and written in ascending order,

(i) what is the 12th term?
(ii) what is the 15th term?
(iii) which term is 89?
(iv) which term is the greatest odd number less than 100?

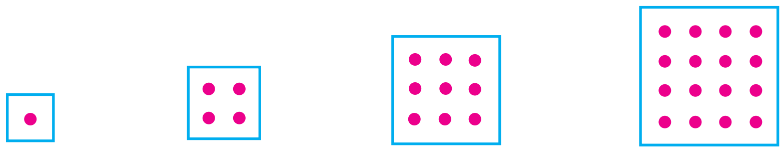
(2) Find the value of the sum of the 34th term of the pattern of the even numbers starting from 2 and the 34th term of the pattern of the odd numbers starting from 1.

• General term of the pattern of the square numbers

You have learnt in Grade 6 that 1, 4, 9, 16, … are the square numbers written in ascending order.

This pattern represented by square shaped figures consisting arrangements of dots is given below.

First term	Second term	Third term	Fourth term



1×1	2×2	3×3	4×4
1^2	2^2	3^2	4^2

The pattern of the square numbers is developed as follows.

First term $= 1 \times 1 = 1^2 = 1$

Second term $= 2 \times 2 = 2^2 = 4$

Third term $= 3 \times 3 = 3^2 = 9$

$\vdots$

Tenth term $= 10 \times 10 = 10^2 = 100$

$\vdots$

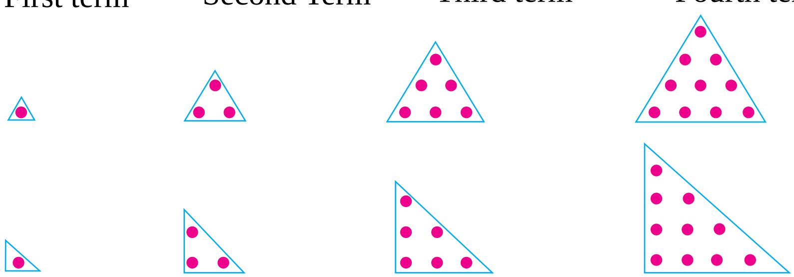
nth term $= n \times n = n^2$

$\therefore$ The general term of the pattern of the square numbers starting from 1 and written in ascending order is n^2.

• The general term of the pattern of the triangular numbers

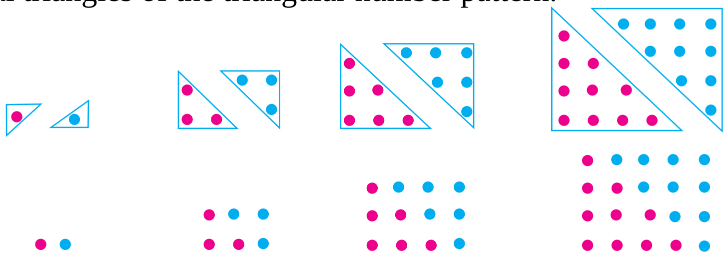
You have learnt in Grade 6 that 1, 3, 6, 10, 15, … are the triangular numbers written in ascending order. They can be represented by dots in both the ways given below.

First term	Second Term	Third term	Fourth term



1	$1 + 2 = 3$	$1 + 2 + 3 = 6$	$1 + 2 + 3 + 4 = 10$

Rectangular shaped arrangements of dots that have twice the number of dots as each corresponding figure in the triangular number pattern can be obtained by joining together two equal triangles of the triangular number pattern.



Number of rows	1	2	3	4
Number of columns	2	3	4	5
Total number of dots	1×2	2×3	3×4	4×5
Triangular number	$\frac{1 \times 2}{2} = 1$	$\frac{2 \times 3}{2} = 3$	$\frac{3 \times 4}{2} = 6$	$\frac{4 \times 5}{2} = 10$

Therefore, the pattern of triangular numbers is as follows.

$$\begin{array}{rcccc}
\text{First term} & = & \frac{1 \times 2}{2} & = & 1 \\
\text{Second term} & = & \frac{2 \times 3}{2} & = & 3 \\
\text{Third term} & = & \frac{3 \times 4}{2} & = & 6 \\
\text{Fourth term} & = & \frac{4 \times 5}{2} & = & 10 \\
\vdots & & \vdots & & \vdots \\
\text{Tenth term} & = & \frac{10 \times 11}{2} & = & 55 \\
\vdots & & \vdots & & \vdots \\
n\text{th term} & = & \frac{n \times (n+1)}{2} & = & \frac{n(n+1)}{2}
\end{array}$$

The general term of the triangular number pattern starting from 1 and written in ascending order is $\frac{n(n+1)}{2}$.

Exercise 1.4

(1) What is the 10th term of the square number pattern starting from 1 and written in ascending order?

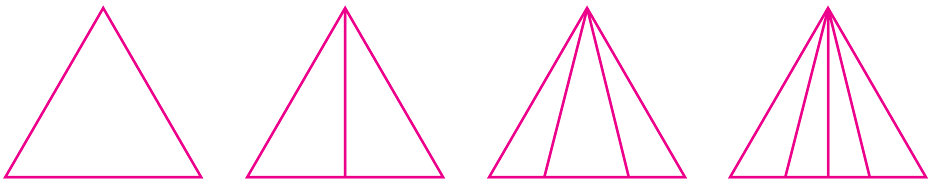
(2) What is the 10th term of the triangular number pattern starting from 1 and written in ascending order?

(3) A certain number greater than 1 and less than 50, which is a term of the square number pattern starting from 1 and written in ascending order, is also a term of the triangular number pattern starting from 1 and written in ascending order.

(i) What is this term?
(ii) Which square number is it?
(iii) Which triangular number is it?

(4) "The sum of the 14th and 15th terms of the triangular number pattern starting from 1 is a square number". Show that this statement is true and find which term it is of the square number pattern.

(5) Write the total number of triangles in each figure in order and see whether you can identify the pattern.



(i) (ii) (iii) (iv)

The pattern of the total number of triangles in the figures in the given order, is identical to the pattern of triangular numbers starting from 1 and written in ascending order. Find the total number of triangles in the 8th figure that is drawn according to this pattern.

(6) Sayuni buys a till and starts saving money by putting one rupee into it on the first day. On the second day she puts 2 rupees, on the third day 3 rupees and so forth. How much money is in the till by the end of the 10th day?

Miscellaneous Exercise

(1) In the pattern of the odd numbers starting from 1, commencing from the first term, if the first two terms, then the first three terms, then the first four terms are added and continued accordingly, a special type of numbers is obtained.

(i) What is the special name given to these numbers?

(ii) Find the number that is obtained if 15 of these terms are added in order starting from the first term.

(2) Milk tins brought to a shop to be sold were arranged on a rack in the following manner.

- 10 tins on the lowest shelf and every other shelf having one tin less than the number on the shelf below it.1 tin on the topmost shelf.

(i) Find the number of milk tins that were brought to the shop.
(ii) All the milk tins on the four topmost shelves were sold within two weeks. Find the number of milk tins that were sold.

(3) What is the sum of the integers from 1 to 30?

What is the difference between a set of numbers and a number pattern?

The pattern of the even numbers between 1 and 9 written in ascending order is 2, 4, 6, 8.

If these four numbers are written in descending order as 8, 6, 4, 2, we obtain a different number pattern.
A is the set of even numbers between 1 and 9.

We can write the set of even numbers between 1 and 9 as follows.
A = {2, 4, 6, 8} = {6, 4, 8, 2} = {8, 6, 4, 2}

Whatever order the numbers 2, 4, 6, 8 are written within brackets, we obtain the same set. Elements of a set are not named as the first element, the second element, etc.

$\therefore$ Although {2, 4, 6, 8} and {8, 6, 4, 2} are the same set, the number pattern 2, 4, 6, 8 is not equal to the number pattern 8, 6, 4, 2.

Summary

- The expression in n obtained for the nth term of a number pattern is called its general term.
- The value of n in the general term of a number pattern should always be a positive integer.
- The general term of the number pattern of the even numbers starting from 2 and written in ascending order is $2n$.
- The general term of the number pattern of the odd numbers starting from 1 and written in ascending order is $2n - 1$.
- The general term of the pattern of the square numbers starting from 1 and written in ascending order is n^2.
- The general term of the triangular number pattern starting from 1 and written in ascending order is $\frac{n(n+1)}{2}$.

Think

(1) Can you construct three different number patterns with 1, 2, 4 as the first three terms?
If you can, then write the next two terms of each of those number patterns.

2 Perimeter

By studying this lesson you will be able to,

- calculate the perimeters of composite rectilinear plane figures composed of two similar or different types of plane figures from equilateral triangles, isosceles triangles, squares and rectangles, and
- solve problems involving the perimeters of composite rectilinear plane figures.

2.1 Perimeter

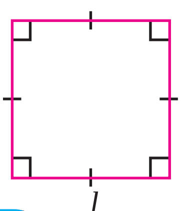
Suppose we need to find the length around a rectangular plot of land. For this we need to obtain the sum of the lengths of all four sides of the plot.

The measurement that is thus obtained is said to be the perimeter of the plot of land.

You have learnt earlier that, the sum of the lengths of all the sides of a closed rectilinear plane figure is called its **perimeter**.

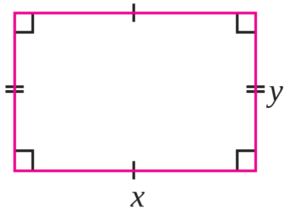
Now let us recall some of the formulae you learnt in Grades 6 and 7 that can be used to find the perimeter of certain plane figures.

- If the perimeter of a square of side length l units is p units, then



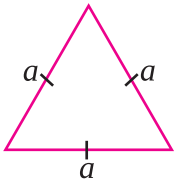
$$p = l + l + l + l$$
$$p = 4l$$

- If the perimeter of a rectangle of length x units and breadth y units is p units, then



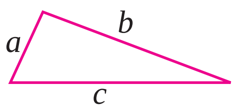
$$p = x + y + x + y$$
$$p = 2x + 2y$$

- If the perimeter of an equilateral triangle of side length a units is p units, then



$$p = a + a + a$$
$$p = 3a$$

- If the perimeter of a triangle with side lengths a, b and c units is p units, then



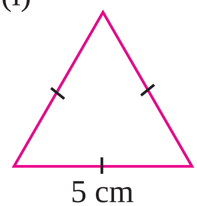
$$p = a + b + c$$

Do the following review exercise to revise what you have learnt.

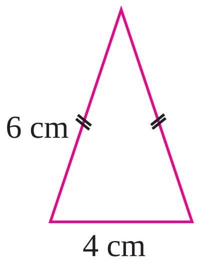
Review Exercise

(1) Find the perimeter of each of the figures given below.

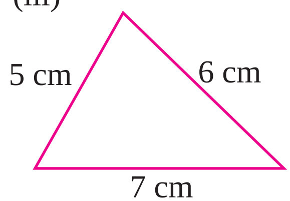
(i)



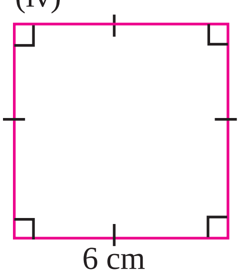
(ii)



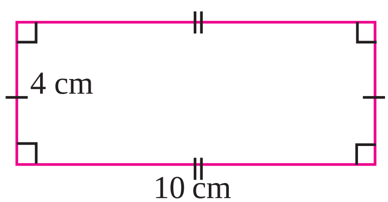
(iii)



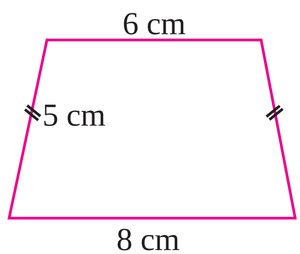
(iv)



(v)



(vi)



(2) The perimeter of a square shaped wall tile is 160 cm. How many such tiles are needed for one lengthwise row of a wall of length 4 m, if the tiles are to be fixed without any gaps between them?

(3) If the perimeter of a rectangular shaped paddy field of length 40 m is 130 m, find its breadth.

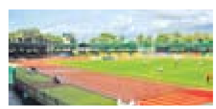
(4) The length of a rectangular shaped wall tile is greater than its breadth by 10 cm. If the breadth of the tile is 15 cm, find its perimeter.

(5) There are two pieces of wire of length 60 cm each. Amali makes an equilateral triangle by bending one of these pieces of wire. Sandamini makes a square with the other piece of wire.

(i) Find the length of a side of the equilateral triangle made by Amali.
(ii) Find the length of a side of the square made by Sandamini.

(6) The length and breadth of a rectangular shaped flower bed are 7 m and 3 m respectively. How many square shaped bricks of length 25 cm each are needed to place one row of bricks around the flower bed without any space left between the bricks?

(7) The length of a rectangular shaped playground is twice its breadth. If the perimeter of the playground is 360 m, find its length and its breadth.



2.2 Perimeter of a composite rectilinear plane figure

You have learnt that a plane figure which is composed of several plane figures is called a composite plane figure. Now let us learn how to find the perimeter of a composite plane figure which is composed of two plane figures.

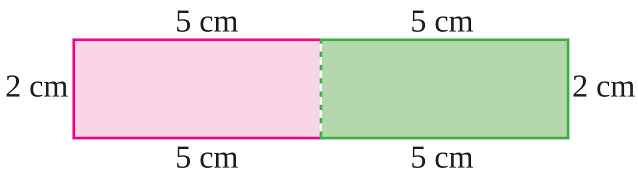
Two rectangular shaped pieces of paper which are 5 cm in length and 2 cm in breadth are given below.

The perimeter of one rectangular shaped piece of paper $= 5 \text{ cm} + 2 \text{ cm} + 5 \text{ cm} + 2 \text{ cm}$
$= 14 \text{ cm}$

The sum of the perimeters of the two rectangular shaped pieces of paper $= 14 \text{ cm} + 14 \text{ cm}$
$= 28 \text{ cm}$

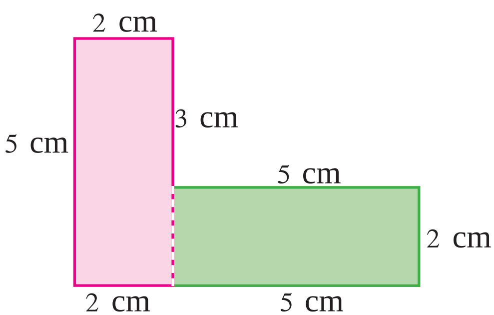
Let us find the perimeter of several composite plane figures formed with these two rectangular shaped pieces of paper.

(i)



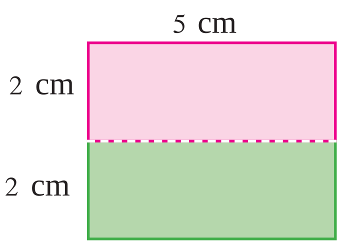
The perimeter of the figure $= 5 \text{ cm} + 5 \text{ cm} + 2 \text{ cm} + 5 \text{ cm} + 5 \text{ cm} + 2 \text{ cm}$
$= 24 \text{ cm}$

(ii)



The perimeter of the figure $= 5 \text{ cm} + 2 \text{ cm} + 3 \text{ cm} + 5 \text{ cm} + 2 \text{ cm} + 5 \text{ cm} + 2 \text{ cm}$
$= 24 \text{ cm}$

(iii)



The perimeter of the figure $= 5 \text{ cm} + 2 \text{ cm} + 2 \text{ cm} + 5 \text{ cm} + 2 \text{ cm} + 2 \text{ cm}$
$= 18 \text{ cm}$

It must be clear to you through these examples, that the perimeter of each of the composite plane figures formed is less than the sum of the perimeters of the two rectangles.

Hence, when calculating the perimeter of a composite rectilinear plane figure, only the lengths of all the straight line segments by which the figure is bounded should be added.

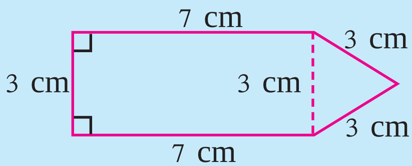
Note :

The perimeter of a composite plane figure cannot be obtained by adding together all the perimeters of the plane figures which the composite figure is composed of.

Example 1

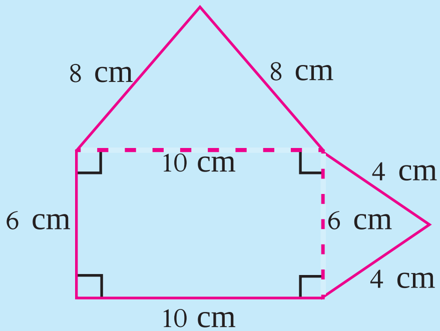
Calculate the perimeter of each of the figures given below.

(i)



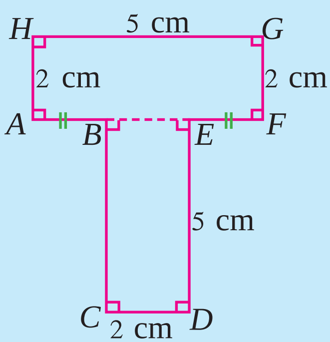
Perimeter $= 7\text{ cm} + 3\text{ cm} + 3\text{ cm} + 7\text{ cm} + 3\text{ cm}$
$= 23\text{ cm}$

(ii)



Perimeter $= 8\text{ cm} + 4\text{ cm} + 4\text{ cm} + 10\text{ cm} + 6\text{ cm} + 8\text{ cm}$
$= 40\text{ cm}$

(iii)



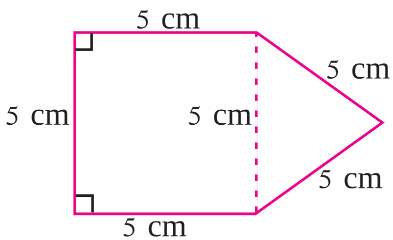
$GH = 5\text{ cm}$
$AB = EF$
$2\ AB = 5\text{ cm} - 2\text{ cm} = 3\text{ cm}$
$\therefore\ AB = 1.5\text{ cm}$

Perimeter of the figure $= 5\text{ cm} + 2\text{ cm} + 1.5\text{ cm} + 5\text{ cm} + 2\text{ cm} + 5\text{ cm} + 1.5\text{ cm} + 2\text{ cm}$
$= 24\text{ cm}$

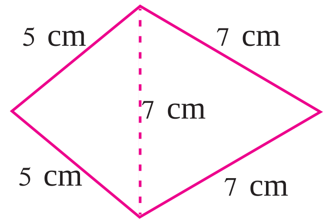
Exercise 2.1

(1) Calculate the perimeter of each of the figures given below.

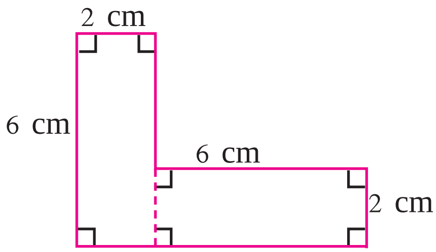
(i)



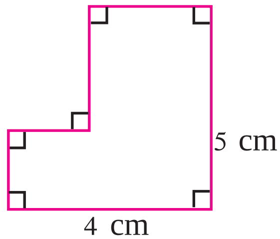
(ii)



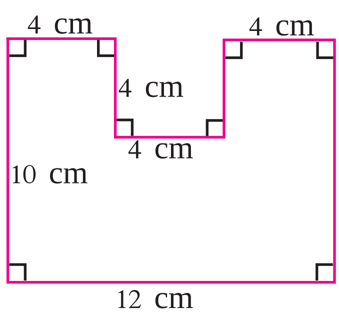
(iii)



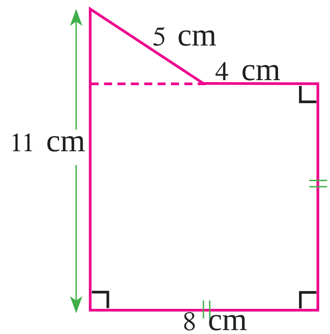
(iv)



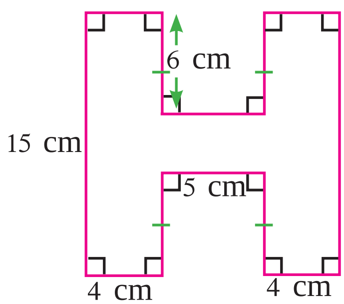
(v)



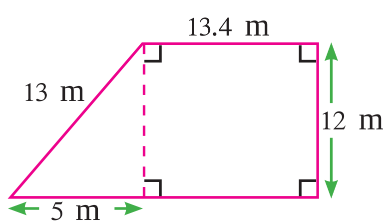
(vi)



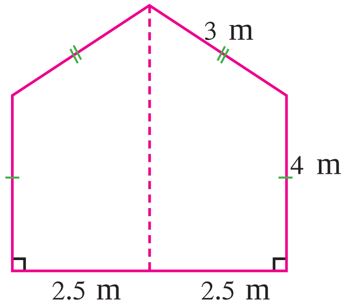
(vii)



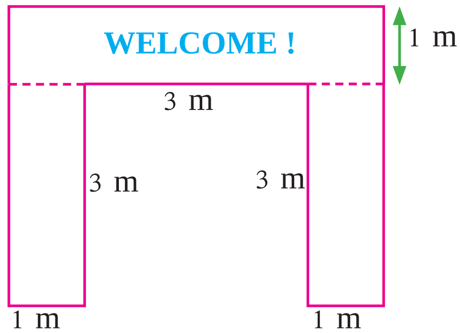
(viii)



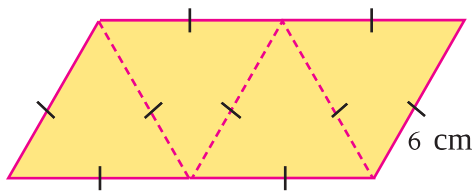
(2) A figure of a gate with two panels is given here. Calculate the perimeter of the gate.



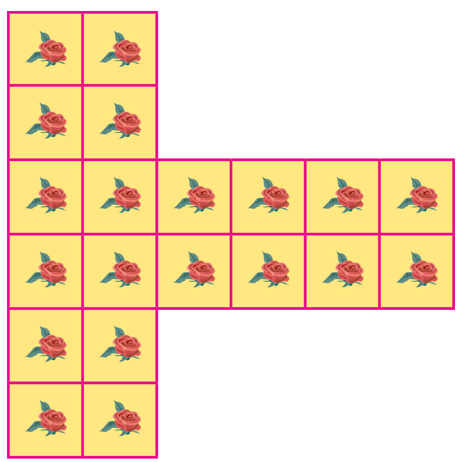
(3) A figure of an entrance structure constructed to welcome the students of grade 1 to a school is given with its measurements. Find the minimum length of the ribbon required to fix around the entrance structure.



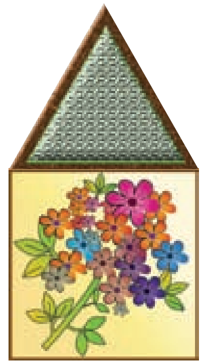
(4) A figure of a net used to construct a solid is shown here. Calculate its perimeter.



(5) A section of a courtyard constructed with square shaped floor tiles of length 40 cm each is shown in the figure. Find the perimeter of this section?



(6) If the perimeter of a wall hanging composed of a square shaped wooden lamina and an equilateral triangular shaped wooden lamina with base equal to a side of the square is 160 cm,

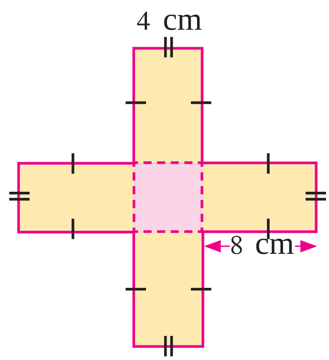


(i) calculate the length of a side of the square shaped wooden lamina.

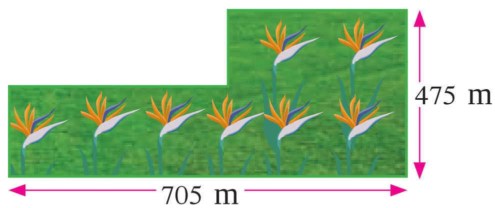
(ii) calculate the perimeter of the equilateral triangular shaped wooden lamina.

(7) What is the least of the perimeters of the composite plane figures that can be made with two rectangles of length 6 cm and breadth 4 cm each?

(8) A composite figure formed with four rectangles of length 8 cm and breadth 4 cm each and a square of side length 4 cm is shown here. Calculate the perimeter of the figure.



(9) Every morning Binuli walks twice around the park shown in the figure. Find the total distance she walks around the park each day.



Summary

- The perimeter of a composite plane figure which is composed of several plane figures is not equal to the sum of the perimeters of the plane figures of which it is composed.

- When calculating the perimeter of a composite rectilinear plane figure, only the lengths of the straight line segments by which it is bounded should be added.

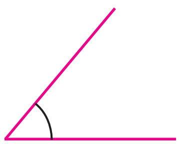
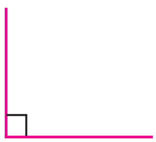
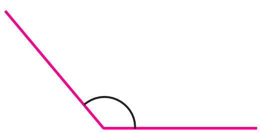
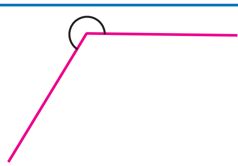
3 Angles

By studying this lesson you will be able to,

- identify pairs of complementary angles, supplementary angles, adjacent angles and vertically opposite angles,
- identify that the sum of the angles which lie around a point on one side of a straight line is 180˚,
- identify that the sum of the angles around a point on a plane is 360˚,
- identify that the vertically opposite angles created by two intersecting straight lines are equal, and
- calculate the magnitudes of angles associated with straight lines.

3.1 Angles

You have learnt in Grade 7 that the standard unit used to measure angles is **degrees** and that one degree is written as 1˚.

Angle	Figure	Note
Acute Angle		An angle of magnitude less than 90° is called an **acute angle**.
Right Angle		An angle of magnitude 90° is called a **right angle**.
Obtuse Angle		An angle of magnitude greater than 90° but less than 180° (that is, an angle of magnitude between 90° and 180°) is called an **obtuse angle**.
Straight Angle		An angle of magnitude 180° is called a **straight angle**.
Reflex Angle		An angle of magnitude between 180° and 360° is called a **reflex angle**.

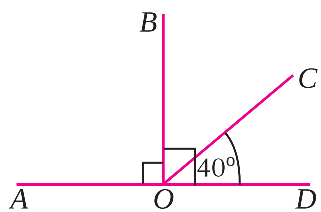
Do the review exercise to recall the above given facts which you learnt in Grade 7 under the lesson on angles.

Review Exercise

(1) Copy the two groups A and B given below and join them appropriately.

A	**B**
135°	Acute angle
90°	Right angle
180°	Obtuse angle
35°	Straight angle
245°	Reflex angle
190°	
280°	

(2) By considering the given figure, find the magnitude of each of the angles given below and write the type of each angle.

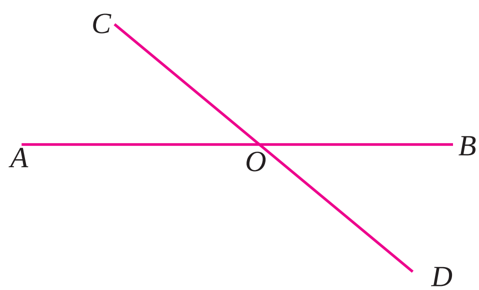


(i) $A\hat{O}B$ (ii) $C\hat{O}D$
(iii) $B\hat{O}D$ (iv) $B\hat{O}C$
(v) $A\hat{O}C$ (vi) $A\hat{O}D$

(3) Draw the following angles using a protractor and name them.

(i) $P\hat{Q}R = 60^{\circ}$ (ii) $A\hat{B}C = 90^{\circ}$ (iii) $X\hat{Y}Z = 130^{\circ}$ (iv) $K\hat{L}M = 48^{\circ}$

(4) As shown in the figure, draw two straight line segments AB and CD such that they intersect each other at O.



(i) Measure the magnitude of each of the angles $A\hat{O}C$, $C\hat{O}B$, $B\hat{O}D$ and $A\hat{O}D$ and write them down.

(ii) What is the value of $A\hat{O}C + C\hat{O}B$?

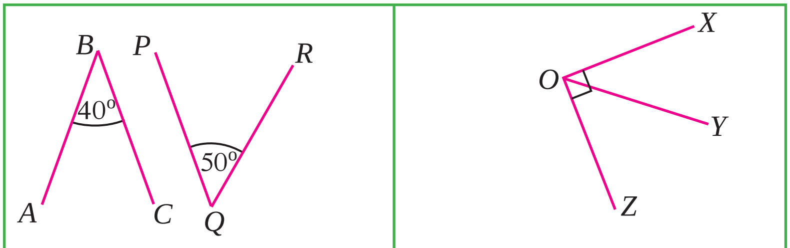
(iii) Are the two angles $A\hat{O}C$ and $B\hat{O}D$ equal to each other?

3.2 Complementary angles and supplementary angles

Let us identify what complementary angles and supplementary angles are.

• Complementary angles

Two pairs of angles are shown in the figure given below. Let us consider the sum of the magnitudes of each pair of angles.



$A\hat{B}C + P\hat{Q}R = 40^\circ + 50^\circ$
$= 90^\circ$

$X\hat{O}Y + Y\hat{O}Z = 90^\circ$

The sum of the magnitudes of the two angles of each pair is obtained as 90°.

If the sum of a pair of acute angles is 90°, then that pair of angles is called a pair of **complementary angles**.

According to this explanation, in the figure given above,

the angles $A\hat{B}C$ and $P\hat{Q}R$ are a pair of complementary angles, and
the angles $X\hat{O}Y$ and $Y\hat{O}Z$ are a pair of complementary angles.

The acute angle which needs to be added to a given acute angle for the sum of the two angles to be 90° is called the **complement** of the given angle.

$30^\circ + 60^\circ = 90^\circ$. Hence, the complement of 30° is 60°.

Example 1

Calculate the complement of 38°.

Since $90^\circ - 38^\circ = 52^\circ$, the complement of 38° is 52°.

Example 2

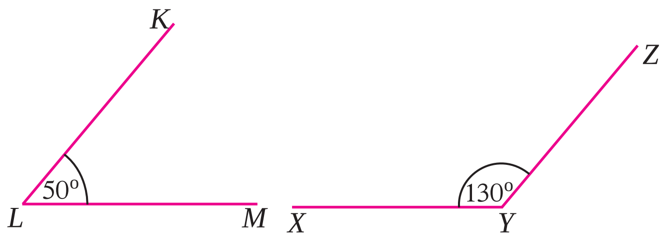
If $A\hat{B}C = 48^{\circ}$, $P\hat{Q}R = 66^{\circ}$, $K\hat{L}M = 42^{\circ}$ and $X\hat{Y}Z = 24^{\circ}$; name the pairs of complementary angles among these angles.

$48^{\circ} + 42^{\circ} = 90^{\circ}$. $\therefore$ $A\hat{B}C$ and $K\hat{L}M$ are a pair of complementary angles.
$66^{\circ} + 24^{\circ} = 90^{\circ}$. $\therefore$ $P\hat{Q}R$ and $X\hat{Y}Z$ are a pair of complementary angles.

• Supplementary angles

Let us consider the sum of the two angles given in the figure.

$$K\hat{L}M + X\hat{Y}Z = 50^{\circ} + 130^{\circ}$$
$$= 180^{\circ}$$



If the sum of a pair of angles is 180˚, then that pair of angles is called a pair of **supplementary angles.**

According to this explanation, $K\hat{L}M$ and $X\hat{Y}Z$ are a pair of supplementary angles.

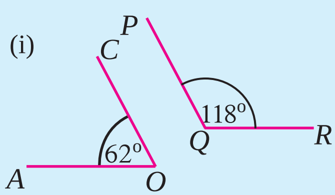
The angle which needs to be added to a given angle of less than 180˚ for the sum to be 180º is called the **supplement** of the given angle.

$60^{\circ} + 120^{\circ} = 180^{\circ}$
$\therefore$ The supplement of 60° is 120°.

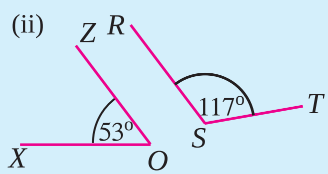
Example 3

Explain whether the pairs of angles given in the figure are supplementary angles.

(i)



(ii)



(i) $A\hat{O}C + P\hat{Q}R = 62^{\circ} + 118^{\circ}$
$= 180^{\circ}$
$\therefore$ $A\hat{O}C$ and $P\hat{Q}R$ are a pair of supplementary angles.

(ii) $X\hat{O}Z + R\hat{S}T = 53^o + 117^o$

$= 170^o$

Since the sum of the two angles is not 180°, $X\hat{O}Z$ and $R\hat{S}T$ are not a pair of supplementary angles.

Exercise 3.1

(1) Copy and complete.

(i) The complement of 60^o is ....
The supplement of 60^o is ....

(ii) The complement of 75^o is ....
The supplement of 75^o is ....

(iii) The complement of 25^o is ....
The supplement of 25^o is ....

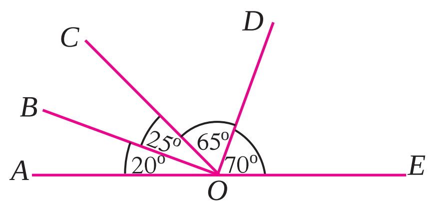
(iv) The complement of 1^o is ....
The supplement of 1^o is ....

(2) From among the angles $A\hat{B}C = 72^o$, $P\hat{Q}R = 15^o$, $X\hat{Y}Z = 28^o$, $K\hat{L}M = 165^o$, $B\hat{O}C = 18^o$, $M\hat{N}L = 108^o$ and $D\hat{E}F = 75^o$, select and write down,

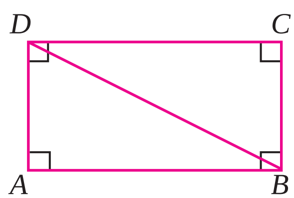
(i) two pairs of complementary angles.
(ii) two pairs of supplementary angles.

(3) According to the figure given here,

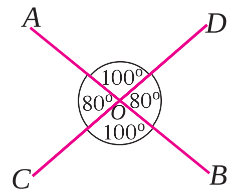
(i) what is the sum of $B\hat{O}C$ and $C\hat{O}D$?
(ii) what is the complement of $B\hat{O}C$?
(iii) what is the magnitude of $A\hat{O}D$?
(iv) what is the sum of $A\hat{O}D$ and $D\hat{O}E$?
(v) what is the supplement of $D\hat{O}E$?
(vi) what is the complement of $D\hat{O}E$?



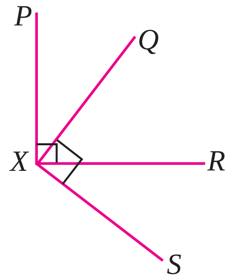
(4) (i) Write two pairs of complementary angles in the given figure.



(ii) The straight line segments AB and CD intersect at O. Write four pairs of supplementary angles in the figure.



(5) Write two pairs of complementary angles according to the information marked in the given figure.



(6) Copy these statements in your exercise book and place a ✓ in front of the correct statements and a ✗ in front of the incorrect statements.

(i) The complement of an acute angle is an acute angle.

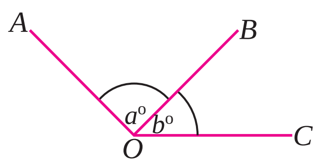
(ii) The complement of an acute angle is an obtuse angle.

(iii) The supplement an obtuse angle is an obtuse angle.

(iv) The supplement of an acute angle is an obtuse angle.

3.3 Adjacent angles

Let us consider the arms and the vertex of the two angles $A\hat{O}B$ and $B\hat{O}C$ in the figure.



The arms of $A\hat{O}B$ are AO and BO. The vertex is O.
The arms of $B\hat{O}C$ are BO and CO. The vertex is O.

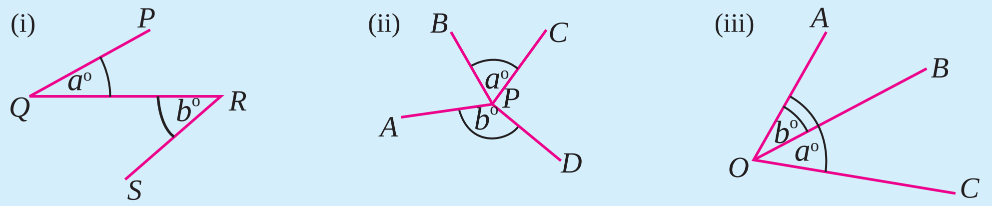
The arm BO belongs to both angles. Hence, BO is a common arm. The vertex of both angles is O. Hence, O is the **common vertex**. Moreover, these two angles are located on either side of the **common arm** OB .

A pair of angles which have a common arm and a common vertex and are located on either side of the common arm is called a pair of **adjacent angles**.

According to this explanation, $A\hat{O}B$ and $B\hat{O}C$ in the figure given above are a pair of adjacent angles.

Example 1

Explain whether the pairs of angles denoted by a and b in the figures given below are pairs of adjacent angles.



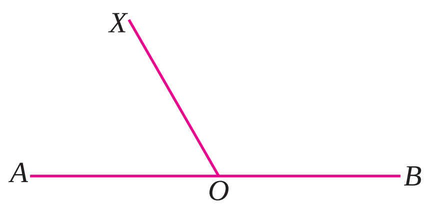
(i) QR is the common arm of both angles. The two angles are located on either side of QR. But there isn't a common vertex. Hence, $P\hat{Q}R$ and $Q\hat{R}S$ are not adjacent angles.

(ii) Both angles have a common vertex. But they do not have a common arm. Therefore, $B\hat{P}C$ and $A\hat{P}D$ are not adjacent angles.

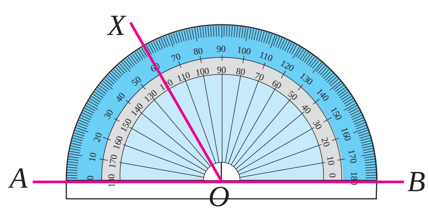
(iii) The angles $A\hat{O}B$ and $A\hat{O}C$ have a common arm and a common vertex. The common arm is AO. However, the two angles are not located on either side of the common arm. Therefore, $A\hat{O}B$ and $A\hat{O}C$ are not adjacent angles.

• Adjacent angles on a straight line

A pair of adjacent angles named $A\hat{O}X$ and $B\hat{O}X$ is created by the straight line XO meeting the straight line AB at O. Let us measure these two angles by using a protractor.



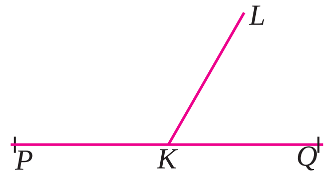
It is clear that in the figure, $A\hat{O}X = 60^\circ$ and $B\hat{O}X = 120^\circ$ (You can read the magnitudes of both angles at the same time by placing the base line of the protractor on the line AOB).



Activity 1

Step 1 - Draw a straight line segment in your exercise book and name it PQ.

Step 2 - Draw the straight line KL with the point K located on PQ.



Step 3 - Measure $P\hat{K}L$ and $Q\hat{K}L$ using the protractor and write down their magnitudes.

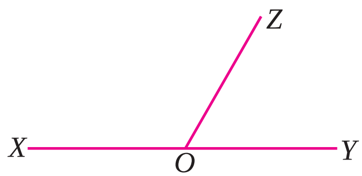
Step 4 - Fill in the blanks below the figure.

$P\hat{K}L + Q\hat{K}L = \ldots\ldots\ldots + \ldots\ldots\ldots$

$= \ldots\ldots\ldots$

Step 5 - As above, engage in the activity for another two figures, and investigate the possible conclusion that can be drawn.

The line segment XY is divided into the two line segments OX and OY by the point O located on XY.



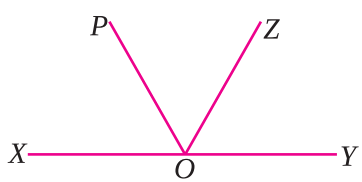
The sum of the two adjacent angles $X\hat{O}Z$ and $Z\hat{O}Y$, where OZ is the common arm, and OX and OY are the other arms, can be shown to be 180° by measuring the two angles separately.

This establishes the fact that a pair of adjacent angles, located on a straight line in this manner is a pair of supplementary angles.

Let us divide the angle $X\hat{O}Z$ into two by the straight line OP in the figure.

Then $X\hat{O}Z = X\hat{O}P + P\hat{O}Z$

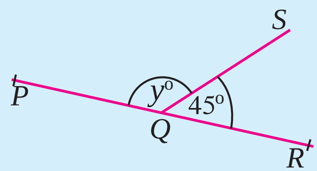
$\therefore\ X\hat{O}P + P\hat{O}Z + Z\hat{O}Y = X\hat{O}Z + Z\hat{O}Y = 180^\circ$.



The sum of the angles around a point on a straight line, located on one side of the straight line is 180°.

Example 2

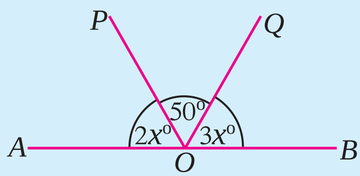
In the given figure, PR is a straight line segment. Find the magnitude of $P\hat{Q}S$.



$$y + 45 = 180$$
$$y + 45 - 45 = 180 - 45$$
$$y = 135$$
$$P\hat{Q}S = 135^\circ$$

Example 3

Find the magnitude of $A\hat{O}P$ according to the information marked in the figure.



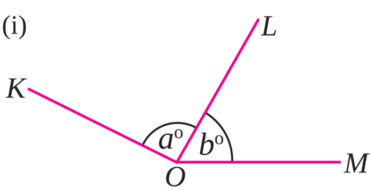
$2x + 50 + 3x = 180$ (the sum of the angles on a straight line is 180°)

$$5x + 50 = 180$$
$$5x + 50 - 50 = 180 - 50$$
$$\frac{5x}{5} = \frac{130}{5}$$
$$x = 26$$
$$\therefore\ A\hat{O}P = 2x^\circ = 2 \times 26^\circ = 52^\circ$$

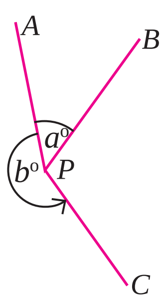
Exercise 3.2

(1) Write whether the pair of angles marked as a and b in each figure is a pair of adjacent angles.

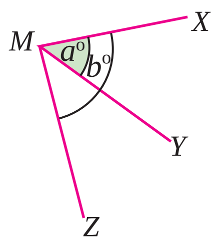
(i)



(ii)

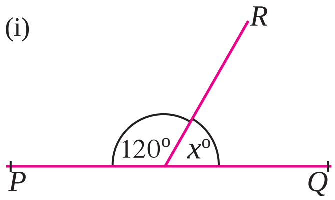


(iii)

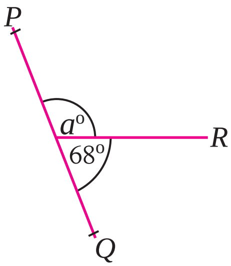


(2) If PQ is a straight line segment in each figure given below, find the magnitude of the angle marked by an English letter.

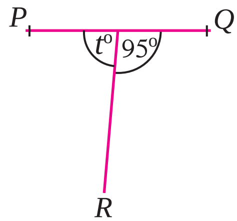
(i)



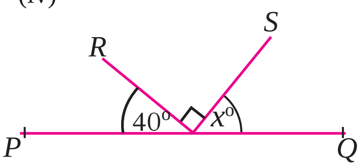
(ii)



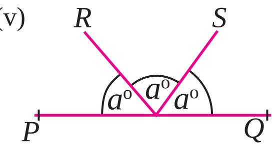
(iii)



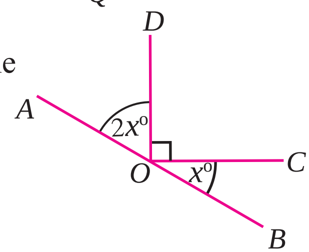
(iv)



(v)



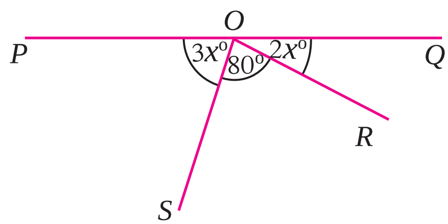
(3) In the figure, if AB is a straight line segment, find the magnitude of $A\hat{O}D$.



(4) PQ is a straight line segment. According to the information marked in the figure,

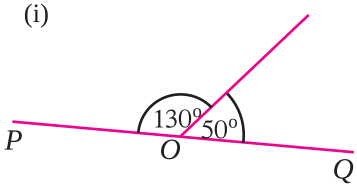
(i) find the magnitude of $P\hat{O}S$.

(ii) find the magnitude of $S\hat{O}Q$.

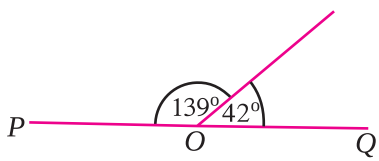


(5) Conclude whether POQ in each of the given figures is a straight line.

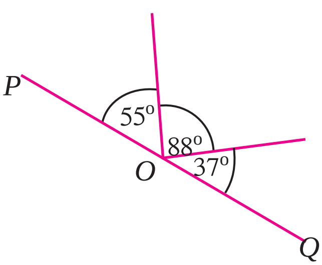
(i)



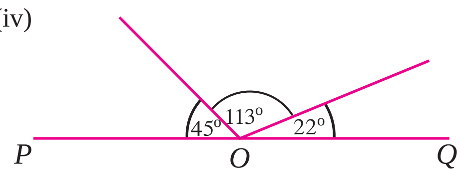
(ii)



(iii)

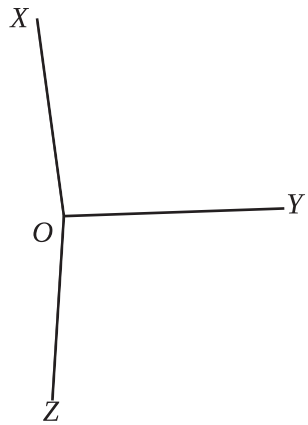


(iv)

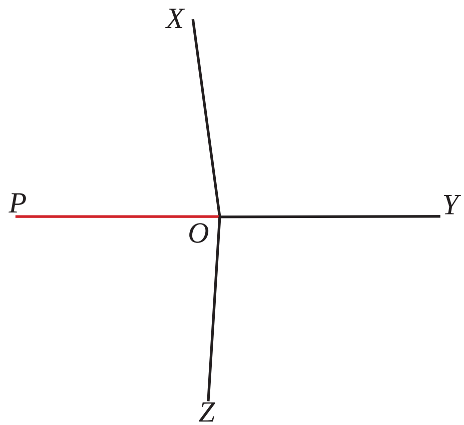


3.4 The sum of the angles around a point on a plane

Consider the angles $X\hat{O}Y$, $Y\hat{O}Z$ and $Z\hat{O}X$ located around the point O in the figure. Let us find the value of $X\hat{O}Y + Y\hat{O}Z + Z\hat{O}X$.



To do this, produce the straight line YO to P.



Method 1

Since POY is a straight line,

$$P\hat{O}X + X\hat{O}Y = 180^{\circ}$$
$$P\hat{O}Z + Z\hat{O}Y = 180^{\circ}$$
$$\therefore\ P\hat{O}X + X\hat{O}Y + P\hat{O}Z + Z\hat{O}Y = 180^{\circ} + 180^{\circ}$$
$$= 360^{\circ}$$

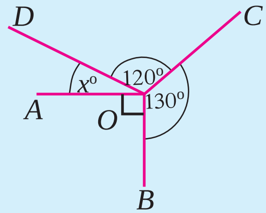
Method 2

$$Z\hat{O}X = Z\hat{O}P + P\hat{O}X$$
$$\therefore\ X\hat{O}Y + Y\hat{O}Z + Z\hat{O}X = X\hat{O}Y + Y\hat{O}Z + Z\hat{O}P + P\hat{O}X$$
$$= \underbrace{X\hat{O}Y + P\hat{O}X}_{\text{Supplementary Angles}} + \underbrace{Y\hat{O}Z + Z\hat{O}P}_{\text{Supplementary Angles}}$$
$$= 180^{\circ} + 180^{\circ} = 360^{\circ}$$

The sum of the angles located around a point on a plane is 360°.

Example 1

Find the magnitude of the angle marked as $A\hat{O}D$ in the given figure.



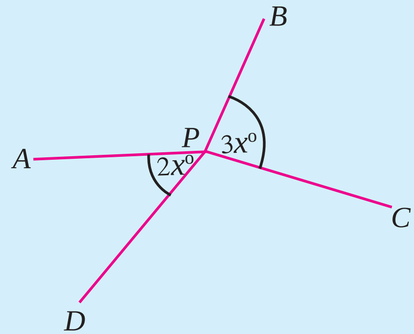
$x + 120 + 130 + 90 = 360$ (the sum of the angles around a point is 360^o)

$$x + 340 = 360$$
$$x + 340 - 340 = 360 - 340$$
$$x = 20$$
$$\therefore A\hat{O}D = 20^o$$

Example 2

If $A\hat{P}B = 150^o$ and $D\hat{P}C = 100^o$ in the figure, find the magnitude of $B\hat{P}C$.

Because the sum of the angles around P is 360^o,

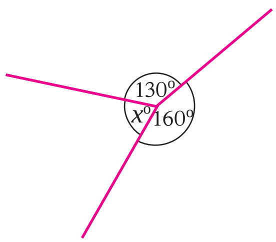


$$2x + 150 + 3x + 100 = 360$$
$$5x + 250 = 360$$
$$5x + 250 - 250 = 360 - 250 = 110$$
$$\frac{5x}{5} = \frac{110}{5}$$
$$x = 22$$

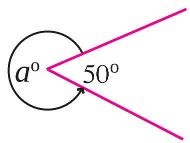
$\therefore B\hat{P}C = 3 \times 22^o = 66^o$

Exercise 3.3

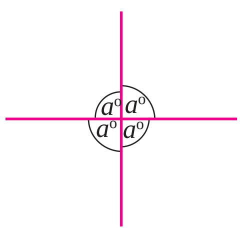
(1) Find the value of x^o.



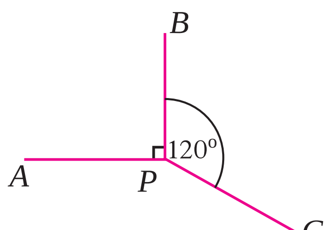
(2) Find the value of a^o.



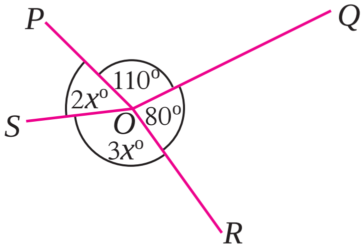
(3) Find the value of a^o .



(4) Find the magnitude of $A\hat{P}C$.

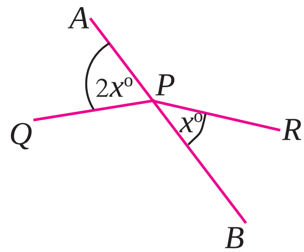


(5) Find the magnitude of $S\hat{O}R$.



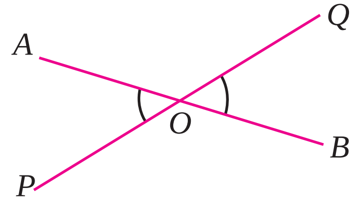
(6) AB is a straight line.

If $A\hat{P}R = 150^{\circ}$, find the magnitude of $Q\hat{P}B$.



3.5 Vertically opposite angles

The two straight lines AB and PQ shown in the figure intersect at point O. The two angles $A\hat{O}P$ and $B\hat{O}Q$ which are located vertically opposite each other as shown here are called **vertically opposite angles**.

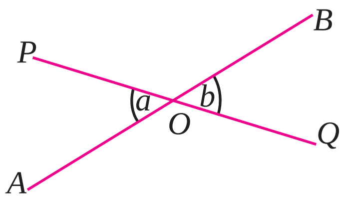


The two angles $A\hat{O}Q$ and $B\hat{O}P$ in the figure are also a pair of vertically opposite angles.

Two pairs of vertically opposite angles are always created by the intersection of two straight lines. Each pair has a common vertex and the two angles are located vertically opposite each other across the common vertex.

Activity 2

Step 1 - In your exercise book, draw two straight lines which intersect each other as shown in the figure and include the information given in the figure.



Step 2 - Copy the figure on a tissue paper and name it also as in the above figure.

Step 3 - Keep the two drawn figures such that they coincide with each other and hold them in place with a pin at point O.

Step 4 - Rotate the tissue paper half a circle around the point O and see whether the two angles a and b coincide with each other.

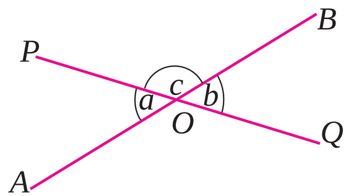
Step 5 - Engage in the activity as above for another two cases and examine whether the vertically opposite angles coincide with each other.

Investigate the conclusion that can be drawn from this activity.

It can be concluded based on the above activity, that vertically opposite angles created by the intersection of two straight lines are equal to each other.

> Vertically opposite angles created by the intersection of two straight lines are equal to each other.

Let us investigate whether this is true by another method.
PQ and *AB* in the figure are straight line segments.



$a + c = 180^{\circ}$ (*AB* is a straight line)

$b + c = 180^{\circ}$ (*PQ* is a straight line)

$\therefore a + c = b + c$

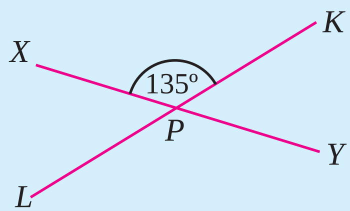
$a + c - c = b + c - c$ (subtracting c from both sides)

$\therefore a = b$

$\therefore$ The vertically opposite angles $A\hat{O}P$ and $B\hat{O}Q$ are equal to each other.

Example 1

Find the magnitude of each angle around the point *P* in the given figure, where *XY* and *KL* are straight line segments.



$L\hat{P}Y = X\hat{P}K$ (vertically opposite angles are equal)

$\therefore L\hat{P}Y = 135^{\circ}$

$X\hat{P}L + 135^{\circ} = 180^{\circ}$ (the sum of the angles on the straight line *LK* is 180˚)

$\therefore X\hat{P}L = 180^{\circ} - 135^{\circ}$

$= 45^{\circ}$

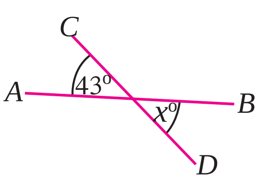
$K\hat{P}Y = X\hat{P}L$ (vertically opposite angles are equal)

$\therefore K\hat{P}Y = 45^{\circ}$

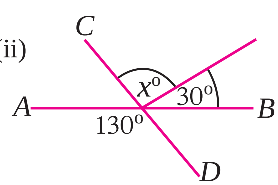
Exercise 3.4

(1) Find the magnitude of each of the angles marked by an English letter in the figures given below (*AB*, *CD* and *EF* are straight lines).

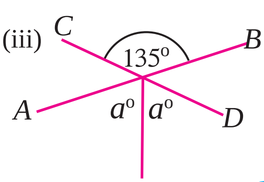
(i)



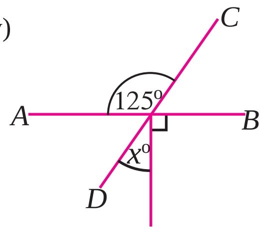
(ii)



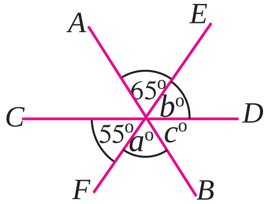
(iii)

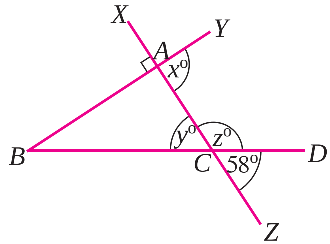


(iv)



(v)





(2) (i) Find the values of the angles denoted by x, y and z in the given figure (BY, BD and XZ are straight lines).

(ii) $A\hat{B}C$ and $A\hat{C}B$ are a pair of complementary angles. What is the magnitude of $A\hat{B}C$?

Summary

- If the sum of a pair of acute angles is 90°, then that pair of angles is called a pair of complementary angles.
- The acute angle which needs to be added to a given acute angle for the sum of the two angles to be 90° is called the complement of the given angle.
- If the sum of a pair of angles is 180°, then that pair of angles is called a pair of supplementary angles.
- The angle which needs to be added to a given angle of less than 180° for the sum to be 180° is called the supplement of the given angle.
- A pair of angles which have a common arm and a common vertex and are located on either side of the common arm is called a pair of adjacent angles.
- The sum of the angles located around a point on one side of a straight line is 180°.
- The sum of the angles located around a point on a plane is 360°.
- Vertically opposite angles created by the intersection of two straight lines are equal to each other.

4 Directed Numbers

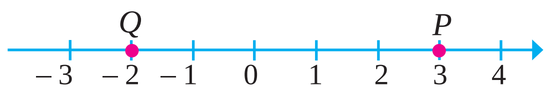
By studying this lesson you will be able to,

- subtract a directed number from another directed number, and
- multiply directed numbers and divide a directed number by a directed number.

4.1 Directed numbers

Let us recall what you learnt in Grade 7 about directed numbers.

Consider the following number line on which the points P and Q are marked.



- On this number line, the point P represents the directed number $(+3)$ and the point Q represents the directed number (-2).
- $(+3)$ is most often written as 3.
- (-2) and $(+3)$, are located on the number line in opposite directions to each other from zero.
- + (positive) sign is used to denote the direction in which the directed number $(+3)$ is located with respect to zero on the number line.
- – (negative) sign is used to denote the opposite direction, in which the directed number (-2) is located.

The **magnitude** of a number represented by a point on the number line is the distance on the number line from zero to that point.

Furthermore, a number gets its sign as + or – according to the position of that number, whether it is to the right or to the left of the point which represents 0.

- Since the distance from zero to point P is 3 units, the magnitude of the directed number $(+3)$ is 3. The magnitude of the directed number (-2) is 2.

In a directed number, the numerical value shows its magnitude and the + or – sign its direction.

$(+3)$, (-7), $(+2.5)$, (-3.4), $\left(+3\frac{1}{2}\right)$, $\left(-5\frac{1}{4}\right)$ are some examples of directed numbers.

Note

- It is important to note that, while using the symbol + or – to denote the direction of a number, the symbol + is also used to denote the addition of two directed numbers and the symbol – is used to denote subtraction of a directed number from another directed number.
- We have to understand that the symbols + and – are used in two different senses here.
- To differentiate this clearly, we write directed numbers within brackets.

• Adding directed numbers

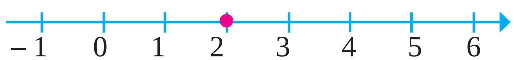
Since the sign of a directed number is important, we should pay special attention to the sign when performing mathematical operations.

You learnt in Grade 7 how the addition of directed numbers can be explained easily by using the number line.

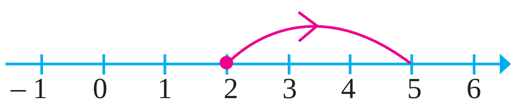
The addition of directed numbers can be explained easily by using the number line in the following manner too.

➤ Let us find the value of (+2) + (+3) by using the number line.

- Mark the directed number (+2) on the number line.

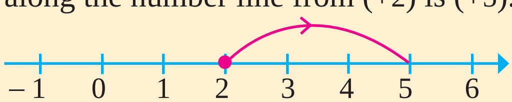


- From this point, move 3 units, which is the magnitude of (+3), to the right, which is the direction of (+3) along the number line.



- The directed number (+5) represented by the ending point is the sum of the above two directed numbers.

That is, the directed number which is obtained when you move 3 units to the right along the number line from (+2) is (+5).



$\therefore$ (+2) + (+3) = (+5)

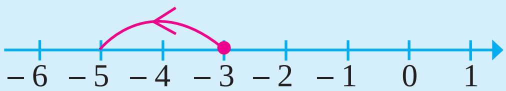
The steps we followed are given below in order.

When adding a directed number to another directed number do the following.

- Mark the point which represents the first directed number on the number line.
- From that point, move a distance **equal to the magnitude of the second directed number** towards the **direction of the second directed number**.
- The directed number which is represented by the ending point is the answer.

Example 1

Find the value of $(-3) + (-2)$ by using the number line.



From (-3), when you move two units to the left, which is the direction of (-2), the directed number you end at is (-5).

$\therefore\ (-3) + (-2) = (-5)$

• Adding directed numbers without using the number line

What you learnt in Grade 7 about the addition of directed numbers without using the number line is presented here.

Let us find the value without using the number line.

When adding two directed numbers of the same sign, first add the two numbers without considering their signs. Include the same sign in the answer.

(i) $(+3) + (+2) = (+5)$

(ii) $(-4) + (-6) = (-10)$

When adding two directed numbers of different signs (positive and negative), first find the difference of their numerical values, without considering their signs. Include the sign of the directed number having the larger magnitude in the answer.

(iii) Let us find the value of $(+8) + (-3)$.

$8 - 3 = 5$

$\therefore\ (+8) + (-3) = (+5)$

(iv) Let us find the value of $(+4.2) + (-6.3)$.

$6.3 - 4.2 = 2.1$

$\therefore\ (+4.2) + (-6.3) = (-2.1)$

Do the following review exercise to recall what you learnt in Grade 7 about directed numbers.

Review Exercise

(1) Find the value of each of the following by using the number line.

(i) $(+2) + (+6)$ (ii) $(+8) + (-5)$ (iii) $(-2) + (+3)$

(iv) $(-3) + (-4)$ (v) $(+4) + (-6)$

(2) Find the value of each of the following.

(i) $(+2) + (+3)$ (ii) $(-4) + (-2)$ (iii) $(-3) + (+5)$

(iv) $(+4) + (-10)$ (v) $(-7) + (+7)$ (vi) $(+2) + (+5) + (+3)$

(vii) $(-3) + (-1) + (-4)$ (viii) $(+2) + (+4) + (-9)$ (ix) $\left(+\frac{5}{7}\right) + \left(-\frac{2}{7}\right)$

(x) $(+3.4) + (-5.2)$ (xi) $(-8.11) + (+8.11)$

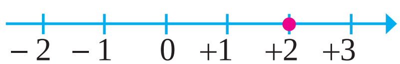
4.2 Subtracting a directed number from another directed number

Now let us consider how to subtract a directed number from another directed number by using the number line. Let us first find out what is meant by the direction opposite to that of a given directed number.

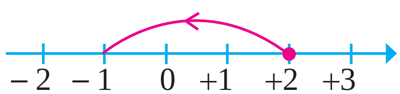
- The magnitude of (+3) is 3 and its direction is towards the right.
  The **direction opposite** to that of (+3) is towards the left.
- The magnitude of (–3) is 3 and its direction is towards the left.
  The **direction opposite** to that of (–3) is towards the right.

➤ Let us find the value of (+2) – (+3) by using the number line.

- First mark the directed number (+2) on the number line.



- From this point, move 3 units which is the magnitude of (+3), towards the left, which is the direction opposite to that of (+3).



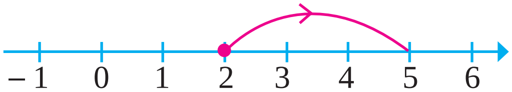
- The answer is the directed number represented by the ending point.

The answer is obtained from the point which is 3 units to the left of (+2).
$\therefore\ (+2) - (+3) = (-1)$

When subtracting a directed number from a directed number, do the following.

- Mark the point which represents the first directed number on the number line.
- From this point, move a distance equal to the magnitude of the second directed number in the direction opposite to that of the second directed number.
- The directed number which is represented by the ending point is the answer.

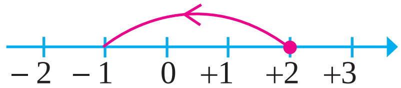
Finding the value of $(+2) + (+3)$.



The directed number which is represented by the ending point, when you move 3 units along the number line **in the direction of (+3)** from (+2) is obtained as the answer.

$\therefore (+2) + (+3) = (+5)$

Finding the value of $(+2) - (+3)$.

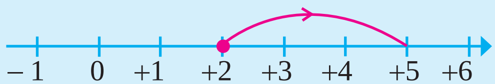


The directed number which is represented by the ending point, when you move 3 units along the number line **in the direction opposite to that of (+3)** from (+2) is obtained as the answer.

$\therefore (+2) - (+3) = (-1)$

Example 1

Find the value of $(+2) - (-3)$ by using the number line.



The magnitude of (–3) is 3 and the direction opposite to that of (–3) is towards the right.

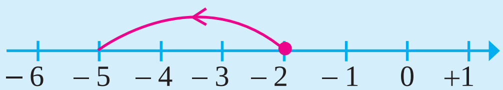
The answer is the directed number which is represented by the point located 3 units to the right of (+2).

$\therefore (+2) - (-3) = (+5)$

Example 2

Find the value of $(-2) - (+3)$ by using the number line.

The magnitude of (+3) is 3 and the direction opposite to that of (+3) is towards the left.



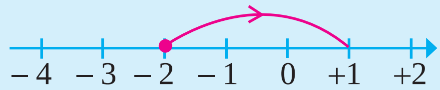
The answer is the directed number which is represented by the point located 3 units to the left of (–2).

$\therefore (-2) - (+3) = (-5)$

Example 3

Find the value of (–2) – (–3) by using the number line.

The magnitude of (–3) is 3 and the direction opposite to that of (–3) is towards the right.



The answer is the directed number which is represented by the point located 3 units to the right of (–2).

$\therefore\ (-2) - (-3) = (+1)$

Exercise 4.1

(1) Find the value by using the number line.

(i) $(+4) - (+2)$ (ii) $(+1) - (-2)$ (iii) $(-2) - (+3)$

(iv) $(-1) - (-3)$ (v) $(-6) - (-5)$ (vi) $(+2) - (-2)$

• More on subtracting a directed number from a directed number

By solving the equation $a + 1 = 0$, let us find the value of a which satisfies this equation.

The value of a cannot be 0 or a positive whole number.

Let us subtract 1 from both sides of the equation $a + 1 = 0$.

$a + 1 - 1 = 0 - 1$

$a = -1$

By taking the value of a in this equation to be (-1),

we obtain the relationship $(-1) + 1 = 0$.

This can also be written as $1 + (-1) = 0$.

(-1) is called the **additive inverse** of $(+1)$.

Furthermore, the additive inverse of (-1) is $(+1)$.

- Likewise, every positive number has a corresponding additive inverse which is a negative number of equal magnitude.
- Similarly, every negative number has an additive inverse which is a positive number of equal magnitude.

The number	The additive inverse of the number
$(+5)$	(-5)
(-5)	$(+5)$
$(+2)$	(-2)
(-2)	$(+2)$
$(+3.5)$	(-3.5)
$\left(-\frac{2}{3}\right)$	$\left(+\frac{2}{3}\right)$

Now let us consider subtracting a directed number from another directed number without using the number line.

$5 - 2 = 3.$

Let us consider subtracting 2 from 5 by 5 and 2 as directed numbers.

Let us write the additive inverse of 2 as a directed number and add it to 5.

The additive inverse of $(+\,2)$ is (-2).

$(+5) + (-2) = 3$

Subtracting a number from another number is the same as adding the additive inverse of the second number to the first number.

Hence, $5 - 2 = (+5) - (+2)$
$= (+5) + (-2)$
$= (+3)$

Example 4

Find the value of $(+2) - (-4)$.

The additive inverse of (-4) is $(+4)$.

$\therefore\ (+2) - (-4) = (+2) + (+4)$
$= (+6)$

Example 5

Find the value of $(-5) - (+2)$.

The additive inverse of $(+2)$ is (-2).

$\therefore\ (-5) - (+2) = (-5) + (-2)$
$= (-7)$

Example 6

Find the value of $(-7) - (-3)$.

The additive inverse of (-3) is $(+3)$.

$\therefore\ (-7) - (-3) = (-7) + (+3)$
$= (-4)$

Example 7

Find the value of $(-12) - (-15) - (+5)$.

$(-12) - (-15) - (+5) = (-12) + (+15) + (-5)$
$= (+3) + (-5)$
$= (-2)$

Example 8

Find the value of $\left(+\frac{3}{5}\right)-\left(+\frac{1}{5}\right)$.

$$\left(+\frac{3}{5}\right)-\left(+\frac{1}{5}\right)=\left(+\frac{3}{5}\right)+\left(-\frac{1}{5}\right)$$
$$=\left(+\frac{2}{5}\right)$$

Example 9

Find the value of $\left(-5\frac{1}{2}\right)-(+2)$.

$$\left(-5\frac{1}{2}\right)-(+2)=\left(-5\frac{1}{2}\right)+(-2)$$
$$=\left(-7\frac{1}{2}\right)$$

Example 10

Find the value of $(-3.2)-(+1.4)$.

$$(-3.2)-(+1.4)=(-3.2)+(-1.4)$$
$$=(-4.6)$$

Example 11

Find the value of $(-8.4)-(-2.1)$.

$$(-8.4)-(-2.1)=(-8.4)+(+2.1)$$
$$=(-6.3)$$

Exercise 4.2

(1) Fill in each cage with the suitable directed number.

(i) $(-5)-(+3)=(-5)+\square$
$=\square$

(ii) $(-3)-(-4)=(-3)+\square$
$=\square$

(iii) $(+7)-(-1)=(+7)+\square$
$=\square$

(iv) $(+7)-(-2)=(+7)+\square$
$=\square$

(2) Find the value of each of the following.

(a) (i) $(+4)-(+1)$ (ii) $(-8)-(-2)$ (iii) $(-3)-(-7)$

(iv) $(+9)-(-6)$ (v) $(-5)-(-5)$ (vi) $0-(+3)$

(vii) $(-11)-(+4)$ (viii) $(+2)+(-1)-(-4)$ (ix) $(-5)-(+2)-(-6)$

(x) $(+4)-(+2)-(+8)$

(b) (i) $\left(+4\frac{1}{2}\right)-(-2)$ (ii) $\left(-6\frac{1}{4}\right)-\left(-\frac{1}{4}\right)$ (iii) $(+15.7)-(-2.3)$

(iv) $(-2)-(+3.5)-(-4.1)$ (v) $\left(+3\frac{1}{2}\right)-(-2)-\left(-\frac{1}{3}\right)$

4.3 Multiplying directed numbers

Now let us consider the multiplication of two directed numbers.

➤ Let us find the value of $(+6) \times (+2)$.

- Obtain the product of the magnitudes of the two directed numbers without considering their signs.

  $6 \times 2 = 12$
- The two directed numbers are of the same sign. Therefore the answer is positive. $\therefore (+6) \times (+2) = (+12)$

➤ Let us find the value of $(-6) \times (+2)$.

- Obtain the product of the magnitudes of the two directed numbers without considering their signs.

  $6 \times 2 = 12$
- The two directed numbers are of opposite signs. Therefore, the answer is negative.

  $\therefore (-6) \times (+2) = (-12)$

When multiplying two directed numbers,

- Find the product of the magnitudes of the two directed numbers without considering their signs.
- If the two directed numbers are of the same sign, include the positive sign in the answer.
- If the two directed numbers are of opposite signs, include the negative sign in the answer.

Example 1

Simplify $(-6) \times (-2)$.

$6 \times 2 = 12$

The two directed numbers are of the same sign. Therefore the answer is positive.

$\therefore (-6) \times (-2) = (+12)$

Example 2

Simplify $(+6) \times (-2)$.

$6 \times 2 = 12$

The two directed numbers are of opposite signs. Therefore the answer is negative.

$\therefore (+6) \times (-2) = (-12)$

Example 3

Simplify the following .

(i) $(+2) \times (+5)$ (ii) $(-2) \times (+3)$ (iii) $(+5) \times (-3)$ (iv) $(-4) \times (-3) \times (+2)$

(i) $(+2) \times (+5) = (+10)$

(ii) $(-2) \times (+3) = (-6)$

(iii) $(+5) \times (-3) = (-15)$

(iv) $(-4) \times (-3) \times (+2) = (+12) \times (+2) = (+24)$

Example 4

Simplify $(+2.5) \times (-5)$.

$2.5 \times 5 = 12.5$

$\therefore (+2.5) \times (-5) = (-12.5)$

Example 5

Simplify $(-3.4) \times (-12)$.

$3.4 \times 12 = 40.8$

$\therefore (-3.4) \times (-12) = (+40.8)$

Exercise 4.3

(1) Find the value.

(i) $(+5) \times (+4)$
(ii) $(-5) \times (+4)$
(iii) $(-10) \times (-5)$
(iv) $(+7) \times (-3)$
(v) $(-1) \times (-4)$
(vi) $(+11) \times 0$
(vii) $(-6) \times (+4)$
(viii) $(+12) \times (-3)$
(ix) $(-2) \times (+2) \times (-5)$
(x) $(-3) \times (-1) \times (+2) \times (-5)$
(xi) $(+2.5) \times (+2)$
(xii) $(+4.1) \times (-23)$

4.4 Dividing a directed number by a directed number

➤ Let us find the value of $(+6) \div (+2)$.

- Let us divide the two directed numbers by considering their magnitudes only, without considering their signs.

  $6 \div 2 = 3$

- The two directed numbers are of the same sign. Therefore the answer is positive.

  $\therefore (+6) \div (+2) = (+3)$

➤ Let us find the value of $(-6) \div (+2)$.

- Let us divide the two directed numbers by considering their magnitudes only, without considering their signs.

  $6 \div 2 = 3$
- The two directed numbers are of opposite signs. Therefore, the answer is negative.

  $\therefore (-6) \div (+2) = (-3)$

When dividing a directed number by another directed number,
- Divide by considering their magnitudes, without considering their signs.
- Include the positive sign in the answer, if the two directed numbers are of the same sign.
- Include the negative sign in the answer, if the two directed numbers are of opposite signs.

Example 1

Simplify $(-6) \div (-2)$.

$6 \div 2 = 3$

The two directed numbers are of the same sign. Therefore, the answer is positive.

$(-6) \div (-2) = (+3)$

Example 2

Simplify $(+6) \div (-2)$.

$6 \div 2 = 3$

The two directed numbers are of opposite signs. Therefore, the answer is negative.

$\therefore (+6) \div (-2) = (-3)$

Example 3

Simplify the following.

(i) $(+15) \div (+5)$ (ii) $(-9) \div (+3)$ (iii) $(+15) \div (-3)$ (iv) $(-9) \div (-3)$

(i) $(+15) \div (+5) = (+3)$
(ii) $(-9) \div (+3) = (-3)$
(iii) $(+15) \div (-3) = (-5)$
(iv) $(-9) \div (-3) = (+3)$

Exercise 4.4

(1) Find the value of each of the following.

(i) $(+10) \div (+2)$ (ii) $(-12) \div (-4)$ (iii) $(+15) \div (-3)$

(iv) $(-21) \div (+7)$ (v) $(-5) \div (+5)$ (vi) $\dfrac{(-20)}{(-4)}$

(vii) $\dfrac{(+2) \times (+8)}{(-4)}$ (viii) $\dfrac{(-36)}{(-6) \times (-2)}$ (ix) $\dfrac{(+5) \times (-4)}{(-2) \times (-2)}$

(x) $\dfrac{(-9) \times (-8)}{(-4) \times (+3)}$

(2) Fill in each cage with the suitable directed number.

(i) $(-20) \div \square = (-10)$ (ii) $(+18) \div \square = (-6)$ (iii) $\square \div (-2) = (+5)$

(iv) $(+4) \div \square = (-4)$ (v) $\dfrac{(+3) \times \square}{(-2)} = (+6)$ (vi) $\dfrac{\square \times (+7)}{(+2) \times \square} = \dfrac{(-28)}{\square} = (+7)$

Summary

- Subtracting a number from another number is the same as adding the additive inverse of the second number to the first number.
- A positive number is obtained, when two directed numbers of the same sign are multiplied or divided.
- A negative number is obtained when two directed numbers of opposite signs are multiplied or divided.

5 Algebraic Expressions

By studying this lesson, you will be able to,

- construct algebraic expressions with three unknown terms,
- multiply an algebraic expression by a number and by an algebraic term,
- simplify algebraic expressions, and
- find the value of an algebraic expression by substituting integers for the unknown terms.

5.1 Algebraic expressions

Let us recall what you learnt in Grade 7 about algebraic expressions.

A certain shop purchases the same amount of milk every day to sell. If we don't know the exact amount, we cannot represent it by a number, although the amount is a constant value.

As in the above situation, when the numerical value of a constant amount is not known, it is called an **unknown constant**.

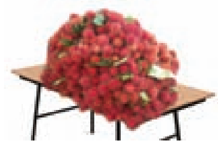
The daily income of Nimal's shop takes different values depending on its daily sales.

Since the daily income of Nimal's shop is not a fixed value, it is a **variable**.

Simple letters of the English alphabet are used to represent unknown constants and variables.

Let us denote the daily income from Nimal's shop by x. Nimal gives Rs.500 to his mother daily from the income from his shop. After giving Rs.500 to his mother, Nimal has an amount of Rs. $x - 500$ remaining.

$x - 500$ is an **algebraic expression**. x and 500 are the **terms of the expression**.



If 350 rambutans are sold at Rs. x each, the income is Rs. $350x$. In the algebraic term $350x$, 350 is called the **coefficient** of x.

Do the review exercise to recall the above facts that you learnt about algebraic expressions in grade 7.

Review Exercise

(1) Complete the table given below.

Algebraic expression	Unknown term of the algebraic expression	Coefficient of the unknown term	Terms of the algebraic expression	Mathematical operations in the order they appear in the algebraic expression
$500 + 3x$	x	3	$500, 3x$	$+ , \times$
$2y + 4$				
$4p - 100$				
$p - 10$				
$3n - 7$				

(2) The length of a table is 2 meters more than its breadth.

(i) Write an algebraic expression for the length of the table by taking its breadth as b meters.
(ii) Write an algebraic expression for the breadth of the table by taking its length as a meters.

(3) A pencil, a pen and an eraser are bought for Rs. a, Rs. b and Rs. 4 respectively.

(i) Write an algebraic expression for the total amount of money needed to buy these three items.

(ii) Write an algebraic expression for the amount of money needed to buy 2 such pencils, 3 such pens and 4 such erasers.

(4) A taxi service charges Rs.100 as an initial fee and Rs.50 for each kilometer travelled. Write an algebraic expression for the total amount that has to be paid for a journey of x meters.

(5) The price of 1 kg of rice is Rs. x and the price of 1 kg of wheat flour is Rs. y.

(i) Write an algebraic expression for the total amount of money required to buy 1 kg of each type.

(ii) Write an algebraic expression for the amount of money required to buy 5 kg of rice and 2 kg of wheat flour.

(iii) Write an algebraic expression for the amount of money required to buy 500 g of each type.

(6) Simplify the algebraic expressions given below.

(a) (i) $a + a + a$ (ii) $4x + 3x$

(iii) $p + 4p - 2p$ (iv) $8a - 5a - a$

(v) $a + 2 + 2a + 3$ (vi) $6x + 10 - 4x + 7$

(b) (i) $3a + 4b + a - 3a + 5$ (ii) $5x - 3y - 4x - 2y$

(iii) $4m - 3n - 4m - n + 8$ (iv) $6x + 7y - 8 - 5x + y - 2$

(v) $2p + 3q + 4r + p - 2q - 3r$

5.2 Constructing algebraic expressions with three unknown terms

In Grade 7 we learnt to construct algebraic expressions with one or two unknown terms. Now let us consider how to construct algebraic expressions with three unknown terms.

- Let us express the total price of 10 books which cost Rs. x each, 3 pens which cost Rs. y each and 5 pencils which cost Rs. z each by an algebraic expression.

Price of the10 books = Rs. $x \times 10$ = Rs. $10x$

Price of the 3 pens = Rs. $y \times 3$ = Rs. $3y$

Price of the 5 pencils = Rs. $z \times 5$ = Rs. $5z$

The price of 10 books, 3 pens and 5 pencils = Rs. $10x + 3y + 5z$

- A cake is made with 500 g of sugar, 1 kg of wheat flour and 500 g of butter. The price of 1 kg of sugar is Rs. x, the price of 1 kg of wheat flour is Rs. y and the price of 1 kg of butter is Rs. z. Let us represent the amount of money required to purchase the items for the cake by an algebraic expression.

Price of 500 g of sugar of which 1 kg is Rs. x = Rs. $\frac{x}{2}$

Price of 1 kg of wheat flour of which 1 kg is Rs. y = Rs. y

Price of 500 g of butter of which 1 kg is Rs. z = Rs. $\frac{z}{2}$

The total amount required = Rs. $\frac{x}{2} + y + \frac{z}{2}$

Example 1

A bus depot uses x number of buses on route 1, y number of buses on route 2, z number of buses on the highway and 12 buses for school services each day. Write an algebraic expression for the total number of buses scheduled to run in a day.

Total number of buses scheduled for route 1, route 2, the highway and school services $\} = x + y + z + 12$

Example 2

Naveen gave Rs. 500 to the shop keeper to buy 2 kg of rice of which 1 kg is Rs. x, 500 g of sugar of which 1 kg is Rs.y and 250 g of flour of which 1 kg is Rs. z. Write an algebraic expression for the balance Naveen received.

Price of 2 kg of rice of which 1 kg is Rs. x $=$ Rs. $2x$

Price of 500 g of sugar of which 1 kg is Rs. y $=$ Rs. $\frac{y}{2}$

Price of 250 g of flour of which 1 kg is Rs z $=$ Rs. $\frac{z}{4}$

Price of 2 kg of rice, 500 g of sugar and 250 g of flour $=$ Rs. $2x + \frac{y}{2} + \frac{z}{4}$

The amount Naveen gave $=$ Rs. 500

Balance Naveen received $=$ Rs. $500 - \left(2x + \frac{y}{2} + \frac{z}{4}\right)$

Exercise 5.1

(1) There are three members in a family. The ages of the mother, the father and the son are given in years by x, y, and z respectively. Using this information, construct algebraic expressions for;

(i) the sum of their ages.

(ii) the sum of their ages after 5 years.

(iii) the difference between the ages of the father and the son.

(iv) the sum of the ages of the mother and the father when the son was born.

(2) The price of a newspaper was Rs. p. If the price increased by Rs. 5, construct algebraic expressions for,

(i) the new price of the newspaper.

(ii) the price of two newspapers after the increase in price.

(iii) the profit gained from a newspaper with the new price, if the cost of printing a newspaper is Rs. q.

(iv) the profit gained from 10 copies, if Rs. r is spent for the distribution of each copy, in addition to the printing cost.

(3) v liters of water is stored in a tank. p liters of water flows out and q liters of water flows into the tank per hour. Construct an algebraic expression for the volume of water in the tank after 3 hours.

(4) There are 700 seats in an auditorium. x number of first class tickets which are Rs.1000 each, y number of second class tickets which are Rs. 500 each and z number of third class tickets which are Rs. 300 each were issued for a drama. Construct algebraic expressions for,

(i) the total number of tickets issued.

(ii) the number of seats which are not occupied.

(iii) the income from the issued tickets.

(iv) the remaining amount when half the income generated from the issued tickets and Rs. 100,000 is paid to the producer of the drama.

5.3 Multiplying an algebraic expression by a number

• Multiplying an algebraic expression by a positive number

Gift parcels are to be prepared for the students in a class. Each parcel is to contain x number of books and y number of pens. Let us find the total number of books and pens needed for 8 such parcels.

Method I

Number of books and pens in a parcel $= x + y$

Number of books and pens needed for 8 such parcels $= (x + y) \times 8$

$(x + y) \times 8$ is also written as $8\ (x + y)$.

Method II

Number of books in a parcel $= x$

Number of books needed for 8 such parcels $= x \times 8$

$= 8x$

Number of pens in a parcel $= y$

Number of pens needed for 8 such parcels $= 8 \times y$

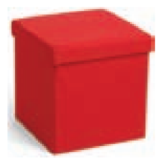
$= 8y$

The number of books and pens needed for 8 parcels $= 8x + 8y$

From this it is clear that $8(x + y) = 8x + 8y$.

$$\therefore 8(x+y) = 8x + 8y$$

- The total mass of a container packed with balls is x kg. The mass of the empty container is y kg. Let us find the total mass of the balls in 5 such containers.



Method I

The mass of the balls in one container $= x - y$

The mass of the balls in 5 such containers $= 5(x - y)$

Method II

The mass of the 5 containers with balls $= 5x$

The mass of the 5 empty containers $= 5y$

The mass of the balls in the 5 containers $= 5x - 5y$

It is clear that $5(x - y) = 5x - 5y$

$$\therefore 5(x - y) = 5x - 5y$$

When multiplying an algebraic expression by a number, each term in the algebraic expression is multiplied by that number.

Example 1

Simplify.

(i) $2(a + b)$ (ii) $3(3x + y)$ (iii) $3(4x - 7)$ (iv) $8(8y - 7x + q)$

(i) $2(a + b) = 2 \times a + 2 \times b$
$= 2a + 2b$

(ii) $3(3x + y) = 3 \times 3x + 3 \times y$
$= 9x + 3y$

(iii) $3(4x - 7) = 3 \times 4x - 3 \times 7$
$= 12x - 21$

(iv) $8(8y - 7x + q) = 64y - 56x + 8q$

Exercise 5.2

(1) Simplify

(i) $5(a + 4)$ (ii) $7(x + 5)$ (iii) $6(2x + 4)$
(iv) $4(4c + 7)$ (v) $5(y - 2)$ (vi) $3(3 - x)$
(vii) $2(m + n - 2p)$ (viii) $4(x - y + 7)$ (ix) $2(x - 2y - q)$

(2) Fill in the blanks.

(i) $2(x + 7) = 2x +$ ..... (ii) $5(6 + a) = 30 +$ ..... (iii) $8(4 - y) = 32 -$ .....
(iv) $6(x - y) =$ ..... $- 6y$ (v) $3(x - 2y + z - 5) =$ ..... $- 6y +$ ..... $-$ .....

(3) The daily wages of a person is Rs. x and overtime payment for an hour is Rs. y. If he did 2 hours of overtime on each of the 5 days he worked,

(i) write an algebraic expression for his salary for the 5 days with overtime payments.

(ii) Due to a loan he has taken, Rs. 150 is deducted from his daily wages. Construct an algebraic expression for the amount he receives in hand for the 5 days and simplify it.

(4) A teacher bought three gift parcels for three students who came first in the third term test. Each parcel contained 5 books and 2 pens.

(i) Write an algebraic expression for the price of one such parcel by taking the price of a book as Rs. a and the price of a pen as Rs. b.

(ii) Write the total price of all three gift parcels as an algebraic expression and simplify it.

(5) On a packet of tea, the mass of the tea is mentioned as p grammes and the mass of the packet as q grammes.

(i) Obtain an algebraic expression for the mass of 20 such packets and simplify it.

(ii) The above 20 packets are packed in a box which is of mass t grammes. Obtain an algebraic expression for 12 such boxes and simplify it.

• Multiplying an algebraic expression by a negative number

When multiplying an algebraic expression by a negative number such as -2 or -1 we have to consider it as a directed number and multiply each term of the algebraic expression by it.

Example 2

Simplify

(i) $-2\,(a+6)$ (ii) $-5\,(6-x)$

(iii) $-\,(2m-3n)$ (iv) $-4\,(2x+3y-2z)$

(i) $-2\,(a+6) = (-2)\times a + (-2)\times 6$
$= -2a - 12$

(ii) $-5\,(6-x) = (-5)\times 6 - (-5)\times x$
$= -30 + 5x$

(iii) $-\,(2m-3n) = (-1)\times 2m - (-1)\times 3n$
$= -2m - (-3)n$
$= -2m + 3n$

(iv) $-4\,(2x+3y-2z) = (-4)\times 2x + (-4)\times 3y - (-4)\times 2z$
$= -8x + (-12y) - (-8z)$
$= -8x - 12y + 8z$

Exercise 5.3

(1) Simplify.

(i) $-3\,(x+5)$ (ii) $-2\,(2x+1)$ (iii) $-2\,(4+x)$
(iv) $-6\,(a-6)$ (v) $-\,(x+5)$ (vi) $-\,(x-3)$
(vii) $-2\,(8+x+y)$ (viii) $-6\,(3b-2+3a)$ (ix) $-\,(a-c-3x)$
(x) $-3\,(6-2x+3b)$

(2) Fill in the blanks.

(i) $-3\ (x + 4) = -3x - \ldots\ldots$	(ii) $-3\ (x - 4) = -3x + \ldots\ldots$
(iii) $-2\ (y + 2) = -2y - \ldots\ldots$	(iv) $-2\ (y - 2) = -2y + \ldots\ldots$
(v) $-\ (m + 2) = \ldots\ldots - 2$	(vi) $-\ (m - 2) = \ldots\ldots + 2$
(vii) $-4\ (2x + 3) = \ldots\ldots -12$	(viii) $-4\ (2x - 3y + 1) = \ldots\ldots + 12y - \ldots\ldots$

(3) Jayamini buys x number of coconuts at Rs. 35 each and y number of mangoes at Rs. 58 each. She gives Rs. 1000 to the vendor. Construct an algebraic expression for the balance she receives and simplify it.

5.4 Multiplying an algebraic term by another algebraic term

Now let us consider multiplying an algebraic term by another algebraic term.

Let us simplify the product of the algebraic terms $5x$ and $3a$.

$$\begin{aligned}(5x) \times (3a) &= 5x \times 3a \\ &= 5 \times x \times 3 \times a \\ &= 5 \times 3 \times x \times a \\ &= 15xa\end{aligned}$$

Similarly, $2p \times 5c = 2 \times p \times 5 \times c = 2 \times 5 \times p \times c = 10pc$

$8r \times 3y = 8 \times r \times 3 \times y = 8 \times 3 \times r \times y = 24ry$

Accordingly, in the algebraic term we get by multiplying an algebraic term by another algebraic term,

- the coefficient is the product of the coefficients of the original two algebraic terms and,
- the product of the unknowns is the product of the two unknowns in the original algebraic terms.

Example 1

Simplify.

(i) $4m \times 3n$	(ii) $8k \times 5y$	(iii) $x \times 5y$
(iv) $2y \times (-2y)$	(v) $2m \times (-7xy)$	(vi) $(-2x) \times 7yz \times 2a$

(i) $4m \times 3n = (4 \times 3) \times (m \times n) = 12mn$

(ii) $8k \times 5y = (8 \times 5) \times (k \times y) = 40ky$

(iii) $x \times 5y = (1 \times 5) \times (x \times y) = 5xy$

(iv) $2y \times (-2y) = (2 \times -2) \times (y \times y) = -4y^2$

(v) $2m \times (-7xy) = (2 \times -7) \times (m \times xy) = -14mxy$

(vi) $(-2x) \times 7yz \times 2a = (-2 \times 7 \times 2) \times (x \times yz \times a) = -28axyz$

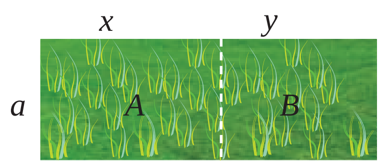
Exercise 5.4

(1) Simplify.

(i) $a \times 2b$	(ii) $2a \times 3b$	(iii) $a \times (-2b)$
(iv) $(-3a) \times 2b$	(v) $(-3x) \times (-4y)$	(vi) $(-5k) \times (-2k)$
(vii) $4p \times (-r)$	(viii) $4y \times (-3y)$	(ix) $ab \times c \times (-4x)$

5.5 Multiplying an algebraic expression by an algebraic term

A rectangular land is divided into two blocks A and B as shown in the figure. Both blocks are rectangular in shape and equal in breadth. Let us find the area of the whole land.



Method I

$$\text{Area of block } A = a \times x = ax$$
$$\text{Area of block } B = a \times y = ay$$

So the area of the whole land $= ax + ay$

We can obtain the area of the land in the following manner too.

Method II

$$\text{The length of the whole land} = (x + y)$$
$$\text{The breadth of the land} = a$$
$$\therefore \text{ the area of the land} = a\,(x + y)$$

Now it is clear that $a\,(x + y) = ax + ay$

$$\therefore\; a\,(x + y) = ax + ay$$

When multiplying an algebraic expression by an algebraic term, every algebraic term of the algebraic expression is multiplied by the given algebraic term.

Example 1

Simplify.

(i) $y(3x + 5)$ (ii) $2y\,(3x + 5)$ (iii) $(-y)\,(3x + 5)$

(iv) $(-2y)\,(3x + 5)$ (v) $2y\,(5y - 3x)$

(i) $y\,(3x + 5) = y \times 3x + y \times 5$
$= 3 \times y \times x + 5 \times y$
$= 3xy + 5y$

(ii) $2y\,(3x + 5) = 2y \times 3x + 2y \times 5$
$= 2 \times 3 \times y \times x + 2 \times 5 \times y$
$= 6xy + 10y$

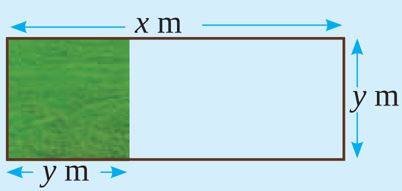
(iii) $(-y)\,(3x + 5) = (-y) \times 3x + (-y) \times 5$
$= (-1) \times 3 \times y \times x + (-1) \times 5 \times y$
$= -3xy - 5y$

(iv) $(-2y)\,(3x + 5) = (-2y) \times 3x + (-2y) \times 5$
$= (-2) \times 3 \times y \times x + (-2) \times 5 \times y$
$= -6xy - 10y$

(v) $2y\,(5y - 3x) = 2y \times 5y - 2y \times 3x$
$= 2 \times 5 \times y \times y - 2 \times 3 \times x \times y$
$= 10y^2 - 6xy$

Example 2

The length of a playground is x meters and its breadth is y meters. Grass is grown on one side, in a square shaped section of side length y meters. Express the area of the remaining land by an algebraic expression and simplify it.



Length of the remaining land $= x - y$
Breadth of the remaining land $= y$
Area of the remaining land $= (x - y)\,y$
$= x \times y - y \times y$
$= xy - y^2$

Exercise 5.5

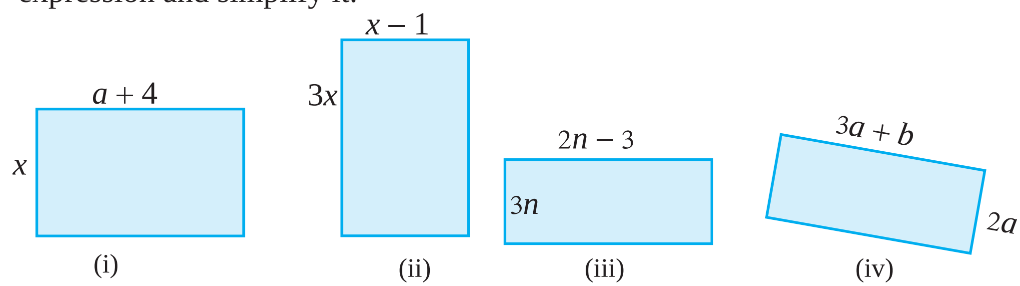
(1) Simplify.

(i) $3x(2y + 1)$ (ii) $3x(2y - 1)$ (iii) $3q(4p - 7)$

(iv) $(-3q)(4p + 8)$ (v) $2x(4p + 5y)$ (vi) $2p(4p + 5y)$

(vii) $2q(xq - z)$ (viii) $(-2q)(x - 4zq)$

(2) Express the area of each rectangular figure given below by an algebraic expression and simplify it.



5.6 Sum of two algebraic expressions

• Like terms

In Grade 7 you learnt that algebraic terms such as x and $2x$ with the same unknown are called **like terms.**

In each of the two terms $3xy$ and $5xy$, the coefficient is multiplied by the common term xy which is the product of the two unknowns x and y. Therefore, they are also known as **like terms**.

• Unlike terms

You learnt in Grade 7 that terms such as $2x$ and $4y$ which have different unknowns are called **unlike terms**.

Let us consider the algebraic terms $3x^2y$ and $5xy^2$.

The coefficient of $3x^2y$ is 3 and the product of the unknowns by which it is multiplied is x^2y.

The coefficient of $5xy^2$ is 5 and the product of the unknowns by which it is multiplied is xy^2.

In these two terms, the products of the unknowns are not the same.

Such algebraic terms are not **like terms**. Therefore they are known as **unlike terms**. Like terms can be added or subtracted and simplified to a single algebraic term.

Example 1

Add $6t + 5$ and $2t + y + 3$ and simplify your answer.

$6t + 5 + 2t + y + 3 = 6t + 2t + y + 5 + 3$

$= 8t + y + 8$

Example 2

Simplify.

(i) $(2x - y + 8) + 2(3y - 10)$

(ii) $(7a - 4b + 2bc) + 2b(4a - 2c + 5)$

(i) $(2x - y + 8) + 2\,(3y - 10) = 2x - y + 8 + 6y - 20$

$= 2x + 5y - 12$

(ii) $(7a - 4b + 2bc) + 2b(4a - 2c + 5) = 7a - 4b + 2bc + 8ab - 4bc + 10b$

$= 7a + 6b - 2bc + 8ab$

Exercise 5.6

(1) Simplify.

(i) $3\,(a + 5b) + a\,(a + 4)$

(ii) $y\,(10 - y) + 3\,(y - 2)$

(iii) $2\,(8a - 5b) + 3\,(5a - 12)$

(iv) $3\,(y - 3) + (8 - 6y + x)$

(v) $a\,(a - 2b) + b\,(b + 2a - c)$

(vi) $5\,(x - y + z) + (4x + 3y)$

5.7 Simplifying the difference of two algebraic expressions

Now let us subtract an algebraic expression from another algebraic expression and simplify it.

Let us subtract $(a + 6)$ from $(2a + 7)$.

$$\begin{aligned}(2a + 7) - (a + 6) &= 2a + 7 + (-1) \times (a + 6)\\ &= 2a + 7 + (-1) \times a + (-1) \times 6\\ &= 2a + 7 + (-a) + (-6)\\ &= 2a + 7 - a - 6\\ &= 2a - a + 7 - 6\\ &= a + 1\end{aligned}$$

Here, the answer is obtained by multiplying each the terms of the algebraic expression which is to be subtracted by (-1) and adding them to the first algebraic expression.

Example 1

Simplify.

(i) $(4x + 3) - (2x - 3)$ (ii) $(3x + 7y) - (2x - 3y - z)$

(iii) $(10a - 8b + \mathrm{c}) - 2\ (4a + b)$ (iv) $a\,(3a + 1) - a(a - 5)$

(i) $(4x + 3) - (2x - 3) = 4x + 3 + (-1) \times (2x - 3)$; [multiplying $(2x - 3)$ by (-1)]

$= 4x + 3 + (-1) \times 2x + (-1) \times (-3)$

$= 4x + 3 + (-2x) + 3$

$= 4x + 3 - 2x + 3$

$= 4x - 2x + 3 + 3$

$= 2x + 6$

(ii) $(3x + 7y) - (2x - 3y - z) = 3x + 7y - 2x + 3y + z$; [multiplying $(2x - 3y - z)$ by (-1)]

$= 3x - 2x + 7y + 3y + z$

$= x + 10y + z$

(iii) $(10a - 8b + c) - 2\ (4a + b) = 10a - 8b + c - 8a - 2b$; [multiplying $(4a + b)$ by (-2)]

$= 10a - 8a - 8b - 2b + c$

$= 2a - 10b + c$

(iv) $a(3a + 1) - a(a - 5) = a \times 3a + a \times 1 - a \times a + a \times 5$

$= 3a^2 + a - a^2 + 5a$

$= 2a^2 + 6a$

Exercise 5.7

(1) Simplify.

(i) $4\ (x + 2) - 2\ (x + 2)$ (ii) $4\ (x - 6) - 6\ (2 + x)$

(iii) $3\ (x - 2) - (x + 2)$ (iv) $4\ (y - 5x) - 2\ (y + 3x + z)$

(v) $4x\ (x + 2) - 3x\ (x - 3)$ (vi) $-6a\ (a - 3) - 3\ (a - 1 + b)$

(2) Simplify.

(i) $-(y + 1) - 3\ (y + 2)$ (ii) $-3\ (y - 2) - 3\ (6 - y)$

(iii) $-(2 - a) - 3\ (a + 8)$ (iv) $-x\ (x + 3) - 2x\ (1 - x)$

(v) $a\ (a + 6) - a\ (a + 2)$ (vi) $a\ (2a - 1) - a\ (6 - a)$

5.8 Substituting given values for each unknown in an algebraic expression up to three unknowns

In Grade 7 you learnt that replacing an unknown term of an algebraic expression by a numerical value is called substitution. By substitution, an algebraic expression takes a numerical value.

Now let us substitute numerical values for the unknown terms of an algebraic expression with three unknowns and find its value.

Let us find the value of the algebraic expression $2p + q - r + 1$ when $p = 4$, $q = 2$ and $r = -3$.

$$\begin{aligned} 2p + q - r + 1 &= 2 \times 4 + 2 - (-3) + 1 \\ &= 8 + 2 + 3 + 1 \\ &= 14 \end{aligned}$$

Now let us find the value of an algebraic expression with brackets by substituting numerical values for the unknowns.

Let us find the value of $3(x + y) + z$ when $x = 2$, $y = 5$ and $z = 10$,

$$\begin{aligned} 3(x + y) + z &= 3(2 + 5) + 10 \\ &= 3 \times 7 + 10 \\ &= 21 + 10 \\ &= 31 \end{aligned}$$

or

$$\begin{aligned} 3(x + y) + z &= 3x + 3y + z \\ &= 3 \times 2 + 3 \times 5 + 10 \\ &= 6 + 15 + 10 \\ &= 31 \end{aligned}$$

Example 1

Find the value of the algebraic expression $2x - y - 2z$ when $x = 4$, $y = 3$ and $z = 2$.

$$\begin{aligned} 2x - y - 2z &= 2 \times 4 - 1 \times 3 - 2 \times 2 \\ &= 8 - 3 - 4 \\ &= 1 \end{aligned}$$

Example 2

Find the value of the algebraic expression $-p + 2q - 3r + 7$ when $p = 5$, $q = -2$ and $r = -3$.

$$\begin{aligned} -p + 2q - 3r + 7 &= -1 \times 5 + 2 \times (-2) - 3 \times (-3) + 7 \\ &= (-5) + (-4) - (-9) + 7 \\ &= (-9) + (+9) + 7 \\ &= 0 + 7 \\ &= 7 \end{aligned}$$

Example 3

Find the value of the algebraic expression $6\,(2a - b) - c$ when $a = 4$, $b = 5$ and $c = 8$.

$$\begin{aligned} 6\,(2a - b) - c &= 6\,(2 \times 4 - 5) - 8 \\ &= 6\,(8 - 5) - 8 \\ &= 6 \times 3 - 8 \\ &= 18 - 8 = 10 \end{aligned}$$

Example 4

Find the value of the algebraic expression $10\,(k - l) + r$ when $k = 4$, $l = 1$ and $r = -3$.

$$\begin{aligned} 10\,(k - l) + r &= 10\,(4 - 1) + (-3) \\ &= 10 \times 3 - 3 \\ &= 30 - 3 = 27 \end{aligned}$$

Example 5

Simplify the algebraic expression $5x + 3y - 4x - y + 8$ and find its value when $x = 2$ and $y = -1$.

$$\begin{aligned} 5x + 3y - 4x - y + 8 &= 5x - 4x + 3y - y + 8 \\ &= x + 2y + 8 \end{aligned}$$

Substituting the given values,

$$\begin{aligned} x + 2y + 8 &= 2 + 2\,(-1) + 8 \\ &= 2 + (-2) + 8 \\ &= 0 + 8 = 8 \end{aligned}$$

Example 6

Simplify the algebraic expression $4\,(a - 2b) + 2\,(b - 3c)$ and find its value when $a = 3$, $b = 1$, $c = -1$.

Expanding the expression,

$$\begin{aligned} 4\,(a - 2b) + 2\,(b - 3c) &= 4 \times a - 4 \times 2b + 2 \times b - 2 \times 3c \\ &= 4a - 8b + 2b - 6c \\ &= 4a - 6b - 6c \end{aligned}$$

Substituting the given values,

$$\begin{aligned} 4a - 6b - 6c &= 4 \times 3 - 6 \times 1 - 6 \times (-1) \\ &= 12 - 6 + 6 \\ &= 12 \end{aligned}$$

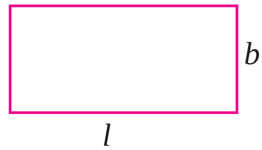
Exercise 5.8

(1) Find the value of each algebraic expression when $x = -3,\ y = -1,\ z = 0$

(i) $x + y$ (ii) $y + 3z + 7$ (iii) $x - 4y + 4z$

(iv) $x + y - z$ (v) $z\ (2x - 3y)$ (vi) $5y - 4z + 3x$

(2) In the given rectangle, the length is l cm and the breadth is b cm.



(i) Write an algebraic expression for its perimeter.
(ii) Find the perimeter of the rectangle if $l = 10$ cm and $b = 7$ cm.
(iii) Find the perimeter of the rectangle if $b = 5$ cm and l is twice b.
(iv) Find the perimeter of the rectangle if $b = 12$ cm and l is 8 cm more than b.

(3) $2x - 9y - 4z + 7$

(i) Find the value of the above algebraic expression when $x = 4,\ y = 3$ and $z = -2$.
(ii) Find the value of the above algebraic expression when $x = 10,\ y = 15$ and $z = -1$.
(iii) Find the value of the above algebraic expression when $x = -4,\ y = -3$ and $z = -2$.
(iv) Find the value of the above algebraic expression when $x = 2,\ y = -3$ and $z = 0$.

(4) Complete the tables given below.

(a)

Expression	Values of the unknowns	Value of the algebraic expression
$3x + 2y + 10$	$x = 4,\ y = 3$	
$2p - 3q - 4r$	$p = 1,\ q = 2,\ r = -3$	
$4a - b + 5c$	$a = 2,\ b = -4,\ c = 1$	

(b)

Expression	Values of the unknowns	Value of the algebraic expression
$3(x + y) + 10z$	$x = -1,\ y = 3,\ z = 2$	
$4(a + 3b) + c$	$a = 5,\ b = 1,\ c = -10$	
$10(m + n) - k$	$m = 3,\ n = -1,\ k = 8$	
$100 - 3(p + 2q)$	$p = 4,\ q = -5$	
$2(a + 2b) + 5(a - b)$	$a = 4,\ b = -1$	

(5) Expand each algebraic expression given below and find its value by substituting the given values for the unknowns.

(i) Find the value of $10(a + 2b) + 3(a - 5b)$ when $a = 7$ and $b = 1$.

(ii) Find the value of $4(m + 3n) + m + 5n$ when $m = 9$ and $n = -2$.

(iii) Find the value of $7(2p - q) - 10p + 3q - 8$ when $p = 2$ and $q = 3$.

(iv) Find the value of $3(2a + 7b) + 3(b + 3c) - 10$ when $a = 1$, $b = 2$ and $c = -3$.

(v) Find the value of $4(x - 5y) - 3(7 - x) + 8l$ when $x = 8$, $y = -1$ and $l = -2$.

Summary

- When multiplying an algebraic expression by a number, each term of the algebraic expression needs to be multiplied by the number. That is, the coefficient of each algebraic term should be multiplied by the given number and simplified.
- When multiplying an algebraic term by an algebraic term, their coefficients are multiplied first and then the unknowns are multiplied.
- When multiplying an algebraic expression by an algebraic term, every algebraic term of the algebraic expression needs to be multiplied by the given algebraic term.
- By substituting values for the unknown terms of an algebraic expression, we obtain a numerical value for the algebraic expression.

6

Solids

By studying this lesson you will be able to,

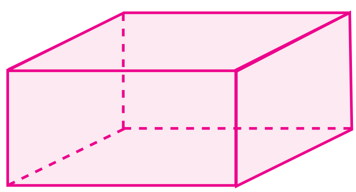
- prepare models of regular octahedrons, regular dodecahedrons and regular icosahedrons,
- verify Euler's relationship for the above solids by considering the number of edges, vertices and faces of these solids, and
- from given, solids identify the platonic solids and describe their characteristics.

6.1 Solids

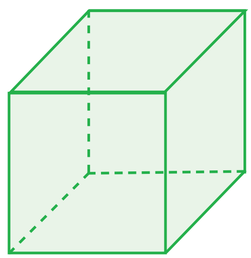
You have learnt that an object which has a specific shape and which occupies a certain amount of space is called a solid object.

You have also learnt that the surfaces of solids objects are plane surfaces or curved surfaces.

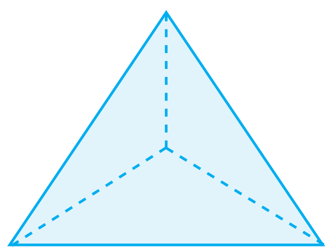
Some solids you have studied in Grades 6 and 7 are illustrated below.



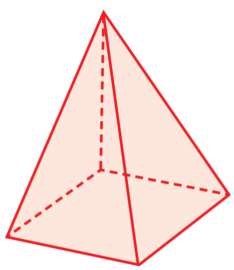
A cuboid



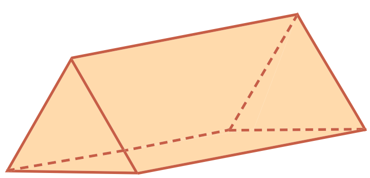
A cube



A regular tetrahedron



A square pyramid



A triangular prism

Do the review exercise to recall what you have learnt about solids in Grades 6 and 7.

Review Exercise

(1) Fill in the blanks in the table given below.

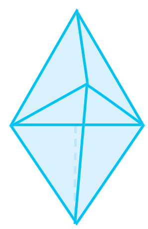
Solid	Number of edges	Number of faces	Number of vertices
Cuboid	12	6	8
Cube			
Regular tetrahedron			
Square pyramid			
Triangular prism			

(2) Draw nets that can be used to construct the following solids.

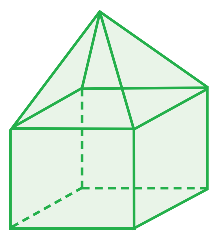
(i) Square pyramid

(ii) Triangular prism

(3) A figure of a solid constructed by pasting together two triangular faces of two identical regular tetrahedrons, one on the other, is given here. Find the number of edges, faces and vertices of this solid.

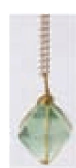


(4) A solid constructed by joining a cube and a square pyramid is shown in the figure. Find the number of edges, faces and vertices of this solid.

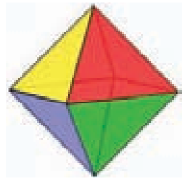


6.2 Octahedron

Diamonds and certain other gems used in jewellery are cut in the shape of an octahedron.



A solid which has 8 faces is called an **octahedron**.

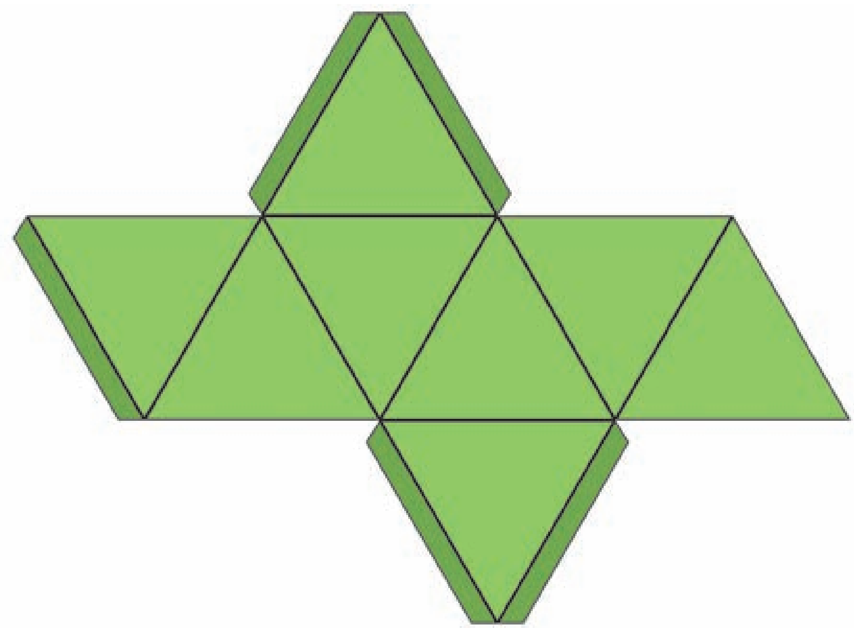


A solid object which has eight identical equilateral triangular shaped faces is called a **regular octahedron**. The figure shows a regular octahedron.

Let us identify the characteristics of a regular octahedron by engaging in the following activity.

Activity 1

Step 1 - Copy the given figure on a thick piece of paper such as a Bristol board, or get a photo copy of the figure and paste it on a thick piece of paper.

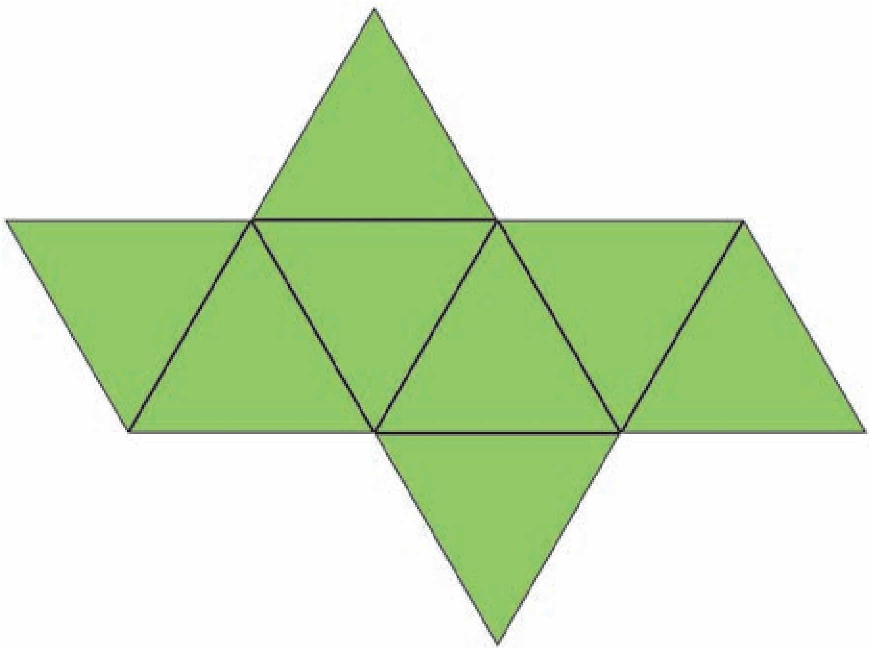


Step 2 - Cut out the figure drawn or pasted on the Bristol board and prepare a model of a regular octahedron by folding along the edges and pasting along the pasting allowances.

Step 3 - By considering the model you prepared, find the number of faces, edges and vertices of a regular octahedron. Examine and identify the special features of the model.

Step 4 - Write the special features you identified in your exercise book.

The figure you obtain by removing the pasting allowances of the above figure, which was used to prepare a model of a regular octahedron, is called a **net of the regular octahedron.**



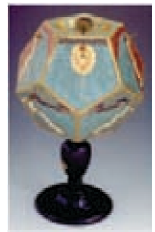
The object you constructed during the above activity is a model of a regular octahedron.

Features you can identify in a regular octahedron

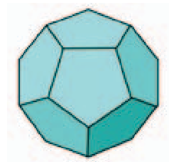
- There are 8 faces in a regular octahedron.
- All faces are the shape of identical equilateral triangles.
- There are 6 vertices in a regular octahedron.
- There are 12 edges in a regular octahedron. All are straight edges.

6.3 Dodecahedron

Models of this shape are used for decorations and ornaments.



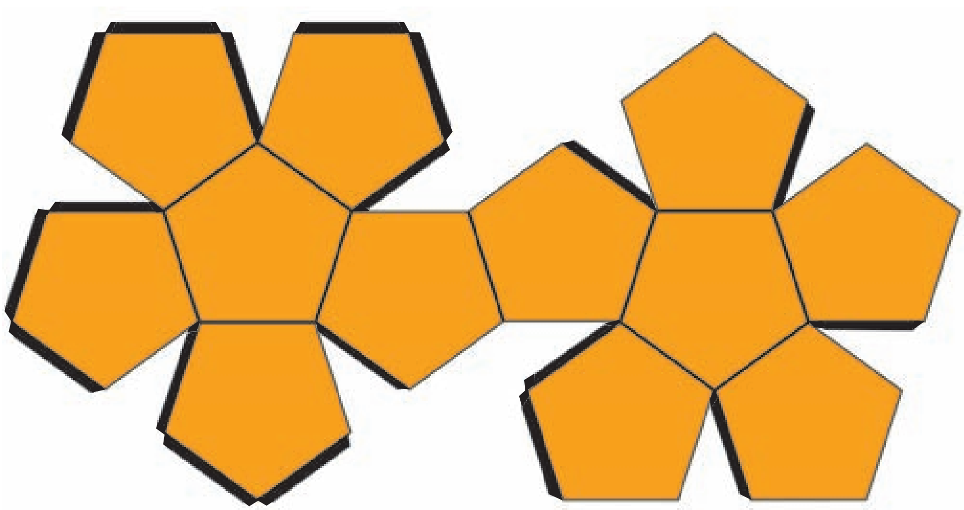
A solid object which has 12 regular pentagonal faces is called a **regular dodecahedron**. The figure shows a regular dodecahedron.



Let us identify the characteristics of a regular dodecahedron by engaging in the following activity.

Activity 2

Step 1 - Copy the given figure on a thick piece of paper such as a Bristol board or get a photo copy of the figure and paste it on a thick piece of paper.

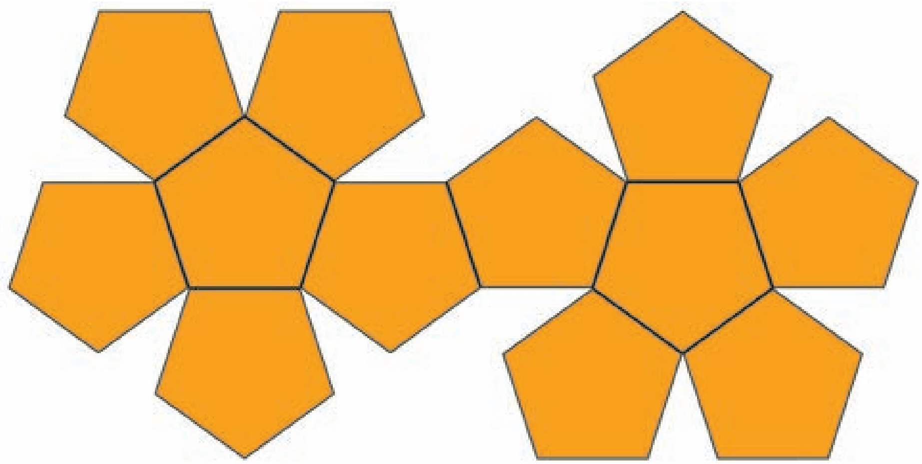


Step 2 - Cut out the figure drawn or pasted on the Bristol board and prepare a model of a regular dodecahedron by folding along the edges and pasting along the pasting allowances.

Step 3 - By considering the model you prepared, find the number of faces, edges and vertices of a regular dodecahedron. Examine and identify the special features of the model.

Step 4 - Write the special features you identified in your exercise book.

The figure you obtain by removing the pasting allowances of the above figure which was used to prepare a model of a regular dodecahedron, is called a **net of the regular dodecahedron**.



The object you constructed during the above activity is a model of a regular dodecahedron.

Features you can identify in a regular dodecahedron

- There are 12 faces in a regular dodecahedron.
- All the faces of a regular dodecahedron take the shape of identical regular pentagons.
- There are 20 vertices in a regular dodecahedron.
- There are 30 edges in a regular dodecahedron. All are straight edges.

6.4 Icosahedron

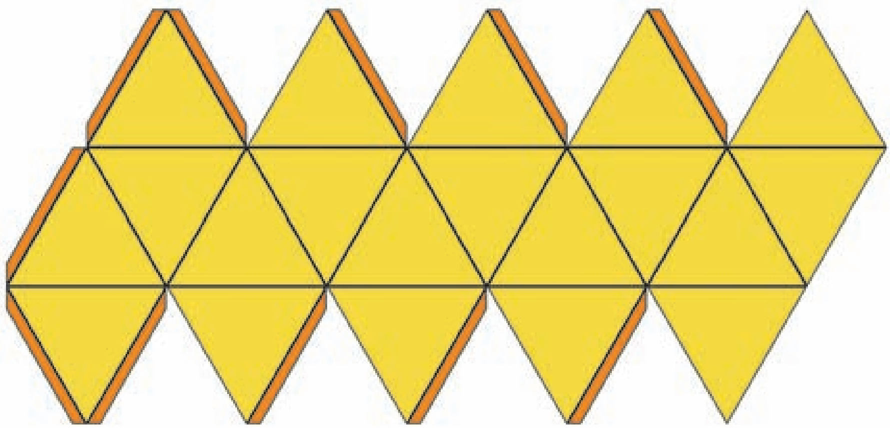
A model which can be used in decorations such as Vesak lanterns is given here. It is known as an icosahedron.

A solid which has twenty equilateral triangular faces is called a **regular icosahedron**. The figure shows a regular icosahedron.

Let us identify the characteristics of a regular icosahedron by engaging in activity 3.

Activity 3

Step 1 - Copy the given figure on a thick piece of paper such as a Bristol board, or get a photo copy of the figure and paste it on a thick piece of paper.

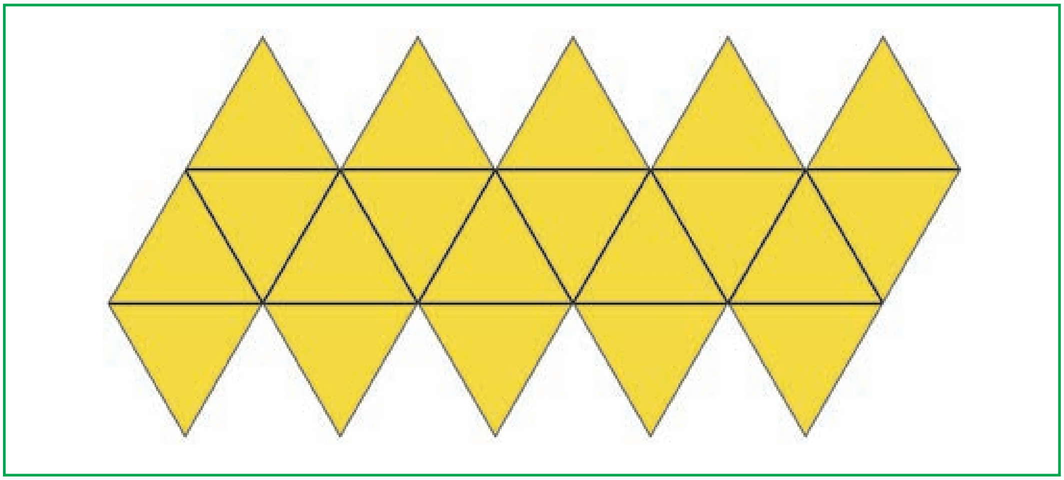


Step 2 - Cut out the figure drawn or pasted on the Bristol board and prepare a model of a regular icosahedron by folding along the edges and pasting along the pasting allowances.

Step 3 - By considering the model you prepared, find the number of faces, edges and vertices of a regular icosahedron. Examine and identify the special features of the model.

Step 4 - Write the special features you identified in your exercise book.

The figure you obtain by removing the pasting allowances of the above figure which was used to prepare a model of a regular icosahedron is called a **net of the regular icosahedron**.



The object you constructed during the above activity is a model of a regular icosahedron.

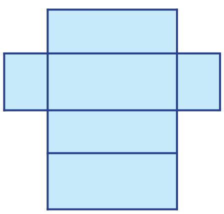
Features you can identify in a regular icosahedron

- There are 20 faces in a regular icosahedron.
- The faces of a regular icosahedron take the shape of identical equilateral triangles.
- There are 12 vertices in a regular icosahedron.
- There are 30 edges in a regular icosahedron. All are straight edges.

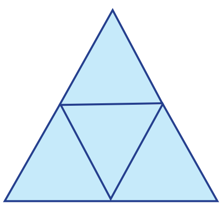
Exercise 6.1

(1) Name the solid which can be constructed using each net given below.

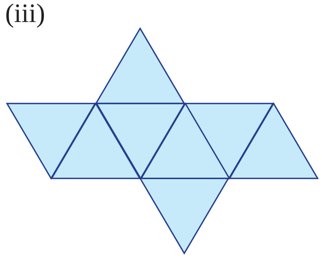
(i)



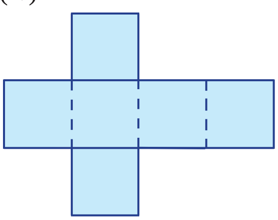
(ii)



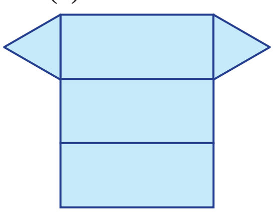
(iii)



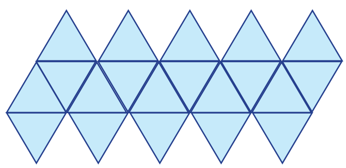
(iv)



(v)



(vi)



6.5 Verification of Euler's relationship for solids

You learnt in Grade 7 about the relationship between the edges, vertices and faces of a solid, which was first presented by the Swiss mathematician Euler. Let us recall what you learnt.

Euler's relationship

In a solid with straight edges, the sum of the number of faces and the number of vertices is two more than the number of edges.

This relationship can be expressed as follows.

Number of Vertices + Number of Faces = Number of Edges + 2

$$V + F = E + 2$$

Activity 4

Fill in the blanks in the table given below by observing the solids you constructed in activities 1, 2 and 3.

Solid	Number of vertices (V)	Number of faces (F)	Number of edges (E)	$V+F-E$	Is Euler's relationship satisfied?
Regular Octahedron					
Regular Dodecahedron					
Regular Icosahedron					

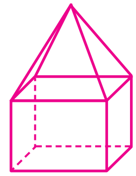
Exercise 6.2

(1) Verify Euler's relationship for a regular tetrahedron by considering the number of faces, vertices and edges it has.

(2) For a square pyramid,
   (i) write down the number of faces, vertices and edges.
   (ii) show that the above values satisfy Euler's relationship.

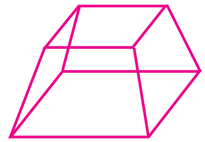
(3) If a certain solid has 9 edges and 6 vertices, and if Euler's relationship is satisfied, find the number of faces it has.

(4) A figure of a composite solid is shown here. Determine with reasons whether Euler's relationship is satisfied for this solid.



(5) A certain solid has 10 edges and 6 faces. Find the number of vertices it has, if Euler's relationship is satisfied.

(6) The figure given here shows a pyramid of which the upper portion has been cut out and removed. Verify Euler's relationship for this solid.

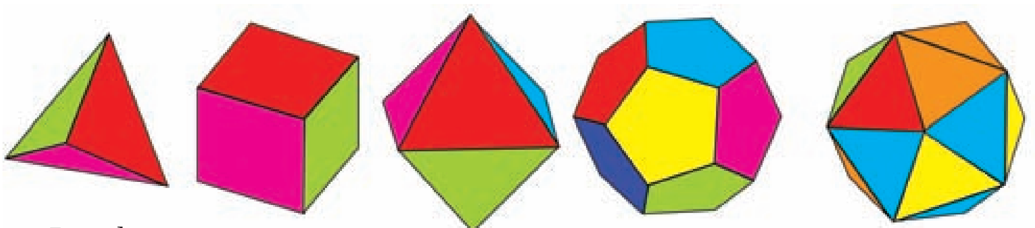


6.6 Platonic solids

Platonic solids are solids having identical regular polygonal faces and with the same number of faces meeting at every vertex.

You have learnt about the five types of solids which are considered as platonic solids. They are the regular tetrahedron, cube, regular octahedron, regular dodecahedron and the regular icosahedron.

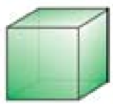
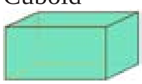
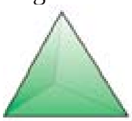
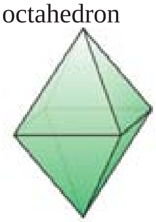
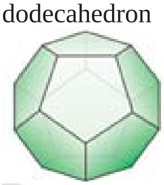
They are called **platonic solids**.

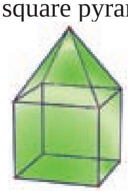


Regular Tetrahedron | Cube | Regular Octahedron | Regular Dodecahedron | Regular Icosahedron

Exercise 6.3

(1) Complete the table given below.

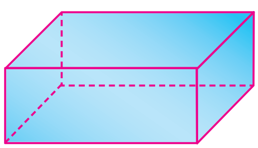
Solid	Shapes of the faces of the solid	Are all the faces regular?	Are the number of faces meeting at each vertex equal?	Number of faces meeting at a vertex	Is the solid a platonic solid?
Cube 	Square	Regular	Equal	3	Yes
Cuboid 					
Regular tetrahedron 					
Regular octahedron 					
Regular dodecahedron 					

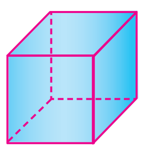
Solid	Shapes of the faces of the solid	Are all the faces regular?	Are the number of faces meeting at each vertex equal?	Number of faces meeting at a vertex	Is the solid a platonic solid?
Regular icosahedron 					
Composite solid consisting of a cuboid and a square pyramid 					

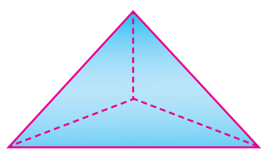
(2) Construct a regular icosahedron and 20 regular tetrahedrons such that the icosahedron and the tetrahedrons have edges of equal length. Construct a composite solid by pasting a tetrahedron on each face of the icosahedron. For the composite figure, find

(i) the number of edges.
(ii) the number of faces.
(iii) the number of vertices.

(3) From the following, select the platonic solids and write down the corresponding numbers.


(i)


(ii)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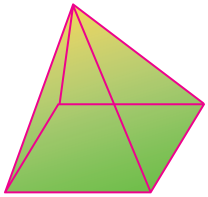

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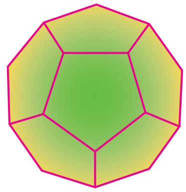

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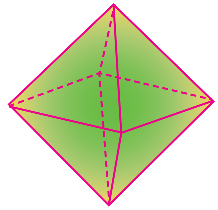

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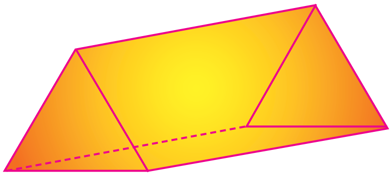
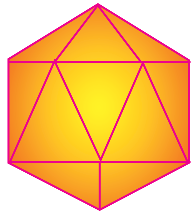


(vii)



(viii)

Summary

- The sum of the number of faces and the number of vertices of a solid with straight edges is 2 more than the number of edges.
- Solids having identical regular polygonal faces and with the same number of faces meeting at every vertex are called platonic solids.
- The five types, regular tetrahedrons, cubes, regular octahedrons, regular dodecahedrons and regular icosahedrons are the only solids that are platonic solids.

Solid	Shape of a face	Number of faces	Number of edges	Number of vertices
Cube	Square	6	12	8
Cuboid	Rectangle	6	12	8
Regular tetrahedron	Equilateral triangle	4	6	4
Square pyramid	One face is square shaped. The other 4 faces take the shape of identical triangles	5	8	5
Triangular prism	3 rectangular faces and 2 triangular faces	5	9	6
Regular octahedron	Equilateral triangle	8	12	6
Regular dodecahedron	Regular pentagon	12	30	20
Regular icosahedron	Equilateral triangle	20	30	12

7 Factors

By studying this lesson you will be able to,

- find the highest common factor of the terms of a set which consists of up to three algebraic terms,
- write an algebraic expression as a product of two factors, where one factor is the highest common factor of the terms of the algebraic expression, and
- establish that an algebraic expression written in terms of its factors is the given algebraic expression, by multiplying the factors.

7.1 The highest common factor (HCF) of several numbers

$6 = 2 \times 3$

You have learnt previously that 2 and 3 are factors of 6.

When a number is written as a product of two whole numbers, those numbers are called **factors** of the original number.

The HCF of two or more numbers is the largest of all the common factors of the given numbers. That is, the largest number by which all the given numbers are divisible is their HCF.

Now let us find the HCF of 6 and 10.

$6 = 1 \times 6$	$10 = 1 \times 10$
$6 = 2 \times 3$	$10 = 2 \times 5$

$\therefore$ the factors of 6 are 1, 2, 3 and 6.
The factors of 10 are 1, 2, 5 and 10.

1 and 2 are the common factors of 6 and 10. Since 2 is the larger factor, the HCF of 6 and 10 is 2.

You learnt in Grade 7 how to find the HCF of several numbers by writing each as a product of prime numbers. Let us recall what you learnt through an example.

Let us find the HCF of 6, 12 and 18.
Let us write each number as a product of prime factors.

$$\begin{array}{r|l} 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$\begin{aligned} 6 &= 2 \times 3 \\ 12 &= 2 \times 2 \times 3 \\ 18 &= 2 \times 3 \times 3 \end{aligned}$$

We obtain the HCF of 6, 12 and 18 by taking the product of the prime factors which are common to these three numbers.

The HCF of 6, 12 and 18 $= 2 \times 3 = 6$

Note

To find the prime factors of a whole number,

- it is sequentially divided by the prime numbers by which it is divisible, starting from the smallest such prime number, till the answer 1 is obtained.

Review Exercise

Find the HCF of each set of numbers given below.

(i) 12, 18 (ii) 30, 24 (iii) 45, 60
(iv) 6, 12, 18 (v) 15, 30, 75 (vi) 36, 24, 60
(vii) 6, 9, 12 (viii) 15, 30, 45 (ix) 11, 13, 5

7.2 The highest common factor of several algebraic terms

Now let us see what is meant by the HCF of several algebraic terms and how to find it.

Let us find the HCF of the algebraic terms $4x$, $8xy$ and $6xyz$.

Let us write each term as a product of its factors.

$$\begin{aligned} 4x &= 2 \times 2 \times x \\ 8xy &= 2 \times 2 \times 2 \times x \times y \\ 6xyz &= 2 \times 3 \times x \times y \times z \end{aligned}$$

Here, the coefficient of each algebraic term is written as a product of its prime factors and the unknowns are separated and written as a product.

The common factors of all three algebraic terms, $4x$, $8xy$ and $6xyz$ are 2 and x.
The HCF of the algebraic terms, $4x$, $8xy$ and $6xyz$ is the product of the factors which are common to all three terms.

$\therefore$ The HCF of $4x$, $8xy$ and $6xyz$ $= 2 \times x$
$= 2x$

Example 1

Find the HCF of the algebraic terms in each part given below.

(i) $2pq,\ 4pqr$ (ii) $7mn,\ 14mnp,\ 28mnq$

(i) $2pq = 2 \times p \times q$

$4pqr = 2 \times 2 \times p \times q \times r$

The HCF of $2pq$ and $4pqr$ $= 2 \times p \times q$

$= 2pq$

(ii) $7mn = 7 \times m \times n$

$14mnp = 2 \times 7 \times m \times n \times p$

$28mnq = 2 \times 2 \times 7 \times m \times n \times q$

The HCF of $7mn$, $14mnp$ and $28mnq$ $= 7 \times m \times n$

$= 7mn$

Exercise 7.1

Find the HCF of the algebraic terms in each part given below.

(i) $xy,\ 3xy,\ 4x$ (ii) $4c,\ 8a,\ 4b$

(iii) $2x,\ 8x,\ 4xy$ (iv) $4p,\ 8pq,\ 12pq$

(v) $8pqr,\ 16qr,\ 7mqr$ (vi) $4x,\ 6xy,\ 8qrx$

(vii) $4x,\ 6abx,\ 10abxy$ (viii) $6mn,\ 12mny,\ 15my$

7.3 Writing an algebraic expression as a product of its factors

Since 2 and 3 are the prime factors of 6, 6 can be written as a product of its prime factors as $6 = 2 \times 3$.

Now let us consider how to write an algebraic expression as a product of its factors.

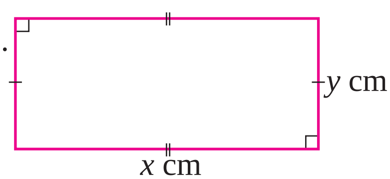
Let us find the perimeter of the rectangle in the figure.

Method I

Let us add the lengths of all four sides of the rectangle.

Perimeter of the rectangle $= x + y + x + y$

$= 2x + 2y$



Method II

Let us find the perimeter by multiplying the sum of the length and breadth of the rectangle by two.

Perimeter of the rectangle $= (x + y) \times 2$
$= 2(x + y)$

Since the perimeter of the same rectangle is found by both methods, the two expressions obtained for the perimeter are equal.

$$\therefore\ 2x + 2y = 2(x + y)$$

Writing the algebraic expression $2x + 2y$ as $2(x + y)$, is called **writing the algebraic expression $2x + 2y$ as a product of factors**.

That is, 2 and $(x + y)$ are two factors of the expression $2x + 2y$.

➢ Now let us write the algebraic expression $12x + 18y$ as a product of two factors.

$12x + 18y$ can be expressed as a product of two factors in several ways.

(i) $12x + 18y = 2 \times 6x + 2 \times 9y$
$= 2(6x + 9y)$
In this instance, 2 is taken as a common factor of the two terms.

(ii) $12x + 18y = 3 \times 4x + 3 \times 6y$
$= 3(4x + 6y)$
In this instance, 3 is taken as a common factor of the two terms.

(iii) $12x + 18y = 6 \times 2x + 6 \times 3y$
$= 6(2x + 3y)$
In this instance, 6 is taken as a common factor of the two terms.

Since there is no common factor in $2x$ and $3y$, which are the terms of the expression within brackets, 6 is the HCF of the terms $12x$ and $18y$.

When writing such an algebraic expression as a product of factors, the convention is to write the first factor as a number which is the HCF of the terms of the given expression, and the other factor as an algebraic expression, where the HCF of its terms is 1.

Accordingly, when writing an algebraic expression as a product of factors,

- first find the highest common factor of the terms of the algebraic expression,
- take this HCF as one factor and the expression which is obtained by dividing each term of the algebraic expression by this HCF as the other factor, and
- write the algebraic expression as a product of these two factors.

Example 1

Write the expression $36a + 60b$ as a product of factors.

$$36a = 2 \times 2 \times 3 \times 3 \times a$$
$$60b = 2 \times 2 \times 3 \times 5 \times b$$

The HCF of the terms $36a$ and $60b$ $= 2 \times 2 \times 3$
$= 12$

$\therefore \; 36a + 60b = 12 \times 3a + 12 \times 5b$
$= 12\,(3a + 5b)$

$36a \div 12 = 3a$
$60b \div 12 = 5b$

Example 2

Write the expression $12x + 20y + 16z$ as a product of factors.

$$12x = 2 \times 2 \times 3 \times x$$
$$20y = 2 \times 2 \times 5 \times y$$
$$16z = 2 \times 2 \times 2 \times 2 \times z$$

The HCF of $12x$, $20y$ and $16z$ $= 2 \times 2$
$= 4$

$\therefore \; 12x + 20y + 16z = 4 \times 3x + 4 \times 5y + 4 \times 4z$
$= 4\,(3x + 5y + 4z)$

$12x \div 4 = 3x$
$20y \div 4 = 5y$
$16z \div 4 = 4z$

Exercise 7.2

(1) Fill in the blanks.

(i) $3x + 12 = 3 \times \square + 3 \times \square = 3\,(\square + \square)$
(ii) $15x + 20y = 5 \times \square + 5 \times \square = 5\,(\square + \square)$
(iii) $12a + \square = 6 \times \square + 6 \times \square = 6\,(\square + 3)$
(iv) $12x + 8y + 20z = 4 \times \square + 4 \times \square + 4 \times \square = 4\,(\square + \square + \square)$
(v) $30x + 24y + 18 = \square\,(5x + \square + \square)$

(2) Write each of the algebraic expressions given below as a product of two factors such that one factor is the HCF of the terms of the expression.

(a) (i) $2x + 6y$ (ii) $8x + 12y$ (iii) $15a + 18b$
(iv) $9x + 27y$ (v) $4p + 20q$ (vi) $12p + 30q$
(vii) $20a - 30b$ (viii) $36a - 54b$ (ix) $60p - 90q$

(b) (i) $5x - 10y + 25$ (ii) $3a + 15b - 12$ (iii) $18 - 12m + 6n$
(iv) $10a - 20b - 15$ (v) $9c - 18a + 9$ (vi) $12d + 6 + 18c$
(vii) $3x + 6y - 3$ (viii) $10m + 4n - 2$ (ix) $12a - 8b + 4$
(x) $9 + 3b + 6c$ (xi) $3a^2 - 6ab + 9b^2$ (xii) $4a^2 - 16ab - 12c$

7.4 Writing an algebraic expression as a product of factors where one factor is a negative number

Since $(-12) = (-6) \times 2$, we have that (-6) is a factor of (-12).

Since $(-12) = 6 \times (-2)$, (-2) is also a factor of (-12).

Since $12 = (-6) \times (-2)$, both (-6) and (-2) are factors of 12.

Example 1

(i) Write (-15) as a product of two factors, such that (-3) is a factor.

$(-15) = (-3) \times 5$

(ii) Write 10 as a product of two factors such that (-2) is a factor.

$10 = (-2) \times (-5)$

Accordingly, (-2) and (-5) are two factors of 10.

Now let us consider an instance where one factor of the algebraic expression is a negative number.

Let us consider the algebraic expression $-2x + 6y$. Here, 2 is a common factor of $-2x$ and $6y$.

Therefore, $-2x + 6y = 2(-x + 3y)$

Since $-2x = (-2) \times x$ and $6y = (-2) \times (-3) \times y$,

(-2) is also a common factor of $-2x$ and $6y$.

$$\begin{aligned} \therefore -2x + 6y &= (-2) \times x + (-2) \times (-3)\, y \\ &= (-2)(x + (-3)\, y) \\ &= -2\,(x - 3y) \end{aligned}$$

$\therefore$ the algebraic expression $-2x + 6y$ can also be written as a product of two factors as $-2\,(x - 3y)$.

Example 2

Write down each of the algebraic expressions given below as a product of two factors such that one factor is a negative number.

(i) $-4x - 16y$ (ii) $-8m + 24n - 16$

(i)
$$\begin{aligned} -4x - 16y &= (-4) \times x + (-16)y \\ &= (-4) \times x + (-4) \times (+4)\, y \\ &= (-4)\,(x + (+4)\, y) \\ &= -4\,(x + 4y) \end{aligned}$$

(ii)
$$\begin{aligned} -8m + 24n - 16 &= -8 \times 1m + (-8) \times (-3)\, n + (-8) \times (+2) \\ &= -8\,(m - 3n + 2) \end{aligned}$$

Note

When one factor is a negative number, the sign of each term of the other factor is opposite to that of the corresponding term in the original algebraic expression.

Exercise 7.3

(1) (i) Write (–20) as a product of two factors such that (–4) is one of the factors.

(ii) Write 12 as a product of two factors such that (–4) is one of the factors.

(2) Write each algebraic expression given below as a product of two factors such that one factor is a negative number.

(i) $12x - 4y$ (ii) $-12x + 4y$ (iii) $-12x - 4y$
(iv) $-3a + 15b - 6c$ (v) $-12a + 18b - 24c$ (vi) $-8p + 40q - 24$

7.5 More on writing an algebraic expression as a product of two factors

Let us consider the algebraic expression $pq + pr$.

$pq = p \times q$

$pr = p \times r$

Since p is a factor of each term of this expression, p is a common factor of the two terms.

$$\therefore\ pq + pr = p \times q + p \times r$$
$$= p(q + r)$$

Accordingly, when writing an algebraic expression as a product of factors,

- first find the HCF of the terms of the algebraic expression,
- take the HCF as one factor and the expression which is obtained by dividing each term of the algebraic expression by the HCF as the other factor, and
- write the algebraic expression as a product of these two factors.

Example 1

Write the expression $18x + 24xy + 12xz$ as a product of two factors.

The HCF of the terms $18x$, $24xy$ and $12xz$ is $6x$

$$\therefore\ 18x + 24xy + 12xz = 6x \times 3 + 6x \times 4y + 6x \times 2z$$
$$= 6x\,(3 + 4y + 2z)$$

Note

- Let us simplify $6 \div 9$.

You have learnt that $6 \div 9 = \frac{6 \div 3}{9 \div 3} = \frac{2}{3}$.

Moreover, this can also be simplified as $\frac{6}{9} = \frac{\cancel{3}^{1} \times 2}{{}_{1}\cancel{3} \times 3} = \frac{2}{3}$.

- Let us simplify $3xy \div 5y$.

$3xy \div 5y = \frac{3xy}{5y} = \frac{3 \times x \times y}{5 \times y}$

Since y represents a number, it can be simplified as above.

$\frac{3 \times x \times \cancel{y}^{1}}{5 \times \cancel{y}_{1}} = \frac{3 \times x}{5} = \frac{3x}{5}$

Example 2

Write the expression $15pq + 45qr + 60q$ as a product of factors.

$$\begin{aligned}15pq &= 3 \times 5 \times p \times q \\ 45qr &= 3 \times 3 \times 5 \times q \times r \\ 60q &= 2 \times 2 \times 3 \times 5 \times q\end{aligned}$$

The HCF of $15pq$, $45qr$ and $60q$ $= 3 \times 5 \times q$

$= 15q$

$\therefore\ 15pq + 45qr + 60q = 15q\ (p + 3r + 4)$

$15pq \div 15q = p$
$45qr \div 15q = 3r$
$60q \div 15q = 4$

Example 3

Write the expression $3a + 6ab + 12ac$ as a product of factors.

$$\begin{aligned}\text{Here } 3a &= 3 \times a \\ 6ab &= 3 \times 2 \times a \times b \\ 12ac &= 2 \times 2 \times 3 \times a \times c\end{aligned}$$

HCF of $3a$, $6ab$ and $12ac$ $= 3 \times a$

When the HCF $3a$ is separated out as a common factor and written we obtain,

$3a + 6ab + 12ac = 3a\ (1 + 2b + 4c)$.

Note that when the expression within brackets is multiplied by $3a$, the original expression, $3a + 6ab + 12ac$ is obtained.

$3a\ (1 + 2b + 4c) = 3a + 6ab + 12ac$

$\therefore\ 3a + 6ab + 12ac$ is the product of the two factors $3a$ and $(1 + 2b + 4c)$.

Exercise 7.4

(1) Write each algebraic expression given below as a product of two factors.

(i) $ab + ac$ (ii) $p + pq$ (iii) $xyz + xpq$

(iv) $3x + 6xy$ (v) $15pq - 20pr$ (vi) $4p - 16pq + 12pr$

(vii) $2a - 8ab - 8ac$ (viii) $5x - 10xy - 5xz$ (ix) $3ab - 9abc$

(2) Write each of the following algebraic expressions as a product of two factors Establish the accuracy of your answer by simplifying the product.

(i) $xyz - 2xyp$ (ii) $12x - 20xy$ (iii) $ab + ac - ad$

(iv) $p + pq + pqr$ (v) $xp - xy - x$ (vi) $6ab - 8ab^2 + 12ac$

(3) Join each algebraic expression in group A with the algebraic expression in group B which it is equal to.

A	B
(i) $2\ (x + 2y + 5)$	$10a - 2ac + 4ab$
(ii) $4\ (2a + b + 3c)$	$15xyz - 25xy + 20xz$
(iii) $5\ (2a - 1 + 3b)$	$4p^2r + 2qr + 2pqr$
(iv) $4\ (3x - 2y + 5z)$	$12x - 8y + 20z$
(v) $4p\ (a + b + 1)$	$2x + 4y + 10$
(vi) $2a\ (5 - c + 2b)$	$12x - 6xy + 9xz$
(vii) $x\ (2 - 3y + 3y^2)$	$8a + 4ab - 4ac$
(viii) $4a\ (2 + b - c)$	$4ap + 4bp + 4p$
(ix) $5x\ (3yz - 5y + 4z)$	$10a - 5 + 15b$
(x) $3x\ (4 - 2y + 3z)$	$8a + 4b + 12c$
(xi) $2r\ (2p^2 + q + pq)$	$2x - 3xy + 3xy^2$

(4) Complete the table given below.

Original expression	After factoring the expression
....	$4\,(3a + 2b + 3a^2)$
$9a + 27ac^2 + 18ab$	....
....	$3a\,(2p + 3r + 6)$
....	$2a\,(a + 3b + 2ac)$
$8xy + 24xp + 40xq$	....
....	$2\,(3ab + 4bc - 5ac)$
....	$3x\,(2pq + 3x + 6p)$
....	$6\,(2xy^2 + 3xy + 4z)$
$3ab - 6ab + 12ac$	....
$8xy - 12px - 20axy$	....

(5) Fill in the blanks in the table.

Algebraic expression	One factor of the algebraic expression	As a product of two factors
$-4x + 12$	4	....
$-4x + 12$	-4	....
$-6x + 8y$	2	....
$-6x + 8xy$	$-2x$	....
$-2a + 4b - 6c$	2	....
$-2a + 4b - 6c$	-2	....
$-3ab - 9b$	$-3b$	....
$2xy - 8xyz$	$2xy$	....
$5xy + 10xy + 10py$	....	....

Summary

When writing an algebraic expression as a product of factors,

- first find the HCF of the terms of the algebraic expression,
- take the HCF as one factor and the expression which is obtained by dividing each term of the algebraic expression by the HCF as the other factor, and
- write the algebraic expression as a product of these two factors.

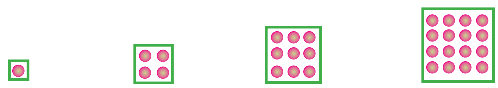
8

Square Root

By studying this lesson, you will be able to,
- identify what a perfect square is,
- write the square of each of the whole numbers from 1 to 20,
- find the square root of the perfect squares from 1 to 1000 by observation and by considering their prime factors.

8.1 Square of a positive integer

A few numbers which can be represented by a square shaped arrangement of dots are given below.

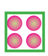
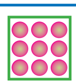
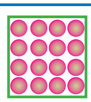


Rows 1 Columns 1 | Rows 2 Columns 2 | Rows 3 Columns 3 | Rows 4 Columns 4

You have learnt earlier that the numbers 1, 4, 9, 16, … which can be represented as above are called square numbers.

We get the square numbers 1, 4, 9, 16, ... by multiplying each positive integer by itself. We can write these square numbers using indices as 1^2, 2^2, 3^2, 4^2, ...

These are read as the "one squared", "two squared" etc.

Representation of the square number	Number of rows/ columns	How the square is obtained	Square of the number, using indices	Square of the number
	Rows 1, Columns 1	1×1	1^2	1
	Rows 2, Columns 2	2×2	2^2	4
	Rows 3, Columns 3	3×3	3^2	9
	Rows 4, Columns 4	4×4	4^2	16

The number we obtain by multiplying a number by itself, is called the **square** of that number. The square of a positive integer is called a **perfect square**.

1, 4, 9, 16, ... are the squares of the numbers 1, 2, 3, 4, ... Therefore they are perfect squares.

Example 1

A square tile is of side length 8 cm. Show that the numerical value of its surface area is a perfect square.

The length of a side of the square tile $= 8$ cm

Its surface area $= 8 \text{ cm} \times 8 \text{ cm}$

$= 64 \text{ cm}^2$

The numerical value of the area of the square tile $= 64 = 8 \times 8$

64, is the square of 8, so the numerical value of the surface area of the tile is a perfect square.

Exercise 8.1

(1) Represent the square of 5 by an arrangement of dots and write down its value.

(2) Complete the table given below and answer the questions accordingly.

Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Square of the number																	

By adding some pairs of perfect squares in the second row, we can obtain another perfect square. Observe the table and write four such relationships.

$$9 + 16 = 25$$
$$\therefore \quad 3^2 + 4^2 = 5^2$$

.... + ... = ...

$\therefore$ .... + ... = ...

.... + ... = ...

.... + ... = ...

(3) (i) Write down the perfect square between 10 and 20, and the reason for it.

(ii) Write down the perfect square between 60 and 70, and the reason for it.

(iii) Write down the perfect square between 80 and 90, and the reason for it.

(iv) How many perfect squares are there between 110 and 160?

(4) Complete the table given below.

Odd numbers added consecutively	Sum	The perfect square in index form
1		
1 + 3	4	2^2
1 + 3 + 5		
1 + 3 + 5 + 7		
1 + 3 + 5 + 7 + 9		

Using the above table, write the special feature of the numbers that are obtained when consecutive odd integers starting from 1 are added together.

8.2 The digit in the units place of a perfect square

The table below shows the squares of the numbers from 1 to 15.

Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Perfect Square	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225
The digit in the units place of the perfect square	1	4	9	6	5	6	9	4	1	0	1	4	9	6	5

- The digit in the units place of the square of a positive integer is the digit in the units place of the square of the digit at the right end (units place) of that positive integer.
- The digit in the units place of a square number is one of the numbers in the 3rd row of the above table.

- It is clear from the 3rd row of the table, that the digit in the units place of a perfect square is one of the digits 0, 1, 4, 5, 6 or 9.
- None of the digits 2, 3, 7 and 8 is ever the digit in the units place of a perfect square.

Example 1

Is 272 a perfect square?

If the digit in the units place of a whole number is 2, 3, 7 or 8, then that number is not a perfect square. In 272, the digit in the units place is 2. Therefore, it is not a perfect square.

Exercise 8.2

(1) By considering the digit in the units place of each of the numbers given below, show that they are not perfect squares.

(i) 832 (ii) 957 (iii) 513

(2) Give an example of a perfect square which has 9 in its units place.

(3) "If the digit in the units place of a whole number is one of 0, 1, 4, 5, 6 or 9, then it is a perfect square". Show with an example that this statement is not always true.

(4) Observe the digit in the units place of each number given below and write the digit in the units place of their respective squares.

(i) 34 (ii) 68 (iii) 45

8.3 The square root of a perfect square

$16 = 4 \times 4 = 4^2$. Since16 is the square of 4, the square root of 16 is 4.

$49 = 7^2$, so the square root of 49 is 7.

$81 = 9^2$, so the square root of 81 is 9.

To indicate the square root of a number, we use the symbol "$\sqrt{\ }$" .

Accordingly; the square root of 16 $= \sqrt{16} = \sqrt{4^2} = 4$,
the square root of 25 $= \sqrt{25} = \sqrt{5^2} = 5$,
the square root of 100 $= \sqrt{100} = \sqrt{10^2} = 10$,

the square root of 4 $= \sqrt{4} = 2$ (because $2^2 = 4$)
the square root of 1 $= \sqrt{1} = 1$ (because $1^2 = 1$)

If $c = a^2$ where a is a positive number, then $\sqrt{c} = a$. That is, a is the square root of c.

If a number is the square of a positive number, then the second number is the square root of the first.

The square roots of perfect squares such as 36, 49, 64 can be expressed quickly from memory. However it is not easy to do the same for every perfect square.

We have to use different methods to find them.
Let us see how we can find the square root,

- by using prime factors, and
- by observation.

• Finding the square root of a perfect square using prime factors

Let us find the value of $\sqrt{36}$ using prime factors.

Let us first write 36 as a product of its prime factors.

$$\begin{aligned} 36 &= 2 \times 2 \times 3 \times 3 \\ 36 &= (2 \times 3) \times (2 \times 3) \\ &= (2 \times 3)^2 \\ \therefore \sqrt{36} &= 2 \times 3 \\ &= 6 \end{aligned}$$

$$\begin{array}{r|l} 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

Example 1

Find the value of $\sqrt{576}$ using prime factors.

$$\begin{aligned} 576 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \\ &= (2 \times 2 \times 2 \times 3) \times (2 \times 2 \times 2 \times 3) \\ &= (2 \times 2 \times 2 \times 3)^2 \quad \text{or} \quad 576 = 24^2 \end{aligned}$$

$$\therefore \sqrt{576} = 2 \times 2 \times 2 \times 3 \quad \text{or} \quad \sqrt{576} = 24$$
$$= 24$$

Exercise 8.3

(1) Find the value of each of the following.

(i) $\sqrt{(2 \times 5)^2}$ (ii) $\sqrt{(2 \times 3 \times 5)^2}$ (iii) $\sqrt{(3 \times 5) \times (3 \times 5)}$

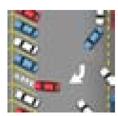
(iv) $\sqrt{3 \times 3 \times 7 \times 7}$ (v) $\sqrt{2 \times 2 \times 3 \times 3 \times 3 \times 3}$

(2) Find the square root using prime factors.

(i) 144 (ii) 400 (iii) 900

(iv) 324 (v) 625 (vi) 484

(3) What is the side length of a square shaped parking lot of area 256 m^2?



(4) The area of a square land is 169 m^2. Find the length of a side of the land.

Finding the square root of a perfect square by observation

Finding the digit in the units place of the square root of a perfect square

Activity 1

(1) Complete the table given below by considering the perfect squares you have identified so far and their square roots.

(i)	Perfect squares with the digit 1 in the units place	1	81	121	361	441
	Square roots of these perfect squares	1	9	11	19	21
(ii)	Perfect squares with the digit 4 in the units place					
	Square roots of these perfect squares					
(iii)	Perfect squares with the digit 5 in the units place					
	Square roots of these perfect squares					
(iv)	Perfect squares with the digit 6 in the units place					
	Square roots of these perfect squares					
(v)	Perfect squares with the digit 9 in the units place					
	Square roots of these perfect squares					
(vi)	Perfect squares with the digit 0 in the units place					
	Square roots of these perfect squares					

(2) Complete the table given below using the information in (i) to (vi) in the above table.

Digit in the units place of the perfect square	Digit in the units place of the square root
1	
4	
5	
6	
9	
0	

According to the above activity, the digit in the units place of the square root of a perfect square, which depends on the digit in the units place of the perfect square, is as follows.

Digit in the units place of the perfect square	Digit in the units place of the square root
1	1 or 9
4	2 or 8
5	5
6	4 or 6
9	3 or 7
0	0

➢ Finding the digit in the tens place of the square root of a perfect square between 101 and 1000

Since $40 \times 40 = 1600$, the square root of a number between 101 and 1000 will be less than 40. Therefore, the square root of such a number will only have digits in the units place and the tens place.

The digit in the tens place of the square root of a perfect square is as follows.

- If the digit in the hundreds place of a number is a perfect square, then the square root of that digit is the digit in the tens place of the answer (the square root of the perfect square).
- If the digit in the hundreds place of a number is not a perfect square, then the square root of the perfect square which is closest and less than the digit in the hundreds place, is the digit in the tens place of the answer.

Example 1

Find $\sqrt{961}$.

- Since the digit in the units place is 1, the digit in the units place of the square root must be 1 or 9.
- The digit in the hundreds place of the given number is 9, so the digit in the tens place of the square root is $\sqrt{9}$, which is 3.

$\therefore \sqrt{961}$ is either 31 or 39.

$$\begin{array}{r} 31 \\ \times\, 31 \\ \hline 31 \\ 93 \\ \hline 961 \\ \hline\hline \end{array} \qquad \begin{array}{r} 39 \\ \times\, 39 \\ \hline 351 \\ 117 \\ \hline 1521 \\ \hline\hline \end{array}$$

Since $31^2 = 961$,

$\therefore \sqrt{961} = 31$

Example 2

Find the square root of 625.

Hundreds place → $\boxed{625}$ ← Units place

- Since the digit in the units place of 625 is 5, the digit in the units place of its square root is 5.
- Since the digit in the hundreds place of 625 is 6, the digit in the tens place of the square root, is the square root of the perfect square closest to 6 and less than it.
- The perfect square closest to 6 and less than it is 4. Its square root is 2.

$\therefore \sqrt{625}$ is 25.

Example 3

Find $\sqrt{784}$.

Method I

Hundreds place → $\boxed{784}$ ← Units place

- Since the digit in the units place of 784 is 4, the digit in the units place of its square root is 2 or 8.
- Since the digit in the hundreds place of 784 is 7, the digit in the tens place of the square root, is the square root of the perfect square closest to 7 and less than it.
- The perfect square closest to 7 and less than it is 4. Its square root is 2.

$\therefore \sqrt{784}$ is either 22 or 28.

$$\begin{array}{r} 22 \\ \times 22 \\ \hline 44 \\ 44 \\ \hline 484 \\ \hline\hline \end{array} \qquad \begin{array}{r} 28 \\ \times 28 \\ \hline 224 \\ 56 \\ \hline 784 \\ \hline\hline \end{array}$$

$\therefore \sqrt{784} = 28$

Method II

The squares of the multiples of 10 which are less than 1000 are 100, 400 and 900. 784 lies between 400 and 900.

When written in order, we get;

$400 < 784 < 900.$

$\therefore \sqrt{400} < \sqrt{784} < \sqrt{900}$ (the square roots of these numbers)

That is, $20 < \sqrt{784} < 30$

Therefore, $\sqrt{784}$ lies between 20 and 30.

The digit in the units place of 784 is 4. So the digit in the units place of the square root is either 2 or 8. Therefore, $\sqrt{784}$ is either 22 or 28.

784 is closer to 900 than to 400.

As shown in the right $28^2 = 784$

$\therefore \sqrt{784}$ is 28.

Let us verify this.

$$\begin{array}{r} 28 \\ \times 28 \\ \hline 224 \\ 56 \\ \hline 784 \\ \hline \end{array}$$

Example 4

Show that 836 is not a perfect square.

Hundreds place — $\boxed{836}$ — Units place

- If 836 is a perfect square, then the digit in the units place of the square root should be 4 or 6.
- The digit in the hundreds place of 836 is 8. Since the closest perfect square less than 8 is 4, the digit in the tens place of the square root is $\sqrt{4}$, which is 2.

Therefore, if 836 is a perfect square, then its square root must be 24 or 26.

But $24 \times 24 = 576$ and $26 \times 26 = 676$. Therefore, 836 is not a perfect square.

Exercise 8.4

(1) Complete the table given below.

Perfect square	Square root of the perfect square
9	$\sqrt{9} = \sqrt{3^2} = 3$
36	
64	
121	
400	
900	

(2) Check whether the given numbers are perfect squares, and find the square root of each number which is a perfect square.

(i) 169 (ii) 972 (iii) 441 (iv) 716
(v) 361 (vi) 484 (vii) 1522 (viii) 529
(ix) 372 (x) 624

(3) The value of $\sqrt{324}$ is a whole number between 15 and 20. Find $\sqrt{324}$ by observing the last digit.

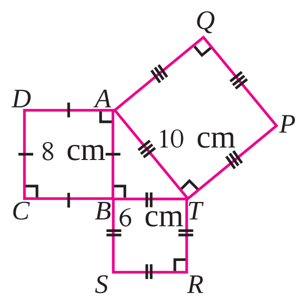
(4) 625 is a perfect square. Its square root is a whole number between 20 and 30. Find $\sqrt{625}$.

(5) Find the square root of each of the numbers given below by observation.

(i) 256 (ii) 441 (iii) 729 (iv) 361 (v) 841

Miscellaneous Exercise

(1) In the given figure, *ABCD* is a square of side length 8 cm, *BTRS* is a square of side length 6cm and *ATPQ* is a square of side length 10 cm.



(i) Find the area of the square *ABCD*.

(ii) Find the area of the square *BTRS*.

(iii) Find the area of the square *ATPQ*.

(iv) Find a special relationship between the areas of the three squares.

(2) The value of $\sqrt{500}$ cannot be found by using prime factors. Explain the reason for it.

(3) Show that $8^2 - 5^2 = (8 + 5)(8 - 5)$ and show the same relationship for another pair of perfect squares.

Summary

- We obtain a perfect square when we multiply a positive integer by itself.
- If a number is a square of a positive integer, then the square root of that number is that positive integer of which is the square.
- The symbol "$\sqrt{\ }$" is used to denote the square root of a positive number.
- The square root of a perfect square can be found by observing the last digit of that number.
- The square root of a perfect square can also be found using prime factors.

9

Mass

By studying this lesson, you will be able to,
- identify metric ton as a unit used to measure mass,
- know the relationship between kilogramme and metric ton.
- solve problems associated with mass which include metric tons.

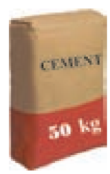
9.1 Units used to measure mass

You have learnt before that milligramme, gramme and kilogramme are units used to measure mass. Now let us identify another unit used to measure mass.

It is mentioned that the mass of the paracetamol in a paracetamol tablet shown in the figure is 500 mg.

It is mentioned that the mass of the margarine in the packet of margarine shown in the figure is 250 g.

It is mentioned that the mass of the cement in the bag of cement shown in the figure is 50 kg.



The approximate mass of the lorry loaded with goods shown in the figure is mentioned as 20 t.

According to the information given above, in order to measure a heavy mass like a lorry, the unit metric ton is used, which is larger than the unit kilogramme (kg). The letter t is used to indicate "metric ton".

One metric ton is equal to a thousand kilogrammes. Accordingly, $1\ \text{t} = 1000\ \text{kg}$

The relationship between the above mentioned units used to measure mass is given below.

$$1\ \text{g} = 1000\ \text{mg}$$
$$1\ \text{kg} = 1000\ \text{g}$$
$$1\ \text{t} = 1000\ \text{kg}$$

9.2 The relationship between kilogramme and metric ton

• Expressing a mass given in metric tons in kilogrammes

Now let us see how to express a mass given in metric tons in kilogrammes.

Since $1 \text{ t} = 1000 \text{ kg}$

$2 \text{ t} = 2 \times 1000 \text{ kg} = 2000 \text{ kg}$

$3 \text{ t} = 3 \times 1000 \text{ kg} = 3000 \text{ kg}$

Accordingly, in order to express a mass given in metric tons in kilogrammes, the amount given in metric tons should be multiplied by 1000.

Example 1

Express 8.756 t in kilogrammes.

$8.756 \text{ t} = 8.756 \times 1000 \text{ kg}$
$= 8756 \text{ kg}$

Example 2

Express 3 t 850 kg in kilogrammes.

$3 \text{ t } 850 \text{ kg} = 3 \text{ t} + 850 \text{ kg}$
$= 3 \times 1000 \text{ kg} + 850 \text{ kg}$
$= 3000 \text{ kg} + 850 \text{ kg}$
$= 3850 \text{ kg}$

Example 3

Express 8.756 t in metric tons and kilogrammes.

$8.756 \text{ t} = 8 \text{ t} + 0.756 \text{ t}$
$= 8 \text{ t} + 0.756 \times 1000 \text{ kg}$
$= 8 \text{ t} + 756 \text{ kg}$
$= 8 \text{ t } 756 \text{ kg}$

Example 4

Express $3\frac{1}{2}$ t in kilogrammes.

$3\frac{1}{2} \text{ t} = 3 \text{ t} + \frac{1}{2} \text{ t}$
$= 3 \times 1000 \text{ kg} + 500 \text{ kg}$
$= 3000 \text{ kg} + 500 \text{ kg}$
$= 3500 \text{ kg}$

• Expressing a mass given in kilogrammes in metric tons

Next let us see how to express a mass given in kilogrammes in metric tons.

Since $1000 \text{ kg} = 1 \text{ t}$

$2000 \text{ kg} = \frac{2000}{1000} \text{ t} = 2 \text{ t}$

$3000 \text{ kg} = \frac{3000}{1000} \text{ t} = 3 \text{ t}$

Accordingly, in order to express a mass given in kilogrammes in metric tons, the amount given in kilogrammes should be divided by 1000.

Example 5

Express 2758 kg in metric tons.

$$2758 \text{ kg} = \frac{2758}{1000} \text{ t}$$

$$= 2.758 \text{ t}$$

Example 6

Express 2225 kg in metric tons and kilogrammes.

$$2225 \text{ kg} = 2000 \text{ kg} + 225 \text{ kg}$$

$$= \frac{2000}{1000} \text{ t} + 225 \text{ kg}$$

$$= 2 \text{ t} + 225 \text{ kg}$$

$$= 2 \text{ t } 225 \text{ kg}$$

When expressing a mass of 1000 kg or more in kilogrammes and metric tons, the number of kilogrammes is written as an addition of a multiple of 1000 and a number less than 1000.

Example 7

Express 3 t 675 kg in metric tons.

$$3 \text{ t } 675 \text{ kg} = 3 \text{ t} + 675 \text{ kg}$$

$$= 3 \text{ t} + \frac{675}{1000} \text{ t}$$

$$= 3 \text{ t} + 0.675 \text{ t}$$

$$= 3.675 \text{ t}$$

Example 8

Complete the table given below.

Mass	The mass in t and kg	The mass in metric tons
2400 kg	2 t 400 kg	2. 400 t
5850 kg	5 t 850 kg	5. 850 t
1050 kg	1 t 050 kg	1. 050 t
600 kg	0 t 600 kg	0. 600 t

Exercise 9.1

(1) Express the masses given below in metric tons.

(i) 2350 kg (ii) 5050 kg (iii) 3 t 875 kg (iv) 13 t 7 kg

(2) Express each mass given below in kilogrammes.

(i) 7 t (ii) 17 t (iii) 3 t 650 kg (iv) 2 t 65 kg

(v) 1.075 t (vi) 7.005 t (vii) 4.68 t (viii) $\frac{3}{4}$ t

(3) Express each mass given below in metric tons and kilogrammes.

(i) 1.275 t (ii) 2.025 t (iii) 5.75 t (iv) 7.3 t (v) 7.003 t

(4) The mass of a fully grown whale is approximately 19 000 kg. Express this mass in metric tons.

(5) Place a ✓ in front of the unit that is used to measure the mass of each item given below.

	The item to be measured	mg	g	kg and g	kg	t
(i)	A mango	…………	…………	…………	…………	…………
(ii)	A comb of plantains	…………	…………	…………	…………	…………
(iii)	A bag of sweet potatoes	…………	…………	…………	…………	…………
(iv)	A loaf of bread	…………	…………	…………	…………	…………
(v)	A lorry	…………	…………	…………	…………	…………
(vi)	Ten travelling bags in a lift	…………	…………	…………	…………	…………

(6) Complete the table given below.

The mass of the given item in metric tons	That mass in metric tons and kilogrammes	That mass in kilogrammes
1.6 t	1 t 600 kg	1600 kg
3.85 t	…………	…………
7.005 t	…………	…………
…………	7 t 875 kg	…………
…………	6 t 5 kg	…………
…………	…………	7008 kg
…………	…………	14 375 kg

9.3 Addition of two masses expressed in metric tons and kilogrammes

The total mass of the passengers and travelling bags in an air plane of mass 181 t 350 kg is 60 t 800 kg. Let us find the mass of the air plane with the passengers and travelling bags.

To do this, let us add the masses of the air plane, passengers and travelling bags.

Method 1

	t	kg
	181	350
+	60	800
	242	150

Let us add the quantities in the kilogrammes column.

350 kg + 800 kg = 1150 kg
1150 kg = 1000 kg + 150 kg
= 1 t + 150 kg
Let us write 150 kg in the kilogrammes column.

Let us carry 1 t to the metric tons column and add the quantities in this column.

1 t + 181 t + 60 t = 242 t

Let us write 242 t in the metric tons column.

Therefore, the total mass is 242 t 150 kg.

Method II

Let us express each mass in metric tons and then simplify.

	t
	181 . 350
+	60 . 800
	242 . 150

181 t 350 kg = 181.350 t
60 t 800 kg = 60.8 t

181.350 t + 60.800 t = 242.150 t
242.150 t = 242 t + 150 kg

Therefore, the total mass is 242 t 150 kg.

Method III

Let us express each mass in kilogrammes and simplify.

181 t 350 kg = 181 350 kg
60 t 800 kg = 60 800 kg
181 350 kg + 60 800 kg = 242 150 kg
242 150 kg = 242 t 150 kg

Therefore, the total mass is 242 t 150 kg.

Example 1

Add 10 t 675 kg and 3 t 40 kg.

	t	kg
	10	675
+	3	040
	13	715

Exercise 9.2

(1) Express the answer in metric tons and kilogrammes.

(i)
$$\begin{array}{rrr} & \text{t} & \text{kg} \\ & 2 & 780 \\ + & 1 & 620 \\ \hline\hline \end{array}$$

(ii)
$$\begin{array}{rrr} & \text{t} & \text{kg} \\ & 3 & 450 \\ & 6 & 065 \\ + & 1 & 275 \\ \hline\hline \end{array}$$

(iii) 10 t 225 kg + 6 t 705 kg

(iv) 150 t 650 kg + 40 t 460 kg

(2) The mass of a grown elephant is 4.75 t. The mass of a baby elephant is 2025 kg.

(i) Express the mass of the baby elephant in metric tons.
(ii) Find the total mass of both elephants in metric tons.
(iii) Express the total mass of both elephants in kilogrammes.

(3) A lorry of mass 3 t 450 kg is loaded with 2 t 700 kg of sugar and 4 t of rice. Find the total mass of the lorry with the goods loaded in it.

9.4 Subtraction of masses expressed in kilogrammes and metric tons

The total mass of a lorry loaded with rice is 10 t 250 kg. The mass of the lorry is 3 t 750 kg. Let us find the mass of the rice loaded in the lorry.

In order to find the mass of the rice loaded in the lorry, the mass of the lorry should be subtracted from the total mass.

Method I

$$\begin{array}{rrr} & \text{t} & \text{kg} \\ & 10 & 250 \\ - & 3 & 750 \\ \hline & 6 & 500 \\ \hline\hline \end{array}$$

Since 750 kg cannot be subtracted from 250 kg, let us carry 1 t from the 10 t in the metric tons column, that is, 1000 kg, to the kilogrammes column and add it to the 250 kg in the kilogrammes column.
Then, 1000 kg + 250 kg = 1250 kg.
1250 kg – 750 kg = 500 kg
Let us write 500 kg in the kilogrammes column.

Let us subtract 3 t from the remaining 9 t in the metric tons column.
Then, 9 t – 3 t = 6 t
Let us write 6 t, in the metric tons column.

Therefore, the mass of the rice is 6 t 500 kg.

Method II

Let us express each mass in metric tons and then simplify.

$10\ \text{t}\ \ 250\ \text{kg} = 10.250\ \text{t}$

$3\ \text{t}\ \ 750\ \text{kg} = 3.750\ \text{t}$

$10.250\ \text{t} - 3.750\ \text{t} = 6.500\ \text{t}$

$6.500\ \text{t} = 6\ \text{t}\ \ 500\ \text{kg}$

$$\begin{array}{r} \text{t} \\ 10\,.\,250 \\ -\ 3\,.\,750 \\ \hline 6\,.\,500 \\ \hline\hline \end{array}$$

The mass of the rice in the lorry is 6 t 500 kg.

Method III

Let us express each mass in kilogrammes and then simplify.

$10\ \text{t}\ \ 250\ \text{kg} = 10\ 250\ \text{kg}$

$3\ \text{t}\ \ 750\ \text{kg} = 3750\ \text{kg}$

$10\ 250\ \text{kg} - 3750\ \text{kg} = 6500\ \text{kg}$

$6500\ \text{kg} = 6\ \text{t}\ \ 500\ \text{kg}$

$$\begin{array}{r} \text{kg} \\ 10\ 250 \\ -\ 3\ 750 \\ \hline 6\ 500 \\ \hline\hline \end{array}$$

The mass of the rice in the lorry is 6 t 500 kg.

Exercise 9.3

(1) Subtract the following.

(i) $\begin{array}{rrr} & \text{t} & \text{kg} \\ & 5 & 000 \\ - & 2 & 750 \\ \hline \hline \end{array}$

(ii) $\begin{array}{rrr} & \text{t} & \text{kg} \\ & 4 & 350 \\ - & 1 & 650 \\ \hline \hline \end{array}$

(iii) 250 t 650 kg – 150 t 105 kg

(iv) 60 t – 25 t 150 kg

9.5 Multiplication of a mass expressed in metric tons and kilogrammes by a number

➢ The mass of a concrete beam used to build a flyover bridge is 6 t 500 kg. Five such beams are placed across two columns. Let us find the total mass borne by the two columns.

The two columns bear 5 beams of mass 6 t 500 kg each. Hence, in order to find the mass borne by the two columns, 6 t 500 kg should be multiplied by 5.

Method I

Let us express 6 t 500 kg in kilogrammes and then multiply it by 5.

6 t 500 kg = 6500 kg

6500 kg × 5 = 32 500 kg

kg
6500
×5
32 500

32 500 kg = 32 t 500 kg

Accordingly, the total mass borne by the two columns is 32 t 500 kg.

Method II

t	kg
6	500
×	5
32	500

First, let us multiply 500 kg by 5.

500 × 5 kg = 2500 kg

2500 kg = 2000 kg + 500 kg = 2 t + 500 kg

Let us write 500 kg in the kilogrammes column.

Let us multiply 6 t by 5. 6 t × 5 = 30 t

Now let us add the 2 t obtained by the multiplication in the kilogrammes column, to the 30 t in the metric tons column.

30 t + 2 t = 32 t

Let us write 32 t in the metric tons column.

➢ Let us simplify 5 t 120 kg × 12.

Method I

t	kg
5	120
×	12
61	440

First let us multiply 120 kg by 12.

120 kg × 12 = 1440 kg = 1 t 440 kg

Now let us multiply 5 t by 12.

5 t × 12 = 60 t

∴ 5 t 120 kg × 12 = 60 t + 1 t 440 kg

= 60 t + 1 t + 440 kg

= 61 t 440 kg

5 t 120 kg × 12 = 61 t 440 kg

Method II

Let us express 5 t 120 kg in kilogrammes and multiply it by 12.

5 t 120 kg = 5120 kg

Let us multiply 5120 kg by 12.

5 120 kg × 12 = 61 440 kg

= 61 t 440 kg

$$\begin{array}{r} \text{kg} \\ 5120 \\ \times\ 12 \\ \hline 10240 \\ 5120 \\ \hline 61440 \\ \hline\hline \end{array}$$

Example 1

(1) The mass of a tin of milk powder is 500 g. The mass of the empty tin is 50 g.

(i) Find the mass of the milk powder in such a tin, in grammes. Express this mass in kilogrammes.

(ii) A container is loaded with 1000 such tins of milk powder. Write the mass of these 1000 tins in kilogrammes and express it in metric tons also.

(i) The mass of a tin of milk powder = 500 g

The mass of the milk powder in the tin = 500 g – 50 g = 450 g

= 450 ÷ 1000 kg = 0.45 kg

(ii) The mass of 1000 tins of milk powder = 500 × 1000 g = 500 000 g

= 500 000 ÷ 1000 kg = 500 kg

= 500 ÷ 1000 t = 0.5 t

Exercise 9.4

(1) Simplify the following.

(i)
$$\begin{array}{rr} \text{t} & \text{kg} \\ 160 & 200 \\ & \times\ 5 \\ \hline & \\ \hline\hline \end{array}$$

(ii)
$$\begin{array}{rr} \text{t} & \text{kg} \\ 165 & 465 \\ & \times\ 4 \\ \hline & \\ \hline\hline \end{array}$$

(iii)
$$\begin{array}{rr} \text{t} & \text{kg} \\ 32 & 45 \\ & \times\ 3 \\ \hline & \\ \hline\hline \end{array}$$

(iv) 16 t 325 kg × 12 (v) 5 t 450 kg × 25 (vi) 64.5 t × 50

(vii) 27.3 t × 25

(2) (i) The approximate mass of a car is 1 t 200 kg. Express the approximate mass of 10 such cars in metric tons.

(ii) The mass of a vehicle which transports these 10 cars is 20 t. Accordingly, express the total mass of the vehicle with these 10 cars, in metric tons.

9.6 Division of a mass by a whole number

➢ If a mass of 6 t 750 kg of rice is loaded equally into 5 lorries, let us find the mass of the rice loaded into one lorry. For this, 6 t 750 kg should be divided by 5.

Method 1

	t	kg
	1	350
5	6	750
	5	
	1 →	1000
		1750
		1750
		0000

First, let us divide the metric ton quantity.
Since there is 1, 5s in 6, let us write 1 in the relevant position of the metric tons column where the answer should be written, and carry the remaining 1 t to the kilogrammes column as 1000 kg.

Next let us find the amount of kilogrammes in the kilogrammes column.
1000 kg + 750 kg = 1750 kg
Let us divide 1750 kg, by 5.
(1750 kg ÷ 5 = 350 kg)

The mass of the rice loaded into one lorry is 1 t 350 kg.

Method II

Let us express 6 t 750 kg in kilogrammes and divide it by 5.

6 t 750 kg = 6750 kg
6750 kg ÷ 5 = 1350 kg

	kg
	1350
5	6750
	5
	17
	15
	25
	25
	000
	000
	0

The mass of the rice loaded into one lorry is 1350 kg.

➢ A mass of 16 t 200 kg of paddy in a storehouse is loaded equally into 9 lorries. Let us find the mass of the paddy loaded into one lorry.

For this, 16 t 200 kg should be divided by 9.

Method I

$$\begin{array}{r|ll}
 & \text{t} & \text{kg} \\
 & 1 & 800 \\ \hline
9 & 16 & 200 \\
 & 9 & \\ \hline
 & 7 \rightarrow & 7000 \\
 & & \overline{7200} \\
 & & 7200 \\
 & & \overline{0000}
\end{array}$$

Let us divide the 16 t in the metric tons column by 9.
Let us carry the remaining 7 t to the kilogrammes column as 7000 kg.
Now let us find the amount of kilogrammes in the kilogrammes column.

7000 kg + 200 kg = 7200 kg

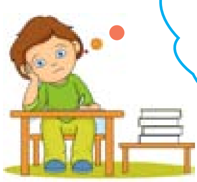
Let us divide 7200 kg by 9.

7200 kg ÷ 9 = 800 kg

The mass of the paddy loaded into one lorry is 1 t 800 kg.

Method II

Let us express 16 t 200 kg in kilogrammes and divide by 9.



16 t 200 kg = 16 t + 200 kg
= 16 000 kg + 200 kg
= 16 200 kg

16 200 kg ÷ 9 = 1800 kg

$$\begin{array}{r|l}
 & \text{kg} \\
 & 1\ 800 \\ \hline
9 & 16\ 200 \\
 & 9 \\ \hline
 & 72 \\
 & 72 \\ \hline
 & \ \ 00 \\
 & \ \ 00 \\ \hline
 & \ \ \ 00 \\
 & \ \ \ 00
\end{array}$$

1800 kg = 1 t 800 kg

The mass of the paddy in one lorry is 1 t 800 kg.

Example 1

A lorry had to make 7 trips in order to transport a quantity of rice of mass 66.5 t. If the lorry carried an equal amount of rice on each trip, find the mass of the rice it carried on one trip.

$$\begin{array}{r|l} & \text{t} \\ & \ \ 9.5 \\ \hline 7 & 66.5 \\ & 63 \\ \hline & \ \ 3\ 5 \\ & \ \ 3\ 5 \\ \hline & \ \ \ \ \ 0 \end{array}$$

The mass of the rice carried by the lorry on 7 trips $= 66.5$ t

The mass of the rice carried by the lorry on one trip $= 66.5$ t $\div 7$

$= 9.5$ t

Exercise 9.5

(1) Simplify the following.

(i) 5 t 200 kg $\div 4$ (ii) 12 t $\div 5$ (iii) 14 t 500 kg $\div 5$

(iv) 15 t $\div 200$ (v) 3 t $\div 40$ (vi) 17 t 200 kg $\div 8$

Summary

- Milligramme (mg), gramme (g), kilogramme (kg) and metric ton (t) are some units used to measure mass.

  1 g = 1000 mg 1 kg = 1000 g 1 t = 1000 kg
- In order to express a mass given in metric tons in kilogrammes, the quantity given in metric tons needs to be multiplied by 1000.
- In order to express a mass given in kilogrammes in metric tons, the quantity given in kilogrammes needs to be divided by 1000.

10 Indices

By studying this lesson you will be able to,

- express a power of a product as a product of powers,
- express a product of powers as a power of a product, and
- find the value of a power of a negative integer by expansion.

10.1 Indices

Let us recall what was learnt in Grade 7 about indices.

You learnt in Grade 7 that 2^3 and x^4 are respectively a power of 2 and a power of x. In 2^3, the base is 2 and the index is 3.

These can be expanded and written as products as $2^3 = 2 \times 2 \times 2$ and $x^4 = x \times x \times x \times x$.

Accordingly, $3x^2y^3 = 3 \times x \times x \times y \times y \times y$ and $3ab = 3 \times a \times b$.
Since $6 = 2 \times 3$, we say that 6 is the product of 2 and 3.
Similarly, since $3ab = 3 \times a \times b$, we say that $3ab$ is the product of 3, a and b.

Do the following review exercise to recall the facts that have been learnt so far regarding indices.

Review Exercise

(1) Complete the following table.

Number	Index Notation	Base	Index
8	2^3	....	....
9	....	....	....
16	....	2	....
....	....	4	2
1000	....	10	....

(2) Expand and write each of the following expressions as a product.

(i) $3x^2$ (ii) $2p^2q$ (iii) $4^2\,x^3$ (iv) $5^2\,x^2\,y^2$

(3) Write down each of the following numbers as a product of powers of which the bases are prime numbers.

(i) 20 (ii) 48 (iii) 100 (iv) 144

(4) Write 64 in index notation (i) with base 2
(ii) with base 4
(iii) with base 8.

10.2 Expressing a power of a product as a product of powers

2×3 is the product of 2 and 3. $(2 \times 3)^2$ is a power of the product 2×3.
Let us write $(2 \times 3)^2$ as a product of powers of 2 and 3.

$$\begin{aligned}(2 \times 3)^2 &= (2 \times 3) \times (2 \times 3)\\ &= 2 \times 3 \times 2 \times 3\\ &= 2 \times 2 \times 3 \times 3\\ &= 2^2 \times 3^2\end{aligned}$$

$\therefore\ (2 \times 3)^2 = 2^2 \times 3^2$

Now let us write $(2 \times 3)^3$ as a product of powers of 2 and 3.

$$\begin{aligned}(2 \times 3)^3 &= (2 \times 3) \times (2 \times 3) \times (2 \times 3)\\ &= 2 \times 3 \times 2 \times 3 \times 2 \times 3\\ &= 2 \times 2 \times 2 \times 3 \times 3 \times 3\\ &= 2^3 \times 3^3\end{aligned}$$

$\therefore\ (2 \times 3)^3 = 2^3 \times 3^3$

Accordingly, the power of a product can be written in this manner as a product of powers of the factors of the given product.

Now let us consider a power of a product that contains unknown terms.

$$\begin{aligned}(ab)^3 &= ab \times ab \times ab\\ &= a \times b \times a \times b \times a \times b\\ &= a \times a \times a \times b \times b \times b\\ &= a^3 \times b^3 = a^3\,b^3\end{aligned}$$

$$\boxed{(ab)^3 = a^3b^3}$$

Let us similarly express $(abc)^3$ as a product of powers of a, b and c.

$$\begin{aligned}(abc)^3 &= (abc) \times (abc) \times (abc)\\ &= (a \times b \times c) \times (a \times b \times c) \times (a \times b \times c)\\ &= (a \times a \times a) \times (b \times b \times b) \times (c \times c \times c)\\ &= a^3 \times b^3 \times c^3 = a^3\, b^3\, c^3\end{aligned}$$

$$\therefore\ (abc)^3 = a^3b^3c^3$$

Accordingly, a power of a product can be written as a product of powers of the factors of the given product.

- Let us express $4a^2$, as a power of a product.

$$\begin{aligned}4a^2 &= 4 \times a^2 = 2^2 \times a^2\\ &= (2 \times a)^2\\ &= (2a)^2\end{aligned}$$

This can be established further through the following examples.

Example 1

Express each of the following powers of products as a product of powers of the factors of the given product.

(i) $(2x)^3$ (ii) $(3ab)^2$

(i) $(2x)^3 = 2^3 \times x^3$
$= 2^3x^3$

(ii) $(3ab)^3$
$(3ab)^3 = 3^3 \times a^3 \times b^3$
$= 3^3a^3b^3$

Example 2

Express $36x^2$, as a power of a product.

Since $36 = 6^2$,

$$\begin{aligned}36x^2 &= 6^2 \times x^2 = (6 \times x)^2\\ &= (6x)^2\end{aligned}$$

Example 3

Express a^3b^3 as a power of a product.

$$\begin{aligned}a^3b^3 &= a^3 \times b^3\\ &= (a \times b)^3\\ &= (ab)^3\end{aligned}$$

Exercise 10.1

(1) Express each of the following powers of products as a product of powers of the factors of the given product.

(a) (i) $(2 \times 5)^2$ (ii) $(3 \times 5)^3$ (iii) $(11 \times 3 \times 2)^3$
(iv) $(a \times b)^2$ (v) $(x \times y)^5$ (vi) $(4 \times x \times y)^3$

(b) (i) $(5a)^2$ (ii) $(6p)^2$ (iii) $(4y)^3$
(iv) $(3a)^3$ (v) $(2y)^4$ (vi) $(2ab)^2$

(2) Find the value of each of the following powers of products. Write each power of a product as a product of powers of the factors of the given product and obtain the value again by simplifying the answer.

(i) $(2 \times 5)^3$ (ii) $(2 \times 3)^3$ (iii) $(11 \times 2)^3$

(iv) $(3 \times 7)^2$ (v) $(5 \times 7)^3$ (vi) $(13 \times 2 \times 3)^2$

(3) Express each of the following products of powers as a power of a product.

(i) $5^2 \times 2^2$ (ii) $5^2 \times 11^2$ (iii) $3^3 \times 4^3 \times 2^3$

(iv) $x^2 \times y^2$ (v) $p^3 \times q^3$ (vi) $a^5 \times b^5 \times x^5$

(vii) $100\ m^2$ (viii) $225\ t^2$ (ix) $8\ y^3$

(4) Show that $1000x^3 = (10x)^3$.

10.3 The power of a negative integer

-1, -2, -3 are negative integers. Do the following activity to find the value of a power of these negative integers.

Activity 1

Complete the following table by using the knowledge on multiplying integers.

Integer	Its second power	Its third power	Its fourth power
2	$2^2 = 2 \times 2 = 4$	$2^3 = 2 \times 2 \times 2 = 8$	$2^4 = 2 \times 2 \times 2 \times 2 = 16$
-1	$(-1)^2 = (-1) \times (-1) = 1$	....	....
-2	....	....	....
-3	....	....	....

- the value of any power of a positive integer is positive.
- the value of an odd power of a negative integer is negative.
- the value of an even power of a negative integer is positive.

Example 1

Find the value of $(-2)^4$.

$$(-2)^4 = 2^4$$
$$= 16$$

Example 2

Find the value of $(-5)^3$.

$$(-5)^3 = -(5)^3$$
$$= -125$$

Exercise 10.2

(1) Find the value.

(a) (i) $(-1)^1$ (ii) $(-1)^2$ (iii) $(-1)^3$ (iv) $(-1)^4$
(v) 1^1 (vi) 1^{1003} (vii) 1^{2018} (viii) 1^{10}

(b) (i) $(-4)^2$ (ii) $(-4)^3$ (iii) $(-4)^4$ (iv) $(-5)^1$
(v) $(-5)^2$ (vi) $(-5)^3$ (vii) $(-1)^{1001}$ (viii) $(-1)^{202}$

(2) Show that $(-1)^8 > (-1)^9$.

Miscellaneous Exercise

(1) Express each of the following products of powers as a power of a product.

(i) $(2x)^2 \times y^2$ (ii) $(3a)^2 \times b^2$ (iii) $p^3 \times (2q)^3$
(iv) $(2x)^3 \times (3y)^3$ (v) $(5a)^3 \times (2b)^3$ (vi) $a^3 \times (2b)^3 \times c^3$

(2) Show that $(3a)^2 \times (2x)^2 = 36a^2x^2$.

(3) Arrange in ascending order of the values.
(i) 2^3, $(-10)^1$, $(-1)^{10}$, 3^2
(ii) $(-2)^4$, $(-2)^5$, $(-1)^4$, $(-1)^5$

(4) If a is a negative integer, show that $a^2 > a^3$.

Summary

- If a, b, c and n are positive integers, then $(ab)^n = a^n \times b^n = a^n b^n$ and $(abc)^n = a^n \times b^n \times c^n = a^n b^n c^n$.
- The value of any power of a positive integer is positive.
- The value of an odd power of a negative integer is negative.
- The value of an even power of a negative integer is positive.

REVISION EXERCISE – FIRST TERM

(1) (i) Find the value of $\sqrt{361}$.

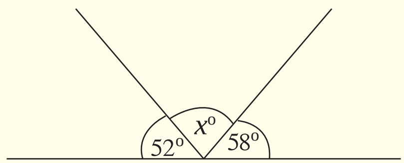
(ii) Evaluate 5t 75 kg $\times$ 12.

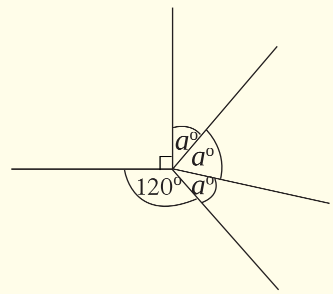
(iii) Write down the value of $(-1)^{11}$.

(iv) What is the complement of the angle 28^{o} ?

(v) What is the supplement of the angle 28^{o} ?

(vi) (*a*) Find the value of *x*. (*b*) Find the value of *a* .





(vii) Write down the number of faces, edges and vertices of a dodecahedron.

(viii) Fill in the blanks.

$12x - 36y + 4 = 4\,(\Box\, x - \Box\, y + \Box\,)$

(2) (*a*) Find the value of each of the following.

(i) (– 5) + (– 3) (ii) (– 7) + 4 (iii) 13 + (– 5)

(iv) (– 5) – (– 2) (v) (– 7) – (– 10) (vi) 0 – (– 5)

(*b*) Find the value of each of the following.

(i) $(-12) \times (-3)$ (ii) $(+8) \times (-5)$ (iii) $(+12) \div (-3)$

(iv) $(-12) \div (-3)$ (v) $(-12) \times 0$ (vi) $0 \div (-100)$

(*c*) Fill in the cages and re-write the following.

(i) $24 \div \Box = (-4)$ (ii) $(-16) \div \Box = (-4)$ (iii) $32 \div \Box = (-4)$

(iv) $(-10) + \Box = -6$ (v) $(-5) + \Box = (-6)$ (vi) $(-2) \times (-4) = \Box$

(3) The general term of the triangular number pattern starting from 1 is $\dfrac{n\,(n+1)}{2}$.

(i) Write the first term of the triangular number pattern.

(ii) Write the 19th and 20th terms of this pattern.

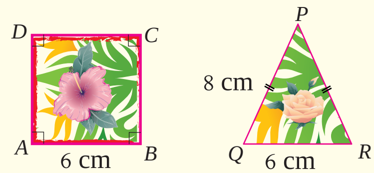
(iii) It is given that $10 \times 11 = 110$. Find which term of the triangular number pattern is 55?

(iv) It is given that $18 \times 19 = 342$. Find which term of the triangular number pattern is 171?

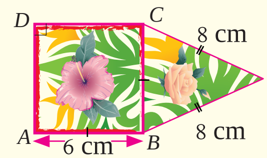
(v) Show that the sum of the 19th and 20th terms of the triangular number pattern is equal to the 20th term of the square number pattern which starts from 1.

(4) (i) Find the perimeter of the square design.

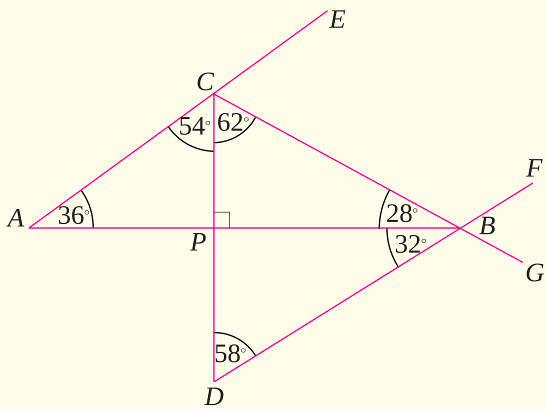
(ii) Find the perimeter of the isosceles triangular design.



(iii) The two designs are pasted together as shown to form a composite figure. Find the perimeter of the composite figure.



(5)



The straight lines AB and CD are drawn such that they intersect each other perpendicularly at P. The given figure is obtained by joining AC, CB and DB and then producing them.

(i) Write three pairs of complementary angles.

(ii) Write three pairs of supplementary angles.

(iii) Write 4 pairs of vertically opposite angles.

(iv) What is the magnitude of $F\hat{B}G$?

(v) $C\hat{B}D$ and $D\hat{B}G$ are a pair of supplementary angles. Write down the magnitude of $D\hat{B}G$.

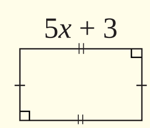
(vi) Name an angle which is a supplement of $C\hat{B}P$.

(vii) Write down the value of the angle you named.

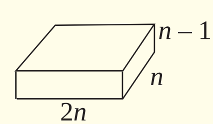
(viii) Find the magnitude of $C\hat{B}F$.

(ix) Find the sum of the angles around the point B and establish the fact that the sum of the angles around a point is 360^0.

(6) (i) The perimeter of a rectangle is $16x + 10$ units. Its length is $5x + 3$ units. Write an algebraic expression for its breadth.

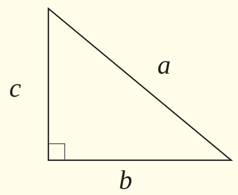


(ii) A cuboid of length, breadth and height equal to $2n$, n and $n - 1$ units respectively is shown in the figure. Show that the sum of the lengths of all its edges is $4\,(4n - 1)$ units.



(7) Simplify the following.

(i) $5(c-2)+12$ (ii) $7(d-9)-d$
(iii) $4(f+5)+2f-3$ (iv) $-2g(h+4)-3g(h-2)$
(v) $4h(i+2)-7(i-1)$

(8) 

If the relationship $a^2 = b^2 + c^2$ is true for this right angled triangle, find a when $b = 8$ cm and $c = 6$ cm.

(9) (i) Express $4y^2$ as a power of a product.
(ii) Write $(8ab)^2$ as a product of powers and simplify it.
(iii) Simplify $(2p)^3 \times (3q)^3$.
(iv) Show that 6^3 is equal to 8×27.
(v) Show that when $(-3)^4$ is simplified, the same value as of 9^2 is obtained .
(vi) Without obtaining the value of $(-15)^3 \times (-27)^4$, explain whether the answer is a positive value or a negative value.

(10) A signboard near an old bridge indicates that the maximum load which the bridge can support is 8 t. A lorry of mass 5.5 metric tons is loaded with 80 bags of cement of mass 50 kg each.

(i) Calculate and show that it is dangerous for the lorry loaded with the bags of cement to cross the bridge.
(ii) What is the minimum number of bags of cement that should be removed for the lorry to safely cross the bridge?

(11) Simplify the following.

(a)	(b)	(c)
(i) $(+7) + (-3)$	(i) $(+10) - (-3)$	(i) $(+4) \times (-3)$
(ii) $(-5) + (-4)$	(ii) $(-7) - (-3)$	(ii) $(-5) \times (-6)$
(iii) $(+12) + (-18)$	(iii) $(-7) - (+20)$	(iii) $(-1) \times (+4.8)$
(iv) $(+5\frac{1}{2}) + (-3)$	(iv) $(+17) - (-12)$	(iv) $(-20) \div (+4)$
(v) $(+3.7) + (-6.3)$	(v) $(+8.7) - (-2.3)$	(v) $(-35) \div (-5)$

(12) Expand the given algebraic expressions and simplify them.

(i) $5(2x-3)-4x+7$ (ii) $x(3y+5)-8xy+2$ (iii) $-3a(5-7b)+5(a-2)$

(13) Simplify the following.

(i) $4a + 7b - 3(a + c)$
(ii) $2(3x - 7) - 2x + 5$
(iii) $3a(a + 7) + 5a^2 - 20a + 4$

(14) Find the value of each algebraic expression when $x = -2$, $y = 3$ and $z = -3$.

(i) $3x + 4y$ (ii) $x^2y + 5y^2$ (iii) $4(2x - 3y - 4z)$

(15) Write down the geometrical shape of the faces of each solid given below.

(i) Regular tetrahedron
(ii) Cube
(iii) Regular octahedron
(iv) Regular dodecahedron
(v) Regular icosahedron

(16) Write down the HCF of each of the following groups of terms.

(i) $3x, 12xy, 15y$ (ii) $12x, 6xy, 9x^2$
(iii) $3a^2b, 15ab, 15y$ (iv) $4x^2y, 6xy, 8xy^2$

(17) Factorize the following expressions.

(i) $8x + 4y + 12$ (ii) $15x^2 + 3xy$
(iii) $6a^2b - 15ab + 18abc$ (iv) $-4mn - 20m^2 + 12m$

(18) (i) Write down the perfect squares that are in the range of values from 1 to 100.
(ii) The digit in the units place of a perfect square is 6. Write two digits that could be in the units place of its square root.
(iii) Which digits do not appear in the units place of a perfect square?
(iv) Find $\sqrt{900}$.

(19) Fill in the blanks.

(i) 3 t = ... kg. (ii) 3500 kg = ... t ....kg.
(iii) 4.05 t = ...kg. (iv) 12 450 kg = ... t.
(v) 10 t 50 kg = ...kg.

(20) Evaluate the following.

(i) $3^2 \times 5$ (ii) $4^3 \times 2^2$ (iii) $2^3 \times 3^2$
(iv) $(-4)^2 \times 5^3$ (v) $(-3)^3 \times 2^2$ (vi) $(-1)^4 \times 5^2 \times 4$

11 Symmetry

By studying this lesson you will be able to,

- identify the rotational symmetry of a plane figure,
- find the order of rotational symmetry of a plane figure that has rotational symmetry, and
- find the relationship between the number of axes of bilateral symmetry and the order of rotational symmetry of a plane figure which is bilaterally symmetrical.

11.1 Bilateral symmetry

You learnt in Grade 7 that if it is possible to fold a plane figure through a line on it to get two identical parts which coincide with each other, then it is called a **bilaterally symmetrical plane figure**. You also learnt that such a line of a bilaterally symmetrical figure is known as an **axis of symmetry** of the figure.

The two parts on either side of an axis of symmetry of a bilaterally symmetrical figure are equal in shape and area.

If by folding a plane figure along a line, it is divided into two parts which are equal in area and shape, but the two parts do not coincide with each other, then that line is not an axis of symmetry of the plane figure.

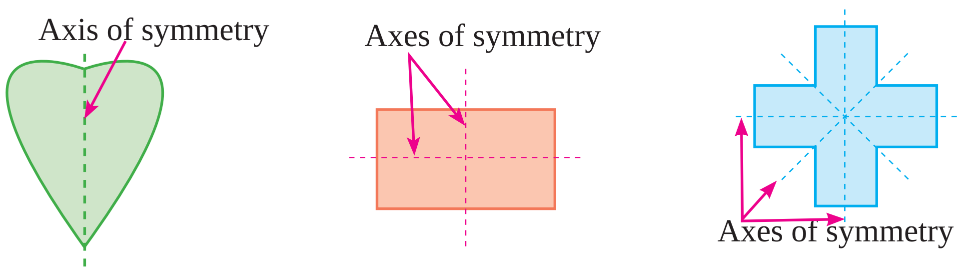


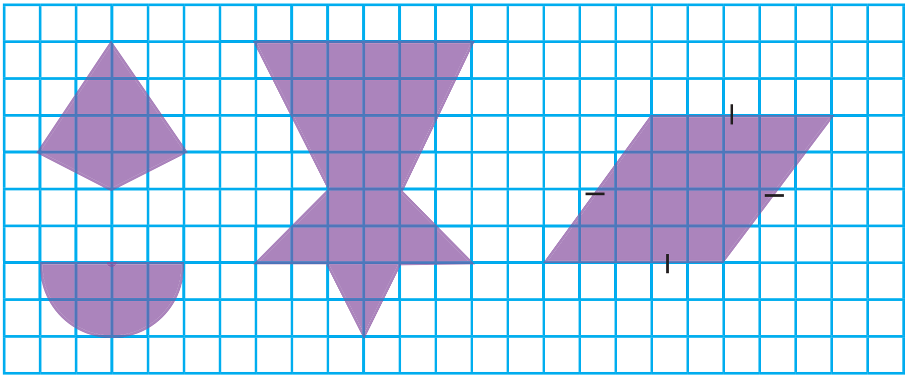
Figure 1 Figure 2 Figure 3

The axes of symmetry are indicated by dotted lines in each figure shown above.

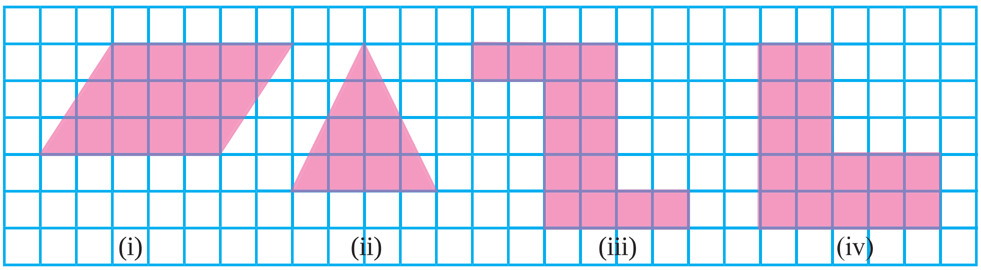
Do the review exercise to recall the facts you learnt in Grade 7 about bilateral symmetry.

Review Exercise

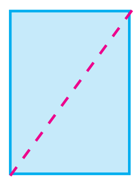
(1) Copy the following plane figures in your exercise book and draw the axes of symmetry of each figure.



(2) Select the figures that are bilaterally symmetrical from the following figures and write down their numbers.

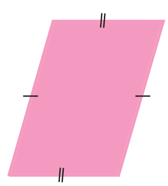


(3) The dotted line shown in the figure divides the given rectangle into two equal parts. Samith says that the dotted line represents an axis of symmetry of the rectangle. Explain why his statement is not true.



(4) (i) Copy the given parallelogram onto a tracing paper and cut it out.

(ii) Can the figure that was cut out be folded along a line so that the two parts on either side coincide?



(iii) Accordingly, show that a parallelogram need not have bilateral symmetry.

11.2 Rotational symmetry

When a plane figure is rotated about a point on it through one complete rotation in the plane of the figure, it coincides with the original position at least once.

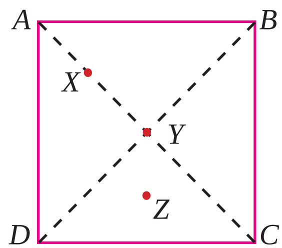
There are some figures, which when rotated about a point on it through one complete rotation, coincide several times with the original position.

The number of times a figure coincides with the original position varies, depending on the point about which it is rotated.

Do the following activity to establish this property further.

Activity 1

Step 1 - Draw a square $ABCD$ and mark the points X, Y and Z as in the given figure.



Step 2 - Trace the figure $ABCD$ onto a transparent sheet like an oil paper or a plastic paper and mark the points X, Y and Z on it.

Step 3 - Overlap the two figures so that they coincide, and keep them in place with a pin fixed at point X.

Step 4 - Rotate the figure on the transparent sheet about the pin point (point X) and observe whether the two figures coincide with each other. Find the number of times the two figures coincide with each other when the transparent sheet is rotated once about X.

Step 5 - As above, rotate the figure on the transparent sheet once about the points Y and Z too, and find the number of times the two figures coincide with each other.

Step 6 - Draw the following table in your exercise book and complete it.

Point	X	Y	Z
Number of times the figures coincide			

While doing the above activity you would have observed that when the figure on the transparent sheet was rotated once about the points X and Z, the two figures coincided with each other only once at the completion of one rotation, and that when the figure was rotated about the point Y, it coincided four times with the original figure during one complete rotation.

When a plane figure is rotated about a fixed point in it through one complete rotation (i.e., 360°), if it coincides with the original position before completing one rotation, then it is said to have **rotational symmetry**. The point about which the figure is rotated is called the **centre of rotation**.

When a plane figure that has rotational symmetry is rotated once about a point which is not the centre of rotation, then it coincides with the original figure only when it completes one complete rotation.

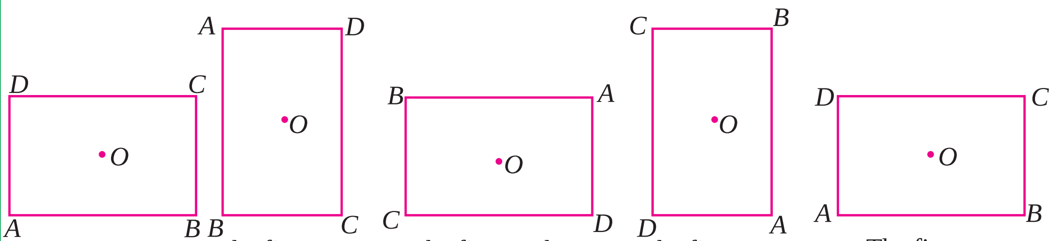
The number of times a figure that has rotational symmetry coincides with itself when it is rotated once about the centre of rotation is called its **order of rotational symmetry**.

From the above activity, it is clear that a square has rotational symmetry, that its centre of rotation is the point at which its axes of symmetry intersect, and that its order of rotational symmetry is 4.

Activity 2

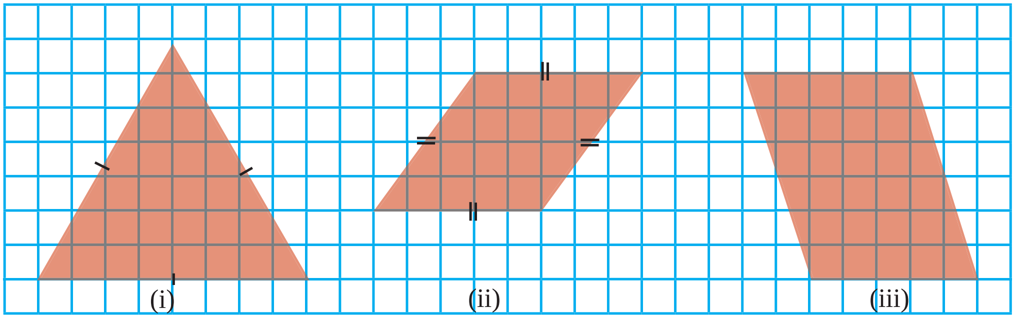
Step 1 - Draw a rectangle in your exercise book and name it *ABCD*.

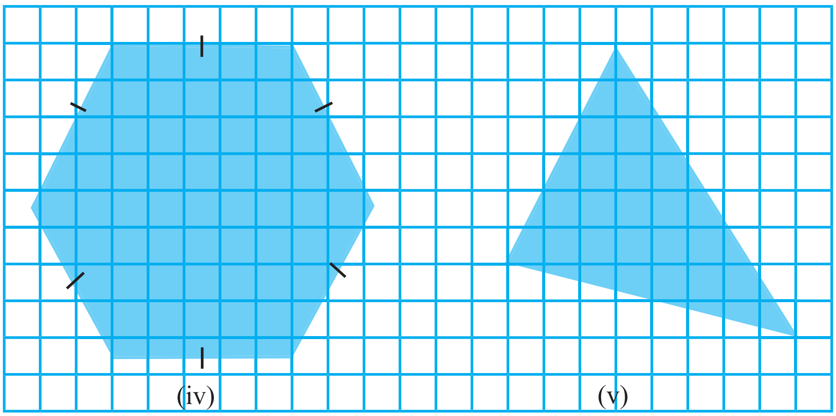
Step 2 - Copy the rectangle *ABCD* onto a plastic paper and place it on the original figure so that they coincide with each other. By using a pin, rotate the plastic paper about *O* as in activity 1 and find out whether the rectangle has rotational symmetry. If it does, find its order of rotational symmetry.



The figure when rotated through 90° | The figure when rotated through 180° | The figure when rotated through 270° | The figure when rotated through 360°

Step 3 - Draw the following figures also in your exercise book, and check whether they have rotational symmetry.





Step 4 - Copy the following table and complete it.

If a given figure has rotational symmetry, then write its order of symmetry.

Plane figure	Number of axes of symmetry	Order of rotational symmetry
Rectangle		
Equilateral triangle		
Rhombus		
Parallelogram		
Regular hexagon		
Scalene triangle		

Examine the table given below.

Plane figure	Number of axes of symmetry	Order of rotational symmetry	Rotational symmetry Yes/No
Equilateral triangle	3	3	Yes
Parallelogram	0	2	Yes
Rhombus	2	2	Yes
Rectangle	2	2	Yes
Square	4	4	Yes
Regular pentagon	5	5	Yes
Regular hexagon	6	6	Yes
Regular octagon	8	8	Yes

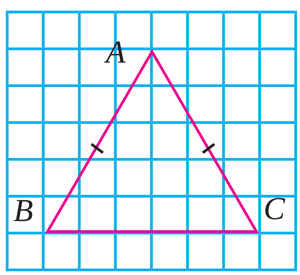
The following facts are clear according to the above table.

- If a **geometrical plane figure** which has rotational symmetry is also **bilaterally symmetrical**, then its order of rotational symmetry is equal to the number of axes of bilateral symmetry.
- A figure which is not bilaterally symmetrical can have rotational symmetry (parallelogram).
- The center of rotational symmetry of a bilaterally symmetrical plane figure which has rotational symmetry, is the point of intersection of its axes of bilateral symmetry.
- If the order of rotational symmetry of a plane figure is 2 or more, then that figure is said to have rotational symmetry.
- The order of rotational symmetry of a figure that has rotational symmetry is greater than 1.

Exercise 11.1

(1) (i) Draw the isosceles triangle ABC in your exercise book as in the figure, and draw its axis of symmetry.

(ii) Copy the triangle ABC onto a plastic paper and find out whether an isosceles triangle has rotational symmetry.



(iii) Does every plane figure which is bilaterally symmetrical have rotational symmetry?

(2) (i) Draw a plane figure which has two or more axes of symmetry.

(ii) Find out whether the figure you have drawn has rotational symmetry.

(iii) If it has rotational symmetry, mark the centre of rotation as *P* and write down the order of rotational symmetry of the figure.

(3) Write down the following statements in your exercise book and mark the true statements with "✓" and the false statements with "×".

(i) Every bilaterally symmetrical plane figure has rotational symmetry.

(ii) Every plane figure which has rotational symmetry is also bilaterally symmetrical.

(iii) If a bilaterally symmetrical plane figure has rotational symmetry, then the number of axes of symmetry is equal to its order of rotational symmetry.

(iv) The point of intersection of the axes of symmetry of a bilaterally symmetrical plane figure which has more than one axis of symmetry is its centre of rotational symmetry.

(v) A scalene triangle does not have rotational symmetry and is also not bilaterally symmetrical.

Summary

- When a plane figure is rotated about a special point in it through an angle of 360^{o}, if it coincides with the original position before completing a full rotation, then it is said to have rotational symmetry.
- The center of rotational symmetry of a bilaterally symmetrical plane figure which has rotational symmetry, is the point of intersection of its axes of bilateral symmetry.
- The order of rotational symmetry of a figure that has rotational symmetry is greater than 1.
- When a plane figure is rotated about its centre of rotation, the number of times it coincides with the original position during a complete rotation is called its order of rotational symmetry.

12 Triangles and Quadrilaterals

By studying this lesson, you will be able to,

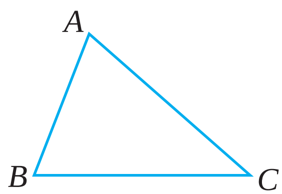
- obtain a value for the sum of the interior angles of a triangle and of a quadrilateral,
- show that the sum of the exterior angles of a triangle and of a quadrilateral is 360°, and
- perform calculations associated with angles of triangles and quadrilaterals.

12.1 Triangles

You have learnt that a polygon formed with three straight line segments is called a **triangle.**

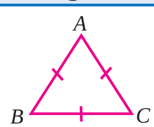
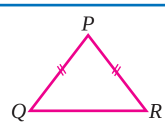
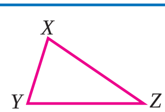
A triangle has three sides and three angles. They are called the elements of the triangle.

The three sides of the triangle ABC are AB, BC and CA. The three angles of the triangle ABC are $A\hat{B}C$, $B\hat{C}A$ and $C\hat{A}B$.

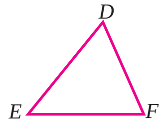
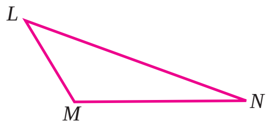
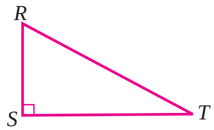


You have learnt in Grade 7 how to classify a triangle according to the lengths of its sides and the magnitudes of its angles.

- ### Classification of triangles according to the lengths of the sides

Triangle	Figure	Note
Equilateral triangle		The lengths of all three sides are equal
Isosceles triangle		The lengths of two sides are equal
Scalene triangle		All three sides are unequal in length

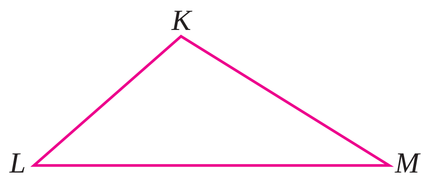
Classification of triangles according to the angles

Triangle	Figure	Note
Acute triangle		The magnitude of each angle is less than 90°.
Obtuse triangle		The magnitude of one angle is greater than 90°.
Right triangle		The magnitude of one angle is 90°.

Do the following review exercise to recall the facts you learnt in Grade 7 on triangles and angles.

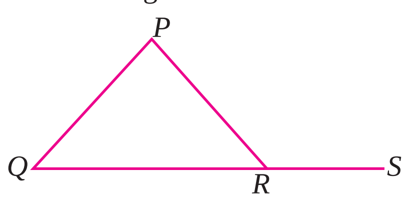
Review Exercise

(1) Name the three sides and the three angles of the triangle shown in the figure.

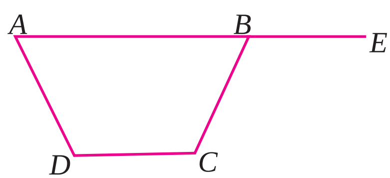


(2) (i) Draw an obtuse triangle and name it ABC.
(ii) Measure $A\hat{B}C$, $B\hat{A}C$ and $A\hat{C}B$ and write down their magnitudes.

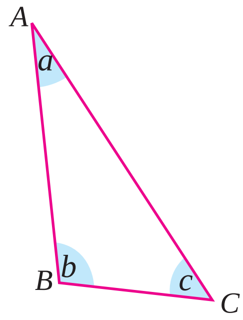
(3) (i) Draw a triangle PQR as in the figure and produce QR to S.
(ii) Measure $P\hat{R}Q$ and $P\hat{R}S$ and write down their magnitudes.



(4) (i) Draw a quadrilateral $ABCD$ and produce AB to E .
(ii) Measure $E\hat{B}C$ and $A\hat{B}C$ and write down their magnitudes.



12.2 The sum of the interior angles of a triangle

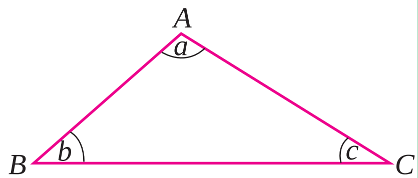


The angles located within the triangle *ABC* are named *a*, *b*, and *c*. Since they are located within the triangle, they are called the **interior angles of the triangle** ***ABC*****.**

Engage in the following activity in order to find the sum of the interior angles of a triangle.

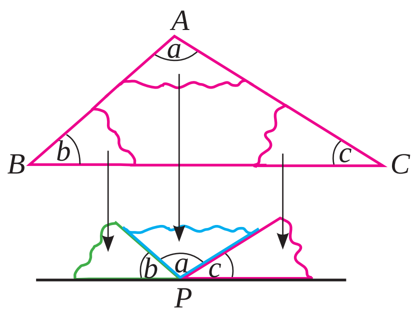
Activity 1

Step 1 - Draw any triangle on a piece of white paper and name its vertices as *A*,*B* and *C* and its interior angles as *a*,*b* and *c* respectively, as shown in the figure.



Step 2 - Cut and separate out the three angles *a*, *b* and *c* as shown in the figure.

Step3 - In your exercise book, paste the three angles *a*,*b* and *c* that were cut out, as shown in the figure, without overlapping them and such that the point *P* on the line is the common vertex.



Step 4 - Establish the fact that the three pasted angles are located on a straight line, by keeping a ruler. Write down the value of $a + b + c$.

- Draw another triangle in your exercise book, measure the three interior angles and find their sum.

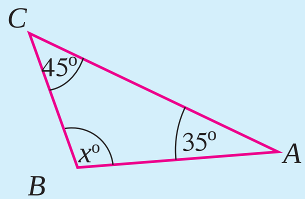
It must be clear to you from the above activity that the sum of the three interior angles of a triangle can be presented as the sum of three angles located on a straight line, completely covering one side of it.

Since the sum of the angles at a point on a straight line is 180°, it can be concluded that the sum of the three interior angles of a triangle is 180°.

The sum of the interior angles of a triangle is 180°.

Example 1

Find the magnitude of $A\hat{B}C$ in the figure.

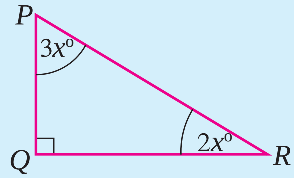


Since the sum of the interior angles of a triangle is 180°,

$$45 + 35 + x = 180$$
$$80 + x = 180$$
$$x + 80 - 80 = 180 - 80$$
$$x = 100$$
$$A\hat{B}C = 100^{\circ}$$

Example 2

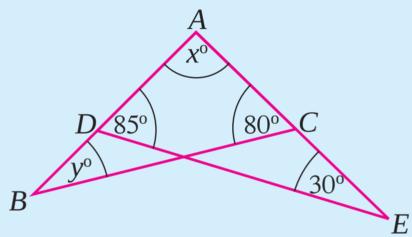
Find the magnitude of $Q\hat{P}R$ in the figure.



$$3x + 2x + 90 = 180$$
$$5x + 90 = 180$$
$$5x + 90 - 90 = 180 - 90$$
$$5x = 90$$
$$\frac{5x}{5} = \frac{90}{5}$$
$$x = 18$$
$$\therefore Q\hat{P}R = 3 \times 18^{\circ} = 54^{\circ}$$

Example 3

Find the values of x and y according to the information marked in the figure.



Since the sum of the interior angles of the triangle ADE is 180°,

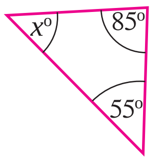
$$85 + 30 + x = 180$$
$$115 + x = 180$$
$$x + 115 - 115 = 180 - 115$$
$$x = 65$$

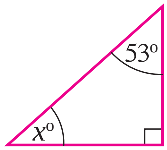
Since the sum of the interior angles of the triangle ABC is 180°,

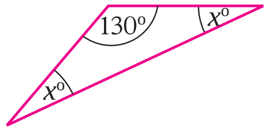
$$x + 80 + y = 180$$
$$65 + 80 + y = 180 \text{ (substituting } x^{\circ} = 65^{\circ})$$
$$y + 145 = 180$$
$$y + 145 - 145 = 180 - 145$$
$$y = 35$$

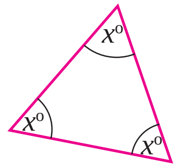
Exercise 12.1

(1) Find the magnitude of the angle marked as x in each figure.

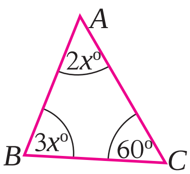
(i)


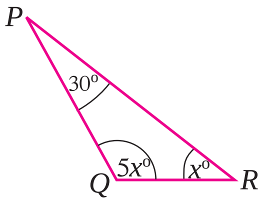
(ii)


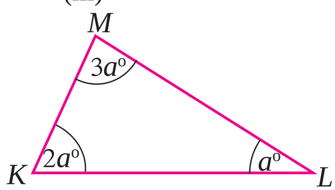
(iii)


(iv)


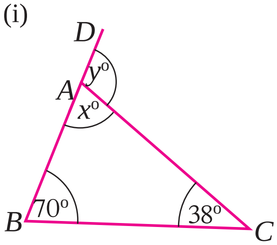
(2) Find the magnitude of each of the angles in each triangle.

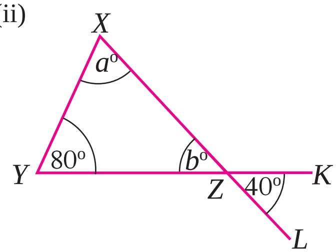
(i)


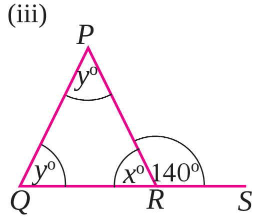
(ii)


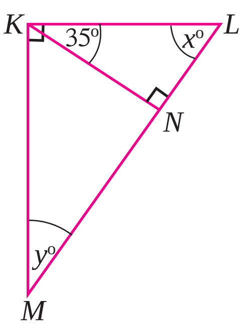
(iii)


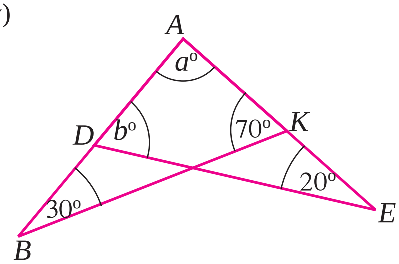
(3) Find the magnitude of each of the angles denoted by an English letter in each figure.

(i)


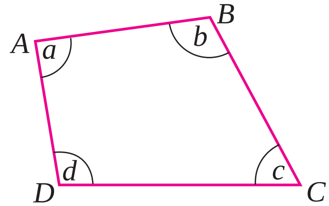
(ii)


(iii)


(iv)


(v)


12.3 The sum of the interior angles of a quadrilateral



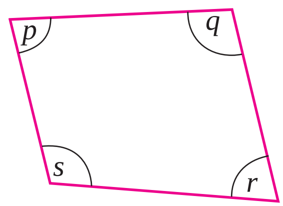
You learnt in Grade 6 that a closed rectilinear plane figure which consists of 4 sides is called a **quadrilateral**. A quadrilateral has 4 sides and 4 angles.

The sides of the quadrilateral *ABCD* are *AB*, *BC*, *CD* and *DA*. Its sides can also be named *BA*, *CB*, *DC* and *AD*.

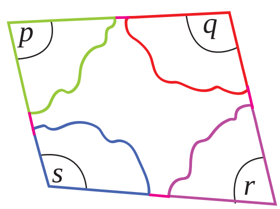
The interior angles of the quadrilateral *ABCD* in the figure are marked as *a*, *b*, *c* and *d*.

Do the activity given below in order to find the sum of the interior angles of a quadrilateral.

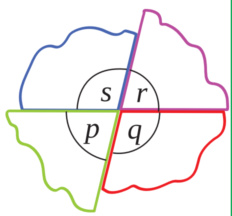
Activity 2



Step 1 - Draw any quadrilateral on a piece of coloured paper and name its interior angles as *p*, *q*, *r* and *s*.



Step 2 - Cut and separate out the angles *p*, *q*, *r*, *s* as shown in the figure.



Step 3 - In your exercise book, paste the angles that were cut out, around a point without overlapping them, such that the vertices of all the angles coincide.

Step 4 - Write down a value for $p + q + r + s$ by considering the sum of the angles around a point.

Step 5 - Draw another quadrilateral in your exercise book, measure its interior angles and obtain a value for their sum.

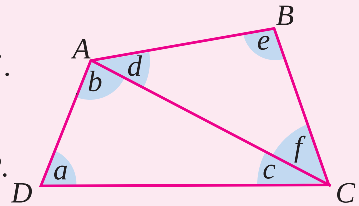
In the above activity, you would have obtained that $p + q + r + s = 360^\circ$.

Since the sum of the angles located around a point is 360°, it can be concluded that the sum of the four interior angles of a quadrilateral is 360°.

The sum of the interior angles of a quadrilateral is 360°.

Note:

The quadrilateral $ABCD$ is shown in the figure. By joining the vertices A and C, the triangles ABC and ADC are created.



The sum of the three angles of the triangle ADC is 180°.

Accordingly, $a + b + c = 180^\circ$.

The sum of the three angles of the triangle ABC is 180°.

Accordingly, $d + e + f = 180^\circ$.

$\therefore$ The sum of the interior angles of the quadrilateral $=$ The sum of the interior angles of the triangle ADC $+$ The sum of the interior angles of the triangle ABC

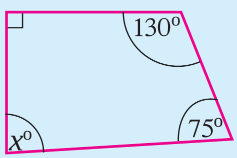
$= (a + b + c) + (d + e + f)$

$= 180^\circ + 180^\circ = 360^\circ$

Accordingly, the sum of the interior angles of a quadrilateral is 360°.

Example 1

Find the value of x in the figure.

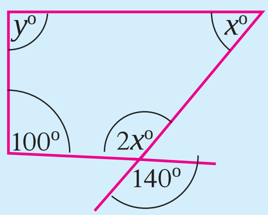


Since the sum of the interior angles of a quadrilateral is 360°,

$$x + 90 + 130 + 75 = 360$$
$$x + 295 = 360$$
$$x + 295 - 295 = 360 - 295$$
$$x = 65$$

Example 2

Find the values of x and y in the figure.



Since vertically opposite angles are equal,

$$2x = 140$$
$$x = 70$$

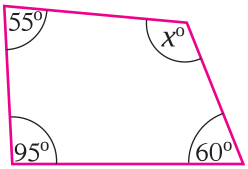
Since the sum of the interior angles of a quadrilateral is 360°,

$$y + 100 + 2x + x = 360$$
$$y + 100 + 140 + 70 = 360$$
$$y + 310 - 310 = 360 - 310 = 50$$

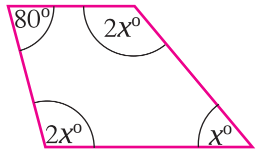
Exercise 12.2

(1) Find the value of x in each figure given below.

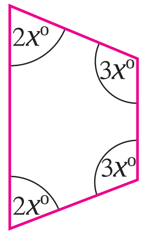
(i)



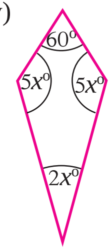
(ii)



(iii)

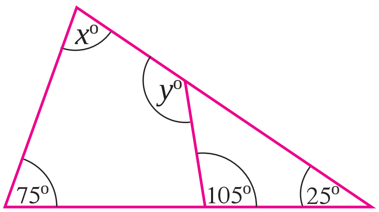


(iv)

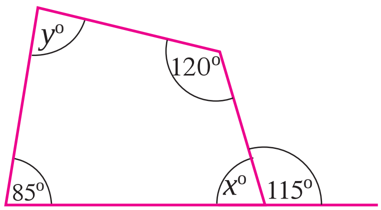


(2) Find the values of x and y in each figure given below.

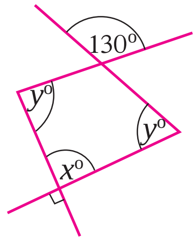
(i)



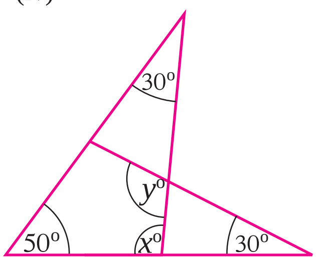
(ii)



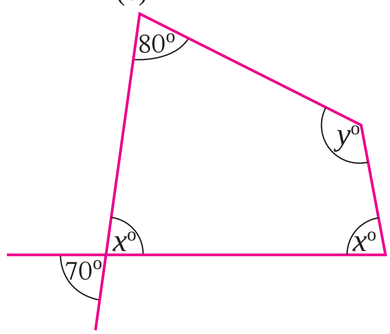
(iii)



(iv)

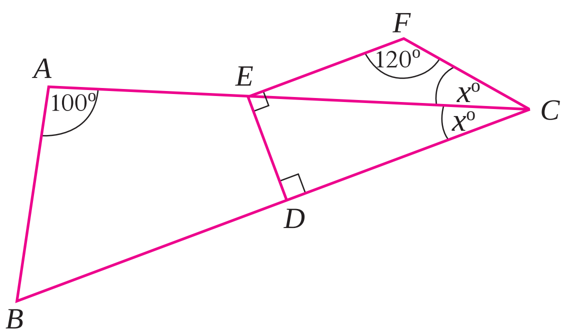


(v)

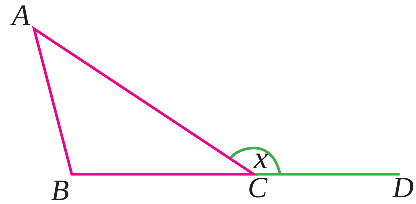


(3) Find the magnitude of each of the following angles, based on the information marked in the figure.

(i) $D\hat{C}F$
(ii) $A\hat{B}C$
(iii) $A\hat{E}D$



12.4 Exterior angles of a triangle

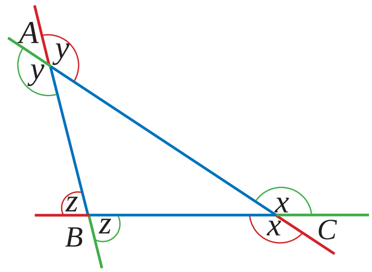


In the triangle ABC, the side BC is produced to D. The angle ACD with arms AC and CD, coloured in green, which is then formed, is **an exterior angle of the triangle *ABC*.**

As shown in the figure, more exterior angles can be created by producing the other sides of the triangle ABC.

Although there are two exterior angles formed at every vertex of a triangle, they are equal in magnitude since they are vertically opposite angles.

When one exterior angle at each vertex is considered, then the sum of these angles is said to be the **sum of the exterior angles of the triangle**.

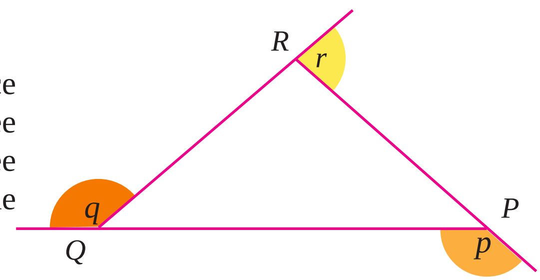


- **The sum of the exterior angles of a triangle**

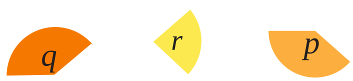
Let us engage in activity 3 in order to obtain a value for the sum of the exterior angles of a triangle.

Activity 3

Step 1 - Draw any triangle on a piece of paper and draw three exterior angles at its three vertices as shown in the figure.

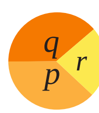


Step 2 - As shown in the figure, cut and separate out laminas of the three exterior angles using a blade.



Step 3 - In your exercise book, paste the three exterior angles (the three laminas) that were cut out, around a point without overlapping them, such that the vertices of the three exterior angles coincide.

Step 4 - Find the sum $p+q+r$ of the exterior angles of the triangle, by using the knowledge on the sum of the angles around a point.

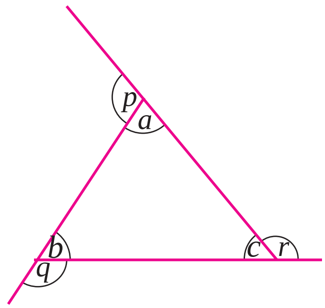


- Draw another triangle in your exercise book, produce the sides to form exterior angles at the three vertices, and by measuring them, obtain the sum of the exterior angles of the triangle.

  It is clear from the above activity, that the three exterior angles of a triangle can be positioned as three angles around a point.

  Since the sum of the angles around a point is 360˚, the sum of the exterior angles of a triangle is also 360˚.

This result is obtained by measuring the angles as well.
According to the given figure,



$$(a+p)+(b+q)+(c+r) = 180^o + 180^o + 180^o$$
$$= 540^o$$
$$\therefore (a+b+c)+(p+q+r) = 540^o$$

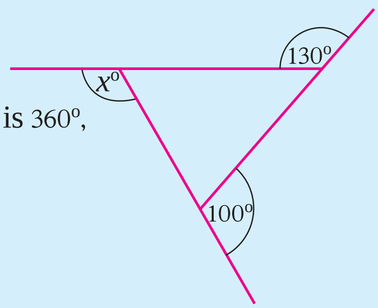
Since, $a+b+c = 180^o$,

$$180^o + (p+q+r) = 540^o$$
$$\therefore p+q+r = 540^o - 180^o$$
$$= 360^o$$

The sum of the exterior angles of a triangle is 360˚.

Example 1

Find the value of x in the figure.

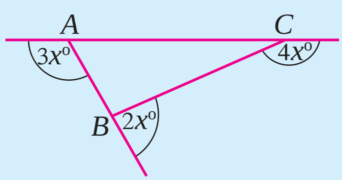


Since the sum of the exterior angles of a triangle is 360^o,

$$130 + 100 + x = 360$$
$$230 + x = 360$$
$$x + 230 - 230 = 360 - 230$$
$$x = 130$$

Example 2

Find the magnitudes of the three exterior and the three interior angles of the triangle ABC.



$$3x + 2x + 4x = 360$$
$$9x = 360$$
$$\frac{9x}{9} = \frac{360}{9}$$
$$\therefore\ x = 40$$

$\therefore$ The exterior angle at vertex $A = 3x^\circ = 3 \times 40^\circ = 120^\circ$

The exterior angle at vertex $B = 2x^\circ = 2 \times 40^\circ = 80^\circ$

The exterior angle at vertex $C = 4x^\circ = 4 \times 40^\circ = 160^\circ$

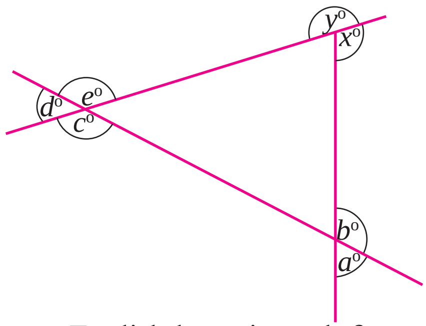
Since the sum of the angles on a straight line is 180˚,

the interior angle at vertex $A = 180^\circ - 120^\circ = 60^\circ$

the interior angle at vertex $B = 180^\circ - 80^\circ = 100^\circ$

the interior angle at vertex $C = 180^\circ - 160^\circ = 20^\circ$

Exercise 12.3

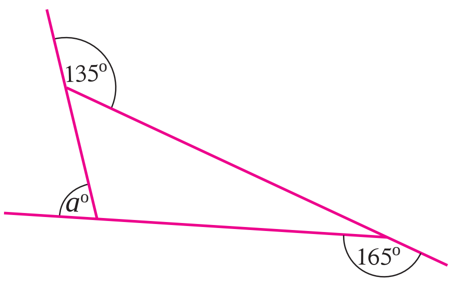


(1) (i) Select and write the exterior angles from among the angles a, b, c, d, e, x and y shown in the figure.

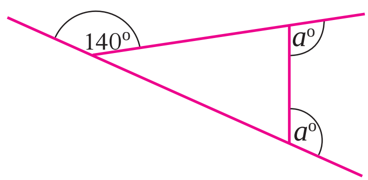
(ii) Explain why the other angles are not exterior angles.

(2) Find the value of each of the angles denoted by an English letter in each figure given below.

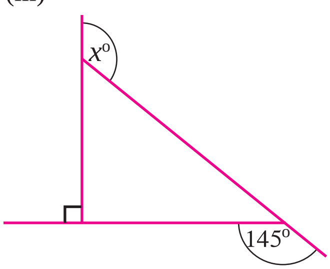
(i)



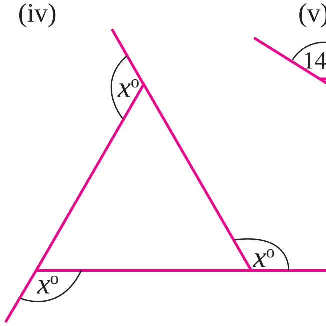
(ii)



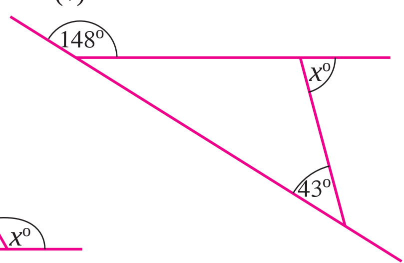
(iii)



(iv)



(v)

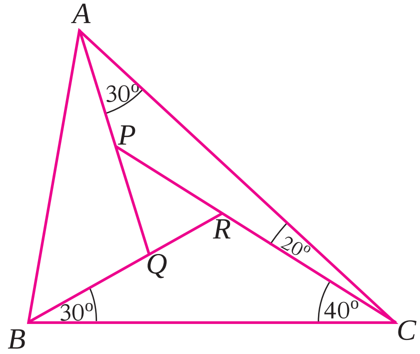


(3) According to the information marked in the figure,

(i) find $B\hat{R}C$.

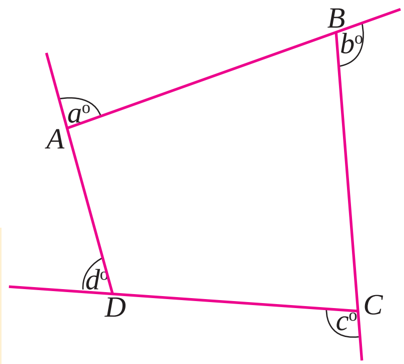
(ii) find $A\hat{P}C$.

(iii) find $B\hat{Q}A$.



12.5 Exterior angles of a quadrilateral

The exterior angles created by producing the sides of the quadrilateral *ABCD* are marked in the figure as *a*, *b*, *c* and *d* .



A quadrilateral has four vertices. Hence, there are four exterior angles.

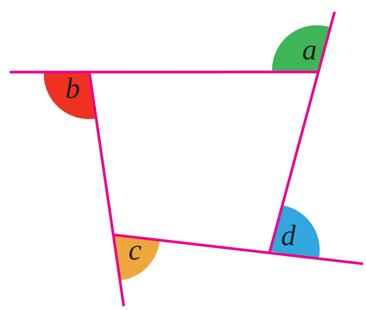
Although there are two exterior angles formed at every vertex of a quadrilateral, they are equal in magnitude since they are vertically opposite angles.

When one exterior angle at each vertex is considered, then the sum of these angles is said to be the **sum of the exterior angles of the quadrilateral**.

Let us engage in the activity given below in order to find the sum of the exterior angles of a quadrilateral.

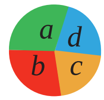
Activity 4

Step 1 - Draw any quadrilateral, and draw four exterior angles at its four vertices.



Step 2 - As shown in the figure, cut and separate out laminas of the exterior angles with a blade.

Step 3 - Obtain a value for $a + b + c + d$, by pasting the four exterior angles that were cut out, around a point without overlapping them, such that their vertices coincide.



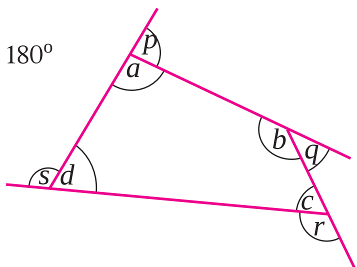
➢ Draw another quadrilateral in your exercise book and obtain a value for the sum of its exterior angles by measuring them.

It is clear from the above activity that the sum of the exterior angles of a quadrilateral is 360˚.

The sum of the exterior angles of a quadrilateral is 360˚.

According to the given figure,

$$\begin{aligned}
a+p+b+q+c+r+d+s &= 180^{\circ}+180^{\circ}+180^{\circ}+180^{\circ}\\
(a+b+c+d)+(p+q+r+s) &= 720^{\circ}\\
\text{Since } a+b+c+d &= 360^{\circ},\\
360^{\circ}+p+q+r+s &= 720^{\circ}\\
&= 720^{\circ}-360^{\circ}\\
&= 360^{\circ}
\end{aligned}$$

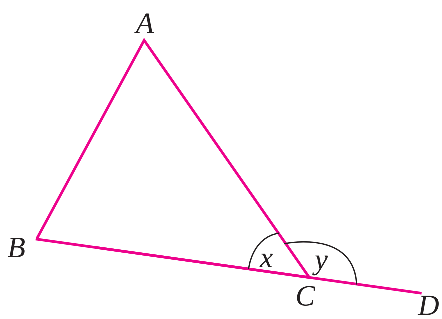


• The sum of an exterior angle and an interior angle at one vertex of a triangle and of a quadrilateral

The interior and exterior angles of a triangle at one vertex are shown in the figure as x and y.

These two angles are located on the straight line BD, at the point C.

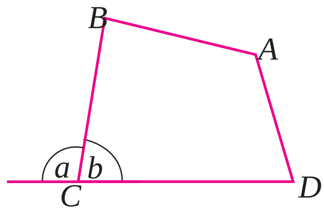
Since the sum of the angles at a point on a straight line is 180˚, $x + y = 180^{\circ}$.



At a vertex of a triangle, interior angle + exterior angle $= 180^{\circ}$.

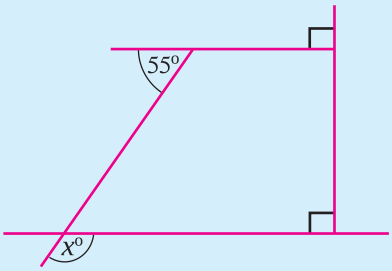
As for a triangle, the sum of the interior angle and the exterior angle at each vertex of a quadrilateral is 180^o.

$\therefore \quad a + b = 180^o$



Example 1

Find the value of x.



$x + 55 + 90 + 90 = 360$

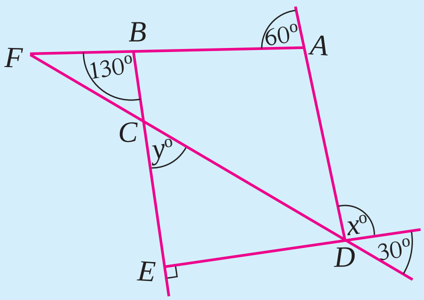
$x + 235 = 360$

$x = 360 - 235$

$x = 125$

Example 2

Find the values of x and y according to the information marked in the figure.



Since the sum of the exterior angles of the quadrilateral $ABED$ is 360^o,

$60 + 130 + 90 + x = 360$

$x + 280 = 360$

$x + 280 - 280 = 360 - 280$

$x = 80$

By taking the sum of the exterior angles of the quadrilateral $ABCD$,

$60 + 130 + y + (30 + x) = 360$

$190 + y + 30 + 80 = 360$

$y + 300 = 360$

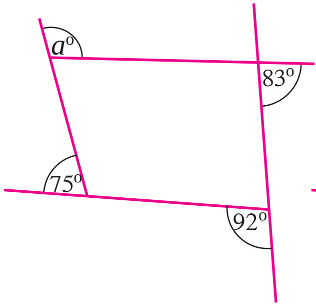
$y = 360 - 300$

$y = 60$

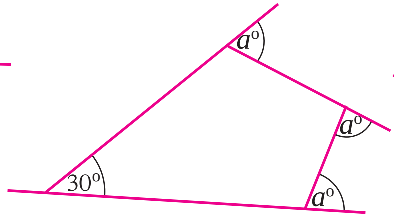
Exercise 12.4

(1) Find the value of a, marked in each figure.

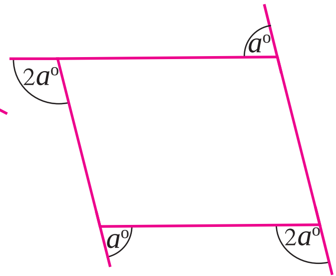
(i)



(ii)

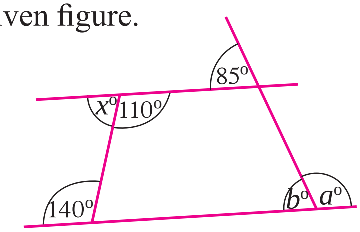


(iii)



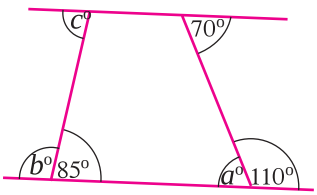
(2) Find the value of each of the angles based on the given figure.

(i) What is the value of x?
(ii) What is the value of a?
(iii) What is the value of b?

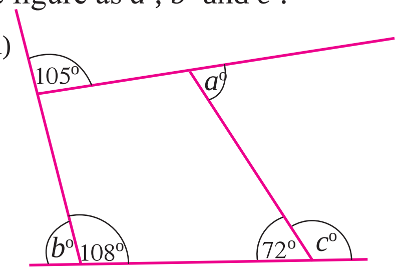


(3) Find the magnitudes of the angles marked in the figure as a°, b° and c°.

(i)

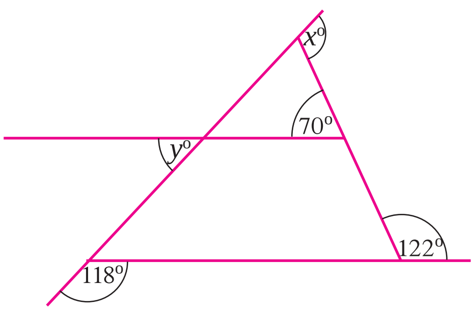


(ii)

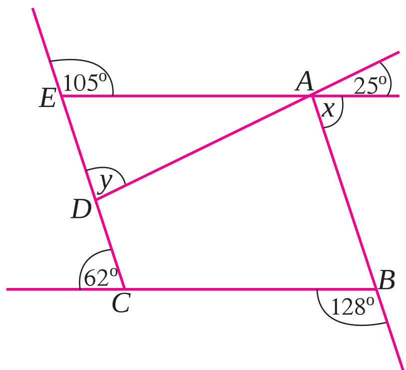


(4) Find the values of x and y in each figure.

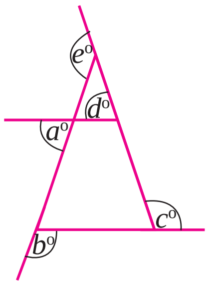
(i)



(ii)



(5)



(i) What is the value of $a+b+c+d$?

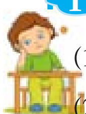
(ii) What is the value of $b+c+e$?

(iii) According to the answers of (i) and (ii), show that $e=a+d$.

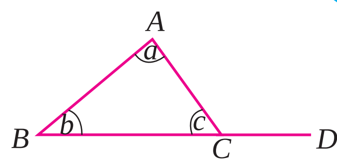
Summary

- The sum of the interior angles of a triangle is 180^o.
- The sum of the interior angles of a quadrilateral is 360^o.
- The sum of the exterior angles of a triangle is 360^o.
- The sum of the exterior angles of a quadrilateral is 360^o.

Think



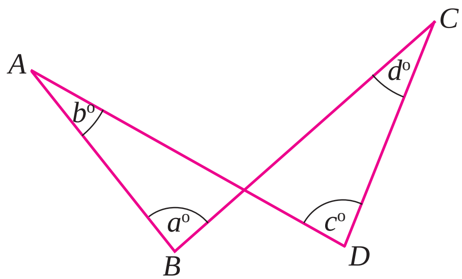
(1) Show that $A\hat{C}D = a + b$.



(2) (i) $ABCD$ is not a polygon. Explain the reason.

(ii) $a + b = c + d$. Explain the reason.

(iii) Show that the value of $a+b+c+d$ is less than 360^o.

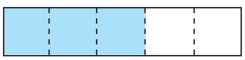


13 Fractions I

By studying this lesson, you will be able to,
- multiply a fraction by a whole number,
- multiply a fraction by a fraction,
- multiply a fraction by a mixed number, and
- multiply a mixed number by a mixed number.

13.1 Fractions

Let us first recall what you have learnt about fractions in Grades 6 and 7. Let us take the area of the figure given below as a unit.



This unit has been divided into five equal parts of which three parts are coloured. You have learnt that the coloured area is $\frac{3}{5}$ of the whole area.

You have also learnt that if a unit is divided into equal parts, then one part or several of these parts is called a fraction of the unit. A portion of a collection is also considered as a fraction of that collection.

In addition, you have learnt that fractions such as $\frac{3}{5}$, $\frac{1}{2}$ and $\frac{2}{3}$ which are less than one and greater than zero are called proper fractions.

A number which has been written by adding together a whole number and a proper fraction is called a **mixed number** or an **improper fraction**, depending on how it has been represented.

Some examples of mixed numbers are $1\frac{1}{2}$, $2\frac{1}{3}$ and $4\frac{2}{5}$.

In the mixed number $4\frac{2}{5}$, the whole part is 4 and the fractional part is $\frac{2}{5}$.

Some examples of improper fractions are $\frac{3}{2}$, $\frac{5}{3}$ and $\frac{11}{7}$.

The numerator of an improper fraction is greater than or equal to the denominator.

A fraction **equivalent** to a given fraction can be obtained by multiplying the numerator and the denominator of the fraction by the same non-zero number.

A fraction **equivalent** to a given fraction can also be obtained by dividing the numerator and the denominator by a non - zero common factor of the numerator and the denominator.

• Representing a mixed number as an improper fraction

By following the steps given below, a mixed number can be represented as an improper fraction.

- Multiply the whole number part of the mixed number by the denominator of the fractional part, and add it to the numerator of the fractional part.
- The denominator of the improper fraction is the same as the denominator of the fractional part of the mixed number.

• Representing an improper fraction as a mixed number

You learnt in Grade 7 how to represent an improper fraction as a mixed number.

Let us represent $\frac{7}{4}$ as a mixed number.

Method I

$$\frac{7}{4} = \frac{4+3}{4}$$

$$= \frac{4}{4} + \frac{3}{4}$$

$$= 1 + \frac{3}{4} = 1\frac{3}{4}$$

Method II

$$\frac{7}{4} = 7 \div 4$$

$$\begin{array}{r|l} & 1 \\ 4 & 7 \\ & 4 \\ \hline & 3 \end{array}$$

The quotient and remainder of $7 \div 4$ are 1 and 3 respectively. The quotient is the whole number part of the mixed number and the remainder is the numerator of the fractional part.

The denominator of the fractional part of the mixed number is the same as the denominator of the improper fraction.

$$\therefore \frac{7}{4} = 1\frac{3}{4}$$

You have learnt how to add and subtract fractions in Grades 6 and 7.

Do the following review exercise to recall what you have learnt previously about fractions.

Review Exercise

(1) Choose the appropriate value from the brackets and fill in the blanks.

(i) $\frac{3}{4}$ is .... $\frac{1}{4}$ths (two, three, five)

(ii) $\frac{2}{5}$ is two .... $\left(\frac{1}{3}\text{rds}, \frac{1}{2}\text{s}, \frac{1}{5}\text{ths}\right)$

(iii) Four $\frac{1}{7}$ths is ...... $\left(\frac{4}{7}, \frac{4}{5}, \frac{4}{9}\right)$

(2) Write down two equivalent fractions for each fraction given below.

(i) $\frac{3}{4}$ (ii) $\frac{2}{5}$ (iii) $\frac{6}{10}$ (iv) $\frac{8}{24}$

(3) Represent each mixed number given below as an improper fraction.

(i) $1\frac{1}{5}$ (ii) $3\frac{3}{5}$ (iii) $6\frac{1}{6}$

(4) Represent each improper fraction given below as a mixed number.

(i) $\frac{14}{5}$ (ii) $\frac{18}{7}$ (iii) $\frac{37}{3}$

(5) Simplify the following.

(i) $\frac{2}{5} + \frac{1}{5}$ (ii) $\frac{1}{3} + \frac{1}{2}$ (iii) $\frac{3}{5} + \frac{1}{3}$ (iv) $\frac{7}{12} + \frac{1}{8}$

(v) $\frac{1}{6} + \frac{5}{8}$ (vi) $\frac{11}{15} + \frac{2}{10}$ (vii) $1\frac{1}{2} + 4\frac{3}{8}$ (viii) $2\frac{1}{4} + 3\frac{5}{9}$

(6) Simplify the following.

(i) $\frac{6}{7} - \frac{2}{7}$ (ii) $\frac{7}{10} - \frac{2}{5}$ (iii) $\frac{1}{3} - \frac{2}{7}$ (iv) $1 - \frac{1}{5}$

(v) $\frac{7}{8} - \frac{5}{6}$ (vi) $3\frac{7}{8} - 1\frac{1}{2}$ (vii) $3 - 1\frac{5}{8}$ (viii) $2\frac{2}{5} - 1\frac{3}{20}$

13.2 Multiplying a fraction by a whole number

The figure depicts a cake, which is divided into five equal parts.

We know that one part of the entire cake is $\frac{1}{5}$ of the cake.
Let us take 3 such parts.

Let us consider how much the total of these three parts is from the entire cake. For this, we have to add these three quantities.

It is $\frac{1}{5}+\frac{1}{5}+\frac{1}{5}=\frac{3}{5}$

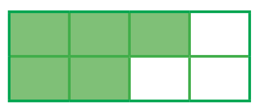
You have learnt previously that addition of the same number repeatedly can be represented as a multiplication.

For example, $2+2+2=2\times 3=6$

Accordingly, we can write $\frac{1}{5}+\frac{1}{5}+\frac{1}{5}=\frac{1}{5}\times 3$

Therefore, $\frac{1}{5}\times 3=\frac{3}{5}$. That is, three $\frac{1}{5}$ is equal to $\frac{3}{5}$.

- The figure depicts a rectangle which has been divided into eight equal parts. One part is $\frac{1}{8}$ of the entire figure.



Let us consider the sum of 5 such parts.
It can be written as $\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{5}{8}$

That is, five $\frac{1}{8}$ s is equal to $\frac{5}{8}$

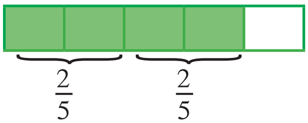
$\frac{1}{8}\times 5=\frac{5}{8}$.

Accordingly,

$\frac{1}{3} \times 1 = \frac{1}{3}$, $\quad \frac{1}{3} \times 2 = \frac{2}{3}$, $\quad \frac{1}{10} \times 7 = \frac{7}{10}$

- Now let us consider a multiplication of the form $\frac{2}{5} \times 2$.

Let us represent this by a figure.



This can be written as $\frac{2}{5} + \frac{2}{5} = \frac{4}{5}$

When this sum is written as a product we obtain,

$\frac{2}{5} \times 2 = \frac{4}{5}$

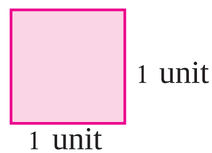
When a fraction is multiplied by a whole number, the numerator of the resultant fraction is the product of the whole number and the numerator of the given fraction, and its denominator is the same as that of the given fraction.

• Multiplying a whole number by a fraction

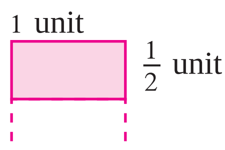
You have learnt that the area of a square shaped lamina of length 1 unit and breadth 1 unit is 1 square unit.

That is, the area of the square shaped lamina = 1 unit × 1 unit

= 1 square unit



Now let us find the area of a rectangular shaped lamina which is of length 1 unit and breadth $\frac{1}{2}$ a unit using two methods.



Method I

Since the area of this rectangular shaped lamina is $\frac{1}{2}$ the area of the square of area 1 square unit, the area of the rectangular lamina is $\frac{1}{2}$ square units.

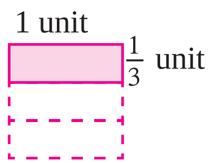
Method II

Since the length of this lamina is 1 unit and its breadth is $\frac{1}{2}$ a unit,

area of the lamina $=$ (length $\times$ breadth) square units

$= 1 \times \frac{1}{2}$ square units

$$\therefore\ 1 \times \frac{1}{2} = \frac{1}{2}$$



Furthermore, the area of the rectangular shaped lamina in the figure which is of length 1 unit and breadth $\frac{1}{3}$ units is $\frac{1}{3}$ square units.

That is, $1 \times \frac{1}{3} = \frac{1}{3}$

You have learnt in the previous section that $\frac{1}{3} \times 1 = \frac{1}{3}$

$$\therefore\ \frac{1}{3} \times 1 = 1 \times \frac{1}{3}$$

Similarly,

$\frac{2}{7} \times 3 = \frac{6}{7}$ and $3 \times \frac{2}{7} = \frac{6}{7}$

$\frac{4}{11} \times 2 = \frac{8}{11}$ and $2 \times \frac{4}{11} = \frac{8}{11}$

$\frac{2}{13} \times 5 = \frac{10}{13}$ and $5 \times \frac{2}{13} = \frac{10}{13}$

$$\therefore\ \frac{2}{7} \times 3 = 3 \times \frac{2}{7}$$
$$\therefore\ \frac{4}{11} \times 2 = 2 \times \frac{4}{11}$$
$$\therefore\ \frac{2}{13} \times 5 = 5 \times \frac{2}{13}$$

When multiplying a fraction by a whole number, and when multiplying the same whole number by the same fraction we obtain the same answer.

Example 1

(i) Simplify $\frac{3}{7} \times 2$.

$\frac{3}{7} \times 2 = \frac{3 \times 2}{7}$

$= \frac{6}{7}$

Example 2

(ii) Simplify $\frac{3}{8} \times 5$.

$\frac{3}{8} \times 5 = \frac{3 \times 5}{8}$

$= \frac{15}{8}$

$= 1\frac{7}{8}$

Example 3

(iii) Simplify $4 \times \frac{2}{5}$.

$4 \times \frac{2}{5} = \frac{4 \times 2}{5}$

$= \frac{8}{5}$

$= 1\frac{3}{5}$

Exercise 13.1

(1) Express the product of each of the following in its simplest form (If the answer is an improper fraction, express it as a mixed number).

(i) $\frac{1}{6} \times 5$ (ii) $\frac{3}{10} \times 3$ (iii) $6 \times \frac{2}{13}$ (iv) $\frac{3}{7} \times 5$

(v) $\frac{2}{7} \times 9$ (vi) $\frac{1}{10} \times 17$ (vii) $5 \times \frac{7}{9}$ (viii) $\frac{3}{4} \times 12$

(ix) $\frac{2}{5} \times 10$ (x) $\frac{7}{8} \times 1$ (xi) $\frac{2}{3} \times 0$ (xii) $4 \times \frac{3}{5}$

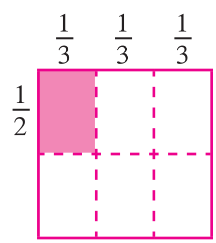
(xiii) $3 \times \frac{1}{4}$ (xiv) $\frac{5}{6} \times 8$ (xv) $10 \times \frac{3}{5}$

(2) A vehicle that travels at a constant speed, journeys $\frac{3}{4}$ kilometers in a minute. How far does it travel in 8 minutes?

(3) A machine produces 600 plastic cups in an hour. How many cups does it produce in $\frac{2}{3}$ hours?

13.3 Multiplying a fraction by a fraction

The figure shows a square shaped lamina of side length 1 unit. It is divided into 6 equal parts, of which one part is shaded as in the figure.



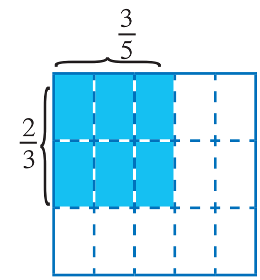
Since the shaded part is $\frac{1}{6}$ of the whole area of the lamina, its area is $\frac{1}{6}$ square units.

Also, the shape of the shaded part is rectangular. Its length is $\frac{1}{2}$ the length of the square lamina and its breadth is $\frac{1}{3}$ the breadth of the square lamina.

The area of the rectangular shaped lamina is calculated by multiplying its length by its breadth.

Therefore, the area of the shaded part can be written as $\frac{1}{2} \times \frac{1}{3}$ square units. Since this is equal to $\frac{1}{6}$ square units,

$$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$



The figure shows a square shaped lamina of side length 1 unit. It is divided into 15 equal parts.

Let us find the area of the shaded part using two different methods.

Method I

Since the area of the shaded part is $\frac{6}{15}$ of the area of the whole lamina, the area of this part is $\frac{6}{15}$ square units.

Method II

The length of the shaded part of the rectangular shape = $\frac{2}{3}$ of the length of the square (that is, $\frac{2}{3}$ units)

Its breadth = $\frac{3}{5}$ of the length of the square (that is, $\frac{3}{5}$ units).

$\therefore$ The area of the shaded part is $\frac{3}{5} \times \frac{2}{3}$ square units.

$$\therefore \frac{3}{5} \times \frac{2}{3} = \frac{6}{15}$$

Let us consider the above two cases.

$$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6} \quad \left(\frac{1 \times 1}{2 \times 3} = \frac{1}{6}\right)$$

$$\frac{3}{5} \times \frac{2}{3} = \frac{6}{15} \quad \left(\frac{3 \times 2}{5 \times 3} = \frac{6}{15}\right)$$

When two fractions are multiplied,

- the numerator of the resultant fraction is the product of the two numerators.
- the denominator of the resultant fraction is the product of the two denominators.

Note

- When any fraction is multiplied by zero, the result is zero.

$$\frac{1}{2} \times 0 = \frac{1}{2} \times \frac{0}{1} = \frac{1 \times 0}{2 \times 1} = \frac{0}{2} = 0$$

- When any fraction is multiplied by 1, the result is the same fraction.

$$\frac{1}{2} \times 1 = \frac{1}{2} \times \frac{1}{1} = \frac{1 \times 1}{2 \times 1} = \frac{1}{2}$$

Example 1

Simplify

(i) $\frac{4}{7} \times \frac{2}{3}$

$$\frac{4}{7} \times \frac{2}{3} = \frac{4 \times 2}{7 \times 3}$$
$$= \frac{8}{21}$$

(ii) $\frac{3}{8} \times \frac{4}{5} \times \frac{1}{2}$

$$\frac{3}{8} \times \frac{4}{5} \times \frac{1}{2} = \frac{3 \times 4 \times 1}{8 \times 5 \times 2} = \frac{12}{80}$$
$$= \frac{12 \div 4}{80 \div 4} \text{ (equivalent fraction)}$$
$$= \frac{3}{20}$$

Note

$$\frac{3}{8} \times \frac{4}{5} = \frac{12}{40}$$

In the fraction $\frac{12}{40}$, since 4 is a common factor of both the numerator and the denominator, let us divide the numerator as well as the denominator by 4.

$$\frac{12}{40} = \frac{12 \div 4}{40 \div 4} = \frac{3}{10}$$

This is written as $\frac{\cancel{12}^{3}}{\cancel{40}_{10}} = \frac{3}{10}$.

$$\frac{3}{8} \times \frac{4}{5} = \frac{\cancel{12}^{3}}{\cancel{40}_{10}} = \frac{3}{10}.$$

Also,

$$\frac{3}{8} \times \frac{4}{5} = \frac{3 \times 4}{8 \times 5} = \frac{3 \times 4}{2 \times 4 \times 5}$$

Now, since 4 is the greatest common factor of the numerator and the denominator, by dividing the numerator and the denominator by 4 we obtain,

$$\frac{3 \times \cancel{4}^{1}}{2 \times \cancel{4}_{1} \times 5} = \frac{3}{10}$$

When simplifying $\frac{3}{8} \times \frac{4}{5}$, it is easy to first divide the numerator and the denominator by their common factors.

$$\frac{3}{\cancel{8}_{2}} \times \frac{\cancel{4}^{1}}{5} = \frac{3 \times 1}{2 \times 5} = \frac{3}{10}$$

Exercise 13.2

(1) Simplify the following.

(a) (i) $\frac{1}{2} \times \frac{1}{4}$ (ii) $\frac{2}{3} \times \frac{1}{5}$ (iii) $\frac{3}{4} \times \frac{5}{7}$ (iv) $\frac{3}{5} \times \frac{2}{7}$

(v) $\frac{3}{8} \times \frac{2}{5}$ (vi) $\frac{7}{10} \times \frac{3}{14}$ (vii) $\frac{5}{12} \times \frac{4}{7}$ (viii) $\frac{6}{7} \times \frac{14}{15}$

(b) (i) $\frac{6}{7} \times \frac{3}{8}$ (ii) $\frac{3}{5} \times \frac{2}{3}$ (iii) $\frac{2}{11} \times \frac{3}{4}$ (iv) $\frac{3}{10} \times \frac{5}{6}$

(v) $\frac{3}{4} \times \frac{2}{3}$ (vi) $\frac{5}{12} \times \frac{3}{10}$ (vii) $\frac{1}{2} \times \frac{1}{4} \times \frac{3}{5}$ (viii) $\frac{2}{3} \times \frac{5}{8} \times \frac{3}{10}$

13.4 Multiplying a fraction by a mixed number

Let us now consider how to multiply a fraction by a mixed number.

Let us multiply $\frac{3}{5}$ by $1\frac{1}{2}$.

That is, let us find the value of $\frac{3}{5} \times 1\frac{1}{2}$.

Let us first represent the mixed number as an improper fraction.

$$\frac{3}{5} \times 1\frac{1}{2} = \frac{3}{5} \times \frac{3}{2}$$
$$= \frac{3 \times 3}{5 \times 2}$$
$$= \frac{9}{10}$$

When simplifying fractions which include mixed numbers, multiplication is made easier by first converting the mixed numbers into improper fractions.

Example 1

Simplify $\frac{2}{3} \times 1\frac{1}{4}$.

$$\frac{2}{3} \times 1\frac{1}{4} = \frac{{}^{1}\cancel{2}}{3} \times \frac{5}{\cancel{4}_{2}} \quad \text{(divide 2 and 4 by 2)}$$
$$= \frac{1 \times 5}{3 \times 2}$$
$$= \frac{5}{6}$$

Example 2

Simplify $1\frac{3}{5} \times \frac{3}{4}$.

$$1\frac{3}{5} \times \frac{3}{4} = \frac{{}^{2}\cancel{8}}{5} \times \frac{3}{\cancel{4}_{1}} \quad \text{(divide 4 and 8 by 4)}$$
$$= \frac{2 \times 3}{5 \times 1}$$
$$= \frac{6}{5}$$
$$= 1\frac{1}{5}$$

Exercise 13.3

(1) Simplify the following.

(i) $\frac{2}{3} \times 1\frac{1}{3}$ (ii) $\frac{3}{5} \times 1\frac{1}{4}$ (iii) $\frac{5}{8} \times 1\frac{2}{3}$ (iv) $\frac{7}{10} \times 2\frac{1}{7}$

(v) $\frac{1}{6} \times 2\frac{1}{5}$ (vi) $\frac{3}{5} \times 3\frac{1}{9}$ (vii) $\frac{7}{10} \times 33\frac{1}{3}$ (viii) $\frac{5}{12} \times 3\frac{3}{11}$

(ix) $2\frac{1}{2} \times \frac{1}{5}$ (x) $3\frac{3}{4} \times \frac{7}{10}$ (xi) $\frac{2}{5} \times \frac{1}{2} \times 2\frac{1}{2}$ (xii) $\frac{3}{4} \times \frac{2}{5} \times 1\frac{1}{6}$

(2) If a vehicle travels a distance of $12\frac{1}{2}$ km on 1 l of fuel, find the distance it travels on $\frac{3}{4}$ l of fuel.

(3) Aheli reads a certain book for $1\frac{3}{4}$ hours each day. She finishes reading the book in 7 days. Find in hours, the time she took to finish the book.

(4) When kamala was hospitalized due to an illness, the doctor instructed her to drink $\frac{1}{10}$ l of liquid once every $\frac{1}{2}$ hour. Calculate the amount of liquid that kamala drinks during $3\frac{1}{2}$ hours in millilitres.

13.5 Multiplying a mixed number by a mixed number

When multiplying a mixed number by a mixed number, first write each mixed number as an improper fraction.

Let us simplify $1\frac{1}{2} \times 1\frac{2}{5}$.

$1\frac{1}{2} \times 1\frac{2}{5} = \frac{3}{2} \times \frac{7}{5}$ (first the mixed numbers need to be written as improper fractions)

$$= \frac{3 \times 7}{2 \times 5}$$

$$= \frac{21}{10} = 2\frac{1}{10}$$

Example 1

Simplify $1\frac{3}{5} \times 2\frac{3}{4}$.

$$1\frac{3}{5} \times 2\frac{3}{4} = \frac{\overset{2}{\cancel{8}}}{5} \times \frac{11}{\underset{1}{\cancel{4}}}$$

$$= \frac{2 \times 11}{5 \times 1}$$

$$= \frac{22}{5} = 4\frac{2}{5}$$

Example 2

Simplify $1\frac{1}{4} \times 3\frac{1}{2} \times \frac{1}{4}$.

$$1\frac{1}{4} \times 3\frac{1}{2} \times \frac{1}{4} = \frac{5}{4} \times \frac{7}{2} \times \frac{1}{4}$$

$$= \frac{35}{32}$$

$$= 1\frac{3}{32}$$

Exercise 13.4

(1) Simplify the following.

(i) $2\frac{1}{2} \times 1\frac{3}{5}$ (ii) $1\frac{1}{2} \times 4\frac{1}{3}$ (iii) $3\frac{3}{4} \times 1\frac{1}{5}$ (iv) $1\frac{2}{3} \times 3\frac{3}{4}$

(v) $6\frac{1}{4} \times 2\frac{2}{5}$ (vi) $10\frac{2}{3} \times 2\frac{1}{4}$ (vii) $1\frac{3}{7} \times 1\frac{1}{100}$ (viii) $5\frac{1}{4} \times 2\frac{2}{7}$

(ix) $3\frac{1}{2} \times 4\frac{4}{5} \times \frac{5}{14}$ (x) $3\frac{3}{10} \times 2\frac{1}{3} \times 4\frac{2}{7}$

Summary

- When a fraction is multiplied by a whole number, the numerator of the resultant fraction is the product of the whole number and the numerator of the given fraction, and its denominator is the same as that of the given fraction
- When a fraction is multiplied by a fraction, the numerator of the resultant fraction is the product of the numerators of the given fractions and its denominator is the product of the denominators of the given fractions.

14 Fractions II

By studying this lesson, you will be able to,

- write the reciprocal of a whole number and of a fraction,
- divide a fraction by a whole number and a whole number by a fraction,
- divide a fraction by a fraction,
- divide a whole number by a mixed number,
- divide a mixed number by a whole number,
- divide a fraction by a mixed number, and a mixed number by a fraction, and
- divide a mixed number by a mixed number.

14.1 Reciprocal of a number

Using the previously gained knowledge on multiplying fractions, let us now examine the following.

$$2 \times \frac{1}{2} = \frac{2}{2} = 1$$

$$\frac{1}{3} \times 3 = \frac{3}{3} = 1$$

$$7 \times \frac{1}{7} = \frac{7}{7} = 1$$

$$\frac{2}{5} \times \frac{5}{2} = \frac{10}{10} = 1$$

$$\frac{3}{8} \times \frac{8}{3} = \frac{24}{24} = 1$$

In each of the cases shown above, the product of the two fractions is 1.

As in the above cases, if the product of two numbers is 1, then each is called the **reciprocal** of the other.

Accordingly,

since $2 \times \frac{1}{2} = 1$,

$\frac{1}{2}$ is the reciprocal of 2. Also, 2 is the reciprocal of $\frac{1}{2}$.

Also, since $3 \times \frac{1}{3} = 1$,

$\frac{1}{3}$ is the reciprocal of 3 and 3 is the reciprocal of $\frac{1}{3}$.

Furthermore, since $\frac{2}{5} \times \frac{5}{2} = 1$,

$\frac{2}{5}$ is the reciprocal of $\frac{5}{2}$ and $\frac{5}{2}$ is the reciprocal of $\frac{2}{5}$.

Note

A whole number can also be written as a fraction, taking the whole number as the numerator and 1 as the denominator as in $3 = \frac{3}{1}$.

Number	Reciprocal
2	$\frac{1}{2}$
$\frac{1}{3}$	3
$\frac{2}{5}$	$\frac{5}{2}$
$\frac{3}{8}$	$\frac{8}{3}$

- The numerator of the reciprocal of a fraction is the denominator of that fraction, while its denominator is the numerator of that fraction.
- It is clear that the reciprocal of a fraction is obtained by interchanging its numerator and its denominator.

• Reciprocal of a mixed number

When finding the reciprocal of a mixed number such as $1\frac{1}{2}$, first the mixed number is written as an improper fraction.

Accordingly, $1\frac{1}{2} = \frac{3}{2}$

Since the reciprocal of $\frac{3}{2}$ is $\frac{2}{3}$, the reciprocal of $1\frac{1}{2}$ is $\frac{2}{3}$.

Note:

Since there is no number which when multiplied by 0 (Zero) gives 1, 0 has no reciprocal.

Exercise 14.1

(1) Fill in the blanks using the correct values.

(i) $\frac{3}{4} \times \frac{\square}{3} = 1$ (ii) $\frac{5}{8} \times \frac{8}{\square} = 1$ (iii) $7 \times \frac{\square}{7} = 1$

(iv) $\frac{1}{5} \times \square = 1$ (v) $1\frac{1}{3} \times \frac{3}{4} = \frac{\square}{3} \times \frac{3}{4} = 1$ (vi) $2\frac{1}{2} \times \frac{2}{\square} = \frac{\square}{2} \times \frac{2}{\square} = 1$

(2) Write down the reciprocal of each of the following numbers.

(i) 6 (ii) $\frac{1}{9}$ (iii) $\frac{5}{7}$ (iv) $\frac{8}{3}$

(v) 1 (vi) $3\frac{1}{3}$ (vii) $2\frac{3}{5}$ (viii) $1\frac{5}{9}$

14.2 Dividing a fraction by a whole number

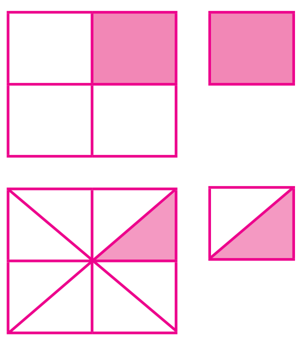
The picture shows a whole cake and $\frac{1}{2}$ a cake.

Suppose we want to share this portion ($\frac{1}{2}$ a cake) equally between Kamal and Amal. Let us consider the share that one person gets from the whole cake, when half the cake is divided equally between them.

This share is $\frac{1}{2} \div 2$.

It is clear from the picture that this share is $\frac{1}{4}$ of the whole cake.

Accordingly, $\frac{1}{2} \div 2 = \frac{1}{4}$



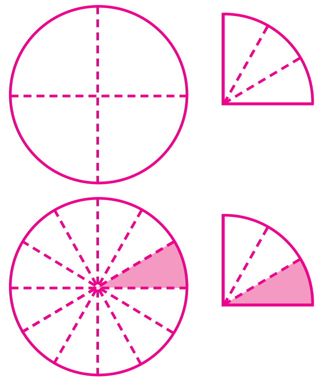
The figure on the right hand side shows a square shaped card of which $\frac{1}{4}$ has been coloured.

If the coloured portion of this card is divided into two equal parts, let us find what fraction of the whole square each of the two parts is.

It is clear from the figure that each part is $\frac{1}{8}$ of the whole square.

This can also be written as $\frac{1}{4} \div 2$.

$$\therefore \frac{1}{4} \div 2 = \frac{1}{8}$$



Consider $\frac{1}{4}$th of the circle shown in the figure. If this is divided further into three equal portions, let us find what fraction of the whole circle each portion is.

It is clear that each portion is $\frac{1}{12}$ th of the whole circle.

$$\therefore \frac{1}{4} \div 3 = \frac{1}{12}$$

Now let us consider each of the above cases one by one.

$\frac{1}{2} \div 2 = \frac{1}{4}$. In addition, $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. $\therefore \frac{1}{2} \div 2 = \frac{1}{2} \times \frac{1}{2}$

$\frac{1}{4} \div 2 = \frac{1}{8}$. In addition, $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$. $\therefore \frac{1}{4} \div 2 = \frac{1}{4} \times \frac{1}{2}$

$\frac{1}{4} \div 3 = \frac{1}{12}$. In addition, $\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$. $\therefore \frac{1}{4} \div 3 = \frac{1}{4} \times \frac{1}{3}$

Dividing a fraction by a number is the same as multiplying the fraction by the reciprocal of that number.

Example 1

Find the value of $\frac{1}{3} \div 2$.

$\frac{1}{3} \div 2 = \frac{1}{3} \times \frac{1}{2}$ (multiplying by the reciprocal of 2)

$= \frac{1}{6}$

Example 2

Find the value of $\frac{4}{5} \div 3$.

$\frac{4}{5} \div 3 = \frac{4}{5} \times \frac{1}{3}$ (multiplying by the reciprocal of 3)

$= \frac{4}{15}$

Exercise 14.2

(1) Find the value of each of the following.

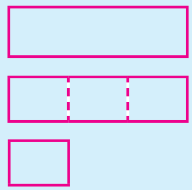
(i) $\frac{1}{5} \div 4$ (ii) $\frac{3}{4} \div 2$ (iii) $\frac{5}{7} \div 3$ (iv) $\frac{9}{10} \div 5$

• Dividing a whole number by a fraction

Let us now consider how a whole number is divided by a fraction. We can study this through examples.

Example 3

Find the value of $1 \div \frac{1}{3}$.



Let us consider the rectangular lamina shown here as one unit.

This unit has been divided into three equal parts. One of these parts is $\frac{1}{3}$.

Accordingly, there are three $\frac{1}{3}$ portions in one unit.

$\therefore\ 1 \div \frac{1}{3} = 3$

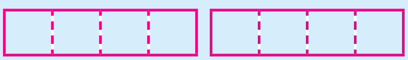
When 1 is multiplied by 3, which is the reciprocal of $\frac{1}{3}$, the same answer is obtained.

$$\therefore\ 1 \div \frac{1}{3} = 1 \times \frac{3}{1} = 3.$$

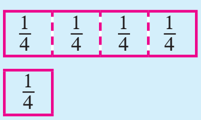
Example 4

Find the value of $2 \div \frac{1}{4}$.

Let us explain this by considering two rectangular shaped laminas of the same size. Let us consider each rectangular lamina as one unit.



When a lamina is divided into four equal parts, there are four $\frac{1}{4}$ in one unit.



Therefore, there are eight $\frac{1}{4}$ in two units.

Accordingly,

$$2 \div \frac{1}{4} = 8$$

$$2 \div \frac{1}{4} = 2 \times \frac{4}{1} = 8$$

Dividing a whole number by a fraction is the same as multiplying the number by the reciprocal of that fraction.

Example 5

Find the value of $3 \div \frac{1}{5}$.

$3 \div \frac{1}{5} = 3 \times 5$ (multiplying by the reciprocal)

$\quad\;\; = 15$

Exercise 14.3

(1) Find the value of each of the following.

(i) $3 \div \frac{1}{4}$ (ii) $2 \div \frac{2}{5}$ (iii) $4 \div \frac{1}{2}$ (iv) $15 \div \frac{3}{5}$

14. 3 Dividing a fraction by a fraction

Consider $\frac{1}{2} \div \frac{1}{4}$.

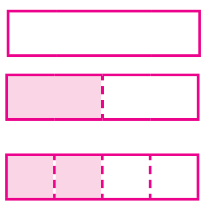
Here we are trying to find out how many $\frac{1}{4}$ there are in $\frac{1}{2}$ a unit.

Let us illustrate this using a figure.

One unit

$\frac{1}{2}$ of the above unit

There are two $\frac{1}{4}$ in $\frac{1}{2}$ a unit.



Accordingly, $\frac{1}{2} \div \frac{1}{4} = 2$. To obtain this answer, $\frac{1}{2}$ should be multiplied by the reciprocal of $\frac{1}{4}$.

That is, $\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1}$ (multiplying by the reciprocal of $\frac{1}{4}$)

$$= \frac{4}{2} = 2$$

Dividing a fraction by another fraction is the same as multiplying the first fraction by the reciprocal of the second fraction.

Example 1

Find the value of $\frac{1}{3} \div \frac{2}{5}$.

$$\frac{1}{3} \div \frac{2}{5} = \frac{1}{3} \times \frac{5}{2} \qquad \text{(multiplying by the reciprocal of } \tfrac{2}{5}\text{)}$$

$$= \frac{5}{6}$$

Example 2

Find the value of $\frac{3}{7} \div \frac{6}{11}$.

$$\frac{3}{7} \div \frac{6}{11} = \frac{\cancel{3}^{1}}{7} \times \frac{11}{\cancel{6}_{2}} \qquad \text{(multiplying by the reciprocal of } \tfrac{6}{11}\text{)}$$

$$= \frac{11}{14}$$

Exercise 14.4

(1) Find the value of each of the following.

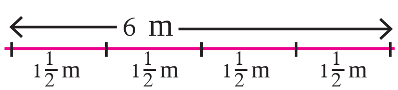
(i) $\frac{3}{8} \div \frac{3}{4}$ (ii) $\frac{15}{16} \div \frac{3}{4}$ (iii) $\frac{15}{28} \div \frac{3}{7}$ (iv) $\frac{10}{11} \div \frac{1}{11}$

(v) $\frac{6}{7} \div \frac{3}{7}$ (vi) $\frac{12}{7} \div \frac{3}{7}$ (vii) $\frac{4}{5} \div \frac{8}{9}$ (viii) $\frac{7}{8} \div \frac{7}{10}$

(ix) $\frac{3}{8} \div \frac{2}{5}$ (x) $\frac{2}{3} \div \frac{5}{7}$

14.4 Dividing a whole number by a mixed number

Let us find out how many pieces of wire of length $1\frac{1}{2}$ m can be cut from a wire of length 6 cm.



According to the figure, four pieces can be cut from the wire.
Accordingly, we can write $6 \div 1\frac{1}{2} = 4$.

Now let us simplify the expression $6 \div 1\frac{1}{2}$.

$$6 \div 1\frac{1}{2} = 6 \div \frac{3}{2} \qquad \text{(writing the mixed number } 1\tfrac{1}{2} \text{ as an improper fraction)}$$

$$= \cancel{6}^{2} \times \frac{2}{\cancel{3}_{1}} \qquad \text{(multiplying by the reciprocal of } \tfrac{3}{2}\text{)}$$

$$= 4$$

• Dividing a mixed number by a whole number

Through the following example, let us establish how a mixed number is divided by a whole number.

Example 1

Find the value of $1\frac{1}{2} \div 6$.

$1\frac{1}{2} \div 6 = \frac{3}{2} \div 6$

$= \frac{{}^{1}\cancel{3}}{2} \times \frac{1}{\cancel{6}_{2}}$ (multiplying by the reciprocal of 6)

$= \frac{1}{4}$

14.5 Dividing a fraction by a mixed number

When dividing a fraction by a mixed number, the mixed number is first written as an improper fraction and then the fraction is multiplied by the reciprocal of this improper fraction.

Example 1

Find the value of $\frac{4}{5} \div 1\frac{1}{3}$.

$\frac{4}{5} \div 1\frac{1}{3} = \frac{4}{5} \div \frac{4}{3}$ (converting the mixed number into an improper fraction)

$= \frac{{}^{1}\cancel{4}}{5} \times \frac{3}{\cancel{4}_{1}}$ (multiplying by the reciprocal of $\frac{4}{3}$)

$= \frac{3}{5}$

• Dividing a mixed number by a fraction

Here, the mixed number is first written as an improper fraction. This improper fraction is then multiplied by the reciprocal of the fraction by which the mixed number is to be divided.

Example 2

Find the value of $1\frac{1}{3} \div \frac{4}{5}$.

$1\frac{1}{3} \div \frac{4}{5} = \frac{4}{3} \times \frac{5}{4}$

$= \frac{5}{3}$

$= 1\frac{2}{3}$

Exercise 14.5

(1) Find the value of each of the following.

(i) $3 \div 1\frac{1}{2}$ (ii) $7 \div 1\frac{1}{8}$ (iii) $15 \div 1\frac{1}{4}$ (iv) $18 \div 1\frac{2}{25}$

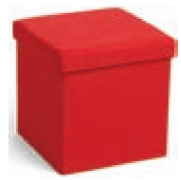
(v) $1\frac{1}{2} \div 3$ (vi) $1\frac{2}{5} \div 14$ (vii) $3\frac{2}{3} \div 22$ (viii) $5\frac{5}{6} \div 21$

(2) Find the value of each of the following.

(i) $\frac{3}{5} \div 2\frac{2}{5}$ (ii) $\frac{6}{7} \div 1\frac{1}{5}$ (iii) $\frac{8}{11} \div 3\frac{1}{5}$ (iv) $\frac{3}{8} \div 2\frac{1}{4}$

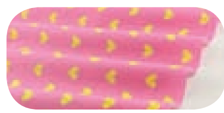
(v) $1\frac{4}{5} \div \frac{3}{5}$ (vi) $2\frac{1}{2} \div \frac{5}{7}$ (vii) $10\frac{2}{3} \div \frac{16}{27}$ (viii) $2\frac{3}{5} \div \frac{1}{2}$

(3) Hasim has packed 10 kg of sweetmeats into packets containing $1\frac{1}{4}$ kg each. Find the number of packets that he has made.



(4) A truck can transport $3\frac{1}{2}$ cubes of soil at a time. What is the minimum number of trips that needs to be made to transport 28 cubes of soil?

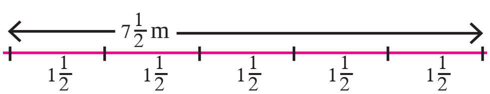
(5) Chalani needs to cut 21 m of fabric into $1\frac{3}{4}$ m pieces. How many such pieces can Chalani cut from this fabric?



(6) A volume of $31\frac{1}{2}$ l of paint in a barrel was poured equally into 7 containers. Find the amount of paint in each container.

14.6 Dividing a mixed number by a mixed number

Let us find out how many pieces of length $1\frac{1}{2}$ m can be cut from a rope of length $7\frac{1}{2}$ m.



It is clear from the figure that five pieces can be cut from the rope.

This can be written as $7\frac{1}{2} \div 1\frac{1}{2} = 5$.

Let us simplify $7\frac{1}{2} \div 1\frac{1}{2}$.

$$7\frac{1}{2} \div 1\frac{1}{2} = \frac{15}{2} \div \frac{3}{2} \quad \text{(converting the mixed number into an improper fraction)}$$

$$= \frac{\cancel{15}^{5}}{\cancel{2}_{1}} \times \frac{\cancel{2}^{1}}{\cancel{3}_{1}} \quad \text{(multiplying by the reciprocal)}$$

$$= 5$$

When dividing a mixed number by a mixed number, the given mixed numbers are first converted into improper fractions, and the answer is obtained by the method of dividing a fraction by a fraction.

Example 1

Simplify $3\frac{1}{2} \div 1\frac{3}{4}$.

$$3\frac{1}{2} \div 1\frac{3}{4} = \frac{7}{2} \div \frac{7}{4}$$

$$= \frac{\cancel{7}^{1}}{{}_{1}\cancel{2}} \times \frac{\cancel{4}^{2}}{\cancel{7}_{1}} \text{(multiplying by the reciprocal)}$$

$$= 2$$

Example 2

Simplify $2\frac{3}{5} \div 1\frac{7}{10}$.

$$2\frac{3}{5} \div 1\frac{7}{10} = \frac{13}{5} \div \frac{17}{10}$$

$$= \frac{13}{\cancel{5}_{1}} \times \frac{\cancel{10}^{2}}{17}$$

$$= \frac{26}{17}$$

$$= 1\frac{9}{17}$$

Exercise 14.6

(1) Simplify each of the following fractions.

(i) $2\frac{1}{4} \div 2\frac{2}{3}$ (ii) $7\frac{7}{8} \div 3\frac{1}{2}$ (iii) $6\frac{3}{5} \div 4\frac{5}{7}$

(iv) $7\frac{5}{8} \div 8\frac{5}{7}$ (v) $11\frac{1}{2} \div 2\frac{3}{4}$ (vi) $5\frac{1}{3} \div 2\frac{1}{2}$

(2) Fabric of length $2\frac{1}{4}$ m is required to sew a dress. What is the maximum number of such dresses that can be sewn from $56\frac{1}{4}$ m of fabric?

(3) The distance between two cities is $57\frac{1}{2}$ kilometers. A van spent $1\frac{9}{16}$ hours to travel from one city to the other. If it took the same amount of time to travel each kilometer, find how many kilometers it travelled in an hour?

(4) Among how many families can $148\frac{1}{2}$ kg of rice be distributed, so that each family gets $8\frac{1}{4}$ kg of rice?

Miscellaneous Exercise

(1) Simplify the following.

(i) $\frac{4}{5} \times 6$	(ii) $\frac{3}{7} \times 3$	(iii) $\frac{3}{8} \div 4$	(iv) $15 \div \frac{3}{10}$
(v) $8 \times \frac{3}{4}$	(vi) $5\frac{1}{4} \times 5$	(vii) $6\frac{3}{5} \div 3$	(viii) $8 \times 1\frac{1}{5}$
(ix) $7 \div 7\frac{1}{2}$	(x) $\frac{2}{3} \times \frac{7}{8}$	(xi) $\frac{3}{7} \times \frac{2}{3}$	(xii) $\frac{5}{9} \div \frac{7}{10}$
(xiii) $\frac{7}{8} \times \frac{4}{5} \times \frac{3}{7}$	(xiv) $\frac{2}{5} \times 1\frac{3}{7}$	(xv) $\frac{4}{9} \div 2\frac{1}{4}$	(xvi) $1\frac{3}{8} \div 1\frac{1}{7}$
(xvii) $1\frac{1}{2} \times 2\frac{2}{3}$	(xviii) $4\frac{2}{3} \div 1\frac{1}{7}$	(xix) $4\frac{1}{2} \times 3\frac{3}{5} \times 1\frac{1}{3}$	(xx) $3\frac{3}{4} \times 1\frac{2}{5} \times 1\frac{1}{7}$

Summary

- If the product of two numbers is 1, then each is the reciprocal of the other.
- Dividing a number by another number is the same as multiplying the first number by the reciprocal of the second number.

Glossary

Adjacent angles	බද්ධ කෝණ	அடுத்துள்ள கோணங்கள்
Algebraic expressions	වීජීය ප්‍රකාශන	அட்சரகணிதக் கோவைகள்
Algebraic terms	වීජීය පද	அட்சரகணித உறுப்புகள்
Angle	කෝණය	கோணம்
Brackets	වරහන	அடைப்புகள்
Centre of rotation	භ්‍රමණ කේන්ද්‍රය	சுழற்சி மையம்
Common factor	පොදු සාධකය	பொதுக் காரணி
Complementary angles	අනුපූරක කෝණ	நிரப்பு கோணங்கள்
Composite plane figures	සංයුක්ත තලරූප	கூட்டுத் தளவுரு
Convex polygon	උත්තල බහු අස්‍රය	குவிவுப் பல்கோணி
Denominator	හරය	பகுதி, பகுதியெண்
Directed numbers	සදිශ සංඛ්‍යා	திசைகொண்ட எண்கள்
Division	බෙදීම	வகுத்தல்
Dodecahedron	ද්වාදසතලය	பன்னிருமுகி
Equilateral triangle	සමපාද ත්‍රිකෝණය	சமபக்க முக்கோணி
Even numbers	ඉරට්ට සංඛ්‍යා	இரட்டை எண்கள்
Exterior angle	බාහිර කෝණය	புறக்கோணம்
Fraction	භාගය	பின்னம்
General term	සාධාරණ පදය	பொது உறுப்பு
Geometric shapes	ජ්‍යාමිතික හැඩතල	கேத்திரகணித வடிவங்கள்
Highest common factor	මහා පොදු සාධකය	பொதுக்காரணிகளுட் பெரியது
Icosahedron	විංසතිතලය	இருபதுமுகி
Improper fraction	විෂම භාගය	முறைமையில்லாப் பின்னம்
Index	දර්ශකය	சுட்டி
Integers	නිඛිල	நிறை எண்கள்
Interior angle	අභ්‍යන්තර කෝණය	அகக்கோணம்
Isosceles triangle	සමද්විපාද ත්‍රිකෝණය	இருசமபக்க முக்கோணி
Kilogramme	කිලෝග්‍රෑම්	கிலோகிராம்
Mass	ස්කන්ධය	திணிவு
Mathematical operations	ගණිත කර්ම	கணிதச் செய்கைகள்
Metric ton	මෙට්‍රික් ටොන්	மெற்றிக் தொன
Mixed number	මිශ්‍ර සංඛ්‍යාව	கலப்பு எண்
Multiples	ගුණාකාර	மடங்குகள்
Multiplication	ගුණ කිරීම	பெருக்கல்
Negative integers	සෘණ නිඛිල	மறை நிறையெண்கள்
Number Line	සංඛ්‍යා රේඛාව	எண் கோடு
Number patterns	සංඛ්‍යා රටා	எண் கோலம்
Numerator	ලවය	தொகுதி, தொகுதியெண்
Octahedron	අෂ්ටතලය	எண்முகி
Odd numbers	ඔත්තේ සංඛ්‍යා	ஒற்றை எண்கள்
Order of rotational symmetry	භ්‍රමක සමමිති ගණය	சுழல் சமச்சீர் வரிசை
Perfect squares	පූර්ණ වර්ග	நிறைவர்க்க எண்கள்
Perimeter	පරිමිතිය	சுற்றளவு
Point	ලක්ෂ්‍යය	புள்ளி
Polygon	බහු අස්‍රය	பல்கோணி
Power	බල	வலு
Quadrilateral	චතුරස්‍රය	நாற்பக்கல்
Reciprocal	පරස්පරය	நிகர்மாற்று
Rectangle	සෘජුකෝණාස්‍රය	செவ்வகம்
Rotational symmetry	භ්‍රමක සමමිතිය	சுழல் சமச்சீர்
Solids	ඝන වස්තු	திண்மங்கள்
Square numbers	සමචතුරස්‍ර සංඛ්‍යා	சதுர எண்கள்
Square root	වර්ග මූලය	வர்க்க மூலம்
Square	සමචතුරස්‍රය	சதுரம்
Statements	ප්‍රකාශ	கூற்றுகள்
Supplementary angles	පරිපූරක කෝණ	மிகைநிரப்பு கோணங்கள்
Triangle	ත්‍රිකෝණය	முக்கோணி
Triangular numbers	ත්‍රිකෝණ සංඛ්‍යා	முக்கோணி எண்கள்
Unknown	අඥාතය	தெரியாக் கணியம்
Vertically opposite angles	ප්‍රතිමුඛ කෝණ	குத்தெதிர்க் கோணங்கள்
Whole numbers	පූර්ණ සංඛ්‍යා	எண்ணும் எண்கள்

Lesson sequence

Content	Competency levels	Number of periods
1st term		
1. Number Patterns	05	2.1
2. Perimeter	05	7.1
3. Angles	05	21.1
4. Directed Numbers	05	1.2
5. Algebraic Expressions	05	14.1
6. Solids	06	22.1
7. Factors	06	15.1
8. Square Root	05	1.1
9. Mass	05	9.1
10. Indices	05	6.1,6.2
	52	
2nd term		
11. Symmetry	05	25.1
12. Triangles	06	23.1
13. Fractions - I	06	3.1
14. Fractions - II	06	3.2
15. Decimals	07	3.3
16. Ratios	06	4.1,4.2
17. Equations	05	17.1
18. Percentages	06	5.1,5.2
19. Sets	04	30.1
20. Area	06	8.1,8.2
21. Time	06	12.1,12.2
	63	
3rd term		
22. Volume and Capacity	06	10.1,11.1
23. Circle	05	24.1
24. Location of a Place	03	13.1
25. Number Line and Cartesian Plane	09	20.1,20.2,20.3
26. Triangle Constructions	06	27.1
27. Data Representation and Interpretation	10	18.1,29.1,29.2
28. Scale Drawings	05	13.2
29. Probability	06	31.1,31.2
30. Tessellation	05	26.1
	55	
Total	**170**	

MATHEMATICS

Grade 8

Part - II

Educational Publications Department

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The National Anthem of Sri Lanka

Sri Lanka Matha
Apa Sri Lanka Namo Namo Namo Namo Matha
Sundara siri barinee, surendi athi sobamana Lanka
Dhanya dhanaya neka mal palaturu piri jaya bhoomiya ramya
Apa hata sepa siri setha sadana jeewanaye matha
Piliganu mena apa bhakthi pooja Namo Namo Matha
Apa Sri Lanka Namo Namo Namo Namo Matha
Oba we apa vidya
Obamaya apa sathya
Oba we apa shakthi
Apa hada thula bhakthi
Oba apa aloke
Apage anuprane
Oba apa jeevana we
Apa mukthiya oba we
Nava jeevana demine, nithina apa pubudukaran matha
Gnana veerya vadawamina regena yanu mana jaya bhoomi kara
Eka mavakage daru kela bevina
Yamu yamu vee nopama
Prema vada sema bheda durerada
Namo, Namo Matha
Apa Sri Lanka Namo Namo Namo Namo Matha

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ආනන්ද සමරකෝන්

ஒரு தாய் மக்கள் நாமாவோம்
ஒன்றே நாம் வாழும் இல்லம்
நன்றே உடலில் ஓடும்
ஒன்றே நம் குருதி நிறம்

அதனால் சகோதரர் நாமாவோம்
ஒன்றாய் வாழும் வளரும் நாம்
நன்றாய் இவ் இல்லினிலே
நலமே வாழ்தல் வேண்டுமன்றோ

யாவரும் அன்பு கருணையுடன்
ஒற்றுமை சிறக்க வாழ்ந்திடுதல்
பொன்னும் மணியும் முத்துமல்ல - அதுவே
யான்று மழியாச் செல்வமன்றோ.

ஆனந்த சமரக்கோன்
கவிதையின் பெயர்ப்பு.

Being innovative, changing with right knowledge
Be a light to the country as well as to the world.

Message from the Hon. Minister of Education

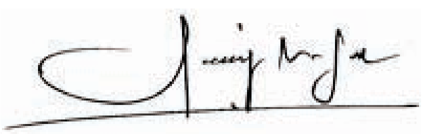
The past two decades have been significant in the world history due to changes that took place in technology. The present students face a lot of new challenges along with the rapid development of Information Technology, communication and other related fields. The manner of career opportunities are liable to change specifically in the near future. In such an environment, with a new technological and intellectual society, thousands of innovative career opportunities would be created. To win those challenges, it is the responsibility of the Sri Lankan Government and myself, as the Minister of Education, to empower you all.

This book is a product of free education. Your aim must be to use this book properly and acquire the necessary knowledge out of it. The government in turn is able to provide free textbooks to you, as a result of the commitment and labour of your parents and elders.

Since we have understood that the education is crucial in deciding the future of a country, the government has taken steps to change curriculum to suit the rapid changes of the technological world. Hence, you have to dedicate yourselves to become productive citizens. I believe that the knowledge this book provides will suffice your aim.

It is your duty to give a proper value to the money spent by the government on your education. Also you should understand that education determines your future. Make sure that you reach the optimum social stratum through education.

I congratulate you to enjoy the benefits of free education and bloom as an honoured citizen who takes the name of Sri Lanka to the world.



Akila Viraj Kariyawasam
Minister of Education

Foreword

The educational objectives of the contemporary world are becoming more complex along with the economic, social, cultural and technological development. The learning and teaching process too is changing in relation to human experiences, technological differences, research and new indices. Therefore, it is required to produce the textbook by including subject related information according to the objectives in the syllabus in order to maintain the teaching process by organizing learning experiences that suit to the learner needs. The textbook is not merely a learning tool for the learner. It is a blessing that contributes to obtain a higher education along with a development of conduct and attitudes, to develop values and to obtain learning experiences.

The government in its realization of the concept of free education has offered you all the textbooks from grades 1-11. I would like to remind you that you should make the maximum use of these textbooks and protect them well. I sincerely hope that this textbook would assist you to obtain the expertise to become a virtuous citizen with a complete personality who would be a valuable asset to the country.

I would like to bestow my sincere thanks on the members of the editorial and writer boards as well as on the staff of the Educational Publications Department who have strived to offer this textbook to you.

W. M. Jayantha Wickramanayaka,
Commissioner General of Educational Publications,
Educational Publications Department,
Isurupaya,
Battaramulla.
2019.04.10

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Message of the Boards of Writers and Editors

This textbook has been compiled in accordance with the new syllabus which is to be implemented from 2017 for the use of grade eight students.

We made an effort to develop the attitude "We can master the subject of Mathematics well" in students.

In compiling this textbook, the necessity of developing the basic foundation of studying mathematical concepts in a formal manner was specially considered. This textbook is not just a learning tool which targets the tests held at school. It was compiled granting due consideration to it as a medium that develops logical thinking, correct vision and creativity in the child.

Furthermore, most of the activities, examples and exercises that are incorporated here are related to day to day experiences in order to establish mathematical concepts in the child. This will convince the child about the importance of mathematics in his or her daily life. Teachers who guide the children to utilize this textbook can prepare learning tools that suit the learning style and the level of the child based on the information provided here.

Learning outcomes are presented at the beginning of each lesson. A summary is provided at the end of each lesson to enable the child to revise the important facts relevant to the lesson. Furthermore, at the end of the set of lessons related to each term, a revision exercise has been provided to revise the tasks completed during that term.

Every child does not have the same capability in understanding mathematical concepts. Thus, it is necessary to direct the child from the known to the unknown according to his / her capabilities. We strongly believe that it can be carried out precisely by a professional teacher.

In the learning process, the child should be given ample time to think and practice problems on his or her own. Furthermore, opportunities should be given to practice mathematics without restricting the child to just the theoretical knowledge provided by mathematics.

We would like to bestow our sincere thanks on Dr. Jayampathi Rathnayake, Department of Mathematics, University of Colombo.

Our firm wish is that our children act as intelligent citizens who think logically by studying Mathematics with dedication.

Boards of Writers and Editors

Contents

15 Decimal Numbers

By studying this lesson you will be able to,
- multiply a whole number by a decimal number,
- multiply a decimal number by a decimal number,
- divide a whole number by a decimal number, and
- divide a decimal number by a decimal number.

15.1 Decimal numbers

In Grades 6 and 7 you learnt how to represent a given fraction as a decimal number and a given decimal number as a fraction.

You also learnt that it is easy to represent a fraction as a decimal number when the denominator of the given fraction can be expressed as a power of 10 such as 10, 100, 1000, ... etc.

➢ Let us consider how a fraction with denominator equal to 10 is written as a decimal number.

$$\frac{1}{10} = 0.1, \quad \frac{9}{10} = 0.9, \quad \frac{17}{10} = 1.7$$

➢ Let us now recall how some fractions with denominators which are not powers of 10 were represented as decimal numbers by using equivalent fractions.

- Let us write $\frac{3}{25}$ as a decimal number.

Since $100 \div 25 = 4$,

$$\frac{3}{25} = \frac{3 \times 4}{25 \times 4} = \frac{12}{100} = 0.12$$

- Let us write $\frac{77}{125}$ as a decimal number.

Since $1000 \div 125 = 8$,

$$\frac{77}{125} = \frac{77 \times 8}{125 \times 8} = \frac{616}{1000} = 0.616$$

- Let us write the improper fraction $\frac{17}{4}$ as a decimal number.

$$\frac{17}{4} = \frac{17 \times 25}{4 \times 25} = \frac{425}{100} = 4.25$$

- Let us write the mixed number $6\frac{33}{40}$ as a decimal number.

$$6\frac{33}{40} = 6 + \frac{33}{40} = 6 + \frac{33 \times 25}{40 \times 25}$$
$$= 6 + \frac{825}{1000}$$
$$= 6 + 0.825$$
$$= 6.825$$

Accordingly, if a number which is a power of 10, such as 10, 100, 1000, is divisible by the denominator of a fraction, then that fraction can easily be written as a decimal number.

You have learnt how to multiply a decimal number by a whole number and how to divide a decimal number by a whole number.

- When multiplying a number which is in decimal form by a number which is a power of 10, the number of places the decimal point in the decimal number is shifted to the right (by adding zeros if necessary), is equal to the number of zeros in the power of ten by which it is multiplied.

  Examples: (i) $3.211 \times 10 = 32.11$ (ii) $2.31 \times 100 = 231$ (iii) $1.11 \times 1000 = 1110$

- When dividing a number which is in decimal form by a number which is a power of 10, the number of places the decimal point in the decimal number is shifted to the left (by adding zeros if necessary) is equal to the number of zeros in the power of ten by which it is divided.

  Examples: (i) $22.31 \div 10 = 2.231$ (ii) $0.4 \div 100 = 0.004$ (iii) $32 \div 1000 = 0.032$

Do the following review exercise to recall the above facts about decimal numbers that you learnt in Grades 6 and 7.

Review Exercise

(1) Represent each of the following proper fractions as a decimal number.

(i) $\frac{3}{10}$ (ii) $\frac{97}{100}$ (iii) $\frac{1}{1000}$ (iv) $\frac{7}{8}$

(2) Write each of the following decimal numbers as a fraction, and express it in its simplest form.

(i) 0.7 (ii) 0.25 (iii) 8.16 (iv) 0.025

(3) Represent each of the following improper fractions and mixed numbers as a decimal number.

(i) $\frac{17}{10}$ (ii) $\frac{308}{25}$ (iii) $3\frac{9}{10}$ (iv) $14\frac{9}{100}$

(4) Find the value of each of the following.

(a) (i) 3.87×10 (ii) 4.08×100 (iii) 0.0456×1000

(iv) 4.09×10^2 (v) 9.45×10^3 (vi) 18.342×10^2

(vii) 3.27×3 (viii) 0.65×11 (ix) 15.08×13

(b) (i) $58 \div 10$ (ii) $34 \div 100$ (iii) $148 \div 1000$

(iv) $7.29 \div 10^2$ (v) $35 \div 10^3$ (vi) $1.785 \div 10^2$

(vii) $78.3 \div 3$ (viii) $0.684 \div 4$ (ix) $30.84 \div 12$

15. 2 Multiplying a whole number by a decimal number

Let us now study how to multiply a whole number by a decimal number. This is similar to multiplying a decimal number by a whole number. We can also convert the decimal number into a fraction and then perform the multiplication.

- Let us find the value of 7×0.8

Let us write the decimal number as a fraction with denominator a power of 10 and then multiply.

$$0.8 = \frac{8}{10}$$

$$\therefore\ 7 \times 0.8 = 7 \times \frac{8}{10} = \frac{7 \times 8}{10}$$

$$= \frac{56}{10} = 5.6$$

That is, to obtain the value of 7×0.8, the value of 7×8 needs to be divided by 10.

$$\therefore\ 7 \times 0.8 = \frac{56}{10} = 5.6$$

- Let us find the value of 8×1.2

Method I

$$8 \times 1.2 = 8 \times \frac{12}{10} = \frac{8 \times 12}{10}$$

$$= \frac{96}{10}$$

$$= 9.6$$

Since $1.2 \times 10 = 12$, to obtain the value of 8×1.2, the value that is obtained for 8×12, by not considering the decimal place in 1.2, should be divided by 10.
That is, $8 \times 1.2 = 9.6$

Method II

Let us first multiply the numbers without considering the decimal places.

$8 \times 12 = 96$

Since there is one decimal place in 1.2, place the decimal point in the answer such that it too has one decimal place.

That is, $8 \times 1.2 = 1.2 \times 8 = 9.6$

Example 1

Find the value of 8×8.73.

Method I

$$8 \times 8.73 = 8 \times \frac{873}{100} = \frac{8 \times 873}{100} = \frac{6984}{100} = 69.84$$

That is, to find the value of 8.73×8, the value of 873×8, must be divided by 100.

Method II

Let us multiply the two numbers without considering the decimal places.

$$\begin{array}{r} 873 \\ \times\ 8 \\ \hline 6984 \\ \hline\hline \end{array}$$

Since 8.73 has two decimal places, the decimal point needs to be placed in the answer such that it too has two decimal places.

$\therefore\ 8 \times 8.73 = 69.84$

Example 2

(1) $7 \times 233 = 1631$. Find the value of each of the following multiplications.

(i) 7×23.3 (ii) 7×2.33 (iii) 7×0.233

(i) $7 \times 233 = 1631$

Since $23.3 \times 10 = 233$,

$7 \times 23.3 = 1631 \div 10$

$= 163.1$

(ii) $7 \times 233 = 1631$

Since $2.33 \times 100 = 233$,

$7 \times 2.33 = 1631 \div 100$

$= 16.31$

(iii) $7 \times 233 = 1631$

Since $0.233 \times 1000 = 233$,

$7 \times 0.233 = 1631 \div 1000$

$= 1.631$

Exercise 15.1

(1) Find the value of each of the following.

(i) 5×8.03 (ii) 12×19.4 (iii) 30×10.53

(iv) 4×3.197 (v) 15×1.91 (vi) 32×24.64

(2) Find the value of 678×4, and hence write down the value of each of the following multiplications.

(i) 4×67.8 (ii) 4×6.78 (iii) 4×0.678

(3) Find the area of a rectangular vegetable plot of length 34 m and breadth 12.8 m.

15. 3 Multiplying a decimal number by a decimal number

The length and breadth of a rectangular shaped bed sheet are 2.7 m and 0.9 m respectively. Find the area of the bed sheet.

Length of the rectangular shaped bed sheet $= 2.7$ m

Breadth of the rectangular shaped bed sheet $= 0.9$ m

$\therefore$ Area of the rectangular shaped bed sheet $= 2.7 \text{ m} \times 0.9 \text{ m}$

$= 2.7 \times 0.9 \text{ m}^2$

Now let us consider how to find the value of 2.7×0.9.

Method I

Let us write each decimal number as a fraction.

$2.7 = \frac{27}{10}$ and $0.9 = \frac{9}{10}$.

$$\therefore\ 2.7 \times 0.9 = \frac{27}{10} \times \frac{9}{10}$$
$$= \frac{27 \times 9}{100}$$
$$= \frac{243}{100}$$
$$= 2.43$$

That is, to find the value of 2.7×0.9, the value of 27×9 must be divided by 100.

Method II

2.7×0.9

multiplicand (2.7) multiplier (0.9)

There are two decimal places in these two decimal numbers (multiplicand and multiplier).

$$\begin{array}{r} 27 \\ \times\ 9 \\ \hline 243 \\ \hline\hline \end{array}$$

Let us multiply the two decimal numbers without considering their decimal places.

$27 \times 9 = 243$

When 243 is written by considering the two decimal places, we obtain 2.43.

That is, $2.7 \times 0.9 = 2.43$

Therefore, the area of the bed sheet is 2.43 m^2.

Example 1

Find the value of 30.8×0.07

Method I

$$30.8 = \frac{308}{10} \text{ and } 0.07 = \frac{7}{100}$$

$$\therefore\ 30.8 \times 0.07 = \frac{308}{10} \times \frac{7}{100} = \frac{2156}{1000} = 2.156$$

Method II

$$\begin{array}{r} 308 \\ \times\ 7 \\ \hline 2156 \\ \hline \end{array}$$

The total number of decimal places in 30.8 (multiplicand) and 0.07 (multiplier) is 3. Therefore let us place the decimal point in the answer so that it has 3 decimal places.

$\therefore\ 30.8 \times 0.07 = 2.156$

Example 2

$172 \times 26 = 4472$. Write the value of each of the following accordingly.

(i) 1.72×2.6 (ii) 17.2×2.6 (iii) 0.172×0.026

(i) $1.72 \times 2.6 = \dfrac{172 \times 26}{100 \times 10} = \dfrac{4472}{1000} = 4.472$

(ii) $17.2 \times 2.6 = \dfrac{172 \times 26}{100} = \dfrac{4472}{100} = 44.72$

(iii) $0.172 \times 0.026 = \dfrac{172 \times 26}{1000 \times 1000} = \dfrac{4472}{1\ 000\ 000} = 0.004472$

Exercise 15.2

(1) Find the value of each of the following.

(i) 0.7×0.6 (ii) 1.2×0.8 (iii) 4.2×2.8 (iv) 1.26×0.9

(v) 1.31×0.91 (vi) 2.78×1.87 (vii) 62.32×3.48 (viii) 59.08×1.42

(ix) $(0.4)^2$ (x) $(0.06)^2$ (xi) $0.3 \times 0.5 \times 0.9$ (xii) $4 + 0.3 \times 0.2$

(xiii) $0.09 - 0.09 \times 0.03$ (xiv) $(1 - 0.7)^2$

(2) The price of 1 kg of potatoes is Rs.76.50. How much does it cost Achala to buy 2.5 kg of potatoes?

(3) The side length of a square shaped stamp is 2.7 cm. Find the area of the stamp.

(4) $273 \times 31 = 8463$. Find the value of each of the following multiplications accordingly.

(i) 27.3×3.1 (ii) 2.73×3.1 (iii) 0.31×2.73

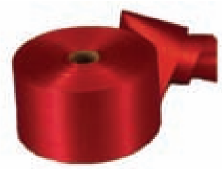
(iv) 3.1×0.273 (v) 0.031×2.73 (vi) 0.031×27.3

(5) The mass of a brick is approximately 2.3 kg. To construct a wall, 2500 such bricks are required.

(i) Estimate the total mass of the bricks.

(ii) A lorry can transport a mass of up to 2 metric tons per trip. Estimate how many such lorries are required to transport these 2500 bricks.

15. 4 Dividing a whole number by a decimal number



Jayamini requires several pieces of ribbon of length 0.8 m each to decorate a classroom. She has a roll of ribbon of length 48 m.

Let us find how many pieces of ribbon of length 0.8 m can be cut from this roll of ribbon.

To obtain the answer, 48 m needs to be divided by 0.8 m.

Method I

$48 \div 0.8 = 48 \div \frac{8}{10}$

Since the reciprocal of $\frac{8}{10}$ is $\frac{10}{8}$

$\therefore\ 48 \div 0.8 = 48 \times \frac{10}{8}$

$= \frac{480}{8} = 60$

Let us find the value of $48 \div 0.8$ without considering the decimal place in 0.8.

Since 0.8 has one decimal place, the answer which is obtained for $48 \div 8$ must be multiplied by 10.

$48 \div 8 = 6$

$\therefore\ 48 \div 0.8 = 60.$

Therefore 60 pieces of ribbon can be cut.

Method II

Multiply both the dividend and the divisor by a power of 10 and convert the divisor into a whole number. Then perform the division in the usual manner.

$$48 \div 0.8 = \frac{48}{0.8} = \frac{48 \times 10}{0.8 \times 10} = \frac{480}{8} = 60$$

Example 1

Divide 63 by 1.2.

Method I

$$63 \div 1.2 = 63 \div \frac{12}{10}$$

$$= 63 \times \frac{10}{12} \quad \left(\text{since the reciprocal of } \frac{12}{10} \text{ is } \frac{10}{12}\right)$$

$$= \frac{63}{12} \times 10$$

$$= \frac{630}{12} = 52.5$$

$$\begin{array}{r|l} & 52.5 \\ \hline 12 & 630.0 \\ & \underline{60} \\ & 30 \\ & \underline{24} \\ & 60 \\ & \underline{60} \\ & 00 \end{array}$$

Let us divide 63 by 12 without considering the decimal places. Since there is one decimal place in 1.2, the answer that is obtained when 63 is divided by 12 must be multiplied by 10.

$$63 \div 1.2 = 5.25 \times 10$$

$$= 52.5$$

$$\begin{array}{r|l} & 5.25 \\ \hline 12 & 63.00 \\ & \underline{60} \\ & 3\,0 \\ & \underline{2\,4} \\ & 60 \\ & \underline{60} \\ & 00 \end{array}$$

Method II

$$\frac{63}{1.2} = \frac{63 \times 10}{1.2 \times 10} = \frac{630}{12}$$

$$= 52.5$$

$$\begin{array}{r|l} & 52.5 \\ \hline 12 & 630.0 \\ & \underline{60} \\ & 30 \\ & \underline{24} \\ & 60 \\ & \underline{60} \\ & 00 \end{array}$$

Example 2

$87 \div 12 = 7.25$. Find the value of each of the following divisions accordingly.

(i) $87 \div 1.2$ (ii) $87 \div 0.12$

(i) $87 \div 12 = 7.25$

$87 \div 1.2 = 7.25 \times 10$

$= 72.5$

(ii) $87 \div 0.12 = \dfrac{87}{0.12}$

$= \dfrac{87 \times 100}{0.12 \times 100}$

$= \dfrac{8700}{12}$

$= \dfrac{87}{12} \times 100$

$= 7.25 \times 100$

$= 725$

Exercise 15.3

(1) Find the value of each of the following.

(i) $7 \div 0.28$ (ii) $11 \div 0.44$ (iii) $82 \div 3.28$

(iv) $12 \div 0.48$ (v) $475 \div 2.5$ (vi) $97 \div 2.5$

(2) $198 \div 11 = 18$. Find the value of each of the following divisions accordingly.

(i) $198 \div 1.1$ (ii) $198 \div 0.11$ (iii) $1980 \div 0.011$

(3) How many pipes of length 2.4 m each are required to construct a pipeline of length 720 m?

(4) A motor car travelled 150.78 km in 4 hours. Find the distance it travelled in an hour by assuming that it travelled an equal distance each hour.

15. 5 Dividing a decimal number by another decimal number

Let us divide 3.72 by 1.2.

Method I

$$3.72 \div 1.2 = \frac{372}{100} \div \frac{12}{10}$$

$$= \frac{372}{10\cancel{0}} \times \frac{1\cancel{0}}{12} \qquad \text{(since the reciprocal of } \frac{12}{10} \text{ is } \frac{10}{12}\text{)}$$

$$= \frac{372}{10 \times 12} = \frac{37.2}{12}$$

$$= 3.1$$

Method II

Multiply the dividend and the divisor by a power of 10 and convert the divisor into a whole number. Then carry out the division in the usual way.

$$\frac{3.72}{1.2} = \frac{3.72 \times 10}{1.2 \times 10} = \frac{37.2}{12} = 3.1$$

$$\begin{array}{r|l} & 3.1 \\ \hline 12 & 37.2 \\ & \underline{36} \\ & 1\,2 \\ & \underline{1\,2} \\ & 0\,0 \end{array}$$

Example 1

Divide 0.648 by 5.4.

Method I

$$0.648 \div 5.4 = \frac{648}{1000} \div \frac{54}{10}$$

$$= \frac{648}{1000} \times \frac{10}{54} \text{ (the reciprocal of } \frac{54}{10} \text{ is } \frac{10}{54}\text{)}$$

$$= \frac{648}{100} \times \frac{1}{54}$$

$$= \frac{6.48}{54}$$

$$= 0.12$$

$$\begin{array}{r|l} & 12 \\ \hline 54 & 648 \\ & \underline{54} \\ & 108 \\ & \underline{108} \\ & 000 \end{array}$$

Method II

$$\frac{0.648}{5.4} = \frac{0.648 \times 10}{5.4 \times 10} = \frac{6.48}{54}$$

$$\therefore\ 0.648 \div 5.4 = 0.12$$

$$\begin{array}{r|l} & 0.12 \\ \hline 54 & 6.48 \\ & \underline{5\ 4} \\ & 1\ 08 \\ & \underline{1\ 08} \\ & 0\ 00 \end{array}$$

Exercise 15.4

(1) Find the value of each of the following.

(i) $0.8 \div 1.6$ (ii) $16.8 \div 0.07$ (iii) $194.3 \div 6.7$ (iv) $1.943 \div 0.67$

(v) $19.43 \div 6.7$ (vi) $0.1943 \div 6.7$ (vii) $1.943 \div 0.067$ (viii) $19.43 \div 670$

(2) (i) Find the value of $336 \div 12$.

(ii) Find the value of each of the following divisions, based on the value of $336 \div 12$.

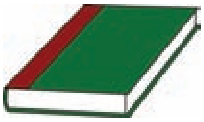
(a) $3.36 \div 0.12$ (b) $33.6 \div 1.2$

(3) (i) Find the value of $3638 \div 17$.

(ii) Find the value of each of the following divisions based on the value of $3638 \div 17$.

(a) $36.38 \div 1.7$ (b) $363.8 \div 0.17$

(4) The price of a book is Rs. 47.25. How many books can be bought for Rs. 425.25?



(5) The area of a rectangular shaped plot of land is $2718.75\ m^2$. The breadth of the plot is 12.5 m. Find its length.

Miscellaneous Exercise

(1) Simplify the following.

(i) 7.18×100 (ii) 9.03×4 (iii) 10.9×7

(iv) 19.2×12 (v) 31.4×15 (vi) 3.07×33

(2) Simplify the following.

(i) 10×8.79 (ii) 100×0.92 (iii) 14×0.21

(iv) 27×0.6 (v) 1.005×40 (vi) 30×4.2

(3) $28 \times 43 = 1204$. Write the value of each of the following multiplications accordingly.

(i) 2.8×43 (ii) 4.3×28 (iii) 0.43×28

(iv) 0.28×43 (v) 0.028×43 (vi) 0.043×28

(4) $188 \div 32 = 5.875$. Write the value of each of the following divisions accordingly.

(i) $18.8 \div 3.2$ (ii) $18.8 \div 0.32$ (iii) $1.88 \div 0.32$

(iv) $0.188 \div 3.2$ (v) $0.188 \div 0.32$ (vi) $1.88 \div 0.032$

(5) Find the value of each of the following.

(i) $5.2 \div 0.4$ (ii) $0.75 \div 0.5$ (iii) $0.075 \div 2.5$

(iv) $3.74 \div 1.1$ (v) $0.195 \div 1.5$

(6) The area of a rectangular sheet is $87.6\ cm^2$. If its breadth is 1.2 cm, find its length.

Summary

- When dividing a decimal number by a decimal number, multiply the dividend and the divisor by a power of 10 and convert the divisor into a whole number. Then carry out the division in the usual manner.
- When multiplying a whole number by a decimal number, write the decimal number as a fraction with a power of ten as its denominator and then do the multiplication.

16 Ratios

By studying this lesson you will be able to,

- represent a ratio as a fraction,
- determine continued ratios, and
- solve problems involving continued ratios.

16.1 Ratios

Let us recall what you learnt about ratios in Grade 7.

You learnt that a ratio is a numerical relationship between two or more quantities which are measured in the same unit.

You also learnt when comparing two groups, that a ratio is a numerical relationship between the magnitudes of the two groups.

Consider the following example,

When preparing a concrete mixture, 1 pan of cement, 3 pans of sand and 4 pans of gravel are mixed together.

The ratio in which cement, sand and gravel are mixed together when preparing this concrete mixture can be expressed as 1 : 3 : 4. This is read as “1 to 3 to 4”. Here, 1, 3 and 4 are the **terms of the ratio**.

By multiplying or dividing each term of a given ratio by a number which is greater than 0, a ratio equivalent to the given ratio can be obtained.

If the terms of a ratio are whole numbers and if the HCF of these numbers is 1, then we say that the ratio is written in its **simplest form**.

- If the terms of a ratio are whole numbers and if they have a common factor greater than 1, then it can be expressed in its simplest form by dividing each term of the ratio by the highest common factor of all the terms.

Do the following review exercise to revise the facts you have learnt previously on ratios.

Review Exercise

(1) Write three equivalent ratios for each ratio given below.

(i) 2 : 5 (ii) 3 : 4 (iii) 9 : 6 : 3 (iv) 8 : 2 : 4

(2) Write each of the ratios given below in the simplest form.

(i) 6 : 15 (ii) 8 : 20 (iii) 30 : 18 : 36 (iv) 40 : 16 : 64

(3) Join the ratio in column *A* to the equivalent ratio in column *B*.

A	*B*
4 : 3	2 : 3
10 : 15	6 : 9 : 3
6 : 5	10 : 35 : 45
2 : 7 : 9	18 : 15
24 : 36 : 12	8 : 6

(4) Rewrite and fill in the blanks.

(i) 3 : 4 = □ : 8 (ii) 8 : 5 = 16 : □ (iii) 1 : 3 = □ : 12

(iv) □ : 6 = 32 : 48 (v) 15 : 25 = □ : 5 (vi) 12 : □ = 36 : 15

(5) The ratio of the price of a pencil to that of a book is 3 : 4. If the price of a pencil is Rs.15, find the price of a book.

(6) The ratio of the mass of Prathapa to that of Nimdiya is 9 : 11. If Nimdiya's mass is 55 kg, find Prathapa's mass.

(7) Saman, Suresh and Kassim are friends. The ratio of their heights is 5 : 4 : 6. If Saman's height is 125 cm, calculate the heights of Suresh and Kassim.

16.2 Representing a ratio as a fraction

The following example shows how a ratio can be expressed as a fraction by writing an equivalent ratio in which one term is equal to 1.

➢ In a race, Dilki ran 30 m in the time that Sayuni took to run 50 m. The ratio of the distance that Dilki ran to the distance that Sayuni ran is 30 : 50. This ratio written in its simplest form is 3 : 5. This means that Dilki ran 3 m in the time that Sayuni took to run 5 m.

- When we divide both the terms in the ratio 3:5 by 5 we obtain $\frac{3}{5} : \frac{5}{5} = \frac{3}{5} : 1$.This means that Dilki runs $\frac{3}{5}$ m in the time that Sayuni runs 1 m. That is, when the distance run by Dilki is expressed as a fraction of the distance run by Sayuni, it is $\frac{3}{5}$.
- By dividing both terms of the ratio 3 : 5 by 3, we can in a similar manner express the distance run by Sayuni as a fraction of the distance run by Dilki as $\frac{5}{3}$.
- Since Sayuni runs 5 m in the time that Dilki runs 3 m, the total distance run by them during this period is 8 m. When we divide both terms in the ratio by 8 we obtain $\frac{3}{8} : \frac{5}{8}$. This means that when the distance run by Dilki is expressed as a fraction of the total distance it is $\frac{3}{8}$, and that the distance run by Sayuni is $\frac{5}{8}$of the total distance.

➢ Sureni and Pradeepa shared a certain amount of money. Sureni received Rs. 35 while Pradeepa received Rs. 25. The ratio in which the money was shared between Sureni and Pradeepa can be expressed as 35 : 25.

When this is expressed in its simplest form it is 7 : 5.

The total amount shared between them $=$ Rs. $35 + 25 =$ Rs. 60

$\therefore$ The amount received by Sureni as a fraction of the total amount $= \frac{35}{60} = \frac{7}{12}$

We can obtain the above fraction as shown below too.

The ratio in which the money was shared between Sureni and Predeepa $= 7 : 5$

The amount received by Sureni as a fraction of the whole amount $= \frac{7}{7+5} = \frac{7}{12}$

Similarly,

the amount received by Pradeepa as a fraction of the whole amount $= \frac{5}{12}$

Example 1

To make a mixed fruit drink, mango juice, pineapple juice and orange juice are mixed in the ratio 2 : 3 : 1. Find the fraction of each type of juice in the drink.

The ratio of mango juice to pineapple juice to orange juice $= 2 : 3 : 1$

$\therefore$ The sum of the terms of the ratio $= 2 + 3 + 1 = 6$

The fraction of mango juice in the drink $= \frac{2}{6}$

The fraction of pineapple juice in the drink $= \frac{3}{6}$

The fraction of orange juice in the drink $= \frac{1}{6}$

Exercise 16.1

(1) Sudesh and Rahim shared some money. Sudesh received Rs. 450 while Rahim received Rs. 500.

(i) Write in the simplest form, the ratio in which the money was divided between them.
(ii) Write the amount Sudesh received as a fraction of the amount Rahim received. Express this in its simplest form.
(iii) What fraction of the total amount did Rahim receive?

(2) A stock of dry rations is distributed among the three families A, B and C in the ratio $A : B : C = 4 : 5 : 3$.

(i) Express separately the quantity of dry rations received by each family as a fraction of the whole stock.
(ii) What fraction of the amount that B received is the amount that A received?
(iii) What fraction of the amount that C received is the amount that A received?

(3) In a running event, Amashi ran 70 m in the time that it took Gayani to run 40 m.

(i) Write the ratio of the distance run by Amashi to that run by Gayani, in its simplest form.

(ii) Using the above ratio, write as a fraction, the distance run by Gayani in the time that Amashi runs 1 m.

(iii) Write as a fraction, the distance run by Amashi in the time that Gayani runs 1 m.

(iv) Express the distance run by Amashi as a fraction of the total distance run by the two of them.

(v) Express the distance run by Gayani as a fraction of the total distance run by the two of them.

(4) The floor area of the bedroom of a house is $\frac{2}{3}$ the floor area of the sitting room.

(i) What is the ratio of the floor area of the bedroom to that of the sitting room?

(ii) What fraction of the total floor area of the bedroom and the sitting room is the floor area of the sitting room?

(iii) What fraction of the total floor area of the bedroom and the sitting room is the difference between the floor areas of these two rooms?

16.3 Dividing in a given ratio

In our day to day life, there are many occasions when we have to share things with each other. Sometimes the sharing is equal while at other times it is not.

Let us recall what we learnt in Grade 7 about dividing something in a ratio.

A, *B* and *C* are three people. If Rs. 2000 was divided among them in the ratio 2 : 3 : 5, let us calculate how much each person received.

The ratio in which money is divided among them $= 2 : 3 : 5$

The total number of parts $= 2 + 3 + 5 = 10$

The amount received by A as a fraction of the whole $= \frac{2}{10}$

The amount received by A $= \text{Rs. } 2000 \times \frac{2}{10}$

$= \text{Rs. } 400$

The amount received by B as a fraction of the whole $= \frac{3}{10}$

The amount received by B $= \text{Rs. } 2000 \times \frac{3}{10}$

$= \text{Rs. } 600$

The amount received by C as a fraction of the whole $= \frac{5}{10}$

The amount received by C $= \text{Rs. } 2000 \times \frac{5}{10}$

$= \text{Rs. } 1000$

- **Dividing the profit when different amounts are invested for the same period of time**

Sandun and Sashika started a business at the beginning of a certain year by investing Rs. 30 000 and Rs. 40 000 respectively. At the end of the year, the profit from the business was Rs. 28 000. They shared it according to the ratio in which they invested money.

Let us consider how to calculate the share of the profit received by each of them.

The ratio in which Sandun and Sashika invested money $= 30\ 000 : 40\ 000$

$= 3 : 4$

The ratio in which the profit should be divided between them $= 3 : 4$

The total number of parts $= 3 + 4 = 7$

Sandun's profit as a fraction of the whole $= \frac{3}{7}$

The profit from the business $= \text{Rs. } 28\ 000$

Sandun's share of the profit $= \text{Rs. } 28\ 000 \times \frac{3}{7}$

$= \text{Rs. } 12\ 000$

Sashika's profit as a fraction of the whole $= \frac{4}{7}$

Sashika's share of the profit $= \text{Rs. } 28\ 000 \times \frac{4}{7}$

$= \text{Rs. } 16\ 000$

- **Dividing the profit when different amounts are invested for different periods of time**

If people invest different amounts in a business for different periods of time, both the amount invested and the period of investment need to be considered when the profit is shared.

Let us now consider such an example.

Kumudu started a business on January 1st of a certain year by investing Rs. 20 000. Sumudu joined the business by investing Rs. 30 000 two months later. At the end of the year, the profit from the business was Rs. 36 000.

Let us consider how the profit should be divided between the two of them.

Observe that in this case, the investments they made and the periods of investment are both different.

Name	Amount invested	Period of investment	Amount $\times$ Period
Kumudu	Rs. 20 000	12 months	$20\ 000 \times 12$
Sumudu	Rs. 30 000	10 months	$30\ 000 \times 10$

In such a situation, it is not fair to divide the profit by considering only the investments. Similarly, since the amounts invested are different, it is not fair to consider only the periods of investment either.

We have to consider both the investments and the periods of investment. This is done by basing the ratio in which the profit should be divided on the product of the amount invested and the period of investment (the last column of the above table).

The ratio in which the profits should be divided between Kumudu and Sumudu $\Big\} = 20\ 000 \times 12 : 30\ 000 \times 10$

$= 240\ 000 : 300\ 000$

$= 4 : 5$

The sum of the parts $= 4 + 5 = 9$

The amount Kumudu should receive $=$ Rs. $36\ 000 \times \frac{4}{9}$

$=$ Rs. $16\ 000$

The amount Sumudu should receive $=$ Rs. $36\ 000 \times \frac{5}{9}$

$=$ Rs. $20\ 000$

Example 1

Siripala starts a business in January by investing Rs. 30 000. His friend Hussain joins the business two months later by investing Rs. 24 000, and his friend Nadaraja joins the business two months after that by investing Rs. 60000. Calculate the ratio in which the profit should be divided between them at the end of a year.

Siripala		Hussain		Nadaraja
$30\ 000 \times 12$	:	$24\ 000 \times 10$	:	$60\ 000 \times 8$
$360\ 000$	:	$240\ 000$	:	$480\ 000$
3	:	2	:	4

Exercise 16.2

(1) The manner in which two people invested money in a joint venture during the same year is shown in the table given below.

Name	Amount invested	Date of investment	Period of investment	Amount $\times$ Period
Sujith	Rs. 18 000	Jan 01	....	....
Vijith	Rs. 20 000	Apr 01	....	....

(i) Fill in the blanks in the above table.

(ii) Find the ratio in which the profit should be divided between Sujith and Vijith after a year.

(2) Kanthi invested Rs.10 000 and started a dressmaking business on January 01st of a certain year.Two months later Nalani joined the business by investing Rs.12 000.

(i) Calculate the ratio in which the profit should be divided between them at the end of the year.

(ii) If the profit for the year was Rs. 25 000, find the amount received by each of them.

(3) Kamal and Sunil started a business on the 01st of January of a certain year by investing Rs. 24 000 and Rs. 30 000 respectively. After 4 months Wimal joined the business by investing Rs. 54 000. The profit from the business for the year was Rs.180 000.

(i) Find the ratio in which the profit should be divided between Kamal, Sunil and Wimal.

(ii) Find separately the amount received by each of them.

(4) Chamara started a spice business by investing Rs. 8000 on the 1st of February. Kumara joined the business by investing Rs. 11 000 on the 1st of June of that year. The profit from the business on December 31st was Rs. 45 000.

(i) Calculate the ratio in which the profit should be divided between them.

(ii) Find separately the amounts received by Chamara and Kumara.

16.4 Continued ratio

A fruit drink is made by mixing pineapple juice, water and mango juice. In this fruit drink, the ratio of pineapple juice to water is 1 : 3 and the ratio of water to mango juice is 3 : 2. Let us find the ratio of pineapple juice to water to mango juice in this drink.

In these two ratios, water is the common substance. It has the same value in both ratios.

The ratio of pineapple juice to water $= 1 : 3$
The ratio of water to mango juice $= 3 : 2$

In both cases, the term related to water is 3.

$\therefore$ the ratio of pineapple juice to water to mango juice $= 1 : 3 : 2$

In a concrete mixture, the ratio of gravel to sand is 5 : 3 and of sand to cement is 2 : 1. Let us consider how to find the ratio of gravel to sand to cement in the mixture.

In both these ratios, sand is the common substance. By making the amount of sand equal in both ratios, we can find the continued ratio of the three substances. We use "equivalent ratios" to do this.

The ratio of gravel to sand $= 5 : 3 = 5 \times 2 : 3 \times 2 = 10 : 6$
The ratio of sand to cement $= 2 : 1 = 2 \times 3 : 1 \times 3 = 6 : 3$

Note

In the ratios 5 : 3 and 2 : 1, the terms corresponding to sand are 3 and 2 respectively. The least common multiple of 3 and 2 is 6. Therefore, equivalent ratios are considered such that the term corresponding to sand in both ratios is equal to 6.

$5 : 3 = 10 : 6$ $\qquad$ $2 : 1 = 6 : 3$

Therefore, the ratio of gravel to sand to cement is 10 : 6 : 3

The ratio of gravel to sand in the concrete mixture is 5 : 3. Therefore, when 10 pans of gravel are used, 6 pans of sand are needed.

The ratio of sand to cement is 2 : 1. Therefore, when 6 pans of sand are used, 3 pans of cement are needed.

Hence the ratio of gravel to sand to cement in the mixture is 10 : 6 : 3.

Example 1

When preparing a sweetmeat, flour and sugar are mixed in the ratio 4 : 3 and sugar and coconut are mixed in the ratio 5 : 3. Find the ratio of flour to sugar to coconut in the sweetmeat.

The ratio of flour to sugar = 4 : 3

The ratio of sugar to coconut = 5 : 3

Sugar is common to both ratios. The terms corresponding to sugar in these two ratios are 3 and 5. Equivalent ratios should be written such that the term corresponding to sugar is the least common multiple of 3 and 5, which is 15.

The ratio of flour to sugar $= 4 : 3 = 4 \times 5 : 3 \times 5 = 20 : 15$

The ratio of sugar to coconut $= 5 : 3 = 5 \times 3 : 3 \times 3 = 15 : 9$

Therefore, the ratio of flour to sugar to coconut $= 20 : 15 : 9$

Example 2

A certain amount of money was divided among *A*, *B* and *C*. The ratio in which it was divided between *A* and *B* is 3 : 4 and between *B* and *C* is 2 : 5. Find the ratio in which the money was divided among *A*, *B* and *C*.

The ratio of *A* to *B* = 3 : 4

The ratio of *B* to *C* = 2 : 5

B is common to both these ratios. The respective terms for *B* are 4 and 2. Their common multiple is 4.

The ratio of *A* to *B* $= 3 : 4$

The ratio of *B* to *C* $= 2 : 5 = 2 \times 2 : 5 \times 2 = 4 : 10$

$\therefore$ The ratio of *A* to *B* to *C* is $= 3 : 4 : 10$

Exercise 16.3

(1) A fertilizer is produced by combining Nitrogen, Phosphorous and Potassium. The ratio of Nitrogen to Phosphorous is 5:3 and the ratio of Phosphorous to Potassium is 6:1. Find the ratio of Nitrogen to Phosphorous to Potassium in this fertilizer.

(2) Coconut oil, sesame oil and margosa oil are combined together to make medicinal oil. Coconut oil and sesame oil are combined in the ratio 5 : 2 and sesame oil and margosa oil are combined in the ratio 3 : 1. Find the ratio of coconut oil to sesame oil to margosa oil in the medicinal oil.

(3) In a certain farm, there are cattle, goats and hens. The ratio of cattle to goats is 4 : 3 and the ratio of cattle to hens is 2 : 7.

(i) Find the ratio of cattle to goats to hens.

(ii) If there are 105 animals of these three types in the farm, find separately the number of cattle, goats and hens in the farm.

(4) In a certain village, Sinhalese, Tamils and Muslims live together. The ratio of Sinhala families to Tamil families is 5 : 3 and the ratio of Tamil families to Muslim families is 4 : 1.

(i) Find the ratio of Sinhalese families to Tamil families to Muslim families.

(ii) How many families are there in the village, if there are 60 Sinhalese families?

(5) Piyadasa, Swaminadan and Nazeer are three friends who set up a joint venture. They shared the profit of their business as follows: Between Piyadasa and Nazeer in the ratio 5 : 6, and between Swaminadan and Nazeer in the ratio 4 : 5.

(i) Find the ratio in which the profit was shared between Piyadasa and Swaminadan.

(ii) If Piyadasa received Rs. 20 000 as profit, calculate how much Swaminadan and Nazeer received.

Miscellaneous Exercise

(1) Ruwani started a sweetmeat business by investing Rs. 5000 at the beginning of a certain year. At the beginning of March of the same year, her neighbours Fathima and Saradha joined the business by investing Rs. 7000 and Rs. 5000 respectively. At the end of the year, the profit from the business was Rs. 54 000. Calculate the amount received by each of them if the profit was divided among them based on their investment and the period of investment.

Summary

- When sharing profits of a joint venture, the invested amount and the period of investment are both taken into consideration.
- When calculating the ratio in which profits should be shared in a joint venture, the invested amount is multiplied by the period of investment.
- When the relationship between three quantities is given by two ratios, we obtain the continued ratio of the three quantities by considering equivalent ratios.

17 Equations

By studying this lesson you will be able to,

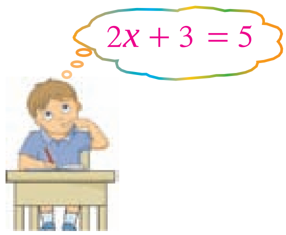
- construct simple equations in one unknown where the coefficient of the unknown is a fraction,
- construct simple equations using one pair of brackets,
- solve simple equations, and
- check the accuracy of the solution of a simple equation.

17.1 Equations

You have learnt that, when the value represented by an algebraic expression is equal to the value of a given number, this can be expressed as, “algebraic expression = number”.

You have also learnt that, when the value represented by an algebraic expression is equal to the value represented by another algebraic expression, this can be expressed as,

"first algebraic expression = second algebraic expression".

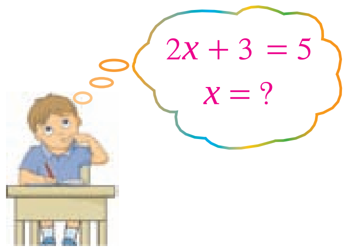


Relationships of the above forms are called **equations**.

$2x + 3 = 5$ is an equation. It has only one unknown term x, of which the index is one. Such equations are called simple equations.

Finding the value of the unknown term for which the value of the left hand side of the equation is equal to the value of the right hand side, is called **solving the equation**.

The value obtained for the unknown by solving the equation, is called **the solution of the equation**. Simple equations have only one solution.



The above equation $2x + 3 = 5$ denotes the following; "5 is obtained by adding 3 to twice the value of the unknown term".

Let us recall the method of solving this equation.

$$
\begin{aligned}
2x + 3 &= 5 \\
2x + 3 - 3 &= 5 - 3 \quad \text{(subtract 3 from both sides, since } 3 - 3 = 0\text{)} \\
2x &= 2 \\
\frac{2x}{2} &= \frac{2}{2} \quad \text{(divide both sides by 2, since } \frac{2}{2} = 1\text{)} \\
\therefore x &= 1
\end{aligned}
$$

$$
\boxed{\begin{aligned} 2x + 3 &= 5 \\ 2x &= 5 - 3 \end{aligned}}
$$

Let us check the accuracy of the solution that we obtained.
When the value obtained for the unknown is substituted in the equation, if we get the same value on the left hand side of the equation as that of the right hand side, then the solution is correct.

When $x = 1$, the left hand side of the equation is, $2x + 3 = 2 \times 1 + 3$

$= 2 + 3$

$= 5$

The right hand side of the equation $= 5$

That is, the left hand side = the right hand side.

Therefore $x = 1$ is the correct solution of the equation $2x + 3 = 5$.

- We obtain the same value on the two sides of an equation when we subtract the same number from both sides.
- We obtain the same value on the two sides of an equation when we add the same number to both sides.
- We obtain the same value on the two sides of an equation when we divide both sides by the same non-zero number.
- We obtain the same value on the two sides of an equation when we multiply both sides by the same number.

Do the following review exercise to recall what you have learnt about constructing simple equations and solving them.

Review Exercise

(1) Construct a simple equation for each of the statements given below and solve it.

(i) When 5 is added to the value of x, the result is 12.
(ii) When 3 is subtracted from the value of a, the result is 8.
(iii) Shashi's age is denoted by x. Her sister who is 2 years older to her is 12 years old.
(iv) I have an amount of money denoted by x rupees. Twice this amount is 60 rupees.
(v) When 5 is subtracted from three times the value of x, the result is 1.
(vi) My father is 44 years old today. His age is 5 years more than 3 times my age. (Take my age today as y years.)

(2) Solve each of the following equations.

(i) $x + 10 = 15$ (ii) $x - 5 = 25$ (iii) $5x = 20$
(iv) $2x + 3 = 13$ (v) $4x - 1 = 19$ (vi) $3x + 22 = 13$

17.2 More on the construction of simple equations

• Construction of simple equations where the coefficient of the unknown is a fraction

Simple equations where the coefficient of the unknown is a whole number have been constructed earlier. Now let us consider how simple equations where the coefficient of the unknown is a fraction are constructed.

My brother's age is 3 years more than one fourth of my age. He is 6 years old now. Let us construct an equation by using this information.

Let x be my age in years.

Then, one fourth of my age $= \frac{1}{4} \times x = \frac{x}{4}$

Since my brother's age is 3 years more than one fourth my age,

My brother's age $= \frac{x}{4} + 3$

Since my brother is 6 years old, $\frac{x}{4} + 3 = 6$

• Constructing simple equations using one pair of brackets

By adding a certain amount of money to the 8 rupees I gave Kasun, he bought 26 olives at the price of two fruits per rupee.
Let us construct a simple equation with this information to find the amount Kasun spent to buy the olives.

Let x rupees be the amount Kasun spent.

Then the total amount spent to buy the olives $= x + 8$ rupees

$$\left.\begin{array}{l}\text{The total number of olives that can be bought}\\ \text{for } x + 8 \text{ rupees at the price of two fruits per rupee}\end{array}\right\} = 2(x + 8)$$

It is necessary to use brackets here because the total amount, which is $x + 8$ rupees, needs to be multiplied by 2.We express the fact that the sum of the two terms x and 8 is multiplied by 2, as $2 \times (x + 8)$, by using brackets.
The number of olives that were bought is 26. Therefore,

$2\,(x + 8) = 26$

Example 1

Nimali plucked some mangoes from the tree in her garden, saved 16 for herself, and by selling the rest at 25 rupees each, made 875 rupees.
Construct a simple equation with this information, to find the total number of mangoes that Nimali plucked.

Let x be the total number of mangoes that were plucked.

The number of mangoes that were sold $= x - 16$

To find the amount of money she received by selling the mangoes at 25 rupees per fruit, $(x - 16)$ should be multiplied by 25.

This amount is $25\,(x - 16)$.

As the amount received by selling the mangoes is 875 rupees,

$25(x - 16) = 875.$

Exercise 17.1

(1) Construct a simple equation for each of the following statements.

(i) When 5 is added to half the value of x, the result is 8.

(ii) A parcel has one book of value x rupees and another book worth 50 rupees. The value of the books in 5 such parcels is 750 rupees.

(iii) Raj's brother's age is one year less than one third of Raj's age. Raj's brother is 3 years old.

(iv) Rashmi has 200 rupees which is 5 times the amount that is obtained when 10 rupees is deducted from twice the amount that Vishmi has.

(v) When 5 is subtracted from half of a certain number, the result is 2.

17.3 Solving equations in one unknown when the coefficient of the unknown is a fraction

Let us consider the method of solving equations in one unknown when the coefficient of the unknown is a fraction.

Let us solve the equation $\frac{x}{2} = 3$.

$\frac{x}{2} \times 2 = 3 \times 2$ (multiply both sides by 2)

$\frac{x \times \cancel{2}^{1}}{\cancel{2}_{1}} = 6$

$\therefore\ x = 6$

Example 1

Solve $\frac{2}{3}x - 1 = 3$.

$$\frac{2x}{3} - 1 = 3$$

$$\frac{2x}{3} - 1 + 1 = 3 + 1 \quad \text{(add 1 to both sides)} \; (-1 + 1 = 0)$$

$$\frac{2x}{3} = 4$$

$$\frac{2x}{3} \times 3 = 4 \times 3 \quad \text{(multiply both sides by 3)} \; \left(\frac{2}{\cancel{3}} \times \frac{\cancel{3}}{1} = 2\right)$$

$$2x = 12$$

$$\frac{2x}{2} = \frac{12}{2} \quad \text{(divide both sides by 2)}$$

$$x = 6$$

Now let us check the accuracy of the solution $x = 6$.

When $x = 6$, the left hand side $= \frac{2x}{3} - 1 = \frac{2 \times 6}{3} - 1$

$= \frac{12}{3} - 1$

$= 4 - 1$

$= 3$

Right hand side $= 3$

That is, the left hand side = the right hand side

$\therefore$ $x = 6$ is the correct solution of the equation $\frac{2x}{3} - 1 = 3$.

Example 2

Solve $2 - \frac{3}{10}a = 5$.

$$2 - \frac{3}{10}a - 2 = 5 - 2 \quad \text{(subtract 2 from both sides)}$$

$$-\frac{3}{10}a = 3$$

$$-\frac{3a}{\cancel{10}_1} \times \cancel{10}^1 = 3 \times 10 \quad \text{(multiply both sides by 10)}$$

$$-3a = 30$$

$$\frac{-3a}{(-3)} = \frac{30}{(-3)} \quad \text{(divide both sides by } (-3))$$

$$a = -10$$

Exercise 17.2

(1) Solve each of the following equations. Check the accuracy of the solution.

(i) $\frac{x}{5} = 2$ (ii) $\frac{a}{3} + 1 = 3$ (iii) $\frac{p}{4} - 1 = 2$

(iv) $\frac{2x}{5} - 1 = 7$ (v) $3 - \frac{2y}{5} = 1\frac{4}{5}$ (vi) $\frac{5m}{16} - 2 = \frac{1}{2}$

17.4 Solving equations having one pair of brackets

Let us solve the equation $2\,(x + 3) = 10$.

Method I

$$2\,(x + 3) = 10$$

$$\frac{\cancel{2}^{1}(x + 3)}{\cancel{2}_{1}} = \frac{10}{2} \quad \text{(divide both sides by 2)}$$

$$x + 3 = 5$$

$$x + 3 - 3 = 5 - 3 \quad \text{(subtract 3 from both sides)}$$

$$\therefore\ x = 2$$

Method II

$$2\,(x + 3) = 10$$

$$2x + 6 = 10$$

$$2x + 6 - 6 = 10 - 6$$

$$2x = 4$$

$$\frac{2x}{2} = \frac{4}{2}$$

$$\therefore\ x = 2$$

We can check the accuracy of the solution by substituting $x = 2$ in the equation $2\,(x + 3) = 10$.

Example 1

Solve $10(1-2x)+1 = 6$.

$$10(1-2x)+1 = 6$$

$$10(1-2x)+1-1 = 6-1 \quad \text{(subtract 1 from both sides)}$$
$$10(1-2x) = 5$$
$$\frac{10(1-2x)}{10} = \frac{5}{10} \quad \text{(divide both sides by 10)}$$
$$1-2x = \frac{1}{2}$$
$$1-2x-1 = \frac{1}{2}-1 \quad \text{(subtract 1 from both sides)}$$
$$-2x = -\frac{1}{2}$$
$$\frac{-2x}{-2} = -\frac{1}{2} \div (-2) \quad \text{(divide both sides by } -2\text{)}$$
$$x = \frac{(-1)}{2} \times \frac{1}{(-2)} = \frac{(-1)}{(-4)}$$
$$\therefore x = \frac{1}{4}$$

Let us check the accuracy of the solution.

When $x = \frac{1}{4}$, the left hand side $= 10\,(1-2x)+1$

$$= 10\left(1-2\times\frac{1}{4}\right)+1$$
$$= 10\left(1-\frac{1}{2}\right)+1$$
$$= 10\times\frac{1}{2}+1$$
$$= 5+1$$
$$= 6$$

That is, the left hand side = the right hand side

$\therefore x = \frac{1}{4}$ is the solution.

Exercise 17.3

(1) Solve each of the following equations. Check the accuracy of the solution.

(i) $2\,(x+3) = 8$ (ii) $3\,(p-2) = 9$ (iii) $2\,(2x-1) = 6$

(iv) $5\,(1-3x) = 20$ (v) $2\,(3-4x)-1 = -19$ (vi) $10\,(2x+1)-5 = 25$

(vii) $2\left(\frac{x}{3}-1\right) = (-6)$ (viii) $2\left(\frac{5x}{2}+1\right) = -18$ (ix) $2-\frac{3x}{4} = (-7)$

(x) $\frac{1}{5}(x-2)=2$ (xi) $\frac{1}{2}(3-x)-1=4$ (xii) $\frac{1}{3}(2p-1)+2=\frac{5}{9}$

(2) An envelope contains x number of 10 rupee notes and five 20 rupee notes. The total amount of money in 5 such envelopes is 750 rupees.

(i) Construct an equation with the information given above.
(ii) By solving the equation, find the number of 10 rupee notes in one envelope.

Miscellaneous Exercise

(1) Let x be a positive integer. When 12 is added to twice the integer right after x, the result is 38.

(i) Express the positive integer right after x in terms of x.
(ii) Construct an equation in terms of x.
(iii) Find the positive integer denoted by x by solving the above equation.

(2) A factory worker receives p rupees as his daily wage and an additional 100 rupees as an allowance for each day he works. He worked 20 days during a certain month and received 20 000 rupees in total. What is his daily wage?

(3) A father's age is a years and his son's age is 31 years. 5 years ago the son's age was one year more than $\frac{1}{2}$ the age of his father at that time.

(i) What was the son's age 5 years ago?
(ii) Write the father's age 5 years ago in terms of a.
(iii) Construct an equation in a using the above information.
(iv) Find the father's present age by solving the equation.

Summary

- When an algebraic expression is equated to a number or another algebraic expression, such a relationship is known as an equation.
- The solution to an equation is the value of the unknown term that satisfies the equation.

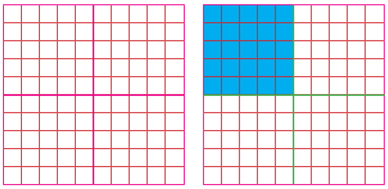
18 Percentages

By studying this lesson you will be able to,

- express a fraction as a percentage,
- express a percentage as a fraction,
- to know the relationship between ratios and percentages,
- calculate a percentage from a given quantity, and
- find the total quantity when a percentage and its corresponding amount are given.

18.1 Expressing a fraction as a percentage

You learnt in Grade 7 that the symbol "%" is known as the percentage sign.



The coloured region of the figure is $\frac{1}{4}$ of the whole figure; that is, $\frac{25}{100}$ of the whole figure.

You have learnt that this is 25%, as a percentage of the whole figure. It is read as twenty five percent. Expressing it as such is called, **expressing a portion of a whole as a percentage**.

A fraction can be expressed as a percentage by writing an equivalent fraction with denominator 100.

We can write the coloured region as a percentage of the whole figure as shown below.

$\frac{1}{4} \times 100\% = 25\%,$

As $\frac{1}{4} = 0.25,$ the coloured region of the figure is 0.25 of the whole figure. As a percentage it is, $0.25 \times 100\% = 25\%$.

By multiplying a given decimal number or fraction by 100%, that decimal number or fraction can be expressed as a percentage.

Example 1

Given that the initial amount is 1, express each of the following quantities as a percentage of the initial amount.

(i) $\frac{3}{8}$ (ii) $\frac{1}{12}$ (iii) 0.068 (iv) $\frac{2}{3}$

(i) $\frac{3}{8} = \frac{3}{8} \times 100\ \% = 37.5\ \%$

(ii) $\frac{1}{12} = \frac{1}{12} \times 100\ \% = \frac{100}{12}\%$

$= 8\frac{4}{12}\ \%$

$= 8\frac{1}{3}\ \%$

(iii) $0.068 = 0.068 \times 100\% = 6.8\ \%$

(iv) $\frac{2}{3} = \frac{2}{3} \times 100\ \% = \frac{200}{3}\ \% = 66\frac{2}{3}\ \%$

Exercise 18.1

Given that the initial amount is 1, express each of the following quantities as a percentage of the initial amount.

(i) $\frac{1}{2}$ (ii) 0.7 (iii) 2.4 (iv) 7.8

(v) 4.025 (vi) 6 (vii) 0.067 (viii) $1\frac{11}{50}$

(ix) $\frac{1}{3}$ (x) $\frac{5}{6}$ (xi) $\frac{9}{11}$ (xii) $1\frac{3}{7}$

18.2 Expressing a percentage as a fraction

Let us consider the following examples in order to learn how to convert a percentage into a fraction.

Example 1

Express each of the following percentages as a fraction.

(i) 20 % (ii) 125 % (iii) $33\frac{1}{3}\%$

(i) $20\ \% = \frac{20}{100} = \frac{1}{5}$

(ii) $125\ \% = \frac{125}{100} = \frac{5}{4} = 1\frac{1}{4}$

(iii) $33\frac{1}{3}\% = 33\frac{1}{3} \div 100 = \frac{100}{3} \div 100 = \frac{100}{3} \times \frac{1}{100} = \frac{1}{3}$

Exercise 18.2

Express each of the following percentages as a fraction and simplify it.

(i) 25%	(ii) 40%	(iii) 16%	(iv) 150%
(v) 120%	(vi) 58%	(vii) 32%	(viii) 175%
(ix) $12\frac{1}{3}$ %	(x) $3\frac{1}{3}$ %	(xi) $1\frac{3}{5}$ %	(xii) 2.25%

18.3 Ratios and percentages

"8% of the eggs in the basket are rotten". This means that 100 such eggs would contain 8 rotten eggs. Accordingly, the ratio of the number of rotten eggs to the total number of eggs is 8 : 100. You learnt this in Grade 7.

• Writing a ratio corresponding to a percentage

Now let us see how to write the ratio corresponding to the percentage, 30%.
30% can be written as 30 : 100.

$30 : 100 = 30 \div 10 : 100 \div 10 = 3 : 10.$

Accordingly, the ratio corresponding to the percentage 30% is 3 : 10.

• Writing a percentage corresponding to a ratio

Now let us see how to express the ratio 1 : 4 as a percentage.

The percentage corresponding to a given ratio can be found by writing the equivalent ratio with 100 as its second term.

$1 : 4 = 1 \times 25 : 4 \times 25 = 25 : 100$

Since the ratio 25:100 can be written as $\frac{25}{100}$, the percentage corresponding to the ratio 1: 4 is 25%.

Example 1

Express 20% as a ratio.
20% can be written as 20 : 100.

Now, 20 : 100 = 20 ÷ 20 : 100 ÷ 20 = 1 : 5.

Accordingly, the ratio corresponding to 20% is 1 : 5.

Note: When writing a ratio, it should be expressed in its simplest form.

Example 2

Express $12\frac{1}{2}\%$ as a ratio.

$$12\frac{1}{2}\% = \frac{12\frac{1}{2}}{100} = 12\frac{1}{2} \div 100 = \frac{25}{2} \times \frac{1}{100} = \frac{25}{200}$$

$\frac{25}{200}$ can be written as 25 : 200. Now,

$$\begin{aligned} 25 : 200 &= 25 \div 25 : 200 \div 25 \\ &= 1 : 8 \end{aligned}$$

$$\boxed{\begin{aligned} &12\frac{1}{2} : 100 \\ &\frac{25}{2} : 100 \\ &25 : 200 \\ &1 : 8 \end{aligned}}$$

Accordingly, the ratio corresponding to $12\frac{1}{2}\%$ is 1 : 8.

Example 3

Express the ratio 2 : 5 as a percentage.

$$\begin{aligned} 2 : 5 &= 2 \times 20 : 5 \times 20 \\ &= 40 : 100 \end{aligned}$$

Accordingly, the percentage corresponding to the ratio 2 : 5 is 40%.

Example 4

Express the ratio 3 : 2 as a percentage.

$$\begin{aligned} 3 : 2 &= 3 \times 50 : 2 \times 50 \\ &= 150 : 100 \end{aligned}$$

Accordingly, the percentage corresponding to the ratio 3 : 2 is 150%.

Example 5

Express the ratio 1 : 3 as a percentage.

$1:3 = \frac{1}{3} : 1 = \frac{1}{3} \times 100 : 1 \times 100 = \frac{100}{3} : 100.$

Accordingly, the percentage corresponding to the ratio 1 : 3 is $\frac{100}{3}\%$ $\left(\text{i.e., } 33\frac{1}{3}\%\right)$.

Exercise 18.3

(1) Write the corresponding ratio for each of the following percentages.

(i) 25% (ii) 40% (iii) 45% (iv) 8%

(v) 125% (vi) 300% (vii) $5\frac{1}{2}\%$ (viii) $33\frac{1}{3}\%$

(2) Write the corresponding percentage for each of the following ratios.

(i) 1 : 2 (ii) 7 : 20 (iii) 13 : 25 (iv) 27 : 50

(v) 3 : 2 (vi) 9 : 4 (vii) 6 : 5 (viii) 13 : 10

(ix) 1 : 7 (x) 3 : 17

(3) 28 males and 22 females participated in a meeting.

(i) Write the ratio of the males to the total participants, and write the corresponding percentage. Describe what this percentage means in words.

(ii) Write the ratio of the females to the total participants and write the corresponding percentage.

18.4 Calculating the corresponding percentage when a certain quantity from a total amount is given

Percentages are used when comparing different quantities of a particular kind, or when comparing amounts in different groups. When such comparisons are made, the relevant quantities should be expressed in the same units.

You learnt in Grade 7 how to calculate the relevant percentage when you are given a certain quantity from a total amount.

When a quantity is given, write it as a fraction of the total amount. You can then obtain the corresponding percentage by multiplying the fraction by 100%.

If 30 of the 200 mangoes brought by a vendor to sell were rotten, let us find the percentage of rotten mangoes in the whole stock.

The total number of mangoes brought by the vendor to sell $= 200$

The number of rotten mangoes $= 30$

The number of rotten mangoes as a fraction of the total number of mangoes $= \frac{30}{200}$

The percentage of rotten mangoes $= \frac{30}{200} \times 100\,\%$

$= 15\,\%$

Example 1

The distance from town A to town B is 50 km. A man leaving town A, travels 20 km by bus and the rest of the distance by train. Express the distance travelled by bus as a percentage of the total distance.

The distance travelled by bus as a fraction of the total distance $= \frac{20}{50}$

The distance travelled by bus as a percentage $= \frac{20}{50} \times 100\,\%$

$= 40\,\%$

Exercise 18.4

(1) Express the first value of each pair given below as a percentage of the second value.

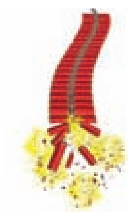
(i) 200g from 1 kg (ii) 25 cm from 1 m

(iii) 750 m from 1 km (iv) 300 ml from 1 l (v) 20 minutes from 1 hour

(2) If 30 of the 50 students in a class are girls, find the number of girls in the class as a percentage of the total number of students.

(3) If a person who borrowed Rs. 2000, pays Rs. 250 as interest at the end of a year, find the annual interest rate he paid.

(4) If 5 from a lot of 25 fire crackers bought by Prathapa to light on New Year's day did not explode, calculate the number of crackers that exploded as a percentage of the total number of crackers.



(5) If Kareem obtained 36 marks for an assignment marked out of 40, express Kareem's marks as a percentage of the total marks allocated for the assignment.

(6) Mr. Perera's monthly salary is Rs. 30 000. He spends Rs. 15 000 on food, Rs. 3000 on transport and the rest of his salary on other expenses.

(i) Find the amount spent on food as a percentage of his salary.

(ii) Find the amount spent on transport as a percentage of his salary.

18.5 Finding the quantity corresponding to a percentage, when the percentage and the total amount are given

The total number of students in a school is 1500. If 48% of the students are boys, let us find the number of boys in the school.

The total number of students in the school $= 1500$

The percentage of boys $= 48\,\%$

The number of boys in the school $= 1500 \times \dfrac{48}{100}$

$= 720$

Example 1

If a man saves 5% from his monthly salary of Rs. 20 000, how much of money does he save?

Monthly salary $=$ Rs. 20 000

The percentage saved $= 5\,\%$

The amount of money saved $=$ Rs. $20\,000 \times \dfrac{5}{100}$

$=$ Rs. 1000

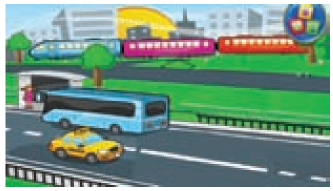
Exercise 18.5

(1) If the prevailing price of Rs. 120 per litre of fuel is increased by 10%, by how many rupees will the price of 1 litre of fuel increase?

(2) If the minimum percentage of marks required to pass an examination marked out of 300 is 60%, what is the minimum mark required to pass the examination?

(3) 15% of the workers in an establishment are men. If the total number of workers in the establishment is 800, how many male workers are there?

(4) A person travels 60% of a journey by train, 35% by bus and the rest of the journey by taxi. The total distance of the journey is 140 km.



(i) Find the distance travelled by train.

(ii) Find the distance travelled by bus.

(5) Mr. Ranasinghe's monthly salary is Rs. 45 000. He puts aside 30% of his salary for food, 20% for transport and the rest of the salary for other expenses.

(i) How much money does he put aside for food?

(ii) How much money does he put aside for transport?

18.6 Finding the total amount, when a certain quantity and its corresponding percentage are given

Let us find the total sum of money, if the value of 10% of the sum is Rs. 250.

$$
\begin{aligned}
10\% \text{ of the sum} &= \text{Rs. } 250 \\
1\% \text{ of the sum} &= \text{Rs. } \frac{250}{10} \\
100\% \text{ of the sum (therefore the total sum)} &= \text{Rs. } \frac{250}{10} \times 100 \\
&= \text{Rs. } 2500
\end{aligned}
$$

Example 1

60% of the students in a class use public transport to travel to school. If the number of students in this class who do not use public transport is 16, find the total number of students in the class.

Percentage of children who do not use public transport = 100% – 60% = 40%

$$
\begin{aligned}
40\% \text{ of the students} &= 16 \\
1\% \text{ of the students} &= \frac{16}{40} \\
100\% \text{ of the students} &= \frac{16}{40} \times 100 \\
\text{Total number of students} &= 40
\end{aligned}
$$

Exercise 18.6

(1) If 30% of a person's salary is Rs. 7200, how much is his salary?

(2) The attendance of the students of a school on a rainy day was 60%. If the number of students who attended school that day was 420, find the total number of students in the school.

(3) After spending 65% of the money he had in hand, if a person had a balance of Rs. 1400, what is the total amount of money he initially had?

(4) A metal alloy is made by mixing iron and zinc. If 36% of the alloy is zinc and the amount of iron in the alloy is 160 g, calculate the total mass of the alloy.

(5) A man gives 5% of the money he obtained by selling his vehicle to a broker. If he is left with Rs. 475 000 there after,

(i) find the selling price of the vehicle.

(ii) find the broker fee paid.

(6) 40% of the employees working in a factory are women. If the number of male employees in the factory is 75, how many employees are there in total?

(7) A doctor gave a diet plan to Rajitha to reduce his mass by 9 kg within 6 months. 9 kg is 10% of his total mass.

(i) How much is Rajitha's mass?

(ii) 12% of his mass was reduced during the said time period. How much is Rajitha's mass now?

Summary

- A fraction can be expressed as a percentage by writing an equivalent fraction with denominator 100.
- A given fraction or decimal number can be converted into a percentage by multiplying it by 100%.
- A percentage corresponding to a given ratio can be found by writing an equivalent ratio with second term equal to 100.

19

Sets

By studying this lesson you will be able to,

- identify the symbols used to denote whether an object is an element of a set or not,
- identify the null set and the symbol used to denote the null set, and
- identify the standard notation used to denote the number of elements in a set.

19.1 Introduction of sets

You learnt in Grade 7 that a set is a collection of identifiable objects. The following are examples of sets.

(i) The set of all districts in the Southern Province of Sri Lanka.
(ii) The set of odd numbers between 0 and 10.
(iii) The set of all the letters in the word 'MATARA".

You have learnt that the objects belonging to a set are called the elements of that set. Sometimes the word “members” is used instead of the word “elements”.

When we are able to list all the elements, we express the set by writing the elements within curly brackets, separated by commas.

Let A be the set of odd numbers between 0 and 10.
This can be expressed as $A = \{1, 3, 5, 7, 9\}$.

When we write a set using curly brackets, we write each element only once within the curly brackets.

Do the review exercise to recall what you have learnt earlier.

Review Exercise

(1) Copy the following expressions in your exercise book. If an expression defines a set, place a ✓ in front of it. Otherwise place a ×.

(i) The multiples of 3 between 0 and 20
(ii) The months of the year
(iii) Beautiful flowers
(iv) Prime numbers
(v) Tall people

(2) Each of the following sets are expressed using a common characteristic of its elements. Rewrite each set by listing all its elements within curly brackets.

(i) $A = \{$square numbers between 0 and 20$\}$
(ii) $B = \{$the letters in the word MAHARAGAMA$\}$
(iii) $C = \{$the months with 31 days$\}$
(iv) $D = \{$the digits in the number 41242$\}$
(v) $E = \{$the provinces of Sri Lanka$\}$

(3) Let A be the set of all multiples of 2 between 1 and 15.

(i) Express the set A using a common characteristic of its elements.
(ii) Write the set A again, listing all its elements within curly brackets.

19.2 Set notation

$X = \{$even numbers between 0 and 10$\}$
Let us write this set by listing all its elements within curly brackets.
$X = \{2, 4, 6, 8\}$
We can write that each of the numbers 2, 4, 6 and 8 is an element of the set X in the following manner by using the symbol "$\in$" in place of "is an element of".

2 is an element of the set X is written as $2 \in X$.
4 is an element of the set X is written as $4 \in X$.
6 is an element of the set X is written as $6 \in X$.
8 is an element of the set X is written as $8 \in X$.
5 is not an element of the set X.

We replace "is not an element of" by the symbol $\notin$ and write $5 \notin X$ to express that 5 is not an element of the set X.

Likewise, 7 is not an element of the set X is written as $7 \notin X$.

Example 1

Write, "4 is an element of the set of square numbers" using set notation.

$4 \in$ {square numbers}

Example 2

Write, "a parrot is not an element of the set of four legged animals" using set notation.

parrot $\notin$ {four legged animals}

Exercise 19.1

(1) Write each of the following in words as it is read.

(i) triangle $\in$ {polygons}
(ii) m $\notin$ {vowels in the English alphabet}
(iii) $8 \in$ {even numbers}
(iv) carrot $\notin$ {varieties of fruits}

(2) Copy the following in your exercise book. Fill in the blanks with $\in$ or $\notin$ as appropriate.

(i) 11 .... {prime numbers}
(ii) 15 .... {multiples of 4}
(iii) blue ...... {colours of the rainbow}
(iv) mango .... {varieties of fruits}
(v) Matara ......{districts in Western Province}

(3) Copy the following statements in your exercise book. Place a ✓ in front of the correct statements, and a × in front of the incorrect statements.

(i) $7 \in \{1, 3, 5, 7, 9\}$
(ii) $5 \notin \{2, 4, 6, 8\}$
(iii) $a \notin \{a, e, i, o, u\}$
(iv) □ $\notin$ {△, □, ⬠, ⬡}
(v) iii $\in$ {i, ii, v, iv, vi, vii, x}

19.3 Number of elements in a set

$A = \{$odd numbers between 0 and 10$\}$

Let us express the set A with its elements written within curly brackets.

$A = \{1, 3, 5, 7, 9\}$

A has 5 elements. The notation $n(A)$ is used to denote the number of elements in the set A.

Accordingly, $n(A) = 5$

Example 1

$P = \{$multiples of 3 between 1and 20$\}$. Find the value of $n(P)$.

$P = \{3, 6, 9, 12, 15, 18\}$

$\therefore\ n(P) = 6$

Example 2

Let P be the set of multiples of 6 between 1 and 20 and Q be the set of even numbers between 1 and 20.

(i) Express each of the sets P and Q with the elements written within curly brackets.

(ii) Copy the following statements and select the ones which are true.

(a) $10 \in P$ (b) $10 \notin Q$ (c) $18 \in P$

(iii) Find $n(P)$ and $n(Q)$.

(i) $P = \{6, 12, 18\}$

$Q = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$

(ii) (a) 10 is not an element of P.

$\therefore\ 10 \in P$ is false.

(b) 10 is an element of Q.

$\therefore\ 10 \notin Q$ is false.

(c) 18 is an element of P.

$\therefore\ 18 \in P$ is true.

(iii) $n(P) = 3$

$n(Q) = 9$

Exercise 19.2

(1) (i) Express each of the following sets with its elements written within curly brackets.

(a) $A = \{$counting numbers that are less than $10\}$
(b) $B = \{$letters in the word "ANURADHAPURA"$\}$
(c) $X = \{$days of the week$\}$
(d) $Y = \{$multiples of 5 between 2 and $8\}$
(e) $P = \{$prime numbers from 32 to 38 $\}$
(f) $Q = \{$the grades in primary school in Sri Lanka$\}$
(g) $M = \{$prime factors of $30\}$

(ii) Write the values of $n(A)$, $n(B)$, $n(X)$, $n(Y)$, $n(P)$, $n(Q)$ and $n(M)$.

(2) Write a set A in terms of a common characteristic of its elements, such that the elements can be identified clearly, where $n(A) = 4$.

(3) Write a set P in terms of a common characteristic of its elements, such that the elements can be identified clearly, where $n(P) = 1$.

19.4 Null set

$A = \{$even prime numbers between 5 and $15\}$
Let us consider the elements of this set.
7, 11 and 13 are the prime numbers between 5 and 15. They are not even prime numbers. Accordingly, A does not consist of any elements. A set such as this, which has no elements, is called the **null set**.

Let us consider each set given below.

$B = \{$whole numbers between 1 and $2\}$
$C = \{$multiples of 10 between 5 and $10\}$
$D = \{$polygons with less than 3 sides$\}$

It is clear that all three sets B, C and D do not have any elements. Therefore, each of them is the null set.

We denote the null set by $\{\}$ or $\varnothing$.
Therefore, since A is the null set, we write $A = \{\}$ or $A = \varnothing$.

Likewise, we write $B = \{\}$ or $B = \varnothing$.
Therefore we can write $A = B = \varnothing$ or $A = B = \{\}$.

Note: The number of elements in the null set is zero. That is, $n(\varnothing) = 0$

Exercise 19.3

(1) Write down whether each of the following sets is the null set or not.

(i) $P = \{$positive multiples of 5 which are less than 5$\}$

(ii) $Q = \{$whole numbers from 0 to 10$\}$

(iii) $R = \{$odd numbers between 1 and 3$\}$

(iv) $S = \{$digits in the number "41242" $\}$

(v) $T = \{$colours of the rainbow$\}$

(vi) $U = \{0\}$

(2) Explain with reasons whether the set {integers such that the square is -1} is the null set.

Miscellaneous Exercise

(1) $M = \{2, 4, 6, 8\}$. Fill in the blanks with $\in$ or $\notin$ as appropriate.

(i) 2 ..... M	(ii) 4 ..... M	(iii) 3 ..... M
(iv) 6 .....M	(v) 7 ..... M	(vi) 8 ..... M

(2) Write down three examples for the null set.

(3) (i) Rewrite each of the following sets by listing the elements within brackets.

(a) $A = \{$prime numbers less than 20$\}$

(b) $B = \{$the letters in the word "university"$\}$

(c) $C = \{$provinces in Sri lanka$\}$

(d) $D = \{$square numbers between 20 and 30$\}$

(e) $E = \{$square numbers which are prime numbers $\}$

(f) $F = \{$whole numbers between 2 and 16 which are divisible by 3 or 5$\}$

(ii) Write the values of $n(A)$, $n(B)$, $n(C)$, $n(D)$, $n(E)$ and $n(F)$ for the above sets.

(4) Write a set P in terms of a common characteristic of its elements, such that the elements can be identified clearly, where $n(P) = 2$.

Summary

- The symbol $\in$ is used to denote that an object belongs to a set.
- The symbol $\notin$ is used to denote that an object does not belong to a set.
- The null set is the set with no element. It is denoted by $\varnothing$ or $\{\}$.
- The notation n(A) is used to denote the number of elements in the set A.

20 Area

By studying this lesson you will be able to,

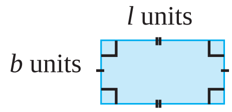
- derive a formula for the area of a triangle,
- solve problems associated with the area of a triangle,
- find the area of composite plane figures, and
- find the surface area of a cube and a cuboid.

20.1 Area

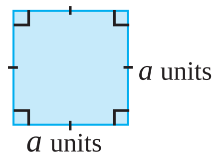
You learnt in Grade 7 that the extent of a surface is called its area.

You also learnt the method of finding the area of a square shaped lamina and of a rectangular shaped lamina.

When the area of a rectangle of length l units and breadth b units is taken as A square units, $A = lb$.



When the area of a square shaped lamina of side length a units is taken as A square units, $A = a^2$.

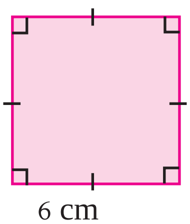


Do the following review exercise to recall these facts.

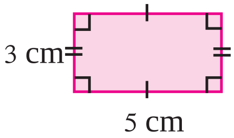
Review Exercise

(1) Find the area of each plane figure given below.

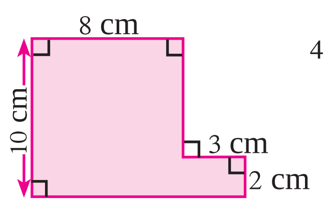
(i)



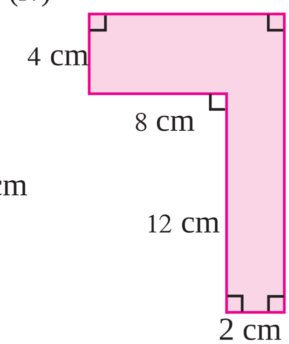
(ii)



(iii)

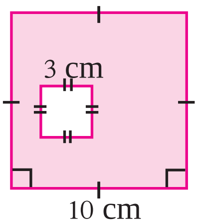


(iv)

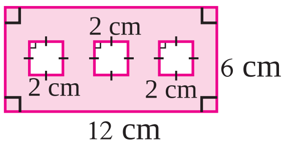


(2) Find the area of the section shaded in pink in each of the plane figures given below.

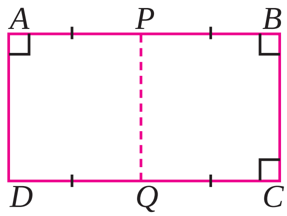
(i)



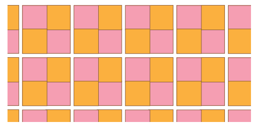
(ii)



(3) The line PQ is drawn such that it divides the area of the rectangle $ABCD$ into two equal parts. Draw three other figures which demonstrate three other ways in which the line PQ can be drawn such that it divides the area of the rectangle $ABCD$ into two equal parts.



(4) The length and breadth of a rectangular floor of a house are 5 m and 3.5 m respectively. This floor is to be tiled with square shaped tiles of side length 25 cm each, without leaving any space between the tiles.



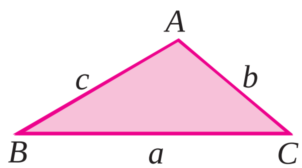
(i) What is the area of a square shaped tile?
(ii) Find the area of the floor.
(iii) How many floor tiles are required to tile this floor?
(iv) If a tile costs Rs. 275, how much money is needed to buy the required tiles?

20.2 Area of a triangle

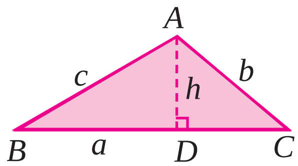
Let us first identify a base of a triangle and the height of the triangle corresponding to that base.

- **Base of a triangle and the height of the triangle corresponding to that base**

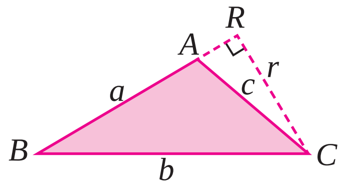
Any side of the triangle ABC can be considered as one of its bases. The way in which the height of the triangle varies according to the base, is explained below.



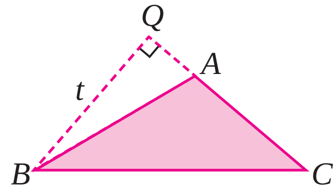
When BC is considered as the base of the triangle ABC, the length of the base is a. In order to find the height of the triangle corresponding to the base BC, a perpendicular line has to be drawn from A to BC. If this perpendicular meets BC at point D, the height of the triangle corresponding to the base BC is the length of AD.



When AB is considered as the base of the triangle, the perpendicular CR should be drawn from C to BA produced, in order to find the height of the triangle corresponding to the base AB. If the length of CR is r, the height of the triangle corresponding to the base AB is r.



According to the above explanation, when CA is considered as the base of the triangle, the height of the triangle corresponding to the base CA is the length of BQ which is t.



- **Area of a right angled triangle**

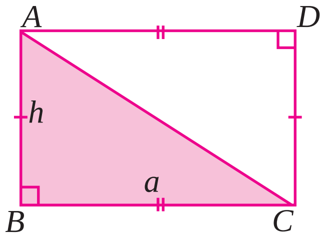
Activity 1

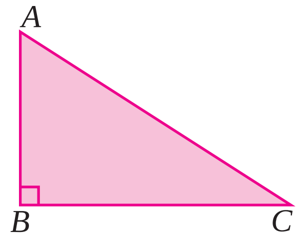
Step 1 - Cut out a rectangular lamina.

Step 2 - Name its vertices as A, B, C and D as shown in the figure.

Step 3 - Join A and C, and cut the lamina along this line. By cutting the rectangular lamina along the straight line AC, two triangles of equal area are obtained.

Step 4 - Find the area of one of these triangles.





The area of the right angled triangle ABC is half the area of the rectangle $ABCD$.

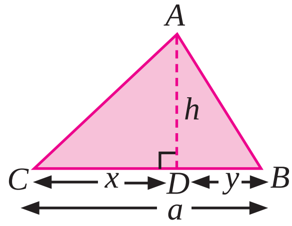
$\therefore$ The area of the right angled triangle ABC $\Big\} = \frac{1}{2} \times$ area of the rectangle $ABCD$

$= \frac{1}{2} \times$ (product of two sides which include a right angle) square units

$= \frac{1}{2} \times (BC \times AB) = \frac{1}{2} \times a \times h = \frac{1}{2}ah$

• Area of a triangle which is not a right angled triangle

➢ Finding the area of the acute angled triangle *ABC* by taking *BC* as the base



To do this, let us draw the perpendicular *AD* from the vertex *A* of the triangle *ABC* to the side *BC*. Now *ADC* and *ADB* are two right angled triangles.

The area of the right angled triangle *ADC* $= \frac{1}{2} \times x \times h$

The area of the right angled triangle *ADB* $= \frac{1}{2} \times y \times h$

The area of the triangle *ABC* = the area of the triangle *ADC* + the area of the triangle *ADB*

$$= \frac{1}{2}xh + \frac{1}{2}yh = \frac{1}{2}h\,(x+y)$$

Since $a = (x+y)$,

$$= \frac{1}{2}h \times a = \frac{1}{2}ah$$

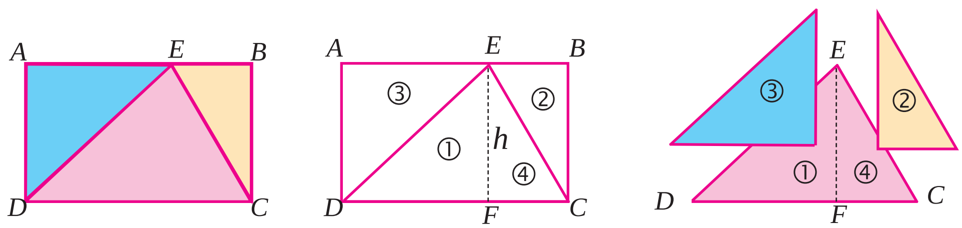
Activity 2

Step 1 - Take a rectangular shaped piece of paper and name it *ABCD* as shown in the figure. Pick any point on the side *AB* and name it *E*.

Step 2 - Join *DE* and *CE*. Then the triangle *DEC* is obtained.

Step 3 - Draw a perpendicular from *E* to *DC* and name the point it meets *DC* as *F*.

Step 4 - Cut the figure along the lines *DE* and *EC*.



Step 5 - Find the area of the triangle *ECD*.

The area of triangle ① is equal to the area of triangle ③.

The area of triangle ② is equal to the area of triangle ④.

∴ Area of rectangle *ABCD* } = Area of rectangle *AEFD* + Area of rectangle *EBCF*

= 2 × area of triangle *DEF* + 2 × area of triangle *ECF*

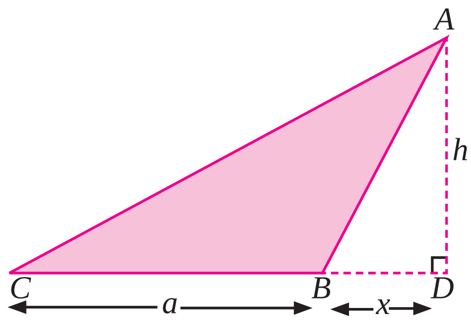
$$\therefore \text{Area of rectangle } ABCD = 2 \times \text{area of triangle } ECD$$

$$\therefore \text{Area of triangle } ECD = \frac{1}{2} \times \text{area of rectangle } ABCD$$
$$= \frac{1}{2} \times DC \times CB$$
$$= \frac{1}{2} \times DC \times EF \ (\text{since } CB = EF)$$

➤ Finding the area of the obtuse angled triangle *ABC* by taking *BC* as the base

The area of triangle $ACD = \frac{1}{2} \times (a + x) \times h$ —— ①

The area of triangle $ABD = \frac{1}{2} \times x \times h$ —— ②



$$\therefore \text{Area of triangle } ABC = \underbrace{\text{Area of triangle } ACD - \text{area of triangle } ABD}$$
$$= \frac{1}{2}(a + x) \times h - \frac{1}{2} \times x \times h$$
$$= \frac{1}{2} h (a + x - x)$$
$$= \frac{1}{2} ha$$
$$= \frac{1}{2} ah$$

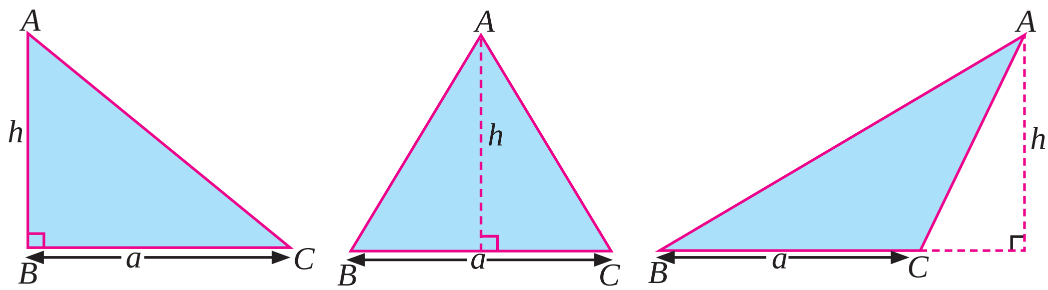
Area of a triangle $= \frac{1}{2} \times$ the length of the base of the triangle $\times$ the perpendicular height of the triangle corresponding to that base

Area of the triangle $= \frac{1}{2} \times$ the length of the base $\times$ height

Note

When selecting the base of a triangle which is not right angled, the perpendicular can be drawn without producing the base, by selecting the side which is opposite the largest angle of the triangle as the base.

The perpendicular drawn from a vertex of a triangle to the opposite side is called as the **altitude** and that opposite side is called as the **base**.



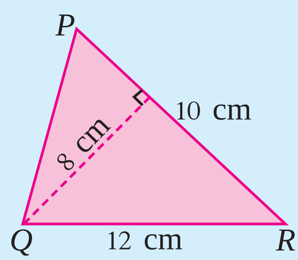
The base of the triangles given above is BC. The perpendicular height (altitude) is marked as h.

The area of the triangle $ABC = \frac{1}{2}ah$

$\therefore$ The area of a triangle $= \frac{1}{2} \times$ base $\times$ perpendicular height (altitude)

Example 1

Find the area of the triangle PQR given in the figure.



The perpendicular is drawn from Q to the side PR.

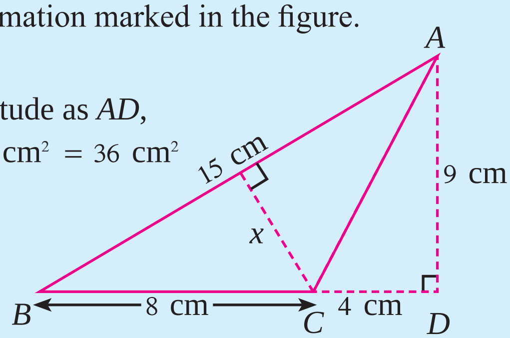
$\therefore$ The base is PR.

$\therefore$ The area of the triangle $PQR = \frac{1}{2} \times 10 \text{ cm} \times 8 \text{ cm}$

$= 40 \text{ cm}^2$

Example 2

Find the value of x according to the information marked in the figure.



When the base is taken as BC and the altitude as AD,
the area of the triangle ABC $= \frac{1}{2} \times 8 \times 9 \text{ cm}^2 = 36 \text{ cm}^2$

When the base is taken as AB and the height is taken as x,

the area of the triangle ABC $= \frac{1}{2} \times 15 \times x \text{ cm}^2$

Therefore, $\frac{1}{2} \times 15 \times x = 36$

$$15x = 36 \times 2$$

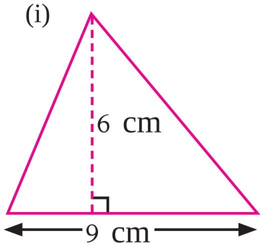
$$x = \frac{36 \times 2}{15}$$

$$\therefore x = 4.8 \text{ cm}$$

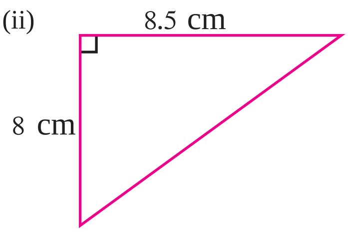
Exercise 20.1

(1) Find the area of each of the triangles given below.

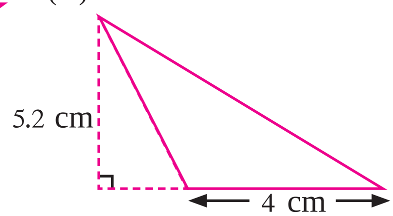
(i)



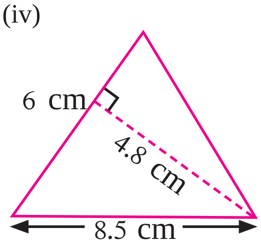
(ii)



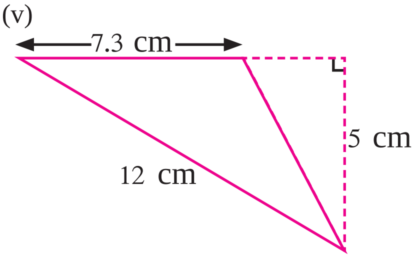
(iii)



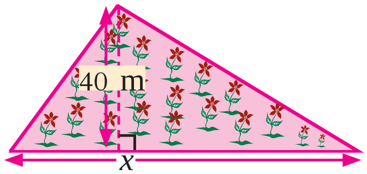
(iv)



(v)

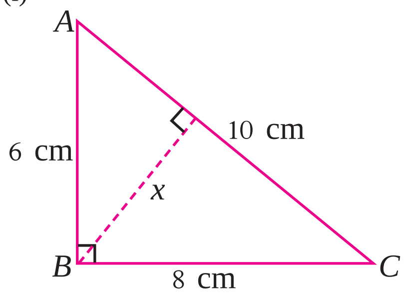


(2) The area of the triangular shaped flower bed in the figure is 800 m^2. Find the length marked as x.

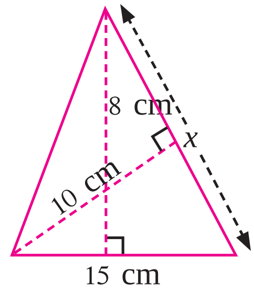


(3) Find the length marked as x in each of the triangles given below.

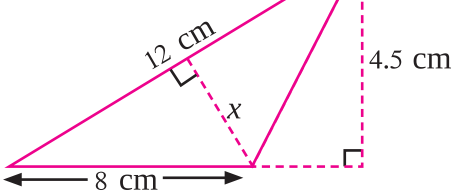
(i)



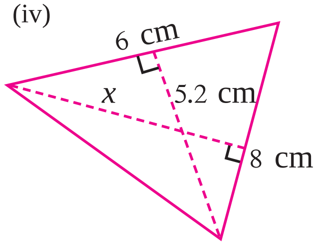
(ii)



(iii)



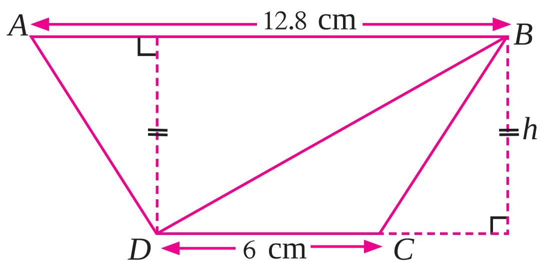
(iv)



(4) The area of the triangle BCD given in the figure is 30 cm^2.

(i) Find the value of h.

(ii) Find the area of the triangle ABD.

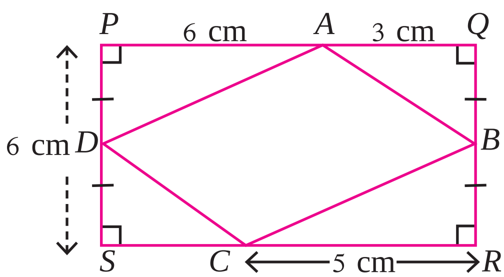


(5) The points A, B, C and D are located on the sides of the rectangle $PQRS$ as indicated in the figure.

(i) Find the area of the rectangle $PQRS$.

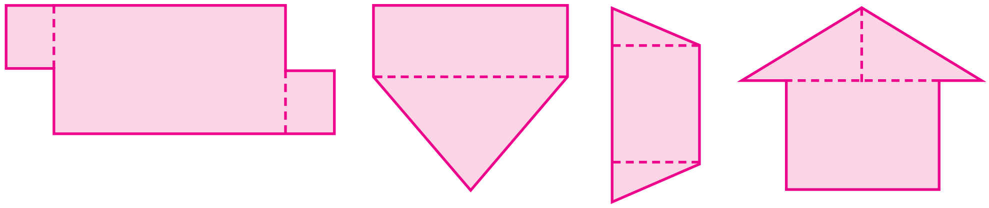
(ii) Find the area of the triangle APD.

(iii) Find the area of the quadrilateral $ABCD$.



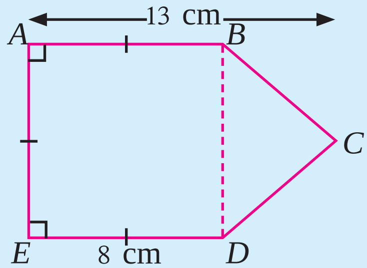
20.3 The area of composite plane figures

When finding the area of a composite plane figure, first divide it into plane figures of which the area can easily be found. Find the area of each of these plane figures and obtain the sum.



Example 1

Find the area of the plane figure *ABCDE* given in the figure.



In this figure, a square and a triangle are obtained by joining *BD*.

The area of *ABDE* $= 8\text{ cm} \times 8\text{ cm} = 64\text{ cm}^2$

The perpendicular distance from *C* to *BD* $= (13 - 8)\text{ cm} = 5\text{ cm}$

$\therefore$ The area of the triangle *BCD* $= \frac{1}{2} \times 8 \times 5\text{ cm}^2$

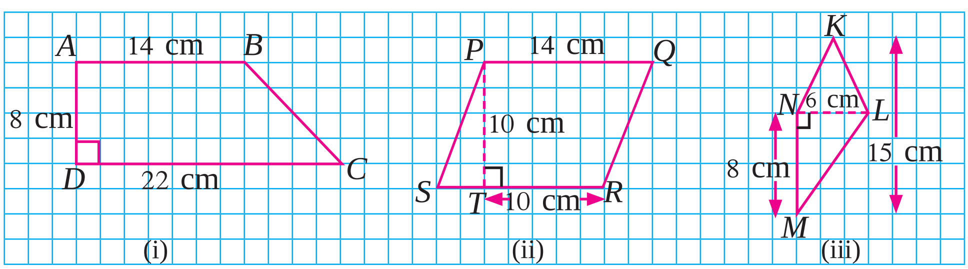
$= 20\text{ cm}^2$

$\therefore$ The area of the whole figure $= 64 + 20\text{ cm}^2$

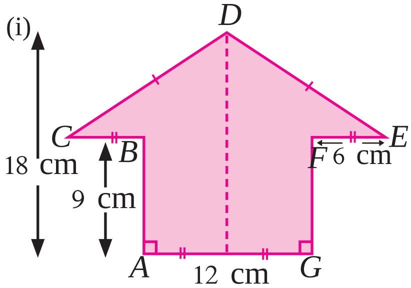
$= 84\text{ cm}^2$

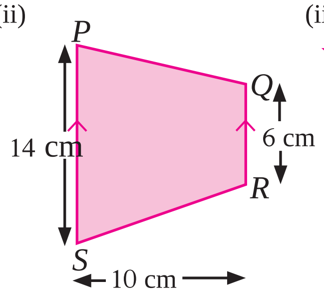
Exercise 20.2

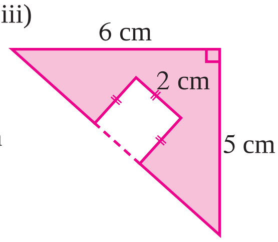
(1) Find the area of each of the plane figures given below.

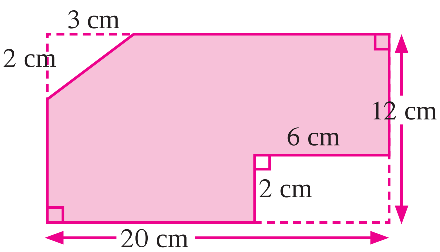


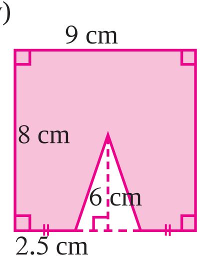
(2) Find the area of the shaded section in each of the figures given below.

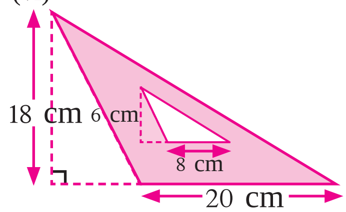
(i)


(ii)


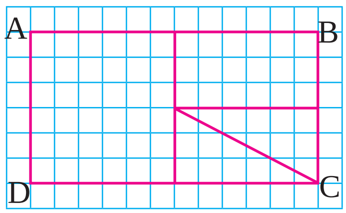
(iii)


(iv)


(v)


(vi)


(4) (i) Copy the rectangle *ABCD* given in the figure onto a coloured paper and cut and separate out the four marked sections.



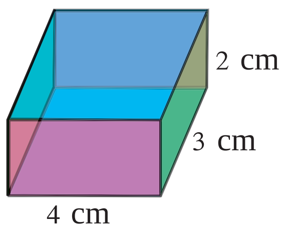
(ii) Construct a composite plane figure using all four sections.

(iii) Cut two other rectangular shaped laminas as above and construct two more composite plane figures and paste them in your exercise book.

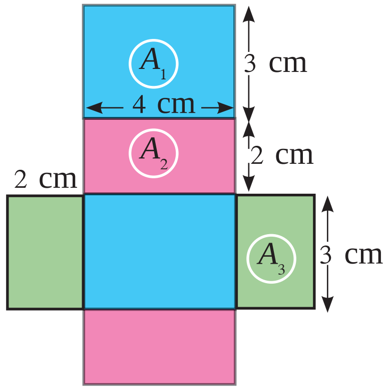
(iv) Write the relationship between the area of each composite plane figure that was constructed and the area of the original rectangular shaped lamina that was used.

20.4 The surface area of a cube and of a cuboid

Let us find the surface area of the cuboid shaped parcel shown in the figure.

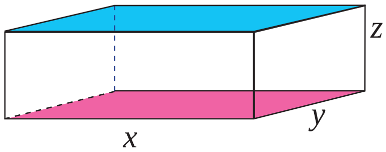


The area of face $A_1 = 4\ \text{cm} \times 3\ \text{cm} = 12\ \text{cm}^2$
The area of face $A_2 = 4\ \text{cm} \times 2\ \text{cm} = 8\ \text{cm}^2$
The area of face $A_3 = 2\ \text{cm} \times 3\ \text{cm} = 6\ \text{cm}^2$
$\therefore$ The total surface area $= 2 \times 12 + 2 \times 8 + 2 \times 6\ \text{cm}^2$
$= 24 + 16 + 12\ \text{cm}^2$
$= 52\ \text{cm}^2$

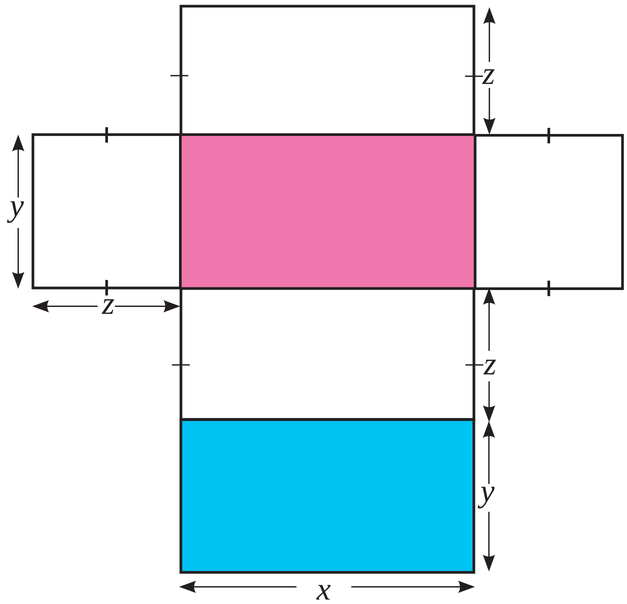


$\therefore$ The total surface area of the cuboid shaped parcel $= 52\ \text{cm}^2$

A cuboid of length, breadth and height equal to x, y and z units respectively, and its net are shown in the given figures.



By observing these figures, it is clear that the base which is coloured pink and the top surface which is coloured blue are equal in area. This feature can be identified by observing any cuboid shaped object such as a brick too.



Accordingly, a cuboid has three pairs of rectangular shaped faces, where each pair is of equal area.

Let us find the surface area of the cuboid by finding the area of each pair of faces which is equal in area.

The area of the base $= xy$

The area of a lengthwise face $= xz$

The area of a breadth-wise face $= yz$

The total surface area $= 2\,xy + 2\,xz + 2\,yz$

$= 2\,(xy + xz + yz)$

Activity 3

(i) Draw a figure of a cube of side length a units and obtain an expression for its surface area in terms of a.

(ii) Obtain an expression for the surface area of a cuboid of length, breadth and height equal to a, b and h units respectively, in terms of a, b and h.

According to the above activity you must have obtained that;

the surface area of a cube of side length a units is $6a^2$ square units, and if the surface area of a cuboid of length, breadth and height equal to a, b and h units respectively is A, then

$$A = 2\,(ab + bh + ah) \text{ square units.}$$

Example 1

Find the minimum quantity of cardboard needed to construct a box the shape of a cuboid, of length, width and height equal to 20 cm, 15 cm and 10 cm respectively.

Here, the minimum quantity of cardboard required is equal to the area of the 6 surfaces of the box.

The area of the 6 surfaces $= 2\,(20 \times 15 + 20 \times 10 + 15 \times 10)$ cm^2

$= 2\,(300 + 200 + 150)$ cm^2

$= 2 \times (650)$ cm^2 $= 1300$ cm^2

$\therefore$ The minimum quantity of cardboard needed $= 1300$ cm^2

Example 2

The height, width and thickness of a door panel are 180 cm, 80 cm and 2 cm respectively. If it costs Rs. 5 to paint 100 cm^2 of the panel, find the total amount of money needed to paint the whole panel.

The surface area of the door panel $= 2\,(180 \times 80 + 180 \times 2 + 80 \times 2)$ cm^2

$= 2\,(14\,400 + 360 + 160)$ cm^2

$= 2\,(14\,920)$ cm^2

$= 29\,840$ cm^2

The total cost of painting the door panel at Rs. 5 per 100 cm^2 $=$ Rs. $\dfrac{29840}{100} \times 5$

$=$ Rs. 1492

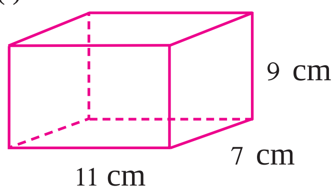
Exercise 20.3

(1) Find the surface area of a cube of side length 10 cm.

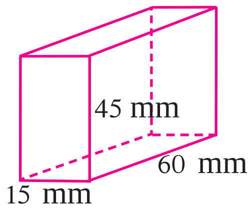
(2) Find the surface area of a cuboid of length, breadth and height equal to 12 cm, 8 cm and 5 cm respectively.

(3) Find the surface area of each of the cuboid shaped solids shown below.

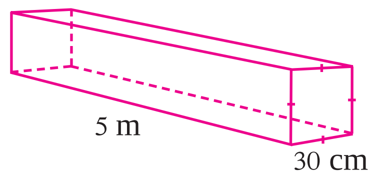
(i)



(ii)

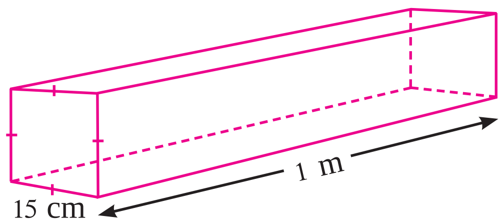


(iii)



(4) It is required to construct a cube shaped iron box without a lid. If the length of a side is 15 cm, find the minimum amount (in cm^2) of iron sheet needed to construct the box.

(5) The measurements of a cuboid shaped wooden pole are given in the figure. Find its surface area.

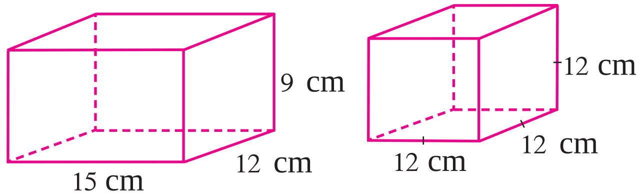


(6) The length, breadth and height of a cuboid shaped closed box are 15 cm, 15 cm and 8 cm respectively.

(i) Draw a sketch of two different faces of this box with its measurements.

(ii) Show that the total surface area of the box is 930 cm^2.

(7) Two cube and cuboid shaped wooden blocks are shown in the figure. Anil says that the amount of paint needed to paint these two blocks are equal. Explain whether you agree or disagree with this statement.



(8) Separately write down the length, width and height of two cuboids of different measurements having the same surface area of 220 cm^2.

Summary

- The area of a triangle $= \frac{1}{2} \times \text{base} \times \text{perpendicular height}$
- The surface area of a cube of side length a units is $6a^2$ square units.
- The total surface area of a cuboid of length, width and height equal to a, b and h units respectively is $2ab + 2ah + 2bh$ square units or $2(ab + ah + bh)$ square units.

21 Time

By studying this lesson you will be able to,
- understand the reason for the difference in time in two different places on earth at the same instant, depending on their locations,
- calculate the standard time at a given location using time zones, and
- identify the International Date Line and understand the change of date associated with it.

21.1 Introduction

Given below is a news item published in a newspaper.

News

The next ODI between England and Sri Lanka begins at 2.30 p.m. England time at Lord's Cricket Ground and will be telecasted live. You will be able to watch this match from 8.00 p.m. onwards Sri Lankan time.

England
2.30 p.m.

Sri lanka
8.00 p.m.

According to the above news item, the time in Sri Lanka is 8.00 p.m. when the time in England is 2.30 p.m. on the same day.

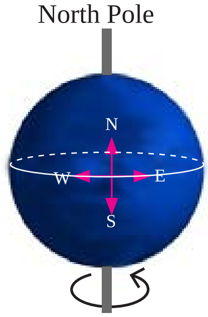
It is clear that the time can be different in two different places on earth at the same instant.

Let us consider why the time is different at different locations in the world at the same instant.

The earth is a spherical object. Land and oceans cover its surface.

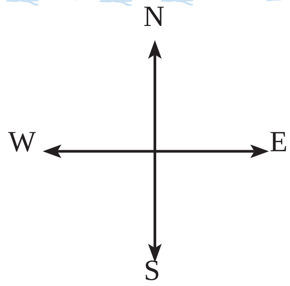
The earth completes one rotation around its axis every 24 hours. This axis is a diameter of the earth. The two end points of the axis are called the North Pole and the South Pole.

North Pole

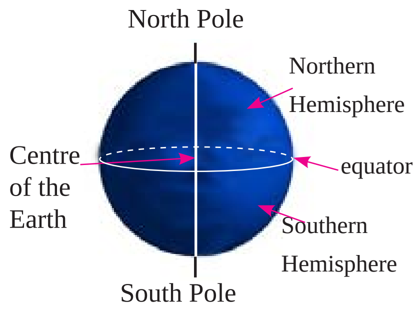


South Pole

The direction in which we observe the sunrise is called the East and the opposite direction is called the West. The direction towards the North Pole is called the North and the direction towards the South Pole is called the South.

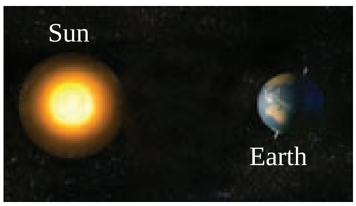


The hemisphere with the North Pole as its topmost point is called the **Northern Hemisphere**, and the hemisphere with the South Pole as its topmost point is called the **Southern Hemisphere**. The imaginary circle on the surface of the earth which separates these two hemispheres is called the **Equator**.



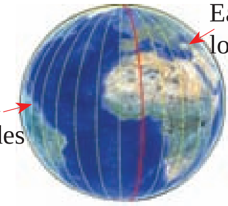
The centre of the Equator is the same as the centre of the earth. The imaginary circles on the earth's surface which are parallel to the Equator are called lines of constant latitude. Latitude is an angle which ranges from 0^o at the Equator to 90^o (North or South) at the poles. It is used to specify the location of a place on earth.

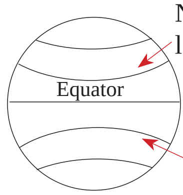
When the earth rotates around its axis, the side exposed to the sun receives sunlight and hence experiences daytime while the other side experiences nighttime. Therefore, the time in different places on earth may be different at the same instant.



21.2 Longitudes

An imaginary semicircle on the surface of the earth, connecting the North Pole and the South Pole, with the same centre as the centre of the earth is called a line of longitude. Longitude is an angular distance usually measured in degrees which is used to specify the East-West position of a location. Longitudes vary from 0^o to $+180^o$ eastward and from 0^o to -180^o westward from the 0^o line of longitude which passes through Greenwich, England.

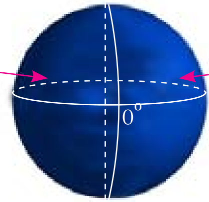




Note
The line of longitude which passes through Greenwich, England is called the **Greenwich Meridian**. The longitude along this line is 0^{o}.

The longitudes between 0^{o} and $+180^{o}$, east of the Greenwich Meridian are called **eastern longitudes** and the longitudes between 0^{o} and -180^{o}, west of the Greenwich Meridian are called **western longitudes**.

The hemisphere with the eastern longitudes and the hemisphere with the western longitudes are called the **Eastern Hemisphere** and the **Western Hemisphere** respectively.



For example, the longitude 23^{o} due east of longitude 0^{o} is written as 23^{o}E and the longitude 105^{o} due west of longitude 0^{o} is written as 105^{o}W.

$$\left.\begin{array}{l}\text{The time taken by the earth to make one complete rotation}\\ (360^{o}) \text{ around its axis}\end{array}\right\} = 1 \text{ day}$$

$$= 24 \text{ hours}$$

$$= 24 \times 60 \text{ minutes}$$

$$\text{The time to rotate } 1^{o} = \frac{24 \times 60}{360} \text{ minutes}$$

$$= 4 \text{ minutes}$$

The time at any location situated on a particular line of longitude is the same.

The time difference between two locations which are 1^{o} of longitude apart from each other is 4 minutes. For example, the time difference between the lines of longitude 20^{o}E and 21^{o}E is 4 minutes.

One rotation of the earth around its axis means a movement of 360^{o}. The earth takes 24 hours for it.

$$\therefore \text{ the number of degrees the earth rotates in an hour} = \frac{360^{o}}{24}$$

$$= 15^{o}$$

Note
The time difference between 1° of longitude is 4 minutes. It takes 1 hour for the earth to rotate 15°. Therefore, the earth is divided into 24 times zones where each time zone is bounded by two lines of longitude 15° apart.

When comparing the time in the Eastern Hemisphere with the Greenwich Meridian Time, the time increases by 4 minutes per degree of longitude, because the sun rises earlier in the east due to the rotation of the earth from west to east. Similarly, the time in the Western Hemisphere decreases by 4 minutes per degree of longitude, from the Greenwich Meridian Line towards the west.

21.3 Local time

The time at different locations in the world is calculated based on the longitude of the location and the time along the Greenwich Meridian. This is called the **local time** of that location.

Assume that Colombo is located at longitude 80°E. Let us find the local time in Colombo when Greenwich time is 6:00.

$$\text{The time difference for } 15^\circ \text{ of longitude} = 1 \text{ hour}$$
$$\text{The time difference for } 80^\circ \text{ of longitude} = \frac{1}{15} \times 80 \text{ hours}$$
$$= 5\frac{1}{3} \text{ hours}$$
$$= 5 \text{ hours and } 20 \text{ minutes}$$

Since Colombo is located east of the Greenwich Meridian, we have to add the above time to Greenwich time.

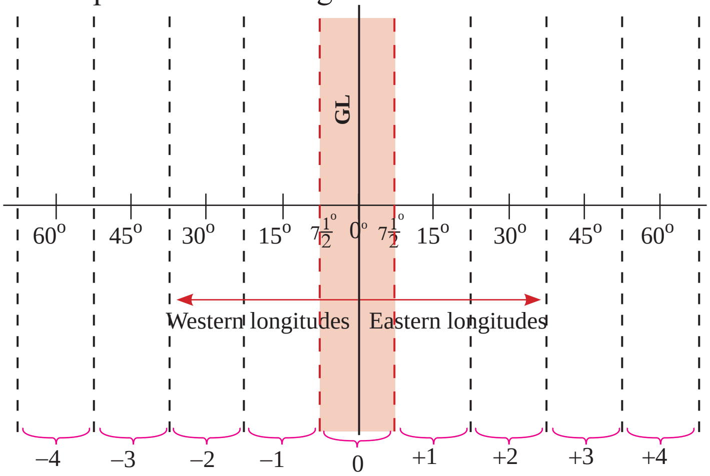
$$\text{The local time in Colombo} = 06:00 + 5 \text{ hours and } 20 \text{ minutes}$$
$$= 11:20.$$

If Batticaloa is considered to be situated at longitude 81° E, we can obtain the local time in Batticaloa as 11:24 when Greenwich time is 6:00 by using the fact that an increase of 1° of longitude results in an increase in time of 4 minutes, or by calculating the time as above.

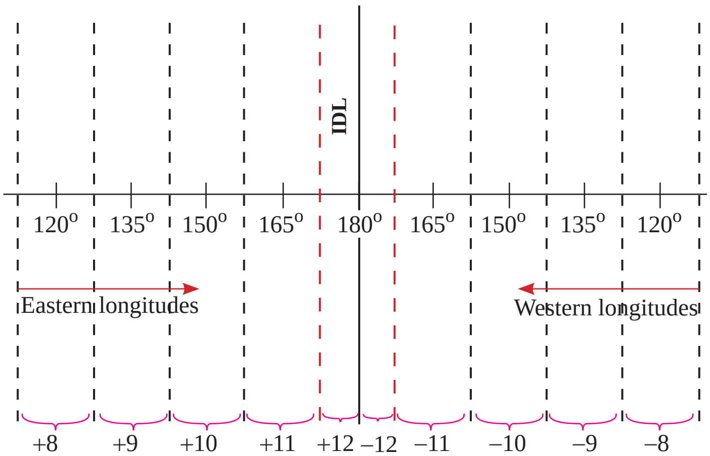
21.4 Standard time based on time zones

As we have already observed, the local time can vary from one city to another city even in a small country like Sri Lanka. It is not practical to have different times at different places in the same country when the country is not too large. To avoid this situation, the earth's surface is divided into several time zones. The time of every location within a time zone is considered to be the same at any given moment.

The earth's surface is divided into several time zones which stretch from the North Pole to the South Pole as shown in the following figure. It is convenient to mark lines of longitude as parallel lines in figures.



- By taking the Greenwich Meridian as the centre line, the region between $7\frac{1}{2}^{\circ}$ W and $7\frac{1}{2}^{\circ}$ E is named the 0 time zone.
- The 11 regions between lines of longitude placed 15º apart, from $7\frac{1}{2}^{\circ}$ E to $172\frac{1}{2}^{\circ}$ E are named the +1 time zone, +2 time zone, +3 time zone, … , + 11 time zone respectively and the region between $172\frac{1}{2}^{\circ}$ E and 180º E is named the +12 time zone.
- The 11 regions between the lines of longitude placed 15º apart from $7\frac{1}{2}^{\circ}$ W to $172\frac{1}{2}^{\circ}$ W, are named the –1 time zone, –2 time zone, –3 time zone, … , –11 time zone respectively and the region between $172\frac{1}{2}^{\circ}$ W and 180º W is named the –12 time zone.



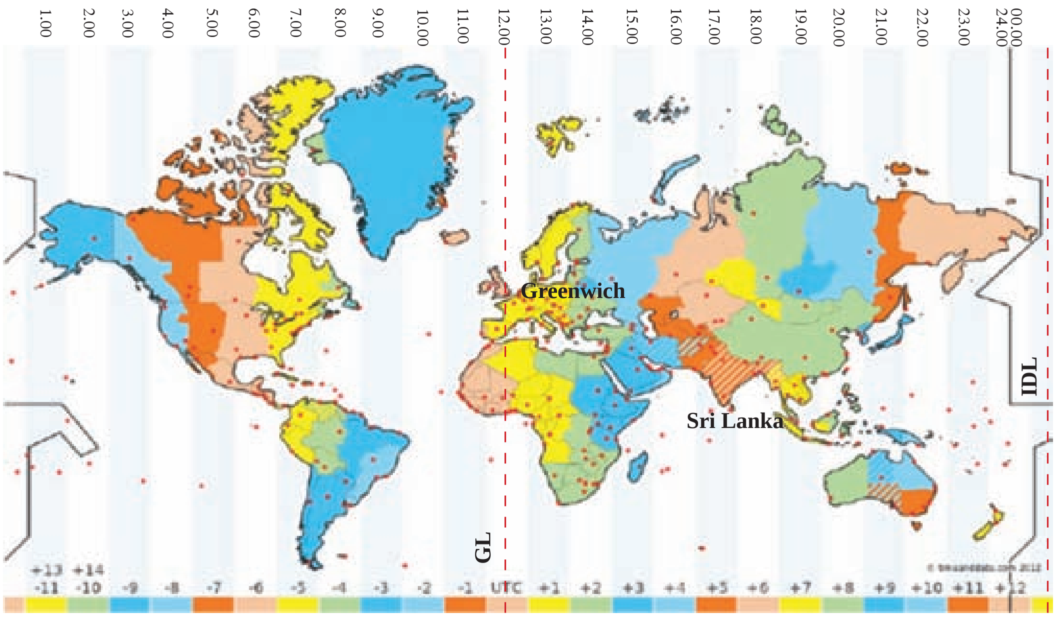
Except for a few special situations,

- at a given moment, every place in a particular time zone has the same time.
- the time in a time zone which is adjacent to another time zone west of it, is one hour ahead of the time in the adjacent time zone. The time in a time zone adjacent to another time zone east of it is one hour behind the time in the adjacent time zone.

- The time in Greenwich city at any particular moment is known as the **Greenwich Mean Time** (GMT) of that moment.
- If the GMT of a particular moment is known, the time at any location in the world can easily be calculated. GMT is often used to express global time.
- • When the time in Greenwich is 11.30 a.m. on Sunday the time in the time zone +12 is 11.30 p.m. on Sunday and the time in the –12 time zone is 11.30 p.m. on Saturday (previous day). Hence the time difference between the two zones +12 and –12 is 24 hours.

• International Date Line

180°W and 180°E are the same longitude. Since the time in the zones +12 and –12 differ by 24 hours, the International Date Line (IDL) is drawn such that it avoids most of the countries so that the date in two locations of the same country will not be different.



A person who travels from the east to the west across the International Date Line gains an additional day since the current day changes into the previous day across the IDL.

Moreover, a person who travels from the west to the east across the IDL loses a day since the current day changes to the next day across the IDL.

Large countries such as the USA and Australia fall into several time zones. The time in the city of Los Angeles in the USA is 4 hours behind the time in Washington DC which is located to its east.

The difference between the time in the city of Greenwich and the time at any other place, which depends on which time zone the place is located in, is mentioned in the world map given above.

Since India, the largest country situated close to Sri Lanka belongs to both the time zones +5 and +6, the difference between Greenwich time and the time at any place in India is taken to be $5\frac{1}{2}$ hours. That is, the Indian Standard Time is $5\frac{1}{2}$ hours ahead of the GMT. Although Sri Lanka belongs to the time zone +5, Sri Lanka also considers the Indian Standard Time as the Sri Lanka Standard Time due to the ease of maintaining international connections.

Example 1

Find the time in Sri Lanka when Greenwich time is 3.24 p.m. on Monday.

Method I

Greenwich time $= 15 : 24$

The time zone to which Sri Lanka belongs $= +5\frac{1}{2}$

Time difference between Greenwich and Sri Lanka $= \left(+5\frac{1}{2}\right) - (0)$

$= \left(+5\frac{1}{2}\right)$

$\therefore$ the time in Sri Lanka $= 15 : 24 + 5$ hours and 30 minutes

$= 20 : 54$ (same day)

The time in Sri Lanka is 20:54 or 8.54 p.m. on Monday.

Method II

	Greenwich	Sri Lanka
Time zone	0	$+5\frac{1}{2}$
Time	15 : 24	?

The time in Sri Lanka = 15 : 24 + 5 hours and 30 minutes

= 20 : 54

The time zone of some of the key cities in the world and the time in those cities when Greenwich time is 12:00 is shown in Table 21.1.

Country/ City	Time Zone + / −	Time in that country	Country/ City	Time Zone + / −	Time in that country
England (London)	0	12:00	Australia (Sydney)	+10	22:00
Bangladesh (Daka)	+6	18:00	Japan (Osaka)	+9	21:00
Thailand (Bangkok)	+7	19:00	Italy (Rome)	+1	13:00
India (Bombay)	+5 1/2	17:30	West indies (Trinidad)	−4	08:00
USA (Los Angeles)	− 8	04:00	Philippines (Manila)	+8	20:00
Sri Lanka (Colombo)	+5 1/2	17:30	Nepal (Thimphu)	+6	18:00
Pakistan (Karachi)	+ 5	17:00	Kuwait (Kuwait)	+ 3	15:00
Malaysia (Kuala Lum- pur)	+ 8	20:00	Norway (Oslo)	+1	13:00

table 21.1

If the time and date of a place A in a particular time zone is known, let us consider how to find the time and date of another place B in a different time zone.

If the time at A according to the 24 hour clock is t and the time difference between A and B is n hours,

Step 1: Time at $A = t$

Step 2: $n = \underset{\text{(as a directed number)}}{\text{Time zone of } B} - \underset{\text{(as a directed number)}}{\text{Time zone of } A}$

Step 3: $T = t + n$

Note

- If T is less than or equal to +24, the time at B is T according to the 24 hour clock, on the same day.
- If T is greater than or equal to +24, the time at B is T –24 according to the 24 hour clock, on the same day.
- If T is 0 or negative, the time at B is 24 + T according to the 24 hour clock, on the previous day.

Example 2

Find the time in Trinidad, West Indies, when Greenwich time is 3.24 p.m. on Monday.

Method I

Greenwich time = 15 : 24

The time zone to which Trinidad belongs = – 4

Time difference between Greenwich and Trinidad = (–4) – 0

= (–4)

$\therefore$ the time in Trinidad = 15 : 24 – 4 hours

= 11 : 24

The time in Trinidad is 11:24 on Monday or 11.24 a.m.

Method II

–4	0
Trinidad	Greenwich
?	15 : 24

The time in Trinidad = 15 : 24 – 4 hours

= 11 : 24

Example 3

Calculate the time in Chile when the time in Sri Lanka is 1.15 a.m. on 2017-08-15.

The time in Sri Lanka = 01:15

The Chile time zone = -5

	Years	Month	Date	Hours	Minutes
	2017	8	15	1	15
$-$				10	30
	2017	8	14	14	45

Method I

The time difference between the two countries $= (-5) - \left(+5\frac{1}{2}\right)$

$$= \left(-10\frac{1}{2}\right)$$

$\therefore$ the time in Chile $= 01:15 - 10$ hours and 30 minutes

$= -9:15$ (previous day)

$= 24 + (-9:15)$

$= 24:00 - 9:15$

$= 14:45$

Therefore, the time in Chile is 14:45 or 2.45 p.m. on 2017-08-14.

Method II

-5	0	$+5\frac{1}{2}$
Chile	Greenwich	Sri Lanka
14 : 45	19 : 45	01 : 15
2017 - 08 - 14	2017 - 08 - 14	2017 - 08 - 15

Example 4

Calculate the time in Sydney when the time in Sri Lanka is 9.15 p.m. on 2017-08-15.

The time in Sri Lanka = 21 : 15

The Sydney time zone = $+10$

	Years	Month	Date	Hours	Minutes
	2017	8	15	21	15
$+$				4	30
	2017	8	16	1	45

Method I

The time difference between Sydney and Sri Lanka $= (+10) - \left(+5\frac{1}{2}\right)$

$$= \left(+4\ \frac{1}{2}\right)$$

$\therefore$ the time in Sydney $= 21:15 + 4$ hours and 30 minutes

$= 25:45$ (next day)

$= 25:45 - 24:00$

$= 01:45$

Therefore, the time in Sydney is 01:45 on 2017-08-16.

Method II

	0	$+5\frac{1}{2}$	+10
	Greenwich	Sri Lanka	Sydney
		21 : 15	01 : 45
		2017 - 08 - 15	2017 - 08 - 16

The time in Sydney = 21 : 15 + 4 hours 30 minutes
= 01 : 45 on 2017-08-16.

Note

- USA, Australia and countries in Europe receive sunlight for more than 12 hours a day during the summer period. During these times, because the sun rises early in these countries, daytime is increased by advancing the clock forward by an hour.
- This time period (Daylight Saving Time – DST) is applicable to countries in the Northern Hemisphere from mid March to early November and to countries in the Southern Hemisphere from early October to early April.
- During these periods, the clock is turned forward one hour, so 1 hour should be added to the usual time.

Exercise 21.1

(1) Complete the table given below stating the time in each time zone when the time in the 0 time zone is 12 noon.

Time zone	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+11	+12
Time	12:00											

Time zone	−12	−11	−10	−9	−8	−7	−6	−5	−4	−3	−2	−1	0
Time													12:00

(2) Write the time and the date in each time zone when Greenwich time is 18:00 on Friday 2016-08-19.

Time zone	−11	−6	−3	0	+4	+7	+10	+11
Time				18:00				
Date				Friday, 2016-08-19				

(3) When the time in Bangkok in the +7 time zone is 16:00, find;
  (i) the time in Auckland, New Zealand in the +12 time zone.
  (ii) the time in Athens, Greece in the +2 time zone.
  (iii) the time in Trinidad in the –4 time zone.

(4) When the time in Nuuk, Greenland in the –3 time zone is 01:00 on 2016-10-20, find;
  (i) the time and date in Chicago in the –6 time zone.
  (ii) the time and date in Bangkok in the +7 time zone.

(5) When the time in Vancouver, Canada in the –8 time zone is 18:00 on 2016-10-29, find;
  (i) the time and date in Greenwich.
  (ii) the time and date in Abu Dhabi in the +4 time zone.

(6) When the time in Philippines in the +8 time zone is 19:00 on 2016-11-02, find;
  (i) the time and date in a country in the +12 time zone.
  (ii) the time and date in a country in the –12 time zone.
  (iii) the time and date in Honolulu located in the –10 time zone.

(7) When the time in Sri Lanka is 09:30 on 2017-05-02, find the time and date in Los Angeles located in the –8 time zone in the USA.

(8) An aeroplane takes off from Dubai located in the +4 time zone at 13:00. It arrives in Manila in Philippines, located in the +8 time zone at 20:00.
  (i) Find the time in Manila when the aeroplane departs from Dubai.
  (ii) Find the time duration of the flight.
  (iii) What is the time in Dubai when the aeroplane arrives in Manila?

Miscellaneous Exercise

(1) Sri Lanka is located in the $+5\frac{1}{2}$ time zone. Dileepa who departs from the Katunayaka airport at 14:30 Sri Lankan time, travels to Trinidad in West Indies through London.
  (i) He arrives in London after a journey of 6 hours. What is the time shown on his wrist watch which indicates Sri Lankan time?
  (ii) If London is located in the 0 time zone, what is the time in London when the flight reaches there?

(iii) After adjusting the time on his wrist watch according to the time in London, Dileepa departs for West Indies by another flight. If he spends one hour at the London Airport prior to departure, and it takes 3 hours for the journey, what is the time in West Indies in the –4 time zone when he reaches there?

(2) An aeroplane departs from Dawson located in the –8 time zone at 6:00 a.m. on Monday, flies across the IDL and arrives in Tokyo (Japan) in the +9 time zone. If the flight arrives in Tokyo at 4.00 p.m. on Tuesday, find the time taken for the journey.

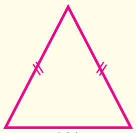
(3) Singapore is located in the +8 time zone. An aeroplane departs from Singapore at 3.00 p.m. (15:00) on Monday and travels across the IDL to Honolulu located in the –10 time zone. If it takes 12 hours for the journey, find the local time and date when it arrives in Honolulu.

Summary

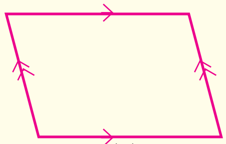
- The 0° longitude line which passes through Greenwich, England is called the Greenwich Meridian.
- The 0 time zone is a region of 15° of longitude with $7\frac{1}{2}^{\circ}$ of longitude lying to the west of the Greenwich Meridian and $7\frac{1}{2}^{\circ}$ of longitude lying to its East.
- Sri Lanka is located in the $+5\frac{1}{2}^{\circ}$ time zone and therefore the time in Sri Lanka is 5 hours and 30 minutes ahead of Greenwich Mean Time.
- The time in the + time zones is ahead of Greenwich Mean Time and the time in the – time zones is behind Greenwich Mean Time.
- The date changes by one day across the International Date Line.

Revision Exercise 2

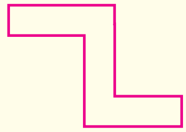
(1)



(A)



(B)



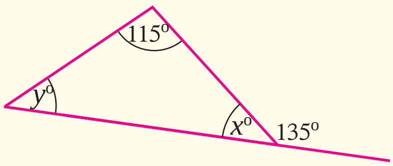
(C)

(i) Which of the above plane figures *A, B, C* have bilateral symmetry?

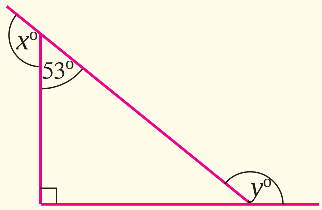
(ii) Which of them have rotational symmetry?

(2) Find the values of the angles represented by x and y in each of the following figures.

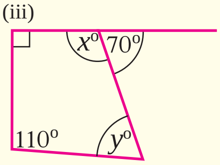
(i)



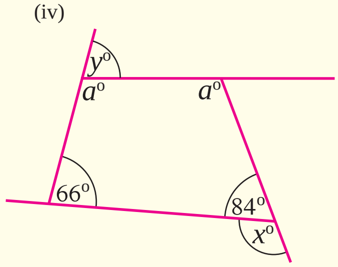
(ii)



(iii)



(iv)

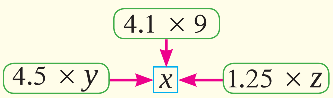


(3) Simplify.

(i) $\frac{3}{5} \times \frac{20}{27}$ (ii) $1\frac{3}{7} \times 14$ (iii) $12 \times 2\frac{3}{8}$ (iv) $4\frac{1}{6} \times 1\frac{3}{5}$

(v) $\frac{6}{7} \div \frac{2}{3}$ (vi) $\frac{7}{12} \div 1\frac{3}{4}$ (vii) $3\frac{2}{11} \div 2\frac{1}{7}$ (viii) $16 \div 4\frac{4}{7}$

(4) Find suitable values for x, y and z in the following chart, where x is the product of three different pairs of numbers.



(5) The mass of a box of biscuits is 1.02 kg. Find the mass of 15 such boxes.

(6) The cost of 1 m of cloth is Rs. 52.75. What is the cost of 12.5 m of this cloth?

(7) The length of a reel of lace is 18.6 m. If it is cut into 6 equal parts, what would be the length of each strip?

(8) What is the maximum number of pieces of rope of length 12.27 m that can be cut from a rope of length 137.43 m?

(9) A golden thread is pasted around the rectangular wall hanging shown in the figure.

(i) What is the total length of the thread that has been pasted?

(ii) Find the minimum length of the thread needed to create 16 such wall hangings.

(iii) If the price of 1m of this thread is Rs. 12.80, find the amount of money needed to buy thread for the above 16 wall hangings.

(10) If A : B = 4 : 3 and B : C = 6 : 5 find A : B : C.

(11) The following table shows the ratios according to which wheat flour, sugar and margarine are mixed to make a type of sweetmeat which is produced by the two companies named *P* and *Q*.

Ratio / Company	wheat flour : sugar	sugar : margarine
P	2 : 1	3 : 2
Q	3 : 2	5 : 4

(i) Find the ratio of wheat flour : sugar: margarine in the sweetmeat made by company *P*.

(ii) Find the ratio of wheat flour : sugar: margarine in the sweetmeat made by company *Q*.

(iii) With reasons indicate which company produces the sweetmeat that tastes sweeter.

(12) When 7 is added to 3 times the answer that is obtained when 2 is subtract from 5 times the number denoted by *x*, the result is 61.

(i) Construct an equation using the above information.

(ii) Solve the equation that was constructed.

(13) The mass of a packet of a certain sweetmeat is *m* grammes. 12 such packets are packed in a box of mass 300g. The total mass of 3 boxes packed as above is $13\frac{1}{2}$ kg. Find the mass of a packet of sweetmeat by constructing an equation and solving it.

(14) Write the following fractions and ratios as percentages.

(i) $\frac{3}{5}$ (ii) $\frac{80}{150}$ (iii) $\frac{1500}{4500}$ (iv) 3 : 2 (iv) 3 : 5

(15) 60% of the students in a class went on a trip. If the total number of students in the class is 45, how many students did not go on the trip?

(16) A bank charges interest of Rs. 10 750 per annum for a loan of Rs. 75 000. Write the interest as a percentage of the loan amount.

(17) 16% of the eggs that were being transported in a vehicle cracked due to an unavoidable circumstance. The number of eggs that cracked was 208.

(i) Find the total number of eggs that were being transported.

(ii) How many eggs did not crack?

(18) $A = \{2, 3, 5, 7, 11, 13\}$

$B = \{$ letters of the word "*POLONNARUWA*" $\}$

$C = \{3, 6, 9, 12, 15\}$

(i) Fill in the blanks using the suitable symbol from $\in$ and $\notin$.

- 5 ..... A
- 9 C
- 18 C
- N ..... B
- 17 ..... A
- B ..... B

(ii) Write down the values of $n(A)$, $n(B)$ and $n(C)$.

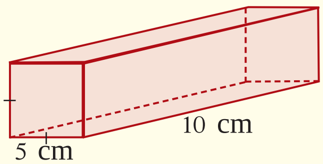
(19) $D = \{$even prime numbers greater than 10$\}$

(i) Write down the set D.

(ii) What is the value of $n(D)$?

(iii) Write the special name given for the set D.

(20) (a) The surface area of a cube is 150 cm^2. Find the length of an edge of the cube.



(b) (i) Find the surface area of the cuboid shaped block of wood shown in the figure.

(ii) The above cuboid shaped block of wood is cut so that two equal cubes are formed. What is the surface area of each one of them?

(iii) Indicate whether the surface area of each of these cubes is exactly half the surface area of the cuboid, according to the answers to (i) and (ii) above.

22 Volume and Capacity

By studying this lesson you will be able to

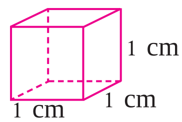
- obtain formulae for the volume of a cube and a cuboid,
- find the volume of a cube and a cuboid by using the formulae,
- solve problems relating to volumes,
- identify what volume and capacity are, and
- estimate capacities.

22.1 Volume

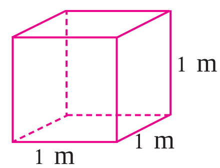
Let us recall the facts you learnt on volume in Grade 7.

The amount of space occupied by an object is called its volume. Cubic centimetre and cubic metre are two units that are used to measure volumes.

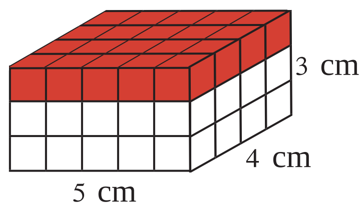
The volume of a cube of side length 1 cm is 1 cubic centimetre ($1\ \text{cm}^3$).



The volume of a cube of side length 1 metre is used as a unit to measure larger volumes. Its volume is 1 cubic metre ($1\ \text{m}^3$).



The upper layer of the cuboid shown in the figure consists of $5 \times 4 = 20$ small cubes. Since there are three such layers, there are $20 \times 3 = 60$ small cubes.



Therefore, the volume of this cuboid is $60\ \text{cm}^3$.

The volume of a cuboid = length × breadth × height

The volume of a cube = length × breadth × height
= side length × side length × side length

When finding the volume of a cube or a cuboid, the length, breadth and height should be written in the same units.

Do the following review exercise to further recall the above facts.

Review Exercise

(1) Find the volume of a cuboid with length, breadth and height equal to 10 cm, 8 cm and 4 cm respectively.

(2) Find the volume of a cube of side length 6 cm.

(3) The length of a box is 1.8 m, its breadth is 1 m and its height is 70 cm. Find the volume of this box in cubic meters.

(4) The length, breadth and height of a cuboid of volume 120 cm^3 are 8 cm, 5 cm and 3 cm respectively. Write the length, breadth and height of three other cuboids of the same volume.

(5) The area of the base of a cuboid of volume 70 cm^3 is 35 cm^2. Find its height.

(6) If the height and length of a cuboid of volume 160 cm^3 are 4 cm and 5 cm respectively, what is its breadth?

(7) The volume of a cube is 8 cm^3. What is the length of each side?

22.2 Formulae for the volume of a cube and a cuboid

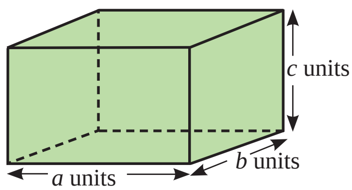
• Formula for the volume of a cuboid

If the volume of a cuboid of length a units, breadth b units and height c units is V cubic units, let us obtain a formula for the volume of the cuboid.

Volume of the cuboid = length × breadth × height

$$\therefore\ V = a \times b \times c$$

$$V = abc$$



If the area of the base of this cuboid is A square units,

$A = a \times b$

$V = a \times b \times c$. Let us substitute A for $a \times b$.

Then $V = A \times c$

Hence, the volume of the cuboid = area of the base × height

If the length, breadth and height of a cuboid are a units, b units and c units respectively, and if the area of its base is A square units and volume is V cubic units, then

$$V = abc \text{ and}$$

$$V = Ac$$

• Formula for the volume of a cube

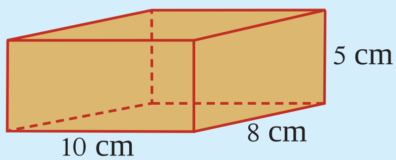
As above, let us obtain a formula for the volume of a cube of side length a units and volume V cubic units.

Since the volume of a cube = side length × side length × side length, the formula for the volume of a cube of side length a units and volume V cubic units is,

$$V = a \times a \times a$$
$$V = a^3$$

Example 1

The length, breadth and height of a cuboid are 10 cm, 8 cm and 5 cm respectively.



(i) Find the volume of this cuboid.

(ii) Another cuboid with a square base is of the same volume as that of the above cuboid. Its height is 4 cm. Find the side length of its base.

(i) Since $V = abc$,

the volume of the cuboid $= 10 \text{ cm} \times 8 \text{ cm} \times 5 \text{ cm}$

$= 400 \text{ cm}^3$

(ii)

Method I

Since $V = A \times c$,

the area of the base × height = volume

$$A \times 4 = 400$$
$$\therefore A = \frac{400}{4} = 100$$

Since the base is square shaped, the length of a side $= \sqrt{100}$ cm

$= 10$ cm

Method II

Since the base of the cuboid is square shaped, if its length and breadth are taken as a,

its volume $V = a \times a \times c$. Since $V = 400$ and $c = 4$

$$a \times a \times 4 = 400$$
$$a \times a = \frac{400}{4} = 100$$
$$a \times a = 10 \times 10$$
$$\therefore a = 10$$

$\therefore$ the side length of the base $= 10$ cm

If the length of a side of a cube is 1 m, then its length in centimetres is 100 cm.
Therefore, its volume $= 100 \text{ cm} \times 100 \text{ cm} \times 100 \text{ cm}$
$= 1\ 000\ 000 \text{ cm}^3$
That is, $1 \text{ m}^3 = 1\ 000\ 000 \text{ cm}^3$

Note;

Cubic feet and cube are two units of volume which are also in use.

100 cubic feet $=$ 1 cube

Exercise 22.1

(1) The following table has measurements of some cubes and cuboids. Copy the table and fill in the blanks.

Length	Breadth	Height	Volume
8 cm	6 cm	5 cm	....
12 cm	....	10 cm	1200 cm^3
1.5 m	0.5 m	0.6 m	....
6 m	6 m	....	216 m^3
$\frac{3}{4}$ m	$\frac{2}{5}$ m	$\frac{2}{3}$ m	....
1 m	$\frac{1}{2}$ m	40 cm	....

(2) The area of one face of a cube is 36 cm^2. Find

(i) the length of an edge,

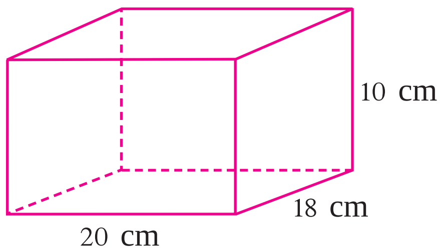
(ii) the volume,

of this cube.

(3) The area of the base of a cuboid is 1300 cm^2. If its volume is 65000 cm^3, find its height in metres.

(4) The volume of a cuboid shaped tank is 3600 cm^3. Its height, breadth and length are three consecutive perfect squares. Find its length, breadth and height. (Write 3600 as a product of prime factors).

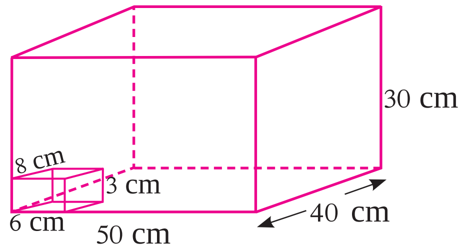
(5) It is required to pack cube shaped wooden blocks of side length 5 cm each in the cuboid shaped box shown in the figure. Find the maximum number of wooden blocks that can be packed in this box.



(6) Find the length, breadth and height of the cuboid shaped box with minimum volume, into which 50 cuboid shaped blocks, each with length, breadth and height equal to 4 cm, 3 cm and 2 cm respectively can be packed.

(7) By melting a solid metal cube of side length 10 cm, 8 identical smaller solid cubes were made without wastage of metal. Find the side length of a small cube.

8) It is necessary to pack soap boxes with measurements 8 cm × 6 cm × 3 cm in a box with measurements 50 cm × 40 cm × 30 cm as shown in the figure. Instructions have been given to pack 10 layers of soap boxes. Find the maximum number of soap boxes that can be packed.

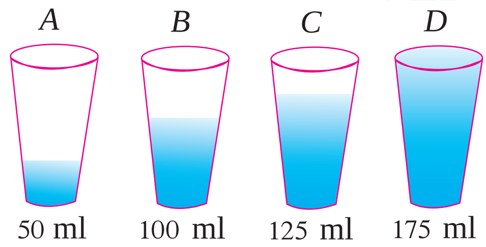


22.3 Capacity

The following figure shows some items that can be observed in day-to-day activities. Each of them indicates a certain amount of milliliters.

In Grade 7, you learnt that millilitres and litres are used to measure liquid amounts and that 1000 ml is equal to 1 l . Since liquids also occupy some amount of space, liquids have a volume.

The figure shows four glass containers A, B, C and D with a certain amount of drink in each of them.



The glasses A, B and C are not completely filled, but glass D is. The volume of drink in glass A is 50 ml and the volume of drink in glass D is 175 ml. The maximum amount of drink that can be poured into glass D is 175 ml. This is the capacity of glass D.

The volume of liquid that is required to fill a container completely is its **capacity**.

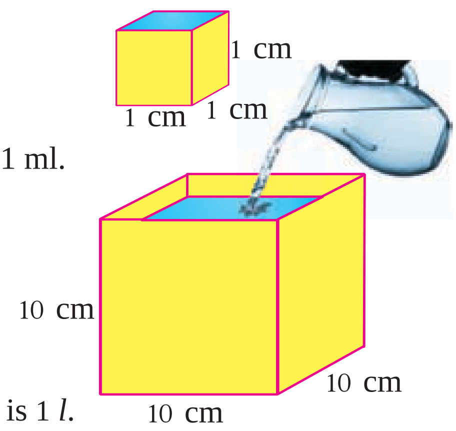
Accordingly, it is clear that the space within a container is its capacity.

Litre and millilitre which are the units used to measure liquid volumes are the units used to measure capacity too. Containers used in everyday activities sometimes have their capacity indicated on them. In some cases, the amount of liquid in the container is mentioned.

- ### The relationship between the units of volume and capacity

There is a close relationship between the units that are used to measure volume and those which are used to measure capacity. The maximum amount of liquid that can be poured into a cube shaped container of side length 1 cm is 1 ml.

$\therefore\ 1\text{ cm} \times 1\text{ cm} \times 1\text{ cm} = 1\text{ ml}$



$1\text{ cm}^3 = 1\text{ ml}$

The capacity of a container of volume 1 cm^3 is 1 ml.

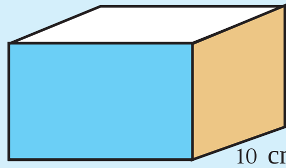
Likewise, $10\text{ cm} \times 10\text{ cm} \times 10\text{ cm} = 1000\text{ ml}$

$1000\text{ cm}^3 = 1\ l$

The capacity of a container of volume1000 cm^3 is 1 l.

Example 1

Find the capacity of the cuboid shaped box in the figure.


20 cm
10 cm
30 cm

Volume of the cuboid shaped box $= 30 \times 10 \times 20 \text{ cm}^3$

$= 6000 \text{ cm}^3$

$\therefore$ Capacity $= 6000$ ml

$= 6\ l$

Example 2

The capacity of a water tank is 6000 *l*. After filling the tank completely, 800 *l* of water is used per day for four days and 1200 *l* of water is used per day for two days. Find the volume of water remaining after these 6 days.

Water used during the first 4 days $= 800\ l \times 4 = 3200\ l$

Water used during the remaining 2 days $= 1200\ l \times 2 = 2400\ l$

$\therefore$ Total volume of water used $= 3200 + 2400\ l$

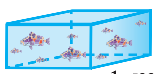
$= 5600\ l$

$\therefore$ Remaining volume of water $= 6000\ l - 5600\ l = 400\ l$

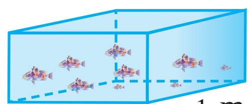
Exercise 22.2

(1) Find the capacity of each fish tank given below in litres.

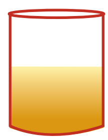
(i)


1 m
1.5 m
1 m

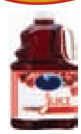
(ii)


75 cm
2 m
1 m

(2) The capacity of an oil tank is 12 *l*. There is 3 *l* 800 ml of oil in it. How much more oil is required to fill the oil tank completely?



(3) The capacity of a container is 150 ml. It is filled completely with a soft drink and then poured into a large bottle. How many litres of soft drink are there in the large bottle if drink is poured into it 10 times in this manner?



(4) A bottle contains 1300 ml of medicinal syrup. From this bottle, 50 ml of syrup is filled into small cups of capacity 65 ml each. Find the maximum number of cups that can be filled in this manner.

(5) A container of capacity 20 l is completely filled with milk. From this amount, 8 l 800 ml of milk is used to make yoghurt and 10 l 800 ml is used to make curd. Find the amount of milk remaining in the container after the above amounts are used.

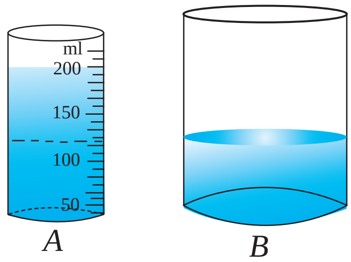
(6) What is the maximum volume of water in millilitres that can be filled into a cube shaped container which is of side length 15 cm?

(7) The base area of a cuboidal shaped container is 800 cm^2. If 4.8 l of water is poured into this container, find the height of the water level.

(8) Find the capacity of a cuboidal shaped container with length, breadth and height equal to 4 m, 2.5 m and 0.8 m respectively.

22.4 Estimating the capacity of a container

The water level in container B is as shown in the figure, after 200 ml of water is poured into it from the calibrated container A. Let us estimate the capacity of B accordingly.



It can be observed that the height of container B is three times the height of the water.

$\therefore$ capacity of container B $= 3 \times 200$ ml

$= 600$ ml

Activity 1

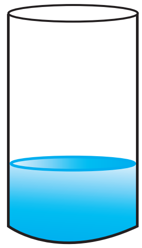
Step 1 - Take a sufficient amount of water as well as calibrated and non-calibrated transparent cylindrical containers from your surroundings (glasses, bottles, plastic cups).

Step 2 - Pour a measured amount of water from a calibrated container into a non-calibrated container. Then examine the height of the water level.

Step 3 - Determine in a suitable way, how many times the height of the water level, the total height of the container is and hence estimate the capacity of the container.

Step 4 - Estimate the capacity of each of the remaining containers in the same manner.

Exercise 22.3



(1) The volume of water in the container in the figure is 150 ml. Estimate the capacity of this container.

(2) 100 lamps have been prepared for a function. 3 litres of oil were required to fill all these lamps completely. Estimate the capacity of a lamp.

(3) A household usually needs 275 litres of water per day. Estimate the minimum capacity of a tank which can store the water required for this house for a week.

Summary

- The volume of a cuboid of length, breadth and height equal to a units, b units and c units respectively is $a \times b \times c$ cubic units. If its volume is V cubic units, then
$$V = abc$$
- The volume of a cube of side length a units is a^3 cubic units. If the volume is V cubic units, then
$$V = a^3$$
- The volume of liquid that is required to fill a container completely is its capacity.

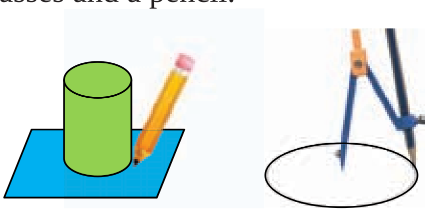
23 Circles

By studying this lesson you will be able to,

- identify that a circle has infinitely many axes of symmetry,
- identify what a chord of a circle is, and
- identify what an arc of a circle, a segment of a circle and a sector of a circle are.

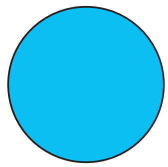
23.1 The axes of symmetry of a circle

In Grades 6 and 7 you learnt to draw circles by using either objects with circular shapes or a pair of compasses and a pencil.

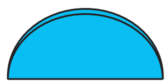


Activity 1

Step 1 - Take a sheet of paper, draw a circle and cut out the circular lamina.



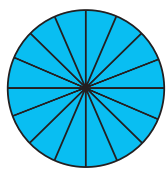
Step 2 - Fold the circular lamina such that you get two equal parts which coincide with each other.



Step 3 - Mark the fold line using a pencil and a ruler.

Step 4 - Unfold the circular lamina, and as above, fold the lamina again along a different fold line to get two equal parts. Repeat this a few times and mark all the fold lines.

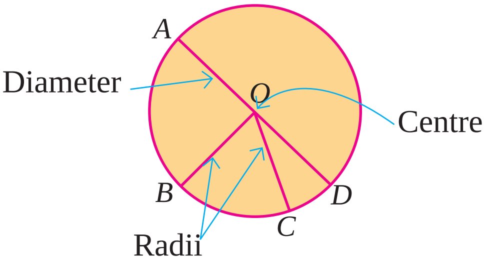
Step 5 - You would have observed that there are many such fold lines and that all of them pass through the same point.



A line which divides a circle into two equal parts is an axis of symmetry of that circle. From the above activity it must be clear to you that a circle has infinitely many axes of symmetry. A straight line segment joining two points which are the points of intersection of an axis of symmetry and the circle, is a **diameter** of the circle.

The point at which the axes of symmetry of a circle intersect is the **centre** of the circle.

A straight line segment joining the center of a circle to any point on the circle is called a **radius** of the circle. The length of this line segment is independent of the point on the circle which is selected. This length is also called the radius of the circle.



In the given circle, the center is O . AD is a diameter of the circle. OA, OB, OC and OD are radii of the circle. If OA = 1.3 cm, then the radius of the circle is 1.3 cm.

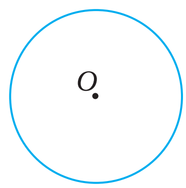
$OA = OB = OC = OD = 1.3$ cm

23.2 Chord of a circle

Activity 2

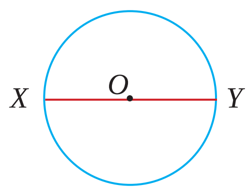
Step 1 - Using a pair of compasses and a pencil, draw a circle of radius 4 cm on a piece of paper.

Step 2 - Mark the centre of the circle and name it O .



Step 3 - Mark a point on the circle and name it X . Join X and O.

Step 4 - Produce XO to meet the circle again and name the point of intersection as Y .

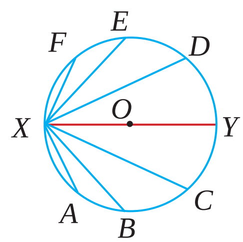


Step 5 - Mark a few points on the circle and name them A, B, C, D, E and F.

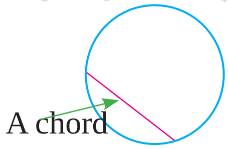
Step 6 - Join the above points to X.

Step 7 - Measure and write the lengths of XA, XB, XC, XD, XE, XF and XY.

Step 8 - Observe that the longest line among them is XY.



XA, *XB*, *XC*, *XD*, *XE*, *XF* and *XY* are known as chords of the circle. A straight line segment joining any two points on a circle is called a **chord of that circle**. The longest chord of a circle is its diameter.

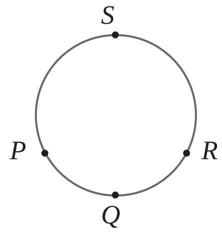


23.3 Arc of a circle

Activity 3

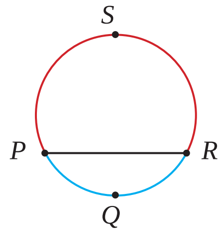
Step 1 - Using a pencil and a pair of compasses, draw a circle of radius 4 cm on a piece of paper.

Step 2 - Mark four points on the circle and name them *P*, *Q*, *R* and *S* respectively.

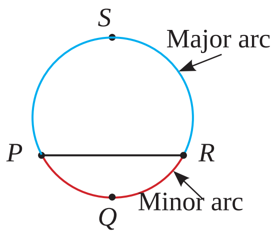


Step 3 - Join *P* and *R*.

Step 4 - Highlight the section *PQR* of the circle in blue and the section *PSR* in red.



In this figure, the line *PR* is a chord of the circle. The sections *PQR* and *PSR* of the circle are called **arcs of the circle**. The section *PQR* is called a **minor arc** and the section *PSR* is called a **major arc**.



Exercise 23.1

(1) Draw a circle of radius 3 cm and name its centre *O* . Draw a diameter of this circle and name it *PQ* . Measure the length of the diameter.

(2) Draw a circle of radius 3.5 cm. Mark a point *A* on the circle. Draw several chords starting from *A* . Find the length of the longest chord you have drawn.

(3) Draw any circle and mark the points *A*, *B*, *C* and *D* on it.

(i) Draw the chord *AC*.

(ii) Name the two arcs separated by the chord *AC*.

(4) (i) Draw a circle of radius 4 cm.

(ii) Draw a chord such that two equal arcs are obtained. Name it *AB*.

(iii) What is the name suitable for the chord *AB*?

(5) (i) Draw a circle of radius 5 cm. Name its center O.

(ii) Draw a chord which is 6 cm in length and name it AB.

(iii) Name the midpoint of AB as P and join OP.

(iv) Measure and write down the magnitudes of $A\hat{P}O$ and $B\hat{P}O$.

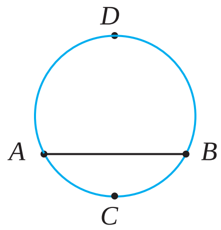
23.4 Segments of circles and sectors of circles

- ### Segment of a circle

Activity 4

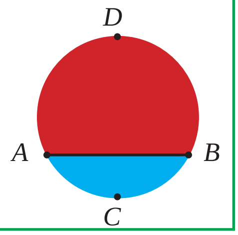
Step 1 - Draw a circle on a sheet of paper using a pair of compasses and a pencil.

Step 2 - Draw a chord AB of the circle, which is not a diameter.



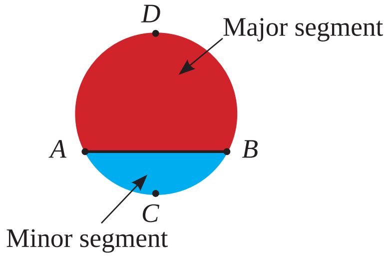
Step 3 - Mark two points C and D on the circle on either side of the chord AB as shown in the figure.

Step 4 - Color the region bounded by the chord AB and the minor arc ACB in blue and the region bounded by the chord AB and the major arc ADB in red.



A region of a circle bounded by a chord and an arc is called a **segment of the circle**.

The region bounded by the chord AB and the minor arc is called a **minor segment of the circle**.



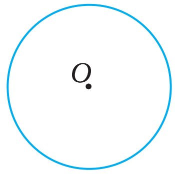
The region bounded by the chord AB and the major arc ADB is called a **major segment of the circle**.

- **Sector of a circle**

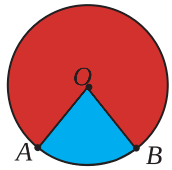
Activity 5

Step 1 - Draw a circle using a pair of compasses and a pencil and name its centre O.

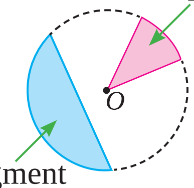
Step 2 - Mark two points A and B on the circle and join AO and BO.



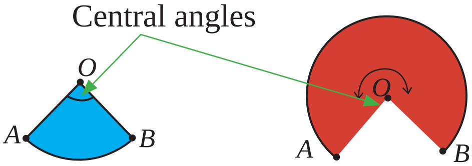
Step 3 - Color in blue, the region bounded by the radii AO and BO and the arc AB where $A\hat{O}B$ is an acute angle. Color in red, the region bounded by the radii AO and BO and the arc AB where $A\hat{O}B$ is a reflex angle.



In a circle, a region bounded by two radii and an arc is called a **sector of the circle**. The angle subtended at the centre of the circle by the arc is called **the central angle**. Two sectors are shown in the given circle.

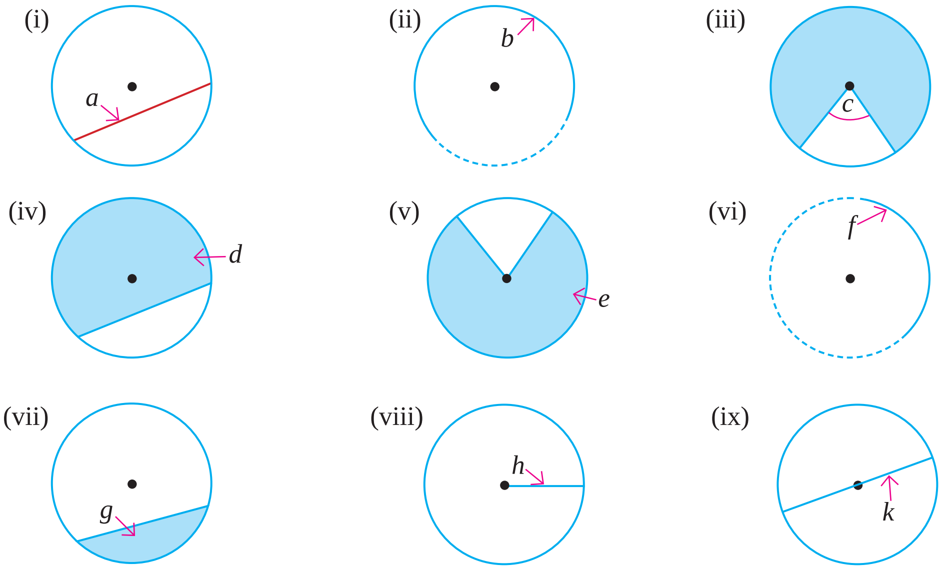


The sector bounded by the minor arc and two radii is called a **minor sector**, and the sector bounded by the major arc and two radii is called a **major sector**. The acute angle $A\hat{O}B$ is the central angle of the minor sector and the reflex angle $A\hat{O}B$ is the **central angle** of the **major sector**.



Exercise 23.2

(1) From the given terms, select and write the most suitable term for each of the regions indicated by an English letter.



(a radius, a sector, a chord, a minor arc, a minor segment, a major segment, a diameter, a major arc, a central angle)

(2) Fill in the blanks.

(i) A straight line segment which joins the centre of a circle to any point on the circle is called a …………………………..of the circle.

(ii) The longest chord of a circle is called a ………………………………….

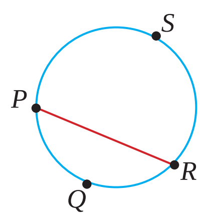
(iii) If the diameter of a circle is 200 mm, then its radius is …………………..c m.

(iv) A region of a circle bounded by a chord and an arc is called a ………………………

(v) A region of a circle bounded by two radii and an arc is called a ………………………………

(3) (i) Name the segments in the figure.

(ii) Shade the minor segment of the circle.



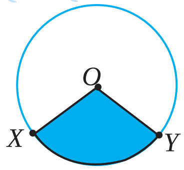
(4) (i) Draw a circle of radius 3.5 cm and name its centre O .

(ii) Draw a chord AB through O.

(iii) What can you say about the two segments you get?

(iv) What is a suitable name for the above two segments?

(5) (i) What is the name given to the shaded part of the circle?
(ii) Write down its boundaries.
(iii) What is the name given to the angle $X\hat{O}Y$?



(6) Draw a circle with centre O . Mark two points M and N on the circle such that a minor arc and a major arc are obtained. Shade the sector with central angle equal to the reflex angle $M\hat{O}N$.

(7) Draw a circle with centre O. Draw a diameter AB. Mark a point X on the circle distinct from A and B.

(i) Shade the sector AXB.
(ii) Measure and write down the magnitude of the central angle of the sector AXB.

(8) (i) Draw a circle of radius 5 cm. Name its centre O.
(ii) Mark a point P on the circle and join OP.
(iii) Using the protractor, draw the sector POQ such that $P\hat{O}Q = 60^0$.
(iv) Draw the sector QOR such that $Q\hat{O}R = 150^o$.
(v) Name the remaining sector and write the magnitude of its central angle.

Summary

- A circle has infinitely many axes of symmetry.
- A line segment which joins any two points on a circle is a chord of that circle. The longest chord is a diameter of the circle.
- A section of a circle between any two points on the circle is known as an arc of the circle.
- A region of a circle, bounded by two radii and an arc is known as a sector of the circle.
- A region of a circle bounded by a chord and an arc is called a segment of the circle.

24 Location of a Place

By studying this lesson you will be able to,

- express the direction of a place with respect to a particular point, based on the direction of north or the direction of south,
- sketch the location of a place with respect to a particular point, based on the direction in which it is located and the distance from the point.

24.1 Introduction

In Grades 6 and 7 you learnt that, when a compass is placed on a flat surface the needle points in the direction of the North. The remaining main directions, South, East and West and the sub directions Northeast, Southeast, Southwest, and Northwest too can be located using a compass.

If a well and a coconut tree are situated due north of our house, one way of finding out their exact locations is to find the direct distance to the well from the house and the direct distance to the coconut tree from the house.

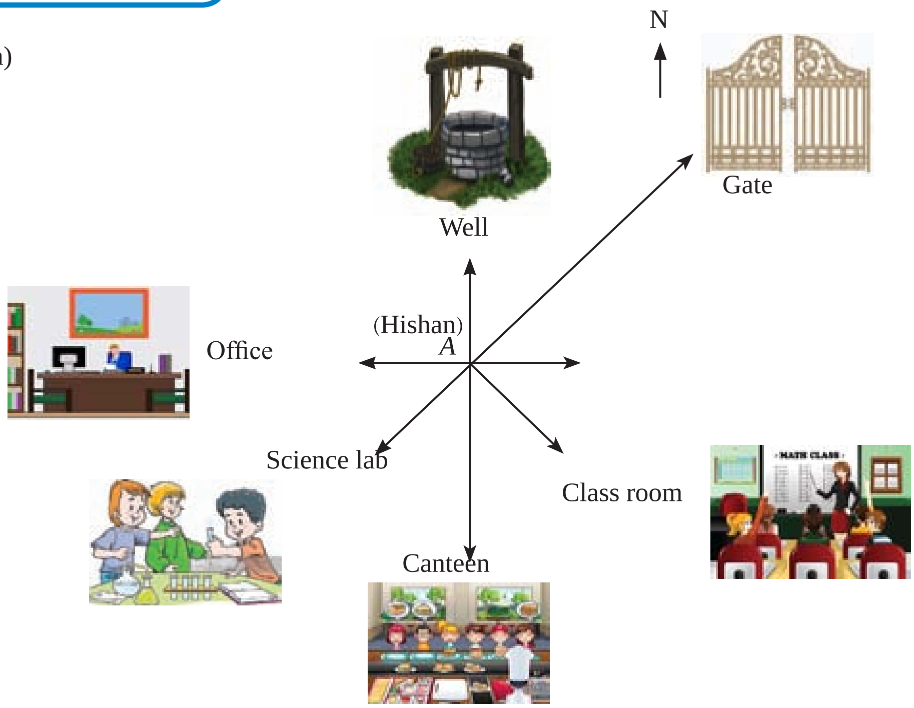
For example, if the distances from the house to the well and to the coconut tree are 105 m and 173 m respectively, then the location of the well is 105 m to the north of the house, and the location of the coconut tree is 173 m to the north of the house. We can find the exact location of the well and the coconut tree in this way.

The location of a place with respect to a particular point can be described exactly by specifying the direction in which it is located with respect to the point and the distance from the point to it.

Do the review exercise to revise what you have learnt in lower grades.

Review Exercise

(1) (a)



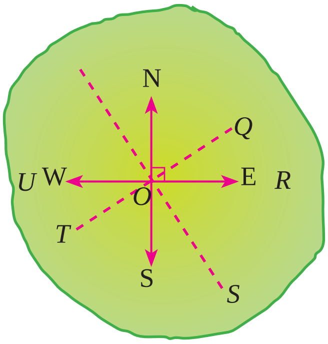
Hishan observed a few places in the school from the point A located in the school grounds. The above figure is a sketch drawn with the information he gathered. Complete the given table using the sketch.

Place that was observed	Direction in which it is located with respect to A
(i)	
(ii)	
(iii)	
(iv)	
(v)	
(vi)	

(b) Complete each sentence using the above sketch.

(i) Hishan is .... of the well.
(ii) Hishan is .... of the office.
(iii) Hishan is .... of the classroom.
(iv) Hishan is .... of the canteen.
(v) The science lab is situated .... of the gate.
(vi) The canteen is situated .... of Hishan.

(2) (a) A flat land is shown in the figure. Write the direction of each place given in the table with respect to the point O, using the main directions and sub directions.

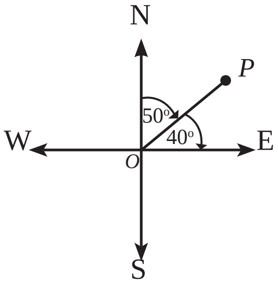


Place	Direction of the place with respect to O
Q	
R	
S	
T	
U	

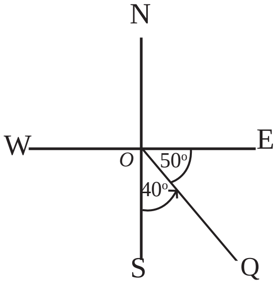
24.2 More on finding the direction of a place with respect to another place, based on the main directions

Let us now consider how we can express the direction of a place which is not in one of the main directions or sub directions from a particular position.

We know that the angle between any two adjacent main directions is a right angle. We describe the direction of a place which is not in either a main direction or a sub direction from a particular point, by means of an angle of less than 90^{o} measured from a main direction



The direction of P as seen from O is 50^{o} east of north. This is written as N 50^{o} E.

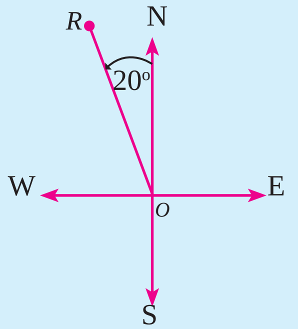


The direction of Q as seen from O is 40^{o} east of south. This is written as S 40^{o} E.

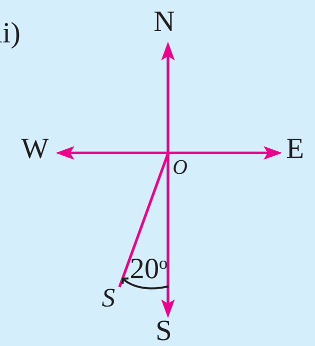
Example 1

Write down the direction of (i) the place R (ii) the place S, from the place O.

(i)



(ii)



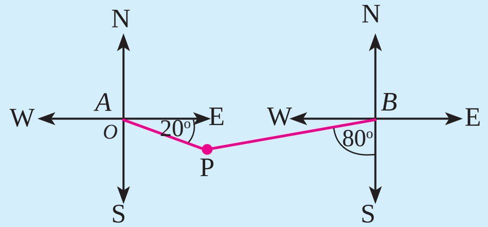
R is located 20º west of north from O.
The direction of R from O is N 20º W.

S is located 20º west of south from O.
The direction of S from O is S 20º W.

Example 2

The figure shows the direction of P where a car is parked, as seen from the points A and B in a field. Write down the direction of the car

(i) as seen from A,
(ii) as seen from B,
using the main directions.

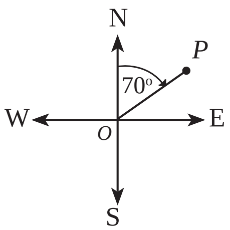


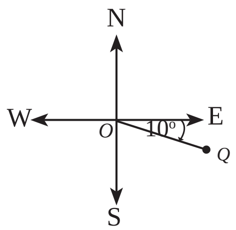
(i) The direction of the car is 70º east of south as seen from A. This is written as S 70º E.

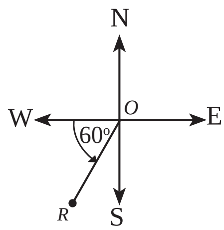
(ii) The direction of the car is 80º west of south as seen from B. This is written as S 80º W.

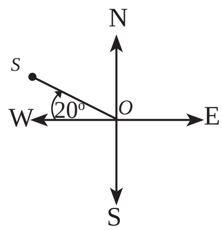
Exercise 24.1

(1) Write the direction in which each of the points P, Q, R and S in each of the following figures is situated with respect to the point O, based on either the direction of north or the direction of south.







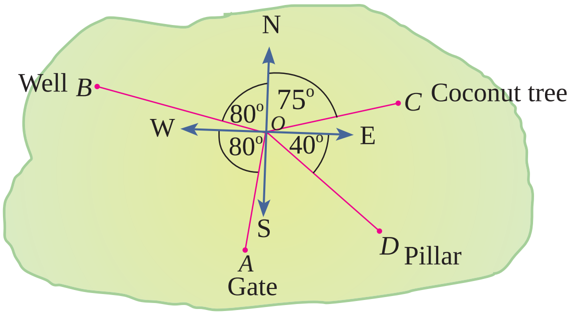


(2) Draw sketches to show each of the directions given below.

(i) N 30° W (ii) S 55° W
(iii) S 30° W (iv) N 30° E
(v) Northeast (NE) (vi) Northwest (NW)

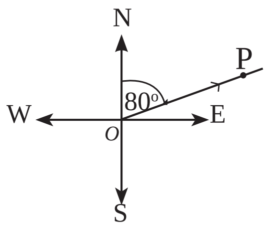
(3) The camp P is situated due west of the camp Q. A fire in a forest is seen by a soldier in camp P in the direction 75° east of south. At the same instant, another soldier in camp Q sees the fire in the direction 20° west of south. Illustrate this information with a sketch.

(4) The information on four places observed by a child from a point O in an open area is given in the figure. Complete the given table using this information.

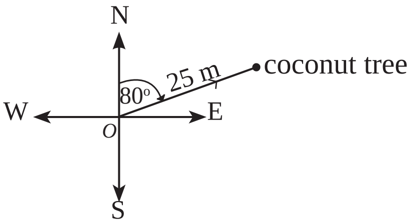


The place which is observed	Direction with respect to O
A - Gate B - Well C - Coconut tree D - Pillar	

24.3 Illustrating the location of a place with respect to another place with a sketch

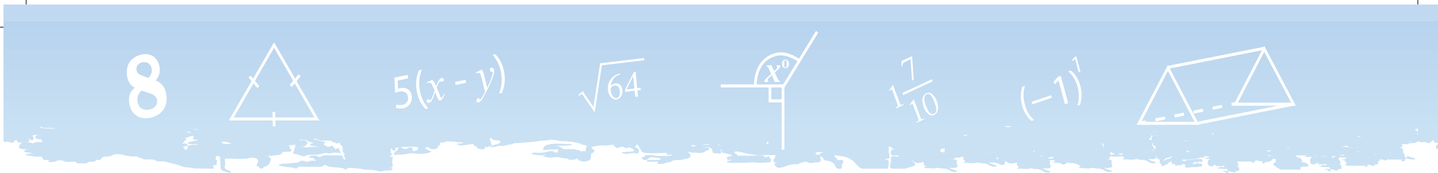


If the direct distance from O to P is known, where P is 80° east of north (N 80° E), then the exact location of P can be identified.



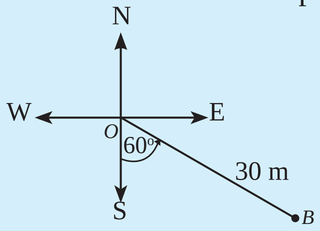
This sketch shows the location of a coconut tree 25 m from O in the direction 80° east of north (N 80° E).

The location of a place with respect to another place can be illustrated with a sketch as shown above.



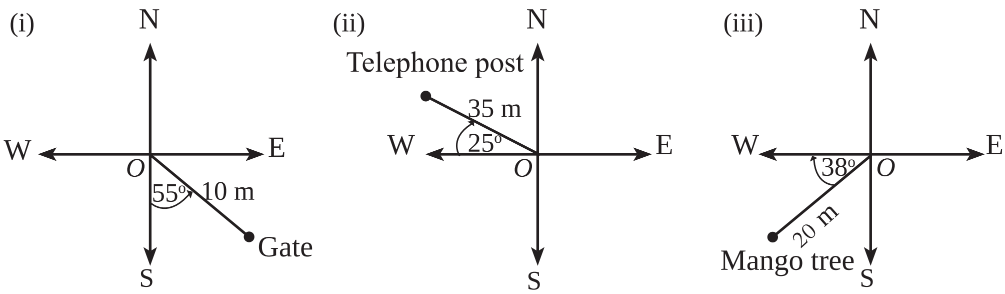
Example 1

Illustrate the location of a place 30 m from O in the direction S 60° E with a sketch.



Exercise 24.2

(1) Using the sketches given below, complete the given table.



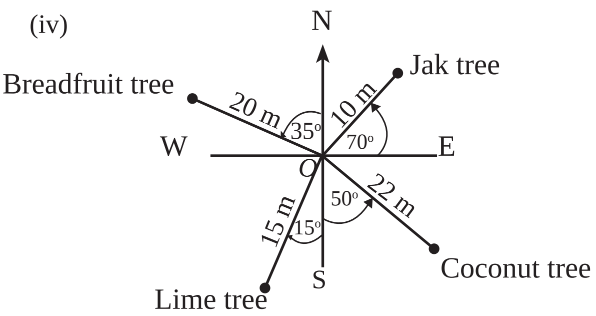


Figure	The place observed from O	Direction with respect to O	Distance from O
(i)	Gate	S 55° E	10 m
(ii)	....	....	....
(iii)	....	....	....
(iv)	Jak tree	....	....
	Coconut tree	....	....
	Lime tree	....	....
	Breadfruit tree	....	....

(2) Draw sketches using the information given below.

(i) B is situated 50 m from A in the direction S 10^{o} W.

(ii) Q is located 25 m from P in the direction N 70^{o} W.

(iii) A child standing at the point K in a playing field sees the gate which is 50 m away in the direction S 20^{o} W.

(iv) Tharushi standing at a point P on flat ground sees Radha 20 m away in the direction S 50^{o} E and Fathima 15 m away in the direction S 25^{o} W.

(3) Kavindu travelled 20 m from O in the direction N 44^{o} E, and from that point he travelled 20 m in the direction S 45^{o} E to reach his destination.

(i) Draw a sketch based on the above information.

(ii) In which direction is Kavindu now with respect to O?

Miscellaneous Exercise

(1) For each of the following, draw a sketch based on the given information.

(i) A person at P walked to a place Q located 100 m away in the direction N 35^{o} E. From there he walked to his work place R, located 75 m away in the direction S 20^{o} E.

(ii) The school that Kavindu attends is situated 125 m away from his home, in the direction S 30^{o} E.

(iii) Bhashitha standing at the location B in a field, can see his school in the direction N 35^{o} W. Thushara who is standing 100 m away to the east of Bhashitha sees the school in the direction N 40^{o} W.

Summary

- The location of a place which lies along a main direction from a particular point can be expressed in terms of its direction and its distance from that point.
- The direction of a place with respect to a particular point can be described based on the direction of north and the direction of south.
- The location of a place with respect to a particular point can be illustrated in a sketch based on the direction and distance to that place from the point.

25 Then umberl ine and Cartesian Plane

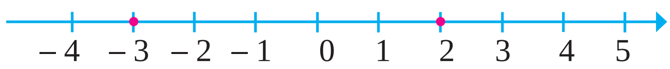
By studying this lesson you will be able to,

- represent fractions and decimal numbers with one decimal place on a number line,
- compare fractions and decimals using the number line,
- represent on a number line, the values that the unknown an inequality with one unknown can take,
- identify a point on a coordinate plane by considering the x - and y - coordinates, and
- identify the nature of the coordinates of the points that lie on a line which is parallel to an axis of the coordinate plane.

25.1 Introduction

In Grade 7 you learnt how to represent an integer on a number line.

Let us find out which of the two numbers 2 and –3 is greater.

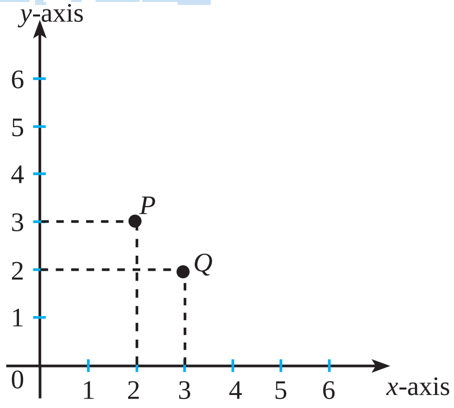


The numbers –3 and 2 have been marked on the above number line.

Any number that is on the right hand side of a given number on the number line is greater than the given number. This property is applicable to the whole number line. Therefore this rule can be applied when comparing two integers using the number line.

Since 2 is on the right hand side of (-3) on the number line, 2 is greater than (-3). This can be written either as $2 > (-3)$ or as $(-3) < 2$.

You have also learnt previously how a Cartesian plane which consists of two number lines drawn perpendicular to each other is used to specify the location of a point on a plane.



- The two number lines which intersect each other perpendicularly are called the x-axis and the y-axis.
- The point of intersection of the x and y axes is called the origin.
- 0 of the two number lines is located at the origin.

- The line drawn from the point P, perpendicular to the x-axis, meets the x-axis at 2. The line drawn from the point P, perpendicular to the y-axis meets the y-axis at 3.

Accordingly, the x-coordinate of point P is 2 and the y-coordinate of P is 3. The coordinates of point P are written as (2,3) by writing the x-coordinate first and the y-coordinate second within brackets. This is written in short as $P(2,3)$.The coordinates of the point Q in the Cartesian plane are (3,2).

Review Exercise

(1) (i) Write all the integers that lie between –3 and 5.
(ii) Mark these integers on a number line.
(iii) Of the numbers mentioned in (i) above, write the least and the greatest numbers.

(2) Write the integers $7, -8, 0, -3, 5, -4$ in ascending order.

(3) Choose the appropriate symbol from $>$ and $<$ and fill in the blanks.

(i) $5 \ .... \ -2$ (ii) $3 \ .... \ 0$ (iii) $-5 \ .... \ 0$
(iv) $-10 \ .... \ -1$ (v) $5 \ .... \ -7$ (vi) $0 \ .... \ -3$

(4) Draw a Cartesian plane and mark the following points on it.

(i) A (3,1) (ii) B (0,5) (iii) C (3,0)
(iv) D (2,3) (v) E (4,1) (vi) F (3,4)

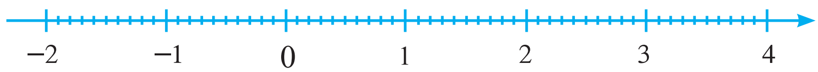
25.2 Representing fractions and decimals on a number line

Fractions and decimals which are not integers can also be represented on a number line. Such a number is located between two consecutive integers on the number line. For example, 1.5 is located between 1 and 2 on the number line, and $-\frac{2}{3}$ is located between -1 and 0.

Do the following activity to learn how to represent on a number line, fractions and decimals that lie between two consecutive integers.

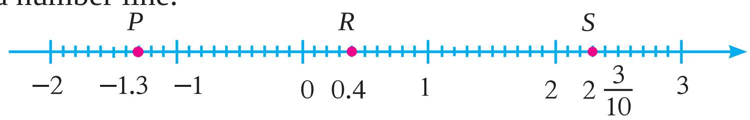
Activity 1

On your square ruled exercise book, draw a number line marked from −2 to 4, taking 1 unit to be 5 squares, as shown below. Divide one unit into 10 equal parts by dividing each square of the exercise book into two equal parts.



- Mark a point on the number line which lies exactly between the two consecutive integers 2 and 3, and name it *P*.
- What is the value of *P*?
- Name the numbers $-\frac{1}{2}$, 1.5 and 0.5 which are on the number line as *Q*, *R* and *S* respectively.
- Mark another point which is not an integer and which does not lie exactly between two consecutive integers on the number line, and write its value.

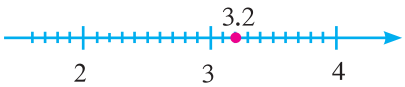
The figure below shows several numbers which are not integers that have been marked on a number line.

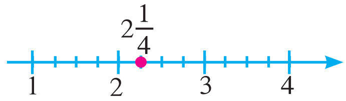


When dividing a unit on the number line into several equal parts to represent a particular number, it is necessary to be careful to select the number of equal parts appropriately, depending on the number that is to be represented.

It is suitable to divide one unit into 10 equal parts when representing a decimal number with one decimal place, and to divide one unit into parts equaling the number in the denominator when representing a fraction.

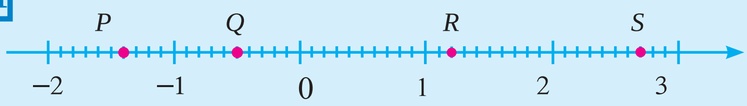
For example, it is suitable to divide one unit into 10 equal parts to represent 3.2 and to divide one unit into 4 equal parts to represent $2\frac{1}{4}$.





Fractions and decimal numbers can be compared using a number line in the same way that integers are compared.

Example 1



(i) Write the numbers that are represented by the points *P*, *Q*, *R* and *S* on the number line shown in the figure.

(ii) Write these numbers in ascending order.

(i) -1.4 , $-\frac{1}{2}$, 1.2, 2.7

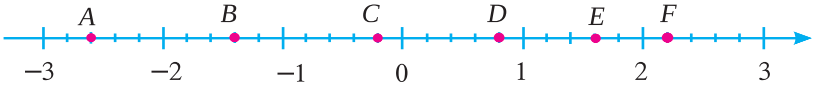
(ii) $-\frac{1}{2} = -0.5$. $-1.4 < -0.5 < 1.2 < 2.7$

$\therefore$ When these numbers are arranged in ascending order we obtain

$-1.4, -\frac{1}{2}$, 1.2 , 2.7.

Exercise 25.1

(1) Write the numbers that are represented by the points *A*, *B*, *C*, *D*, *E* and *F* on the given number line.



(2) (i) Mark the numbers 1.8, 3.5, 2.6 and 4.1 on a number line.
(ii) Mark the numbers 13.2, 14.7, 15.5 and 16.3 on a number line.

(3) Arrange each of the following groups of numbers in ascending order using a number line.

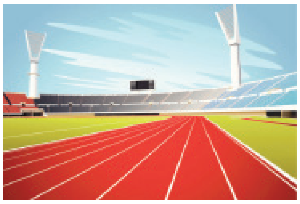
(i) -2, $1\frac{1}{2}$, -1.5, -3

(ii) 2.5, -0.5, -5.2, $3\frac{1}{4}$

(iii) $1\frac{1}{4}$, 0, $-2\frac{2}{5}$, -4.1

(iv) 2.7, -10.5, $5\frac{1}{4}$, -1.3

25.3 Representing inequalities containing an algebraic term on a number line



According to the rules of a certain competition, only children of height greater than 120 cm are allowed to participate. If the height of a competitor is denoted by h cm, this means that $h > 120$. Accordingly, anyone of height greater than 120 cm such as 121 cm, 125 cm or 127 cm can participate in the competition.

$x > 2$ is an inequality. This means that the values that x can take are only those which are greater than 2. On the other hand the inequality $x \geq 2$ means that the values that x can take are those which are greater than or equal to 2.

- The symbol $>$ is used to denote 'greater than',
- the symbol $<$ is used to denote 'less than',
- the symbol $\geq$ is used to denote 'greater than or equal', and
- the symbol $\leq$ is used to denote 'less than or equal'.

Accordingly, $8 > x$ can also be written as $x < 8$, and $2 \geq y$ can also be written as $y \leq 2$.

Therefore, $h > 120$ means that the values that h can take are only those values which are greater than 120.

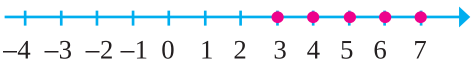
The set of all values that the algebraic term in an inequality with one algebraic term can take is called the set of solutions of the inequality.

- The integral solutions of the inequalities (i) $x > 2$ and (ii) $x \geq 2$ are represented on the number lines given below.

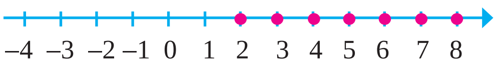
The integers belonging to the set of integral (whole number) solutions of the inequality $x > 2$ are 3, 4, 5, 6, ...

The integers belonging to the set of integral (whole number) solutions of the inequality $x \geq 2$ are 2, 3, 4, 5, 6, ...

(i) $x > 2$ where x is an integer



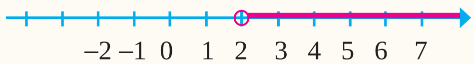
(ii) $x \geq 2$ where x is an integer



- However, when all the solutions of $x > 2$ or $x \geq 2$ are represented on a number line, we obtain a section of the number line.

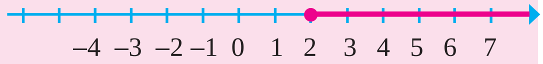
(i) $x > 2$

The set of all solutions of the inequality $x > 2$ is the set of all the numbers greater than +2. This includes all the fractions and decimals which are greater than 2 too. Therefore the solutions of this inequality are marked as follows.



Since 2 does not belong to the set of all solutions of this inequality, the point on the number line representing 2 is not shaded. An un-shaded circle is drawn on 2. Since all the numbers greater than 2 belong to this set, it is represented by a dark line drawn to the right of 2 as shown above.

(ii) $x \geq 2$



Since 2 belongs to the set of all solutions of the inequality, a shaded circle is drawn on 2 as shown in the figure.

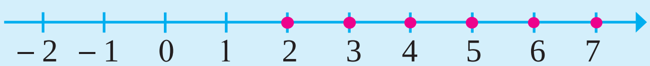
Example 1

(1) Mark the set of integral solutions of the inequality $x > 1$ on a number line.

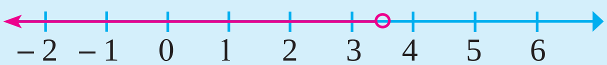
(2) Represent the set of all solutions of each of the following algebraic inequalities on a separate number line.

(a) $x < 3\frac{1}{2}$ (b) $x > 3\frac{1}{2}$ (c) $x \leq 3\frac{1}{2}$ (d) $x \geq 3\frac{1}{2}$

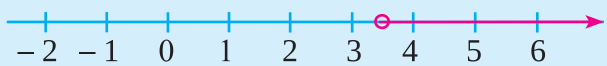
(i)



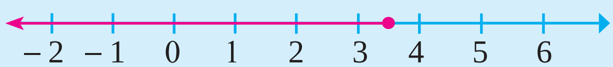
(ii) (a)



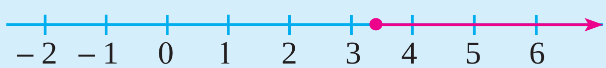
(b)



(c)



(d)



Exercise 25.2

(1) Mark the set of integral solutions of each of the following inequalities on a separate number line.

(i) $x > 0$ (ii) $x < 3.5$ (iii) $x \geq -2\frac{1}{2}$

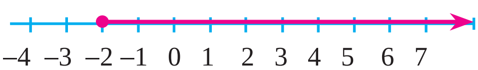
(2) Represent the set of all solutions of each of the following algebraic inequalities on a separate number line.

(i) $-\frac{1}{2} \leq m$ (ii) $2.5 \leq m$ (iii) $1.5 < m$

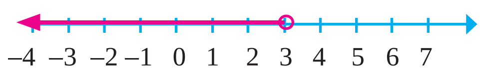
25.4 More on representing inequalities on a number line

- To find the values which satisfy both the inequalities $x \geq -2$ and $x < 3$ at the same time, let us first represent the solutions of the two inequalities on separate number lines.

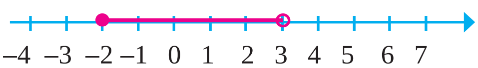
(i) $x \geq -2$



(ii) $x < 3$



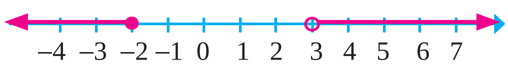
Now let us represent the values of x which satisfy both these inequalities on a number line.



When two inequalities are combined in this manner, by writing it as, $\boldsymbol{x \geq -2}$ **and** $\boldsymbol{x} < 3$, we express the fact that both inequalities have to be satisfied simultaneously.

We can express the region on the number line consisting of all the values that satisfy both these inequalities as $-2 \leq x < 3$.

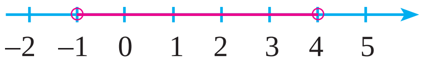
- Now let us represent the values of x which satisfy at least one of the two inequalities $x \leq -2$, $x > 3$ on a number line.



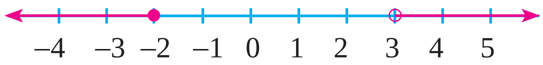
Any number in the shaded region of the number line satisfies at least one of the given inequalities.

When two inequalities are combined in this manner, by writing it as $\boldsymbol{x \leq -2}$ **or** $\boldsymbol{x > 3}$, we express the fact that at least one of the two inequalities should be satisfied.

The values in the shaded region of the number line satisfy both the inequalities $x > -1$ and $x < 4$. This region can be expressed algebraically by the inequality $-1 < x < 4$.

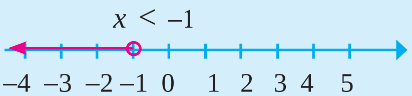


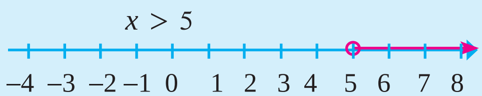
The figure given below shows the values satisfying $x \leq -2$ or $x > 3$ represented on a number line.



Example 1

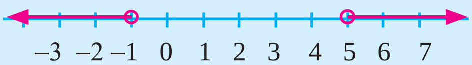
(i) Indicate the values of x which satisfy both of the inequalities $x < -1$ and $x > 5$ on a number line.





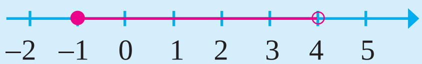
No number satisfies both these inequalities at the same time. Therefore the set of values that satisfy both the inequalities $x < -1$ and $x > 5$ is the empty set.

(ii) Represent the values of x which satisfy at least one of the two inequalities $x < -1$ and $x > 5$ on a number line.



Example 2

Write the inequality represented on the number line in algebraic form.



$-1 \leq x < 4$

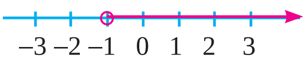
Exercise 25.3

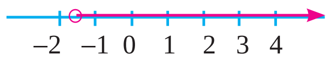
(1) Represent each of the following inequalities on a separate number line.

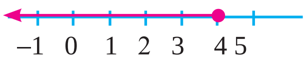
(i) $-2 < x < 3$ (ii) $-3 < x \leq 2$ (iii) $0 \leq x < 6$

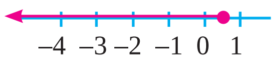
(iv) $-1 \leq x \leq 4$ (v) $x \leq -1$ or $x \geq 5$ (vi) $x \leq -1$ or $x \geq 4$

(2) Write the inequality represented on each of the following number lines in algebraic form.

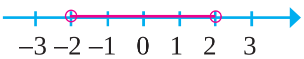
(i)


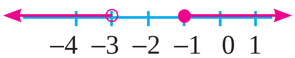
(ii)


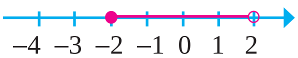
(iii)


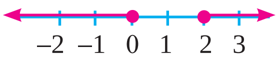
(iv)


(3) For each of the following cases, write the inequalities represented on the number line.

(i)


(ii)


(iii)


(iv)


(4) Write the set of integral values that satisfy both the inequalities $x > -1$ and $x < 5$.

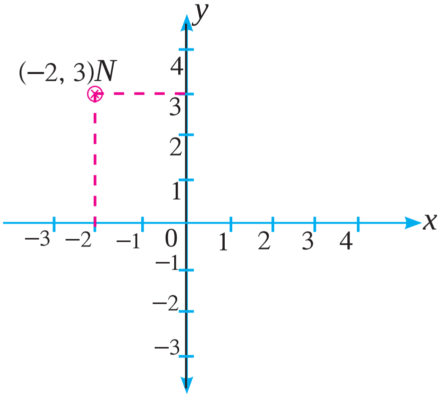
25.5 Plotting points on a Cartesian plane

You have learnt previously how points with coordinates equal to either 0 or positive integers are marked on a Cartesian plane. Let us now study how coordinates with negative numbers also are marked on a Cartesian plane. Let us consider how the point N(–2,3) is marked on a Cartesian plane.

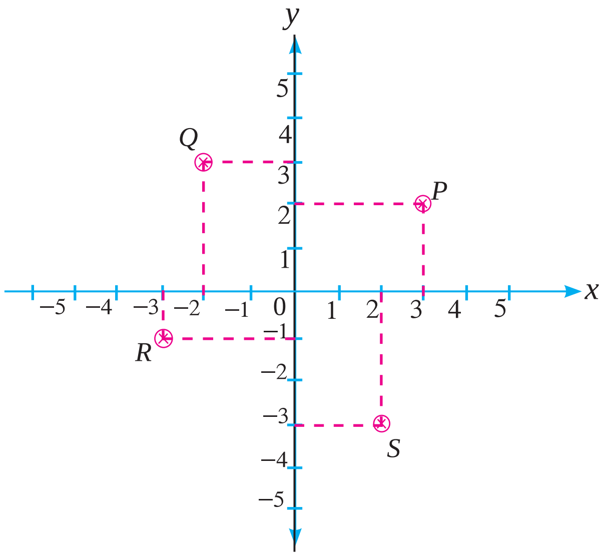
➢

The point N with coordinates (–2, 3) on the

Cartesian plane is the intersection point of the line drawn perpendicular to the x-axis through the point –2 on the x-axis and the line drawn perpendicular to the y-axis through the point 3 on the y-axis.



Now let us consider how the coordinates of points on the Cartesian plane are identified. The line drawn from point *R* perpendicular to the *x*-axis, meets the *x*-axis at –3. The line drawn from point *R* perpendicular to the *y*-axis, meets the y-axis at –1. Accordingly, the *x*-coordinate of *R* is –3 and the *y*-coordinate of *R* is –1. Therefore the coordinates of *R* are written as $(-3, -1)$.



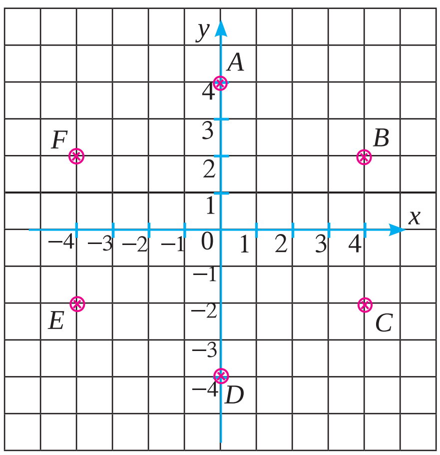
Point	*x* - coordinate	*y* - coordinate	coordinates
P	3	2	(3, 2)
Q	−2	3	(−2, 3)
R	−3	−1	(−3, −1)
S	2	−3	(2, −3)

Exercise 25.4

(1) Mark each of the following points on a Cartesian plane where the *x*-axis and the *y*-axis are marked from –5 to 5.

A (2, −5) , *B* (−3, 4) , *C* (−3, −3) , *D* (−4, −1) , *E* (−2, 0) , *F* (0, −4)

(2) Write the coordinates of the points which are marked on the Cartesian plane given below.



(3) Mark the points with the following coordinates on a Cartesian plane where the x-axis and the y-axis are marked from -5 to 5. Identify the figure that is obtained by joining all the points in the given order.

(0, 4), (1, 1), (4, 0), (1, −1), (0, −4), (−1, −1), (−4, 0), (−1, 1), (0, 4)

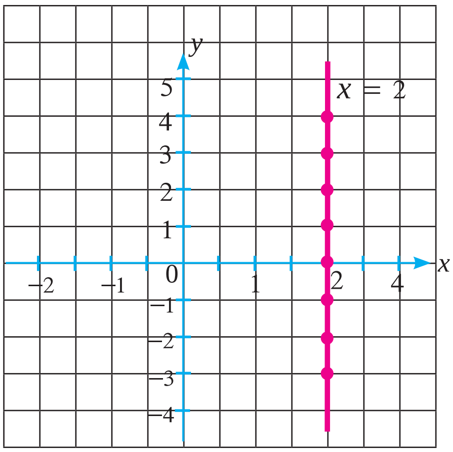
25.6 Straight lines parallel to the two axes

Observe each of the following coordinates carefully.

(2, 4), (2, 3), (2, 2), (2, 0), (2, −1), (2, −2), (2, −3)

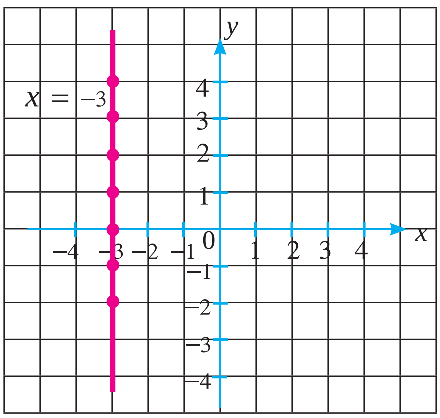
The x-coordinate of each of these pairs is 2.

When the points with these coordinates are marked on the Cartesian plane, they are as follows.

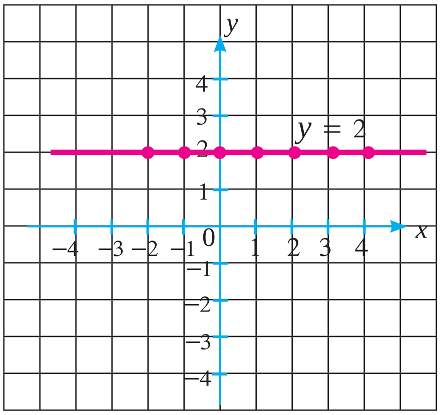


All these points lie on the straight line which is parallel to the y-axis and intersects the x-axis at the point 2. That is, they all lie on the straight line given by $x = 2$. Furthermore, the x-coordinate of every point on this line is equal to 2.

➢ $x = -3$ is the straight line on which all the points with x-coordinate equal to -3 lie.



➤ The straight line given by the equation $y = 2$ is shown in the Cartesian plane given below. This line is parallel to the x-axis and intersects the y-axis at 2.



Example 1

(a) (i) Write the coordinates of 5 points which lie on the straight line given by $x = -3$.

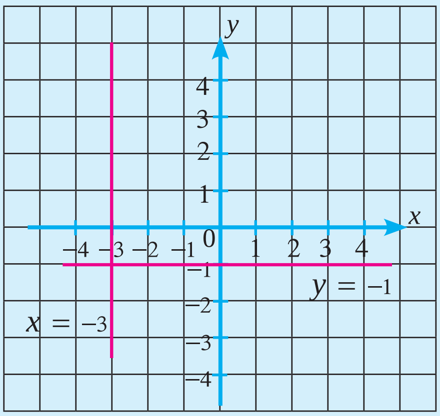
(ii) Write the coordinates of 5 points which lie on the straight line given by $y = -1$.

(b) Draw the straight lines given by $x = -3$ and $y = -1$ on the same Cartesian plane.

(a) (i) The points with coordinates $(-3, -1)$, $(-3, 0)$, $(-3, 1)$, $(-3, 2)$, and $(-3, 3)$ lie on the straight line given by $x = -3$.

(ii) The points with coordinates $(-3, -1)$, $(-2, -1)$, $(-1, -1)$, $(0, -1)$, and $(2, -1)$ lie on the straight line given by $y = -1$.

(b)



Exercise 25.5

(1) Copy each of the following statements in your exercise book. Place a "✓" next to the correct statements and a "×" next to the incorrect statements.

(i) (0, 5) are the coordinates of a point that lies on the straight line given by $x = 5$. ( )

(ii) The straight line given by $y = 3$ is parallel to the x-axis. ( )

(iii) The coordinates of the point of intersection of the straight lines given by $x = 2$ and $y = 1$ are (2, 1). ( )

(iv) The straight line given by $y = 0$ is identical to the x-axis of the Cartesian plane. ( )

(v) From among the ordered pairs, (3, 1), (–2, 1), (1,–1) and (0,1), the pair which is not the coordinates of a point which lies on the straight line given by $y = 1$ is (1 , –1). ( )

(2) (i) Draw the straight lines $x = 3$ and $y = -3$ on the same Cartesian plane.

(ii) Write the coordinates of the point of intersection of the two lines.

(3) (i) Draw a Cartesian plane with both the x-axis and the y-axis marked from –5 to 5.

(ii) On this Cartesian plane, draw the four straight lines which are the graphs of the following equations.

(a) $y = 2$ (b) $y = -2$ (c) $x = 4$ (d) $x = -2$

(iii) What is the special name that is given to the figure which is obtained by the intersection of these lines?

(iv) Write the coordinates of the points of intersection of each pair of lines which intersect.

(v) Draw the axes of symmetry of the closed plane figure that was obtained in (iii) above and write their equations.

Miscellaneous Exercise

(1) Represent the set of integral solutions of the inequality $-2 \leq x \leq 3$ on a number line.

(2) (i) Mark the points $A(-1,\ 1)$, $B(\ 2,\ 1)$ and $C(\ 1,\ -1)$ on a Cartesian plane which has both axes marked from –3 to 3.

(ii) Mark the point D on the Cartesian plane such that $ABCD$ forms a parallelogram and write it coordinates.

(iii) Write the equations of the sides AB and DC of the parallelogram.

(3) Arrange each of the following groups of numbers in ascending order using a number line.

(i) $-5, \ -1\frac{3}{4}, \quad -3\frac{1}{3}, \quad -0.2$

(ii) $3.8, \ -5\frac{1}{2}, \quad 0.5, \quad -7.5$

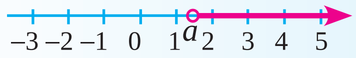
(iii) $1.2, \ -0.3, \ 1\frac{2}{5}, \ 2$

(iv) $-1\frac{3}{4}, \ -2, \ 1\frac{5}{8}, \ 0$

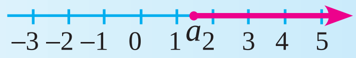
Summary

- Fractions and decimal numbers can be represented on a number line as numbers which lie between integers.
- The inequalities (i) $x > a$ and (ii) $x \geq a$ can be represented on a number line as follows.

(i)

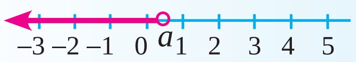


(ii)

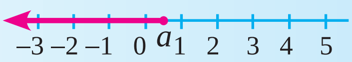


- The inequalities (i) $x < a$ and (ii) $x \leq a$ can be represented on a number line as follows.

(i) $x < a$



(ii) $x \leq a$



- The inequality $b \leq x \leq a$ can be represented on a number line as follows.

- All points on a straight line of the form $x = a$,which is parallel to the y - axis has 'a' as their x-coordinate.
- All points on a straight line of the form $y = b$, which is parallel to the x-axis has b' as their y-coordinate.

26 Construction of triangles

By studying this lesson you will be able to,

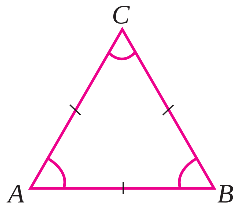
- identify that the sum of the lengths of any two sides of a triangle is greater than the length of the remaining side, and
- construct a triangle when the lengths of all the sides are given.

26.1 Introduction

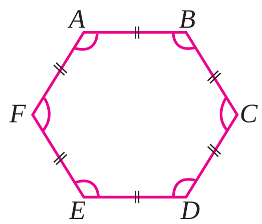
In geometry, we need to draw and construct plane figures. When constructing a plane figure, we have to construct it according to the given conditions.

In Grade 7 you learnt to construct a straight line segment of given length, an equilateral triangle of given side length, and a regular hexagon by means of equilateral triangles or a circle.

A straight line segment



An equilateral triangle



A regular hexagon

- Let us recall the steps that need to be performed to construct an equilateral triangle.
  - Construct a straight line segment.
  - Taking the same length as that of the straight line segment onto the pair of compasses, construct an arc by placing the point of the pair of compasses at one end of the above line segment.
  - Construct an arc from the other end point using the same length as above, such that it intersects the earlier arc.
  - Join the intersection point of the arcs to the end points of the straight line segment.

- A regular hexagon can be constructed by performing the following steps.
  - Construct a circle.
  - Divide the circle into 6 equal parts by intersecting the circle with arcs of the same length as the radius of the circle
  - Join the points of intersection.

Do the following review exercise to recall these facts which you learnt in Grade 7.

Review Exercise

(1) Construct the straight line segment AB of length 7.9 cm.

(2) Construct an equilateral triangle of side length 5.4 cm.

(3) (i) Construct a circle of radius 4 cm and centre O.

(ii) Construct the regular hexagon $ABCDEF$ of side length 4 cm such that its vertices lie on the above constructed circle.

(4) Construct a regular hexagon of side length 5 cm.

26.2 Identifying the condition for three line segments to be the sides of a triangle

The figure represents a paddy field bounded by AB, BC and CA which are straight paths around it. Nimali is at A and her puppy is at B. Nimali has two routes to get from A to her puppy at B. Identify the two routes and determine which is shorter.

It is easy to establish that the shorter route is along AB. This means that the sum of the distances AC and CB is greater than the distance AB.

Do the following activity to find a condition to determine whether three line segments of given length can be the sides of a triangle.

Activity 1

Step 1 - Take pieces of ekel of length 3 cm, 4 cm, 5 cm, 7 cm and 9 cm respectively.

Step 2 - Pick any three pieces of ekel, place them on a table and see whether a triangle can be formed with the three pieces, such that their endpoints meet.

Step 3 - Complete a row of the following table by first noting down the lengths of the three pieces of ekel you picked.

Step 4 - Repeat the above steps several times over.

Lengths of the three piece of ekel (in cm)	The sum of the lengths of two pieces (in cm)	Length of the third piece (in cm)	Relationship between the values in the second and the third columns	If a triangle can be formed place a ✓. If not, place a ×.
3, 4, 5	7 9 8	5 3 4	$7 > 5$ $9 > 3$ $8 > 4$	✓
3, 4, 9	7 13 12	9 3 4	$7 < 9$ $13 > 3$ $12 > 4$	×
3, 7, 9				
4, 5, 7				

According to the table you have completed, it is clear that it is not always possible to construct a triangle with the three pieces of ekel that were selected.

However, it is possible to construct a triangle with three pieces of ekel, if the sum of the lengths of any two of them is greater than the length of the third.

It is clear that the sum of the lengths of any two sides of a triangle is greater than the length of the remaining side, and that given three straight line segments, if the sum of the lengths of any two of them is less than that of the third, then we cannot construct a triangle with line segments of these lengths as the sides.

Exercise 26.1

(1) Choose the triples from the following groups that can be the lengths of the sides of a triangle.

(a) For each triple that was selected, write the reason for your choice.
(b) For the triples that were not selected, write the reason for not selecting them.

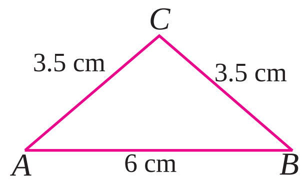
(i) 5 cm, 6 cm, 7 cm (ii) 4 cm, 4 cm, 4 cm (iii) 4 cm, 4 cm, 8 cm
(iv) 3 cm, 3 cm, 7 cm (v) 5 cm, 5 cm, 8 cm (vi) 6 cm, 4 cm, 10 cm

26.3 Construction of triangles

In Grade 7 you learnt how to construct an equilateral triangle. Let us consider how to construct an isosceles triangle.

- **Construction of an isosceles triangle**

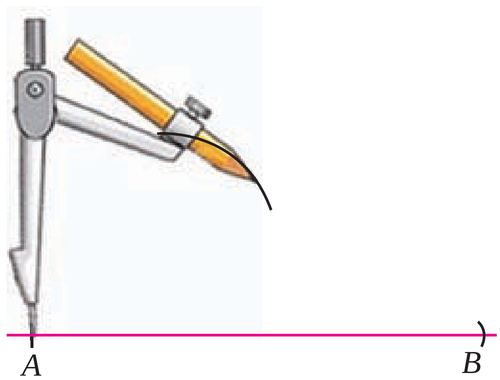
Let us construct the isosceles triangle *ABC* with *AB* = 6 cm, and *BC* and *CA* equal to 3.5 cm each.
Let us draw a sketch of the triangle first.



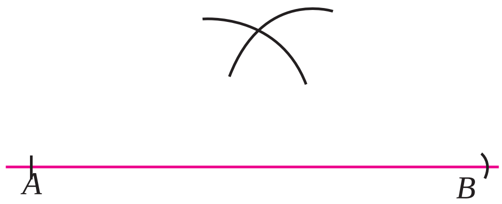
Sketch

Step 1 - Construct a straight line segment *AB* of length 6 cm using a pair of compasses and a ruler.

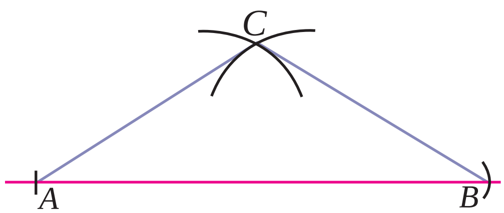
Step 2 - Set the pair of compasses so that its point and the pencil point are at a distance of 3.5 cm apart. Place the point of the pair of compasses on *A* and construct an arc as shown in the figure.



Step 3 - Place the point of the pair of compasses at the point B, and without changing the length on the pair of compasses, construct another arc such that it intersects the first arc. If the arcs do not intersect, place the point of the pair of compasses again at A and lengthen the first arc sufficiently until the two arcs intersect. Name the point of intersection of the two arcs as C.



Step 4 - Join AC and BC.



Step 5 - After completing the triangle ABC by drawing the straight line segments AC and BC, measure the magnitudes of the interior angles by using the protractor, and write them down.

By this we can establish the fact that we have constructed an isosceles triangle with side lengths 6 cm, 3.5 cm and 3.5 cm.

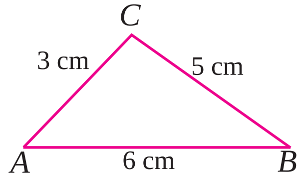
(i) Construct an isosceles triangle of side lengths 7.6 cm, 5.2 cm and 5.2 cm.
(ii) Measure and write down the magnitudes of the angles.
(iii) Write what type of triangle this is according to the angles.

• Construction of a scalene triangle

Let us now construct a scalene triangle.

If all three sides of a triangle are different in length, then it is called a scalene triangle.

Let us construct a scalene triangle *ABC*, with side lengths *AB* = 6 cm, *BC* = 5 cm and *AC* = 3 cm

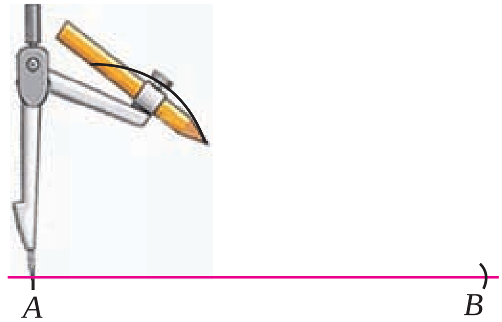


Sketch

Let us draw a sketch of the triangle first.

Step 1 - Construct a straight line segment *AB* of length 6 cm using a pair of compasses and a ruler.

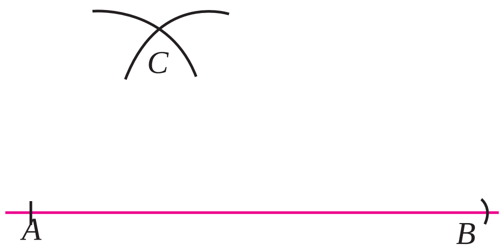
Step 2 - Set the pair of compasses so that its point and the pencil point are at a distance of 3 cm apart. Place the point of the pair of compasses on *A* and construct an arc as shown in the figure.



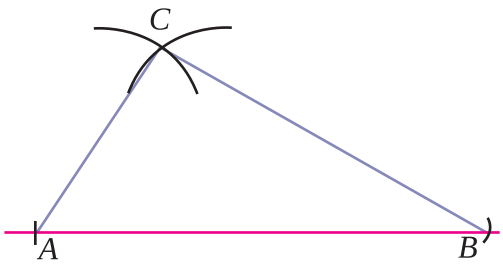
Step 3 - Set the pair of compasses so that its point and the pencil point are at a distance of 5 cm apart.

Place the point of the pair of compasses on the point *B* and construct another arc so that it intersects the first arc.

If the two arcs do not intersect, place the point of the pair of compasses again on *A* and lengthen the first arc sufficiently until the two arcs intersect.
Name the point of intersection of the two arcs as *C*.



Step 4 - Join AC and BC



Step 5 - After completing the triangle ABC by drawing the straight line segments AC and BC, measure the magnitudes of the interior angles by using the protractor, and write them down.

You have now constructed the triangle ABC of side lengths 3 cm, 5 cm and 6 cm.

$C\hat{A}B = 55^{o}$, $A\hat{B}C = 30^{o}$ and $B\hat{C}A = 95^{o}$. Therefore, $C\hat{A}B + A\hat{B}C + A\hat{C}B = 180^{o}$.

This triangle is a scalene triangle according to the lengths of the sides.

➤ (i) In the triangle PQR, $PQ = 4$ cm, $QR = 3$ cm and $PR = 5$ cm. Construct this triangle.

(ii) Measure and write down the magnitude of the largest angle of this triangle. Write what type of triangle is this according to its angles.

Exercise 26.2

(1) (i) Construct two equilateral triangles, one of side length 4 cm and the other of side length 5.7 cm.

(ii) Measure and write down the magnitudes of the angles of the two triangles.

(2) (i) Construct triangles with the given side lengths by using a pair of compasses and a ruler.

(a) 6 cm, 8 cm, 10 cm

(b) 4.5 cm, 6 cm, 7.5 cm

(c) 5 cm, 5 cm, 4 cm

(ii) Show that the sum of the angles of each of the triangles you constructed is equal to 180^{o} by measuring them.

(iii) Categorize the triangles according to the largest angle.

Summary

- To construct a triangle when the lengths of the three sides are given, the following steps are performed.
  - Constructing a straight line segment of length equal to the length of one of the sides of the triangle.
  - Constructing an arc of length equal to the length of another side of the triangle by placing the point of the pair of compasses at one end point of the above straight line segment.
  - Constructing another arc from the other end point of the straight line segment, of length equal to the length of the remaining side, so that it intersects the above drawn arc.
  - Joining the point of intersection of the two arcs to the end points of the straight line segment.
- The sum of the lengths of any two sides of a triangle is greater than the length of the remaining side.

27 Data Representation and Interpretation

By studying this lesson you will be able to,
- represent data in a stem and leaf diagram,
- find the maximum value, minimum value and the range of a collection of data using a stem and leaf diagram, and
- find the mode, median, mean and range of a collection of raw data.

27.1 Stem and leaf diagram

In Grades 6 and 7, you learnt to represent and interpret data using picture graphs, bar graphs and multi bar graphs. Now we will consider what a stem and leaf diagram is and how data is represented in a stem and leaf diagram.

A stem and leaf diagram is a standard method of organizing numerical data to enable us to interpret the data easily.

When data is organized according to this method,

- if the values of the data are from 0 to 99, the value in the units place of a datum is indicated as the leaf and the value in the tens place is indicated as the stem.
- if the values of the data are from 100 to 999, the value in the units place is indicated as the leaf and the values in the tens and hundreds places considered together is indicated as the stem.

- Only the digit in the units place is indicated as the leaf.
- For values from 0 to 9, the stem takes the value 0.
- If a row has more than one leaf value, the values are written leaving a gap between the digits.

Example 1

(i) Write the stem and leaf of each of the numbers 2, 43 and 225.

(ii) Write the datum of which the stem is 3 and the leaf is 0.

(i)

Data	Stem	Leaf
2	0	2
43	4	3
225	22	5

(ii) 30

The marks obtained by 25 students in a certain class for a mathematics test paper marked out of 50 are given below.

5	7	9	11	13
16	19	20	21	22
24	25	26	26	29
31	33	35	36	38
40	43	45	48	49

Let us represent this data in a stem and leaf diagram.

In a stem and leaf diagram, the first column is called the stem and the second column is called the leaf.

Stem	Leaf
0	5 7 9
1	1 3 6 9
2	0 1 2 4 5 6 6 9
3	1 3 5 6 8
4	0 3 5 8 9

Key: 3|1 means 31.

- All the numbers are written in ascending order such that the stems of the numbers are in the first column (stem column) and the leaves of the numbers are in the second column, and with the numbers from 0 to 9 in the first row, the numbers from 10 to 19 in the second row and the numbers from 20 to 29 in the third row etc.
- The numbers in the fourth row of the above stem and leaf diagram have 3 as the stem and 1, 3, 5, 6, 8 respectively as the leaves. Their corresponding values are 31, 33, 35, 36, and 38.

  The numbers represented in the other rows can also be written as shown above.
- It is easier to understand information related to the above 25 data when they are represented in a stem and leaf diagram than when they are written in a row.

* If the students who obtained less than 20 marks failed the test, then we can easily say that the number of students who failed is 3 + 4 = 7.
* If an “A” pass is given to those who have obtained 40 or more marks, then we can easily say by considering the stem and leaf diagram that there are 5 such students.

Therefore, a stem and leaf diagram can be considered as a simple method of representing and understanding data.

Now let us consider through an example how data is organized in ascending order.

Example 2

The heights of some students in a class are given below in centimetres.

141	148	142	130	152	135	157	146	140	160
151	173	139	135	144	134	151	138	137	137
169	136	143	154	146	166	131	150	145	143

(i) Represent this data in a stem and leaf diagram.

(ii) What is the least value of this collection of data?

(iii) What is the greatest value of this collection of data?

Stem	**Leaf**
13	0 5 9 5 4 8 7 7 6 1
14	1 8 2 6 0 4 3 6 5 3
15	2 7 1 1 4 0
16	0 9 6
17	3

Key : 14|1 means 141.

The stem and leaf diagram prepared with the data values in ascending order is given below.

Stem	**Leaf**
13	0 1 4 5 5 6 7 7 8 9
14	0 1 2 3 3 4 5 6 6 8
15	0 1 1 2 4 7
16	0 6 9
17	3

(ii) 130 (iii) 173

Now let us consider through the following examples how a collection of data consisting of decimal numbers is represented in a stem and leaf diagram.

Example 3

The birth weights of 25 animals of a certain species are given below in kilogrammes.

6.1	9.8	6.7	8.1	5.6	6.4	7.5	8.6
8.5	7.2	9.5	6.8	8.9	7.3	6.8	7.7
9.3	9.0	8.4	7.6	8.2	8.5	7.9	8.3
9.5							

(i) Represent this data in a stem and leaf diagram.
(ii) What is the minimum birth weight?
(iii) What is the maximum birth weight?

(i) In these decimal numbers, the whole number parts take values from 5 to 9. These are taken as the stems and the decimal parts are taken as the leaves.

Stem	Leaf
5	6
6	1 4 7 8 8
7	2 3 5 6 7 9
8	1 2 3 4 5 5 6 9
9	0 3 5 5 8

Key : 7|3 means 7.3

(ii) 5.6 kg
(iii) 9.8 kg

Exercise 27.1

(1) The period of service of a group of employees of a certain company are given below in months. Represent this data in a stem and leaf diagram.

120	145	164	156	134	129	132	145	158	162

(2) The mass in kilogrammes of the bags of 30 pilgrims who flew to their destination in Dambadiva are given below. Represent this data in a stem and leaf diagram.

30	29	27	28	19	22	18	21	20	24
28	12	23	30	09	21	17	25	27	26
26	10	29	25	24	20	15	29	29	28

(3) The masses of the water melons for sale in a certain shop on a particular day are given below in kilogrammes.

6.5	7.8	5.7	4.3	5.8	6.2	4.3	6.9	7.8	7.2
6.9	5.5	7.7	7.8	5.2	6.7	5.7	6.1	6.0	7.3
7.1	6.7	7.7	4.3	6.5	7.3	6.7	5.8	6.8	5.4

(i) Represent this data in a stem and leaf diagram.

(ii) How many water melons are there for sale in this shop on this day?

(iii) What is the mass of the heaviest water melon for sale in this shop?

(iv) What is the mass of the water melon with the least mass?

27.2 Distribution of data represented in a stem and leaf diagram

The number of customers who bought gift items from a certain shop on each day of a period of 30 days is given below.

Stem	**Leaf**
0	8 9
1	2 8 9
2	3 2 6 6 9
3	0 5 6 8
4	0 1 1 4
5	3 4 6 7
6	2 5 8
7	2 4 6
8	0 1

Key :4|0 means 40.

- The minimum value of this collection of data is 8.

  This is the minimum number of customers who bought items from the shop on a day in that period of 30 days.
- The maximum value of this collection of data is 81.

  This is the maximum number of customers who bought items from the shop on a day in that period of 30 days.
- Accordingly, this data is distributed from 8 to 81. To find the range of this data, we use;

$$\begin{aligned}\text{Range} &= \text{Maximum value} - \text{Minimum value}\\ &= 81 - 8\\ &= 73\end{aligned}$$

- When the groups of ten from 0 to 90 are considered, the maximum number of data, that is 5 data, is in the group 20 – 29. The minimum number of data, that is 2 data, is in the groups 0 – 9 and 80 – 89.

Exercise 27.2

(1) A cyclist had a training schedule for a month. The distance he cycled each day is given below in kilometres.

Stem	Leaf
1	5 5 8
2	0 1 3 4 6 7
3	2 4 5 6 6 8 8
4	0 2 4 4 5 6 8 8
5	1 2 4 6
6	3 5

Key : 5|1 means 51.

(i) What is the minimum value of this data?
(ii) What is the maximum distance he cycled in a day during this period?
(iii) Find the range of this data.

(2) 30 students in Grade 8 were given 40 English words to read and then write down. The number of incorrect words written by each student is given below.

16	24	12	15	10	23
23	15	13	19	14	25
26	21	31	24	19	27
35	12	17	29	18	29
32	18	27	31	21	31

(i) Represent this data in a stem and leaf diagram.
(ii) How many incorrect words were written by the student who wrote the least number of incorrect words?
(iii) How many incorrect words were written by the student who wrote the most number of incorrect words?
(iv) Find the range of the incorrect words written by this group of students.
(v) Write the groups of ten to which the greatest and least values belong.

(3) The number of fish buns and bottles of fruit juice sold by a mobile food truck during a period of 30 days are given in the following two stem and leaf diagrams.

Fish buns sold

Stem	Leaf
5	4 5 6 8 8 9
6	0 3 3 5 8 8
7	2 3 3 5 9 9
8	0 0 3 4 5 7
9	0 1 3 4 4 5

Key :6|3 means 63.

Bottles of fruit juice sold

Stem	Leaf
0	8 9
1	0 2 5
2	0 1 3 5 8 9
3	5 6
4	3 4 5
5	0 2 6 8
6	1
7	0 2 5
8	1 4
9	0 2 4 6

Key :8|1 means 81.

(i) What is the minimum number of fish buns sold in a day?
(ii) What is the maximum number of fish buns sold in a day?
(iii) Find the range of the sales of fish buns.
(iv) What is the minimum number of bottles of fruit juice sold in a day?
(v) What is the maximum number of bottles of fruit juice sold in a day?
(vi) Find the range of the sales of bottles of fruit juice.
(vii) Compare the sales of fish buns with the sales of fruit juice and write your conclusions.

(4) The marks obtained for a mathematics test paper marked from 100, by the students of two parallel classes A and B are given below.

Class A

Stem	Leaf
5	0 2 6
6	0 1 3 5 6 6 8
7	2 2 3 5
8	0 2

Key :7|2 means 72.

Class B

Stem	Leaf
0	5 9
1	0 2 5 6
2	1
3	2 3
4	4 5 8
5	1 3
6	0 8

Key :5|1 means 51.

(i) Write separately the number of students in class A and the number of students in class B.

(ii) Find the minimum mark, maximum mark and the range of the marks obtained by the students in class A.

(iii) Find the minimum mark, maximum mark and the range of the marks obtained by the students in class B.

(iv) Compare the level of achievement in mathematics of the students in classes A and B for this mathematics paper based on the above data and write your conclusions.

27.3 Interpreting a collection of numerical data

Now let us consider how a given collection of data is interpreted.

- During a pluck, we can usually get 8 coconuts from each coconut tree in an estate.
- The average mark of a student for 8 subjects is 73.6.
- The average runs scored per over in a certain cricket match was 5.3.
- On a certain day, the price of 1 kg of beans displayed by most of the vendors in a market was 120 rupees.

A single value that is used to give an idea regarding a collection of data, as in the above examples is called a **representative value**.

We will now consider a few representative values that are used.

• Mode

The marks obtained by the 13 students in a certain class for a mathematics question paper are given below.

96, 81, 78, 45, 71, 57, 71, 83, 95, 68, 94, 71, 79

The 'number of data in a collection' is the total number of data in that collection.

The number of data in the above collection is 13.

Let us write these values in ascending order.

45, 57, 68, 71, 71, 71, 78, 79, 81, 83, 94, 95, 96

The mark that has been obtained the most is 71. Three students have obtained this mark.

In a collection of data, some of the values could be identical. The value which occurs most often is called the **mode** of that collection of data.

In the above collection of data, since 71 is the value that occurs most often, 71 is the mode.

Note:
It is not necessary to write the data in ascending order to find the mode.

Example 1

The ages of 10 students in Grade 8 are given below in years. Find the mode of this collection of data.

13	14	15	14	15	14	14	14	13	14

In the above collection of data, 14 years is the value that occurs most frequently. Therefore the mode of the ages of these grade 8 students is 14.

Example 2

The number of employees who took leave on each of the 15 working days of a certain month is given below. Find the mode of this collection of data.

12	14	20	16	15
16	21	19	16	18
17	15	18	19	18

Here, the values 16 and 18 have each occurred 3 times. The other values have occurred less than 3 times. Therefore, we can take 16 or 18 as the mode of this collection of data.

Such a distribution of data is known as a bimodal distribution. A collection of data may have more than two modes too.

- **Median**

➢ **Let us consider a collection of data with an odd number of values.**

The median of a collection of data is the value of the datum in the centre, when the data is arranged in ascending order.

3, 9, 9, 11, 15, $\boxed{22}$, 24, 25, 31, 37, 40

There are 11 data. The 6th datum is in the centre. Its value is 22. There are 5 data values less than 22 and 5 data values greater than 22.

If the number of values in a collection of data is an odd number, then the value in the centre, when the values are arranged in ascending order, is the median of the collection of data.

Accordingly, the median of the above collection of data is 22.

When the values of the above collection of data are arranged in ascending order, the value in the centre is the $\frac{11+1}{2}$ = 6th value. Therefore, the median of this set of data is 22.

➢ **Now let us consider a collection of data with an even number of values.**

3, 9, 9, 11, 15 22, 24, 25, 31, 37, 40, 41

There are 12 values in this collection of data, which is an even number. Here, we cannot find a datum in the centre. The two data in the centre are the 6th and 7th data. Their values are 22 and 24 respectively.

- If there is an even number of data, the median is half the sum of the values of the two data in the centre, when the data are arranged in ascending order.

- If there is an even number of data, and if the values of the data are arranged in ascending order, then the data in the centre are the $\left(\frac{\text{number of data}}{2}\right)$th datum and the $\left(\frac{\text{number of data}}{2}+1\right)$ th datum.
- There fore the median of the above data is $\frac{22+24}{2}$; that is, 23. There are 6 data less than 23 and 6 data greater than 23.

Example 3

The number of soft drink bottles sold at a certain shop on each day of a week is given below. Find the median of the number of bottles sold during a day.

32 60 52 44 48 41 40

When this data is organized in ascending order we obtain

32 40 41 $\boxed{44}$ 48 52 60

The median is 44.

The median of this collection of data is 44.

Example 4

The number of athletes who came to a certain sports centre for training each day of a period of 16 days is given below. Find the median of the number of athletes who came for training to the sports centre each day.

18 09 14 26 22 12 16 23 36 15 18 25 20 21 20 15

When this data is organized in ascending order we obtain,

09 12 14 15 15 16 18 $\boxed{18\ \ 20}$ 20 21 22 23 25 26 36

There are two values in the centre.

There are two values in the centre because there are 16 scores in total. They are the value of the $\frac{16}{2}$ = 8th datum and the value of the $\frac{16}{2} + 1$ = 9th datum.

The value of the 8th datum $= 18$

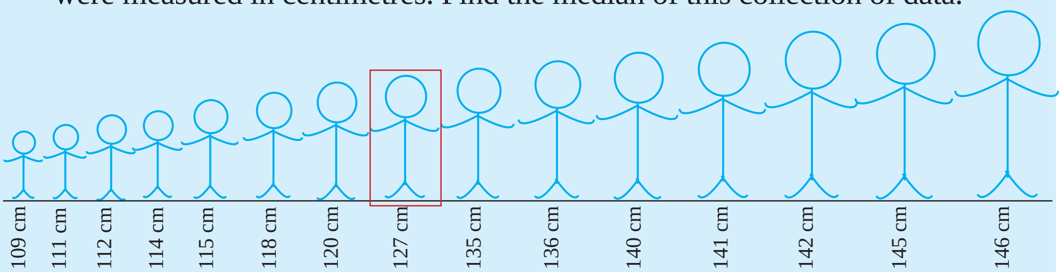
The value of the 9th datum $= 20$

$\therefore$ the median = $\frac{18 + 20}{2} = 19$

The median number of athletes who came for training to the sports centre each day is 19.

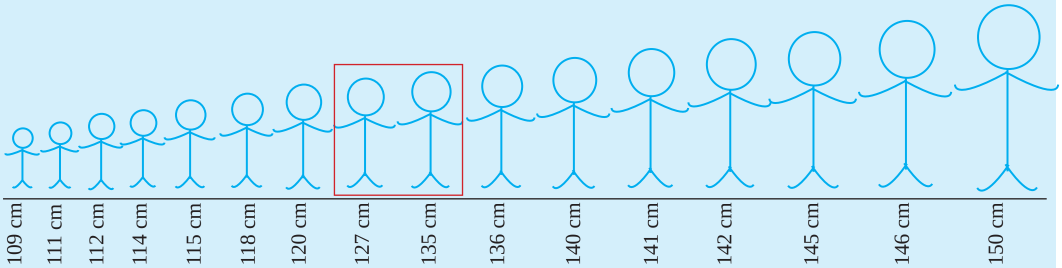
Example 5

(i) A group of 15 students who are members of a drill display, are made to stand in a row as shown in the figure in ascending order of their heights, after their heights were measured in centimetres. Find the median of this collection of data.



In the figure, the student in the centre is caged. This is the 8th student. We can easily obtain the value in the centre when there is an odd number of data. The median of this distribution is the value of the $\frac{15 + 1}{2}$ = 8th datum. The height of the 8th student is 127 cm. Therefore, the median of this collection of data is 127 cm.

(ii) Assume that a new student of height 150 cm joined the end of this row of students. Find the median of this collection of data.



Now there are 16 values in this collection of data. If we order these students as previously, two students are in the centre. They are the 8th and 9th students. Accordingly, the median is the value that is obtained when the heights of the 8th and 9th students are added together and divided by 2. Therefore the median is $\frac{127 + 135}{2}$ cm; that is, 131 cm.

- ## Mean

The average value of a collection of data is considered as its **mean**.

The mean of a collection of data is the value that is obtained when the sum of all the values of the collection of data is divided by the number of values.

$$\text{Mean} = \frac{\text{The total sum of the values of the collection of data}}{\text{number of data}}$$

Example 6

The marks obtained by 13 students of a certain class for a mathematics test paper marked out of 100 are given below in ascending order. Find the mean of this data.

45, 57, 69, 71, 71, 71, 78, 79, 81, 81, 94, 95, 96

$$\text{Mean} = \frac{\text{The total sum of the values of the collection of data}}{\text{number of data}}$$

The mean of the data $= \frac{45 + 57 + 69 + 71 + 71 + 71 + 78 + 79 + 81 + 81 + 94 + 95 + 96}{13}$

$= 76$

By comparing the value 76 with the total marks of 100 allocated for this mathematics test paper, we can assess these students' knowledge and skills in mathematics.

• Range

The marks obtained by the students in three classes for a mathematics paper are given below.

$\boxed{A}$	57	58	60	60	60	62	63

Median of the marks obtained by the students in class A = 60
Mean of the marks obtained by the students in class A = 60

$\boxed{B}$	35	45	55	60	65	75	85

Median of the marks obtained by the students in class B = 60
Mean of the marks obtained by the students in class B = 60

$\boxed{C}$	31	42	55	60	69	73	90

Median of the marks obtained by the students in class C = 60
Mean of the marks obtained by the students in class C = 60

The marks of the students vary across the classes, but the medians and the means are the same for all three classes.

In such situations, interpreting the data using only the median and the mean is insufficient. We need to consider the dispersion (spread) of the data too. For this we use measurements related to the dispersion of the data.

In this lesson we learn about one measurement of dispersion, namely the range.

The marks obtained by 8 students in a certain class for a mathematics test paper marked out of 100 are given below.

12, 28, 56, 48, 32, 64, 80, 92

Let us write the above values in ascending order.

12, 28, 32, 48, 56, 64, 80, 92

The maximum value of the above data set is 92 and the minimum value is 12.
The difference between the maximum value and the minimum value is 92 - 12 = 80.
It indicates that the dispersion of the data is 80 units.

The difference between the greatest value and the least value of a collection of data is called **its range**.

Range = Greatest value – Least value

The range of the above set of data is 80.

- The difference between the maximum mark of 100 and the minimum mark of 0 that can be scored by as student for the above mathematics test paper is 100.

- When the value of the range is comparatively low, the data take values which are close to each other. The range 80 in the above example is relatively large, when compared with 100. Therefore we can conclude that the marks are not close to each other.

Example 7

The marks obtained by another class of 8 students for a mathematics test paper are written below in ascending order. Find the range of these marks.

46, 48, 49, 50, 50, 51, 52, 54

The range of the above marks = 54 – 46 = 8

The range 8 in this example is relatively small when compared with 100. Therefore we can say that the marks are approximately at the same level, and conclude that these students' knowledge tested in this paper is at about the same level.

The most suitable representative value:

The runs scored by a cricketer during 8 overs of a cricket match are given below.

3, 8, 9, 12, 5, 3, 5, 3

The total number of runs he scored is 48. When the runs he scored in each of the 8 overs are written in ascending order we obtain

3, 3, 3, $\boxed{5, 5,}$ 8, 9, 12

The mode of this collection of data is 3.

$$\text{The median} = \frac{5+5}{2} = 5$$

$$\text{The mean} = \frac{48}{8} = 6$$

- ➢ The mode value of 3 indicates that the runs he scores in an over is most often 3.
- ➢ The median value of 5 indicates that the likelihood of scoring 5 or less runs per over is the same as the likelihood of scoring 5 or more runs per over.
- ➢ The mean value of 6 indicates that the rate at which he scores runs is 6 per over.

Exercise 27.3

(1) Find the mode, the median, the mean and the range of each collection of data.

(i) 8, 9, 12, 10, 12, 7, 8, 6, 10, 5, 8

(ii) 33, 32, 18, 33, 45, 23, 53, 32, 33

(iii) 78, 78, 80, 70, 78, 65, 69, 70

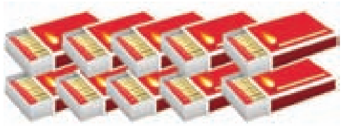
(iv) 3.5, 2.5, 4.8, 1.3, 3.9

(v) 12.5, 32.4, 23.6, 8.3

(2) The number of matchsticks in 10 boxes of matches is given here.

49, 50, 48, 47, 49, 50, 49, 50, 47, 51.

For these boxes of matchsticks, find



(i) the mode,

(ii) the median,

(iii) the mean number of matchsticks in a box.

(3) The temperature in the 9 provinces of Sri Lanka on a certain day are given here.

26^{o}C, 27^{o}C, 28^{o}C, 32^{o}C, 29^{o}C, 28^{o}C, 30^{o}C, 29^{o}C, 28^{o}C.

What was the mean temperature on that day?

(4) The masses of a group of children of the same age who arrived at a clinic on a certain day are given here. 15 kg, 16 kg, 18 kg, 12 kg, 14 kg, 16 kg, 17 kg, 20 kg.

(i) What is the mode of the masses of this group of children?

(ii) If the children are kept in a row in an ascending order of their masses, what would be the mass of the child in the centre?

(iii) According to the given data, what is the mean mass of a child in this group?

(5) The number of runs scored by each of the 11 batsmen in each of the two teams that played a cricket match against each other is given in the following table.

Batsman	1	2	3	4	5	6	7	8	9	10	11
Team *A*	34	42	58	5	32	21	16	0	9	3	12
Team *B*	8	0	12	33	31	60	44	36	24	12	6

(a) By considering the runs scored by the batsmen in team *A*, find;

(i) the minimum value (ii) the maximum value (iii) the range

(iv) the median (v) the mean

(b) By considering the runs scored by the batsmen in team *B*, find;

(i) the minimum value (ii) the maximum value (iii) the range

(iv) the median (v) the mean

(c) Using the above information, fill in the table given below.

Team	Range	Mean	Median
A			
B			

(d) From which representative value is the total marks of a cricket team obtained accurately? Show how you get this answer.

(6) The mean of the masses of 4 children is 34 kg. When another child joined, the mean mass increased to 38 kg.

(i) Find the total mass of the 4 children.

(ii) What is the mass of the child who joined later?

(iii) Show that the mean mass of 34 kg does not change if the mass of the child who joined later is also 34 kg.

Miscellaneous Exercise

(1) A bowler allows the opponent batsmen to score 52 runs in his 10 overs. Find the mean number of runs he gave per over.

(2) A group of 15 pilgrims are in an airplane. The mean mass of their luggage is 29 kg. Each person can carry up to 30 kg of luggage. If this is exceeded, an additional fee is charged.

(i) What is the total mass of the luggage carried by this group?

(ii) What is the total mass allowed for this group?

(3) The marks that Malitha and Dilitha got in the previous term test are as follows.

Subject	**Sinhala**	**English**	**Mathematics**	**Science**	**Buddhism**	**Geography**	**Art**	**Agriculture & Food Technology**	**History**
Malitha	39	40	65	60	56	64	70	65	54
Dilitha	64	55	42	58	70	68	49	70	45

(i) Complete the table given below.

	Malitha	Dilitha
Mode of the marks	....	....
Mean of the marks	....	....
Number of subjects for which the mark exceeds 50	....	....

(ii) Find separately the median mark of each student.

(iii) What is the most suitable representative value to compare two collections of data? Give reasons for your answer.

(4) The total marks obtained in a term test for all the subjects offered is given below for a group of students in a certain class. Represent this data in a stem and leaf diagram.

481	706	609	689	273	538	386	525	720	356
529	513	634	713	673	224	736	281	613	496
671	381	524	591	613	729	681	673	571	351

(5) The number of ready-made garments released to the market by a factory during the 26 working days of a month is given below.

Stem	Leaf
25	0 2 5
26	1 4 6 8
27	0 0 0 5 6 7 8 9
28	0 1 5 5 5
29	0 1 2
30	0 0 0

Key : 28|1 means 281.

(i) What is the minimum value of this data?
(ii) What is the maximum value?
(iii) Find the range.

Summary

- Data can be represented easily using a stem and leaf diagram. Understanding the data is facilitated by using a stem and leaf diagram.
- In a collection of data, some of the values could be identical. The value which occurs most often is called the mode of that collection of data.
- If the number of values in a collection of data is an odd number, then the value at the centre, when the values are arranged in ascending order, is the median of the collection of data.
- If there is an even number of data, the median is half the sum of the values of the two data at the centre, when arranged in ascending order.
- The mean of a collection of data is the value that is obtained when the sum of all the values of the collection of data is divided by the number of values.
- The difference between the greatest value and the least value of a collection of data is called its range.

  Range = Greatest value – Least value

28 Scale Diagrams

By studying this lesson you will be able to,

- identify what a scale diagram is,
- calculate the actual lengths of a rectilinear plane figure which has been drawn according to a given scale, and
- draw a scale diagram according to a given scale, when the actual measurements of a rectilinear plane figure are given.

28.1 Scale diagrams

It is often difficult to draw the various objects in the environment according to the actual measurements.

In such situations, for every rectilinear plane figure,

(i) a figure drawn to represent the shape of the original figure is called a **sketch**, and

(ii) when a rectilinear plane figure is drawn such that every measurement of length is increased or decreased by the same ratio, the drawn figure is called a **scale diagram** of the given figure.

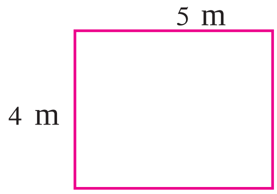
The shape of a figure in a scale diagram is exactly the same as the shape of the original figure, and only the size is different.

The floor plan of a house is drawn by decreasing the measurements

Let us recall the facts that were learnt on scale diagrams of rectangular shapes by considering the sketch given below.

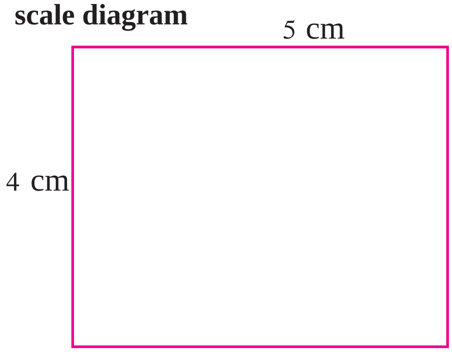
The length and breadth of the floor of a rectangular room is 5 m and 4 m respectively. A sketch of it is given here.

A sketch of the room



A scale diagram of the floor of the room can be drawn in your exercise book by representing an actual measurement of 1 m by 1 cm. Since 1 m is 100 cm, 1 cm in the scale diagram represents 100 cm of the floor. This is represented as a ratio by 1: 100. This ratio is known as the scale of the scale diagram.

scale diagram



Scale 1 : 100

The actual length of 5 m is represented by 5 cm in the scale diagram, and the actual length of 4 m is represented by 4 cm in the scale diagram.

Do the following review exercise to recall these facts which you have learnt previously.

Review Exercise

(1) Write the scale given in each of the following parts as a ratio.

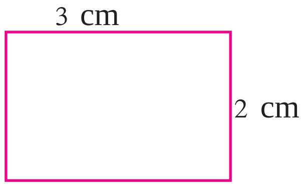
(i) Representing 50 cm of the actual length by 1 cm in the scale drawing.

(ii) Representing 2 m of the actual length by 1 cm in the scale drawing.

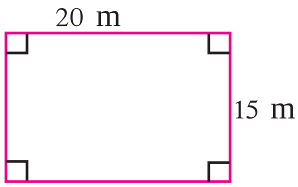
(iii) Representing 100 m of the actual length by 2 cm in the scale drawing.

(iv) Representing 1 mm of the actual length by 5 cm in the scale drawing.

(2) A scale diagram drawn to the scale 1:200 is given here.
   (i) Find the actual length represented by 1 cm according to the given scale.
   (ii) Find the length and breadth of the actual figure that is represented by this scale diagram.



(3) A sketch of the floor plan of a rectangular building is shown in the figure.
   (i) Write as a ratio, a suitable scale to draw a scale diagram of this floor plan.
   (ii) Draw a scale diagram of the floor plan using this scale.



28.2 Calculating the lengths corresponding to actual lengths when the scale of a scale diagram is given

Suppose we want to draw a scale diagram of the rectangular floor of a hall of length 6 m and breadth 4 m, using the scale 1 : 200. Let us find the length of each side in the scale diagram.

In the scale diagram,

an actual length of 200 cm $= 1$ cm

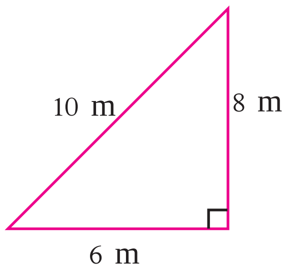
an actual length of 600 cm $= \frac{1}{200} \times 600$ cm $= 3$ cm

an actual length of 400 cm $= \frac{1}{200} \times 400$ cm $= 2$ cm

Since, 400 cm = 4 m and 6 m = 600 cm, the length of the floor in the scale diagram is 3 cm and the breadth is 2 cm.

i.e., the lengths 6 m and 4 m are represented by 3 cm and 2 cm respectively in the scale diagram.

The figure shows a sketch of a right triangular vegetable plot.

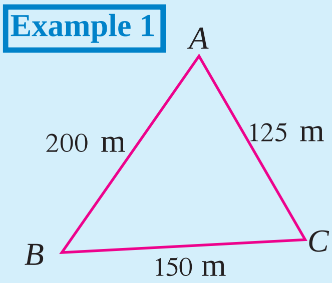


Let us find the lengths of the boundaries of this plot in the scale diagrams which are drawn according to the different scales given below.

Actual lengths of the boundaries of the vegetable plot	**Lengths in the scale diagram according to the given scale**		
	1 m is represented by 1 cm (1 : 100)	**2 m is represented by 1 cm (1 : 200)**	**$\frac{1}{2}$ m is represented by 1 cm (1 : 50)**
10 m	$\frac{1000}{100}$ cm = 10 cm	$\frac{1000}{200}$ cm = 5 cm	$\frac{1000}{50}$ cm = 20 cm
8 m	$\frac{800}{100}$ cm = 8 cm	$\frac{800}{200}$ cm = 4 cm	$\frac{800}{50}$ cm = 16 cm
6 m	$\frac{600}{100}$ cm = 6 cm	$\frac{600}{200}$ cm = 3 cm	$\frac{600}{50}$ cm = 12 cm

The scale 1 : 50 can be used if a larger scale diagram is required, and for a smaller scale diagram, the scale 1 : 100 can be used.

Example 1



A sketch of a triangular plot of land is shown in the figure. If a scale diagram of this plot is to be drawn to the scale 1 : 2500, find the length of each boundary of this plot in the scale diagram.

The scale is 1 : 2500.

Since 2500 cm = 25 m, an actual length of 25 m is represented by 1 cm in the scale diagram.

$\therefore$ In the scale diagram,

a length of 200 m is represented by $\frac{200}{25}$ cm, that is by 8 cm,

a length of 150 m is represented by $\frac{150}{25}$ cm, that is by 6 cm,

a length of 125 m is represented by $\frac{125}{25}$ cm, that is by 5 cm.

Example 2

By what length is an actual length of 250 m represented in a scale diagram which has been drawn to the scale 1 : 10 000?

1 cm represents 10 000 cm.

1 cm in the scale diagram represents an actual length of 100 m.

That is, an actual length of 100 m is represented by 1 cm in the scale diagram.

$\therefore$ In the scale diagram, an actual length of 250 m is represented by $\frac{250}{100} = 2.5$ cm.

Exercise 28.1

(1) The sketches of two flower beds *ABCD* and *PQRST* are shown in the figure. Complete the table using the given information.

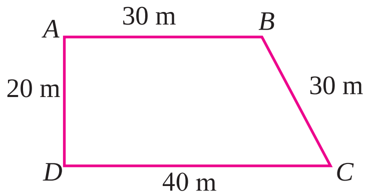


Figure (i)

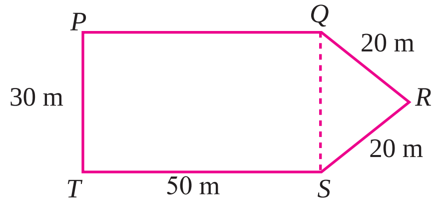
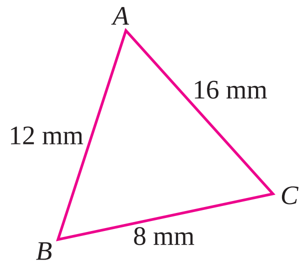


Figure (ii)

Figure	Scale	Actual length	Lengths in the scale diagram
(i)	1 : 1000	30 m	....
		20 m	....
		40 m	....
	1 : 500	30 m	....
		20 m	....
		40 m	....
(ii)	1 : 10	20 m	....
		50 m	....
		30 m	....

(2) (i) Write as a ratio the scale of 4 mm represented by 1 cm.

(ii) The measurements of a small triangular opening which is to be drawn according to the above scale is given in the sketch. Find the length of each side of the triangle in the scale diagram.



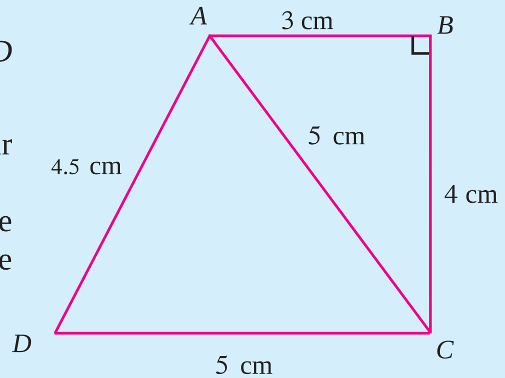
sketch

28.3 Determining the actual lengths when a scale diagram is given

You learnt in Grade 7 how to find the actual lengths when a scale diagram is given. Now, let us study this further.

Example 1

The scale diagram of a flower bed *ABCD* drawn to the scale 1 : 500 is given here.



(i) Find the actual lengths of all four sides.

(ii) Calculate the actual length of the drain *AC* that has been cut across the flower bed.

The scale is given as 1: 500.

$\therefore$ 1 cm in the scale drawing represents an actual length of 500 cm, which is 5 m.

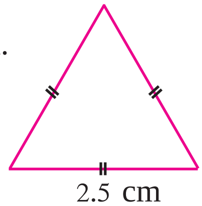
$\therefore$ Actual length of $AB = 3 \times 5\text{ m} = 15\text{ m}$
Actual length of $BC = 4 \times 5\text{ m} = 20\text{ m}$
Actual length of $DC = 5 \times 5\text{ m} = 25\text{ m}$
Actual length of $AD = 4.5 \times 5\text{ m} = 22.5\text{ m}$

(ii) The length of the drain $AC = 5 \times 5\text{ m} = 25\text{ m}$

Exercise 28.2

(1) A scale diagram of an equilateral triangular flower bed is drawn to the scale 1 : 100.

(i) Find the actual length indicated by 1 cm in the scale diagram.
(ii) Find the actual length of a boundary of the flower bed.
(iii) Find the actual perimeter of the flower bed.

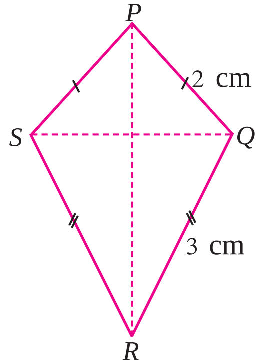


(2) A map of Sri Lanka has been drawn to the scale 1 : 50 000. What is the actual distance in kilometers between two towns which are 4 cm apart in the scale diagram?

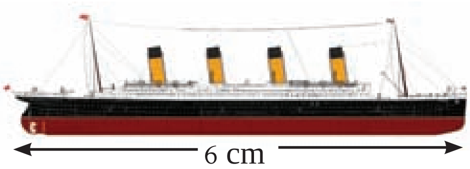
(3) The figure shows a scale diagram which has been drawn based on the measurements of a playground. The scale of the drawing is 1 : 20 000.

(i) Calculate the actual length of the side PQ of the play ground.

(ii) How much longer is the side QR than the side PQ of the actual playground?



(4) A scale diagram of a ship drawn to the scale 1: 1000 is shown here. Find the actual length of the ship.

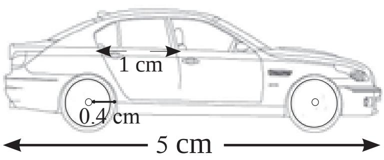


(5) A scale diagram of a car drawn to the scale 1:60 is shown here.

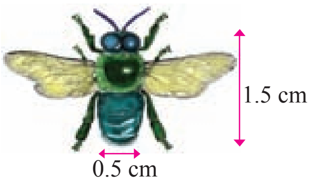
(i) Find the actual length of the car.

(ii) Find the actual diameter of a wheel of the car.

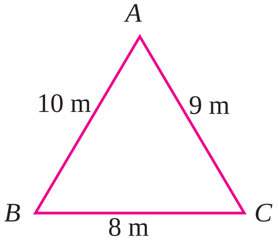
(iii) Find the actual breadth of a door.



(7) A scale diagram of an insect drawn to the scale 1 : 0.25 is given in the figure. Find the actual lengths represented by the lengths in the scale diagram.



28.4 Drawing scale diagrams

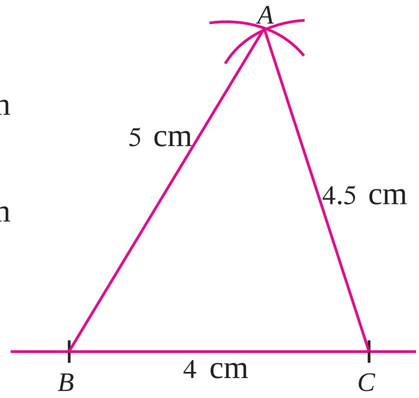


A sketch of a triangular flower bed ABC is shown in the figure. Let us select a suitable scale to draw a scale diagram of it. If 1 cm represents 2 m of an actual length, the scale is 1:200.

Let us follow the steps given below to draw the scale diagram.

The actual length represented by 1 cm in the scale diagram = 200 cm = 2 m

Step 1 - Let us calculate the length of each side of the scale drawing.



An actual length of 10 m is equal to $\frac{10}{2}$ cm $= 5$ cm in the scale diagram.

An actual length of 8 m is equal to $\frac{8}{2}$ cm $= 4$ cm in the scale diagram.

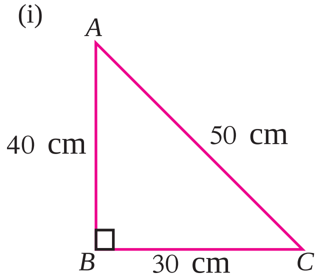
An actual length of 9 m is equal to $\frac{9}{2}$ cm $= 4.5$ cm in the scale diagram.

Step 2 - Using the knowledge gained in the lesson on the construction of triangles, construct the triangle *ABC* with sides of length 5 cm, 4 cm and 4.5 cm.

Exercise 28.3

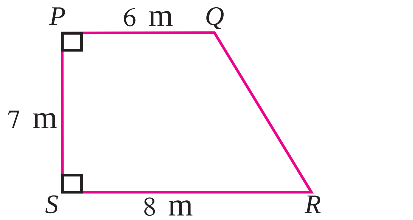
(1) Draw scale diagrams of each of the figures given in the following sketches, to the given scale.

(i)



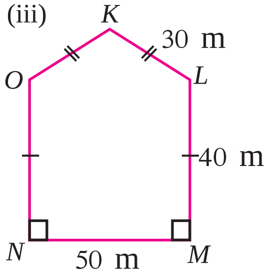
Scale 1 : 10

(ii)



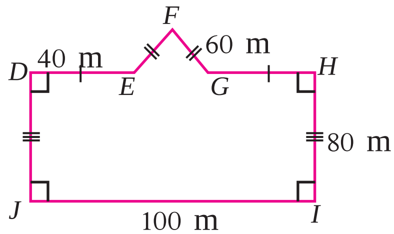
Scale 1 : 100

(iii)



Scale 1 : 1000

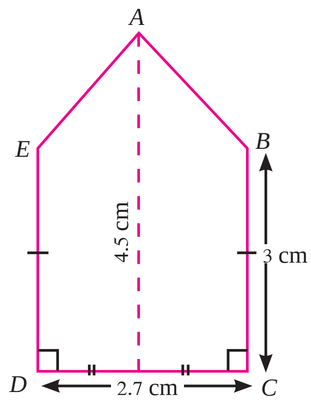
(iv)



Scale 1 : 2000

Miscellaneous Exercise

(1) A scale diagram of a side wall of a building is shown in the figure. It has been drawn to the scale 1 : 600.



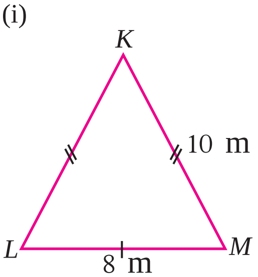
(i) Find the actual breadth of the wall.

(ii) Calculate the distance to the top of the building from ground level.

(iii) It costs Rs. 45 to paint 1 m^2 of the wall. Find the total cost of painting one side of the wall completely.

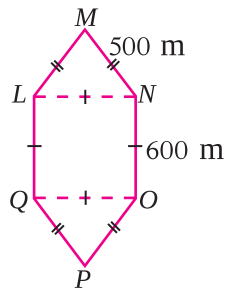
(2) Draw scale diagrams of each of the figures given in the following sketches, to the given scale.

(i)



Scale 1 : 200

(ii)



Scale 1 : 10 000

Summary

- The scale of a scale diagram indicates the actual length that is represented by a unit length in the scale diagram. This scale is often given as a ratio.
- For every rectilinear plane figure,

  (i) a figure drawn to represent the shape of the original figure is called a **sketch**, and

  (ii) when a rectilinear plane figure is drawn such that every measurement of length is increased or decreased by the same ratio, the drawn figure is called a **scale diagram** of the given figure.

29 Probability

By studying this lesson you will be able to,

- identify what the fraction of success of an outcome of a random experiment is,
- identify what the experimental probability of an event is, and
- identify what the theoretical probability of an event is.

29.1 Likelihood of an event occurring

Let us consider some events that occur in the environment. “The rising of the sun from the east” is an event that definitely occurs.

“The appearance of a full moon on a new moon day” is an event that definitely does not occur.

Let us consider the event of “a coin landing heads up when it is flipped”. We cannot say definitely which of the two events “landing heads up” and “landing tails up” will occur. Therefore this is a **random event.**

You learnt in grade 7 that events can be classified into three groups. They are;

- Events which definitely occur
- Events which definitely do not occur
- Random events

Consider flipping a coin.

- ➢ The experiment is flipping the coin and observing the side that lands up.
- ➢ The outcomes of this experiment are; “the coin landing heads up” and “the coin landing tails up”.
- ➢ If the coin is a fair (unbiased) coin, then there is an equal likelihood of each of these two outcomes occurring.

- The likelihood of an event which definitely does not occur is 0.
- The likelihood of an event which definitely occurs is 1.
- The likelihood of a random event occurring takes a value between 0 and 1, based on its tendency to occur.

Accordingly,

the likelihood of the sun rising from the west is 0,

the likelihood of the sun rising from the east is 1 and

the likelihood of a coin landing heads up when it is flipped takes a value between 0 and 1.

When a fair coin is flipped, there is an equal likelihood of it landing heads up and not landing heads up. Therefore we consider the likelihood of it landing heads up as $\frac{1}{2}$ and not landing heads up (landing tails up) as $\frac{1}{2}$.

- If there is an equal chance of an event occurring and of not occurring, then the likelihood of each is $\frac{1}{2}$.
- If the chance of an event occurring is greater than the chance of it not occurring, then the likelihood of that event occurring is a value between $\frac{1}{2}$ and 1.
- If the chance of an event not occurring is greater than the chance of it occurring, then the likelihood of that event occurring is a value between 0 and $\frac{1}{2}$.
- If p is the likelihood of a random event occurring, then the likelihood of it not occurring is $1 - p$.

When rolling an unbiased die with its faces marked with the numbers 1 to 6 (one number on each face), since there is an equal chance of each of the numbers from 1 to 6 showing up, the likelihood of 1 showing up is taken as $\frac{1}{6}$. Accordingly, the likelihood of 1 not showing up is $1 - \frac{1}{6} = \frac{5}{6}$.

Exercise 29.1

(1) Write three events which definitely occur.

(2) Write three events which definitely do not occur.

(3) Write three random events.

(4) An unbiased regular tetrahedral die with its faces numbered 1, 2, 3, 4 is tossed. Write the outcomes of the experiment of observing the number on the face which lands down.

(5) Complete the table given below.

	Event	Likelihood or the interval in which the likelihood lies (0, 1, $\frac{1}{2}$, between 0 and $\frac{1}{2}$, between $\frac{1}{2}$ and 1)
1	A fruit dropping from a tree landing on the ground	1
2	The sun rising from the east	....
3	If today is Monday, tomorrow being Wednesday	....
4	A bead drawn from a bag containing 10 red beads and 2 blue beads of equal size and shape, being a red bead	....
5	A face marked 1 showing up when a fair die with the numbers 1, 1, 1, 2, 2, 2 marked on its six faces is rolled	....
6	Winning the toss in a match	....
7	Getting a value greater than 2 when a fair die marked 1 to 6 is rolled	....
8	The sum of two odd numbers being an even number	....
9	The birthday of a child picked randomly from your class, falling on January 2	....
10	A person passing away on a Monday	....

29.2 Experimental Probability

- ### Random experiment

Let us consider again the event of a coin landing tails up when flipped. When the coin is being flipped, we cannot say with certainty whether it would land heads up or tails up. You have learnt that such an event is called a random event.

The experiment is flipping a coin and observing the side that lands up.

The outcomes of this experiment are "the coin landing heads up" and "the coin landing tails up".

An experiment of which the possible outcomes are known, but the actual outcome cannot be stated with certainty before the experiment is conducted is called a **random experiment**.

A random experiment and its possible outcomes are given in the following table.

Random Experiment	Possible Outcomes
Rolling a die with its faces numbered 1 to 6, and observing the number which shows up	1 showing up, 2 showing up 3 showing up, 4 showing up 5 showing up, 6 showing up

A random experiment has the following common features.

- Can be repeated any number of times under the same conditions
- The actual outcome cannot be stated with certainty before the experiment is conducted
- All the possible outcomes of the experiment are known before the experiment is conducted

• Fraction of Success (Relative frequency)

A Rs. 2 coin was flipped 20 times and the side that landed up was observed. The following are the outcomes.

The coin landed heads up 11 times.
The coin landed tails up 9 times.

The fraction of success of the coin landing heads up is

$$\frac{\text{the number of times the coin landed heads up}}{\text{the number of time the coin was flipped}}$$

$\therefore$ The fraction of success of the coin landing heads up $= \frac{11}{20}$

The fraction of success of the coin landing tails up is

$$\frac{\text{the number of times the coin landed tails up}}{\text{the number of time the coin was flipped}}$$

$\therefore$ The fraction of success of the coin landing tails up $= \frac{9}{20}$

Let A be one possible outcome of a random experiment. If we conduct the experiment several times over under the same conditions, then

$$\text{the fraction of success of } A = \frac{\text{number of times the outcome } A \text{ occurs}}{\text{total number of times the experiment is conducted}}$$

- **Obtaining the probability through observations**

Consider a random experiment with several possible outcomes. Then the likelihood of each outcome is called the probability of that outcome.

The outcome resulting from flipping a fair coin once cannot be stated with certainty before the experiment is conducted. Let us consider the outcomes of this experiment when it is repeated a large number of times under the same conditions.

The outcomes that were obtained by performing the random experiment of flipping a Rs. 2 coin 20 times and observing the side that lands up have been recorded in the table given below and the table has been completed.

Number of times the experiment was conducted	Total number of times Heads occurred	Total number of times Tails occurred	Fraction of success of getting heads $= \frac{\text{number of heads}}{\text{number of times the coin was flipped}}$	Fraction of success of getting tails $= \frac{\text{number of tails}}{\text{number of times the coin was flipped}}$
1	1	0	$\frac{1}{1} = 1$	$\frac{0}{1} = 0$
2	1	1	$\frac{1}{2} = 0.5$	$\frac{1}{2} = 0.5$
3	1	2	$\frac{1}{3} = 0.33$	$\frac{2}{3} = 0.67$
4	2	2	$\frac{2}{4} = 0.5$	$\frac{2}{4} = 0.5$
5	2	3	$\frac{2}{5} = 0.4$	$\frac{3}{5} = 0.6$
6	2	4	$\frac{2}{6} = 0.33$	$\frac{4}{6} = 0.67$
7	3	4	$\frac{3}{7} = 0.43$	$\frac{4}{7} = 0.57$
8	4	4	$\frac{4}{8} = 0.5$	$\frac{4}{8} = 0.5$
9	4	5	$\frac{4}{9} = 0.44$	$\frac{5}{9} = 0.56$
10	5	5	$\frac{5}{10} = 0.5$	$\frac{5}{10} = 0.5$
11	5	6	$\frac{5}{11} = 0.45$	$\frac{6}{11} = 0.55$
12	5	7	$\frac{5}{12} = 0.42$	$\frac{7}{12} = 0.58$
13	5	8	$\frac{5}{13} = 0.38$	$\frac{8}{13} = 0.62$
14	6	8	$\frac{6}{14} = 0.43$	$\frac{8}{14} = 0.57$

Number of times the experiment was conducted	Total number of times Heads occurred	Total number of times Tails occurred	Fraction of success of getting heads $= \dfrac{\text{number of heads}}{\text{number of times the coin was flipped}}$	Fraction of success of getting tails $= \dfrac{\text{number of tails}}{\text{number of times the coin was flipped}}$
15	7	8	$\frac{7}{15} = 0.47$	$\frac{8}{15} = 0.53$
16	8	8	$\frac{8}{16} = 0.5$	$\frac{8}{16} = 0.5$
17	9	8	$\frac{9}{17} = 0.53$	$\frac{8}{17} = 0.47$
18	10	8	$\frac{10}{18} = 0.56$	$\frac{8}{18} = 0.44$
19	10	9	$\frac{10}{19} = 0.53$	$\frac{9}{19} = 0.47$
20	11	9	$\frac{11}{20} = 0.55$	$\frac{9}{20} = 0.45$

Activity 1

Complete the table by flipping a coin 40 times with your friends.

Number of times the coin was flipped	Number of times tails occurred	Number of times heads occurred	$\dfrac{\text{Number of times tails occurred}}{\text{Number of times the coin was flipped}}$	$\dfrac{\text{Number of times heads occurred}}{\text{Number of times the coin was flipped}}$

An important conclusion that we can arrive at through this experiment is that the fractions,

$$\frac{\text{number of times tails occurred}}{\text{number of times the coin was flipped}} \text{ and } \frac{\text{number of times heads occurred}}{\text{number of times the coin was flipped}}$$

approach the value $\frac{1}{2}$ as the number of times the coin is flipped is increased.

What this means is that when a fair coin is flipped, the likelihood of the coin landing heads up, that is the experimental probability of the coin landing heads up is $\frac{1}{2}$. In this case, the experimental probability of the coin landing tails up is also $\frac{1}{2}$.

- Since the number of times an outcome of an experiment occurs is always less than or equal to the total number of times the experiment is repeated, the fraction of success of an outcome takes a value from 0 to 1. If the fraction of success of the outcome A approaches a constant value when the number of times the experiment is repeated (n) is increased, then that value is called the **experimental probability** of the outcome A occurring when the experiment is conducted once.

The sun always rises from the east, and does not depend on the number of times the sunrise is observed.
Therefore, the probability of the sun rising from the east is 1. The probability of the sun rising from the south is 0 because it never rises from the south.

- If an outcome will definitely occur, then irrespective of the value n (the number of times the experiment is conducted) takes, its fraction of success is $\frac{n}{n} = 1$. In this case, the probability of the outcome occurring is 1. **Accordingly, the probability of an event that will definitely occur is 1.**
- If an outcome cannot occur when an experiment is conducted, then, irrespective of the value n takes, its fraction of success is $\frac{0}{n} = 0$. Therefore, the probability of that outcome occurring is 0. **Accordingly, the probability of an event that will definitely not occur is 0.**

Apart from these two special cases, the probability of a possible outcome of a random experiment occurring is a value between 0 and 1.

★ When the probability of an outcome of a random experiment occurring is not known, then, the fraction of success that is obtained by increasing n, that is by conducting the experiment a large number of times, is **value that is suitable to be used as an estimate of the probability** of that outcome occurring.

Exercise 29.2

(1) There are three identical beads in a bag. They are red, blue and yellow in colour. A bead is drawn, its colour is recorded and is put back in the bag. Another bead is drawn, its colour is also recorded and is put back. The outcomes of this experiment which was repeated 50 times are given in the following table.

Bead	Number of times it was drawn
Red	18
Blue	17
Yellow	15

(i) Find the experimental probability of drawing the red bead.

(ii) Find the experimental probability of drawing the blue bead.

(iii) Find the experimental probability of drawing the yellow bead.

(2) An unbiased tetrahedral die with the numbers 1 to 4 marked on its four faces (one number on each face), was tossed 40 times and the number on the face that landed down was recorded. The results of this experiment are shown below.

Number	Number of times it occurred
1	8
2	11
3	10
4	11

(i) Find the experimental probability of getting the number 2.
(ii) Find the experimental probability of getting an even number.
(iii) Find the experimental probability of getting a prime number.
(iv) Find the experimental probability of getting a number greater than 1.

29.3 Theoretical Probability

Let us find the probability of the occurrence of each possible outcome of a random experiment with equally likely outcomes.

- In the random experiment of flipping an unbiased coin and observing the side that lands up, the outcomes are "the coin landing heads up" and "the coin landing tails up". The likelihood of each of these two outcomes occurring is the same.

- When an unbiased die is rolled, the number on the face that shows up is either 1 or 2 or 3 or 4 or 5 or 6. There is an equal likelihood of each of these outcomes occurring.

The probability of 2 showing up when an unbiased die is rolled can be calculated as follows.

The outcome can be either 1 or 2 or 3 or 4 or 5 or 6.
Since the die is unbiased, there is an equal likelihood of each of these six numbers showing up. Therefore, the probability of any one of the six numbers from 1 to 6 showing up is $\frac{1}{6}$.

Accordingly, the probability of 2 showing up $= \frac{1}{6}$

- Three of the six numbers on the die are even numbers. Therefore, the probability of an even number showing up is $\frac{3}{6} = \frac{1}{2}$.

When there is an equal likelihood of each of the possible outcomes of a random experiment occurring, the theoretical probability of a selected outcome occurring

$$= \frac{1}{\text{total number of possible outcomes of the random experiment}}.$$

The method of finding the theoretical probability of each outcome of a random experiment, when the probabilities of the possible outcomes of the experiment occurring are different to each other, is described in the following example.

Example 1

In an opaque bag, there are 4 red balls, 5 blue balls and 2 green balls which are identical in all aspects except the colour. Find the probability of a ball drawn from the bag

(i) being red.
(ii) being blue.
(iii) being green.

$$\text{Probability of being red} = \frac{\text{number of red balls}}{\text{total number of balls}} = \frac{4}{11}$$

$$\text{Probability of being blue} = \frac{\text{number of blue balls}}{\text{total number of balls}} = \frac{5}{11}$$

$$\text{Probability of being green} = \frac{\text{number of green balls}}{\text{total number of balls}} = \frac{2}{11}$$

Exercise 29.3

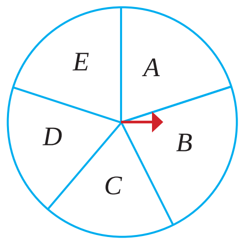
(1) An unbiased die with its faces marked from 1 to 6 is rolled. Find the probability of each of the following events.

(i) The number 5 showing up.
(ii) An even number showing up.
(iii) A square number showing up.

(2) There are 3 white beads 2 black beads and 1 blue bead in a bag, which are identical in size and shape. A bead is drawn randomly from the bag. Find the probability of each of the following events.

(i) Drawing a white bead
(ii) Drawing a black bead.
(iii) Drawing a blue bead.
(iv) Drawing a white bead or a black bead.
(v) Drawing a bead which is not black in colour.
(vi) Drawing a red bead.

(3) As shown in the figure, a circular lamina is divided into five equal parts and an indicator is fixed at the centre. The five parts are named A, B, C, D and E. When the indicator is rotated freely, it stops in one of the five parts. Accordingly, find the probability of each of the following events.



(i) The indicator stopping in D.
(ii) The indicator stopping in A or D
(iii) The indicator stopping in B, C or E.

Summary

- The likelihood of an event occurring is its probability.
- Let A be one possible outcome of a random experiment. If the experiment is repeated several times under the same conditions, then

$$\text{the fraction of success of } A = \frac{\text{number of times the outcome } A \text{ occurs}}{\text{total number of times the experiment is conducted}}.$$

- When there is an equal likelihood of each of the possible outcomes of a random experiment occurring, the theoretical probability of a selected outcome occurring is

$$\frac{1}{\text{total number of possible outcomes of the random experiment}}.$$

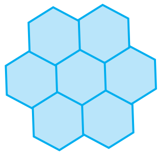
30 Tessellation

By studying this lesson, you will be able to,
- identify what regular tessellations and semi-regular tessellations are,
- select suitable polygons to create regular and semi-regular tessellations, and
- create regular and semi-regular tessellations.

30.1 Tessellation

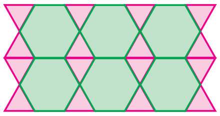
Let us recall what was learnt in Grade 7 about tessellation.

Covering a certain space using one or more shapes, in a repeated pattern, without gaps and without overlaps is called **tessellation**. An arrangement of shapes of this form is also called a tessellation.



If a tessellation consists of one shape only, it is called a **pure tessellation.**

If a tessellation consists of two or more shapes, it is called a **semi-pure tessellation**.



In tessellations where rectilinear plane figures are used, the sum of the angles around each vertex point is 360^{o}.

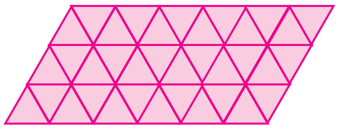
Therefore, the shapes that are selected for such tessellations should be such that the 360^{o} around a point on a plane can be covered without gaps and without overlaps with the selected shapes.

Do the following review exercise to revise the facts you have learnt previously on tessellation.

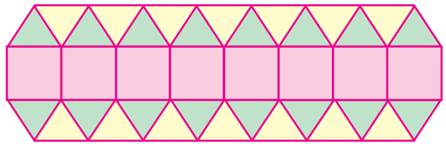
Review Exercise

(1) In your exercise book, draw a tessellation consisting of only equilateral triangular shapes.

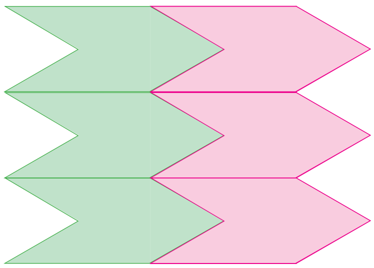
(2) For each of the following tessellations, write with reasons whether it is a pure tessellation or a semi-pure tessellation.



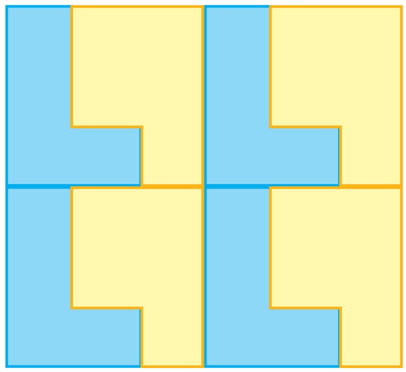
(a)



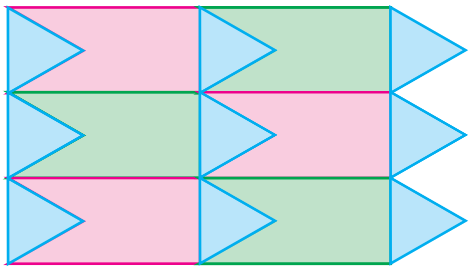
(b)



(c)

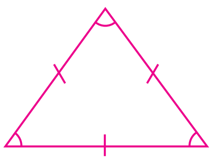


(d)

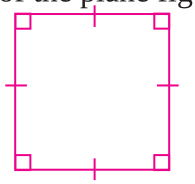


(e)

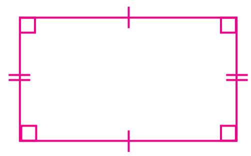
(3) Select and write the numbers of the plane figures which are regular polygons.



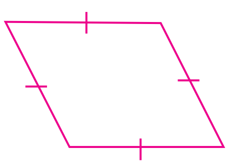
(i)



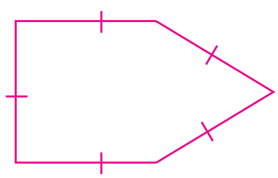
(ii)



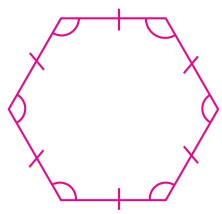
(iii)



(iv)



(v)



(vi)

30.2 Regular tessellation

We know that a polygon with sides of equal length and interior angles of equal magnitude is a regular polygon. Equilateral triangles, squares, regular pentagons and regular hexagons are examples of regular polygons.

A tessellation created using only one regular polygonal shape is known as a **regular tessellation**.

When creating a regular tessellation,

- a vertex of one geometrical shape should not be on a side of another geometrical shape.

In the tessellation in Figure 1 created with equilateral triangles, all the shapes are identical regular polygons. A vertex of any triangle is not located on a side of another triangle. Therefore this is a regular tessellation.

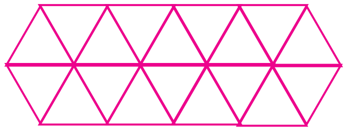


Figure 1

In the creation in Figure 2, although identical regular polygons have been used, the vertices of some polygons lie on the sides of other polygons. Therefore, this is not a regular tessellation.

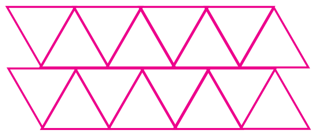
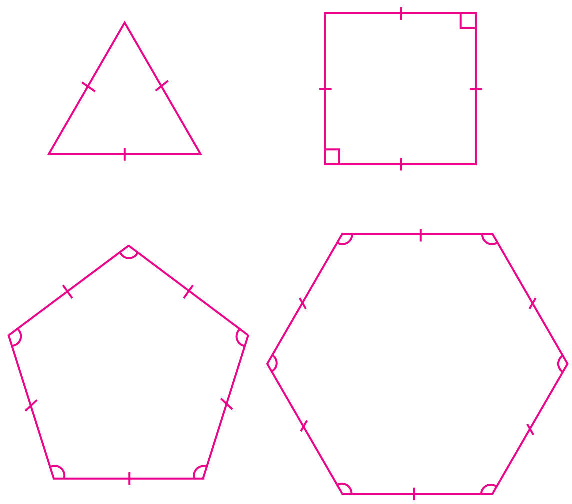


Figure 2

Activity 1

Step 1 - Trace these regular polygonal shapes onto coloured paper using a tissue paper and cut out 10 shapes of each kind.

Step 2 - Create a regular tessellation using only the triangular shapes and paste it in your exercise book.



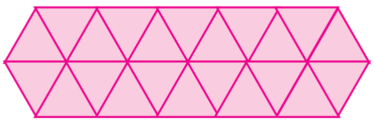
Step 3 - Examine each of the other shapes carefully and check whether a regular tessellation can be created.

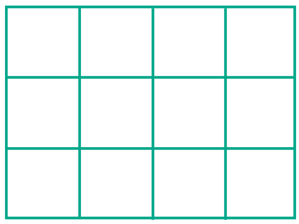
Step 4 - Using the shapes that were identified above as those with which regular tessellations can be created, create regular tessellations and paste them in your exercise book.

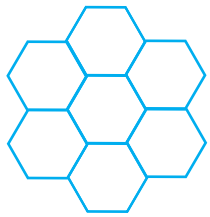
Step 5 - Find out how many types of regular polygons can be used to create regular tessellations.

Step 6 - Investigate the condition that needs to be satisfied by an interior angle of a regular polygon, to be able to create a regular tessellation with that polygon.

According to the above activity, we can create regular tessellations by using either equilateral triangles or squares or regular hexagons only.







In the creation of regular tessellations, the vertices of the regular shapes used should meet at particular points. These are called the vertices of the tessellations.

The sum of the angles around each vertex point of a tessellation is 360^{o}.

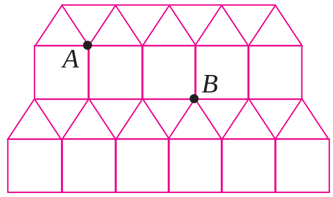
It must be clear to you through the above activity that a regular tessellation can be created by using a particular regular polygon, only if 360^{o} is a multiple of the magnitude of an interior angle of that polygon.

An interior angle of a regular pentagon is 108^{o}. Since 360^{0} is not divisible by 108^{o}, we cannot create a regular tessellation by using a regular pentagon.

30.3 Semi-regular tessellation

Tessellations created using two or more regular polygons, and such that the same polygons in the same order (when considered clockwise or anticlockwise) surround each vertex point are called **semi-regular tessellations.**

Given here is a semi-regular tessellation created using squares and equilateral triangles.

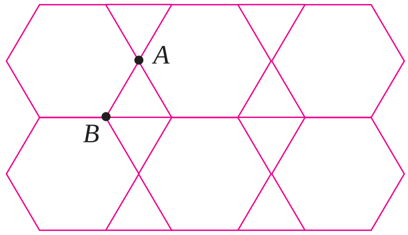


Observe how the polygons are positioned at the vertex points *A* and *B*. You can see that three triangular shapes and two square shapes meet at each of these two points. At both points, the three triangles and the two squares are positioned in the same order, with the three triangles together followed by the two squares next to each other.

This feature can be observed in the whole tessellation.

This is a feature of a semi-regular tessellation. That is, in a semi-regular tessellation, the same polygonal shapes should surround each vertex point and they should be positioned in the same order around these points.

This tessellation is made up of equilateral triangles and regular hexagons. Observe the vertex points *A* and *B* carefully. We can clearly see that the orders in which the polygons are positioned around these two points are different to each other.



Since the orders in which the shapes are positioned at different vertex points are not identical, this tessellation is **not a semi-regular tessellation**.

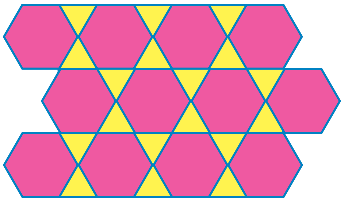
Activity 2

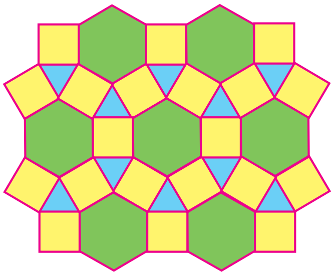
Step 1 - Cut out the shapes used in Activity 1 again using coloured paper.

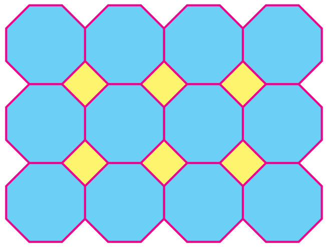
Step 2 - Create semi-regular tessellations using two types of shapes and paste them in your exercise book.

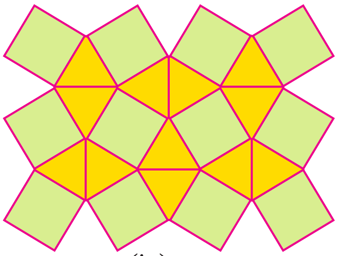
Step 3 - Create semi-regular tessellations using three types of shapes and paste them in your exercise book.

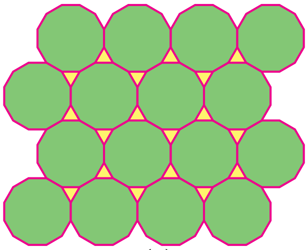
There are 8 types of semi-regular tessellations that can be created on a plane. They are given below.

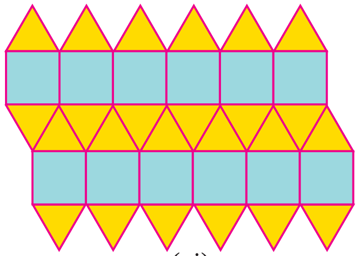

(i)

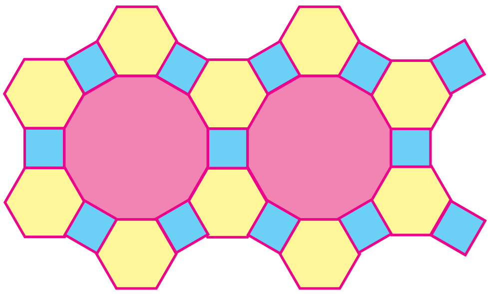

(ii)

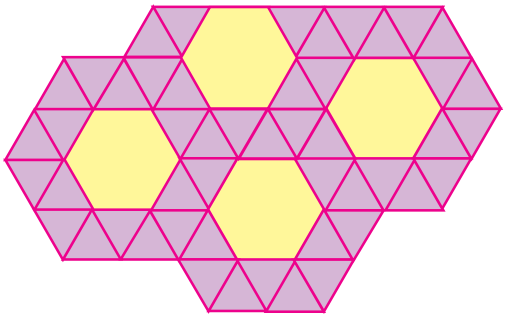

(iii)


(iv)


(v)


(vi)


(vii)


(viii)

Exercise 30.1

(1) (i) What are the regular polygons that can be used to create regular tessellations?

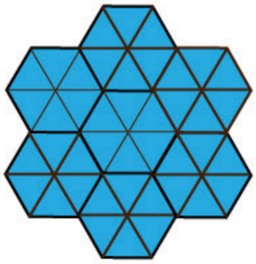
(ii) How many types of regular tessellations are there?

(iii) Each interior angle of a certain regular polygon is 98^{o}. Explain whether a regular tessellation can be created using this polygon.

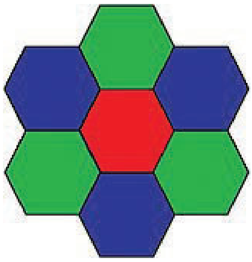
(2) Some figures are given below.

(i) Select and write the letters corresponding to the figures which are regular tessellations.

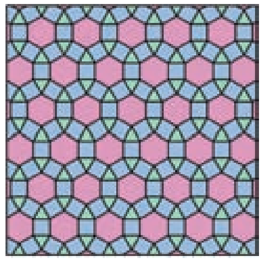
(ii) Select and write the letters corresponding to the figures which are semi-regular tessellations.



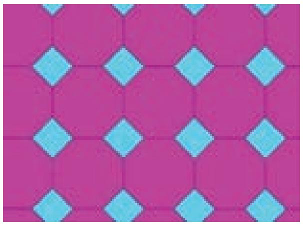
(a)



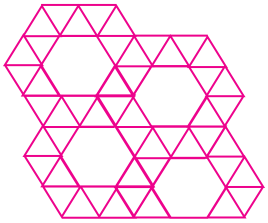
(b)



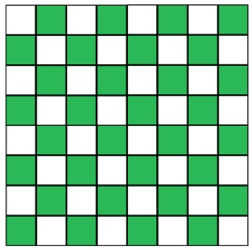
(c)



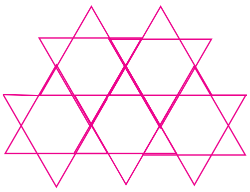
(d)



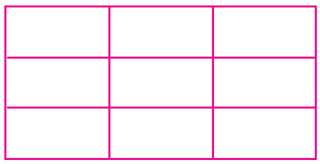
(e)



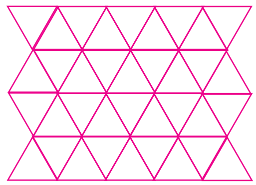
(f)



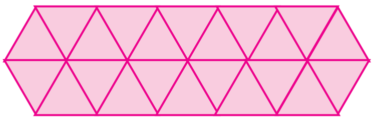
(g)



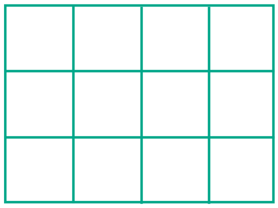
(h)



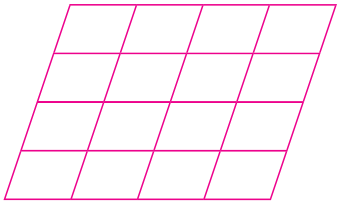
(i)



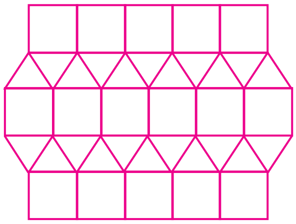
(j)



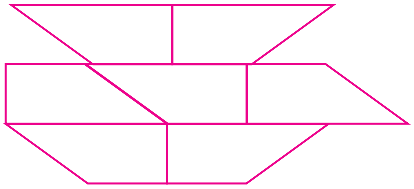
(k)



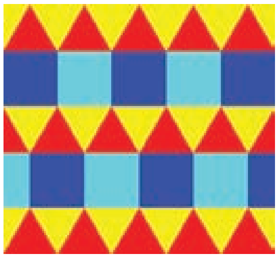
(*l*)



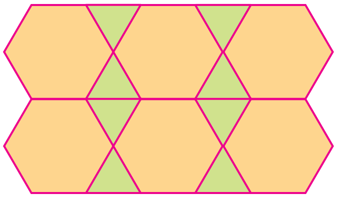
(m)



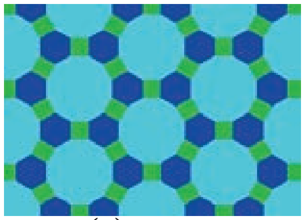
(n)



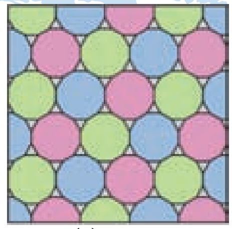
(o)



(p)

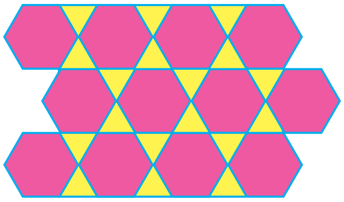


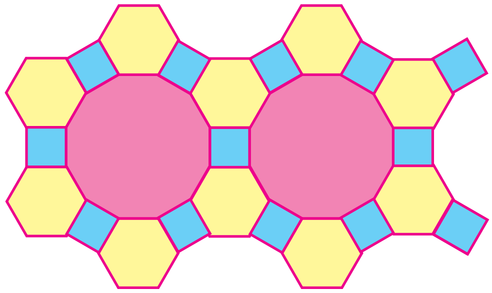
(q)



(r)

(3) Explain with reasons whether each of the following tessellations which have been created using regular polygons is a semi-regular tessellation or not.





Miscellaneous Exercise

Prepare several regular/semi-regular tessellations that are suitable for wall hangings.

Summary

- A tessellation created using only one regular polygonal shape is known as a regular tessellation.
- Tessellations created using two or more regular polygonal shapes, and such that the same polygons in the same order (when considered clockwise or anticlockwise) surround each vertex point are called semi-regular tessellations.

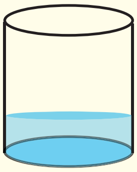
Revision Exercise 3

(1) Represent each of the following inequalities on a separate number line.

(i) $x > 2$ (ii) $x < -1$ (iii) $x \leq 3$ (iv) $-2 < x \leq 3$

(v) $0 \leq x < 5$

(2) The shaded portion of the cylindrical container in the figure contains 550 ml of water. Estimate the capacity of the container.



(3) The length, breadth and height of a cuboidal shaped container are 8 cm, 6 cm and 10 cm respectively. Find the following.

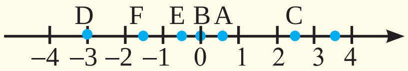
(i) The capacity of the container.

(ii) The volume of water in the container if water is filled up to a height of 6 cm.

(4) With the aid of figures, explain the terms given below which are related to circles.

- Chord
- Arc
- Sector
- Segment

(5) For each part given below, select the correct answer from within the brackets by considering the given number line.



(i) The number indicated by A is

$(1\frac{1}{2}\ ,\ -0.5,\ \frac{1}{2})$

(ii) The number indicated by F is

$(-2.5,\ -1.5,\ -3\frac{1}{2})$

(iii) According to the numbers indicated by B and D,

(B > D, D > B)

(iv) According to the numbers indicated by C, D and E,

(C > E and D > E, D > E > C, D < E < C)

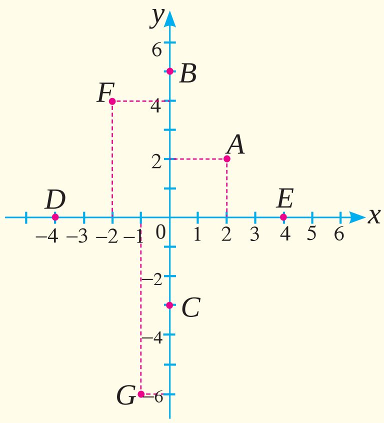
(6) A wax cube of side length 6 cm is given.

(i) Find the volume of the wax cube.

(ii) Write the above answer as a product of prime factors.

(iii) The given wax cube is melted and eight equal size cubes are made without wastage. If the side lengths of the two cubes are integral values, write the side length of each cube separately.

(7) Write the coordinates of the points A, B, C, D, E, F and G that are marked on the Cartesian plane.



(8) Draw a Cartesian plane with the x and y axes marked from -5 to 5.

(i) Draw the graphs of the straight lines given by $x = -2$, $y = 3$, $x = 5$ and $y = -4$ on the above Cartesian plane.

(ii) Write the coordinates of the points of intersection of the above graphs.

(9) From the sets of length measurements given below, write the sets that could be the lengths of the sides of a triangle.

(i) 4.2 cm , 5.3 cm, 6 cm

(ii) 12.3 cm , 5.7 cm, 6.6 cm

(iii) 8.5 cm , 3.7 cm, 4.3 cm

(iv) 15 cm , 9 cm, 12 cm

(10) Construct triangles with the following measurements as side lengths.

(i) 8 cm , 6 cm, 10 cm

(ii) 6.3 cm , 3.5 cm, 8.2 cm

(11) (i) Construct the triangle ABC such that $AB = 7.2$ cm, $BC = 5$ cm and $AC = 6.7$ cm.

(ii) Measure and write the magnitude of $A\hat{B}C$ in the above triangle.

(12) The lengths of the calls received on a certain day by a person who uses a mobile phone are given below to the nearest minute.

3, 2, 5, 10, 1, 3, 7, 3, 4, 6, 2, 4, 3, 8, 11, 4, 3, 2

(i) Write the range of the given set of data.

(ii) What is the mode?

(iii) Write the median.

(iv) Using the mean, estimate the time in hours and minutes that could be expected to be spent on 100 calls that are received by this person.

(13) Write the scales given below using a different method.

(i) Representing 100 m by 1 cm.

(ii) Representing 0.25 km by 1 cm.

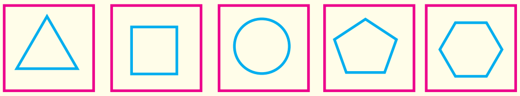
(iii) 1 : 50000

(iv) Representing $\frac{3}{4}$ km by 1 cm.

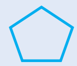
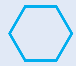
(14) (i) In a scale diagram drawn to the scale 1: 50000, what is the actual distance in kilometres represented by 3.5 cm?

(ii) The scale selected to draw a scale diagram is 1: 0.5. Find the length of the straight line segment that needs to be drawn to represent 3.5 km.

(15) Three points *A*, *B* and C are located on a flat ground. *B* is situated 800 m away from *A* is 60^{o} east of north and *C* is situated 600 m away from *B* is 30^{o} east of south. Illustrate this information with a sketch.

(16) 

The figure shows five types of plane figures printed on 5 identical cards. The cards are mixed well and one card is picked randomly. The plane figure on the picked card is recorded and the card is re-placed. Another card is picked randomly as before, and again the plane figure on it is recorded. The results obtained by conducting this experiment repeatedly are given in the following table.

Figure					
Tally marks	$\cancel{||||}$ $|||$	$\cancel{||||}$ $||$	....	$\cancel{||||}$ $\cancel{||||}$	$\cancel{||||}$ $||||$
Number of outcomes	....	....	9	....	....

(i) Copy the table and complete it.

(ii) How many times was this experiment repeated?

(iii) Write the fraction of success of obtaining the shape $\square$.

(iv) Draw the shape of the plane figure with the highest fraction of success.

(v) Draw the shapes of the plane figures with equal fractions of success and write this fraction.

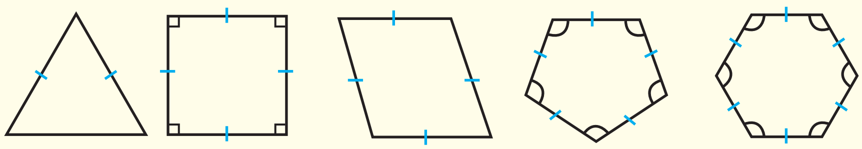
(17) A bag contains 2 red pens, 3 blue pens and 1 black pen of identical shape and size. A pen is taken out randomly. Find the probability of it being,

(i) a black pen.

(ii) a blue pen or a black pen

(iii) a green pen.

(18) From the given figures, select the shapes that can be used to create regular tessellations and write their corresponding letters.



(a) (b) (c) (d) (e)

(19) Copy each of the statements given below and place a "✓" before the statement if it is correct and a "✗" if it is incorrect.

(i) A circle has no rotational symmetry.

(ii) Only rectilinear plane figures have rotational symmetry.

Lesson sequence

Content	Competency levels	Number of periods
1st term		
1. Number Patterns	2.1	05
2. Perimeter	7.1	05
3. Angles	21.1	05
4. Directed Numbers	1.2	05
5. Algebraic Expressions	14.1	05
6. Solids	22.1	06
7. Factors	15.1	06
8. Square Root	1.1	05
9. Mass	9.1	05
10. Indices	6.1, 6.2	05
		52
2nd term		
11. Symmetry	25.1	05
12. Triangles	23.1	06
13. Fractions - I	3.1	06
14. Fractions - II	3.2	06
15. Decimals	3.3	07
16. Ratios	4.1, 4.2	06
17. Equations	17.1	05
18. Percentages	5.1, 5.2	06
19. Sets	30.1	04
20. Area	8.1, 8.2	06
21. Time	12.1, 12.2	06
		63
3rd term		
22. Volume and Capacity	10.1, 11.1	06
23. Circle	24.1	05
24. Location of a place	13.1	03
25. Number line and Cartesian plane	20.1, 20.2, 20.3	09
26. Triangle Constructions	27.1	06
27. Data representation and Interpretation	28.1, 29.1, 29.2	10
28. Scale Drawings	13.2	05
29. Probability	31.1, 31.2	06
30. Tessellation	26.1	05
		55
	Total	**170**

Glossary

Arc of a circle	වෘත්ත චාපය	வட்டவில்
Area	වර්ගඵලය	பரப்பளவு
Base	ආධාරකය	அடி
Capacity	ධාරිතාව	கொளள் ளவு
Cartesian co-ordinate plane	කාටීසීය ඛණ්ඩාංකතලය	தெகக் ாடீன்ஆளகூறுதத்ளம்
Centre	කේන්ද්‍රය	மையம்
Chord	ජ්‍යාය	நாண்
Circle	වෘත්තය	வட்டம்
Closed figures	සංවෘත රූප	மூடிய உரு
Commiunication	සන්නිවේදනය	தொடர்பாடல்
Continued ratios	සංයුක්ත අනුපාත	கூட்டுவிகிதம்
Compound solids	සංයුක්ත ඝනවස්තු	கூட்டுத்திண்மங்கள்
Construction	නිර්මාණය	அமைப்பு
Conversion	පරිවර්තනය	வகுப்பு எல்லை
Cube	ඝනකය	சதுரமுகி
Cuboid	ඝනකාභය	கனவுரு
Data	දත්ත	தரவு
Decimal numbers	දශම සංඛ්‍යා	தசம எண்கள்
Denomínator	හරය	பகுதி
Direction	දිශාව	திசை
Distance	දුර	தூரம்
Elements	අවයව	மூலகம
Events that do not occur	සිදු නොවන සිද්ධි	நடகக்ும்நிகழச்சிகள்
Events that definitely occur	ස්ථිර වශයෙන් සිදු වන සිද්ධි	
Events	සිද්ධි	நிகழ்ச்சிகள்
Experiment	පරීක්ෂණය	பரிசோதனை
Experimental probability	පරීක්ෂණාත්මක සම්භාවිතාව	பரிசோதனை முறை நிகழ்சசிகள்
Flow chart	ගැලීම් සටහන	பாய்ச்சற் கோட்டுப்படம்
Formula	සූත්‍රය	சூத்திரம்
Fraction of success	සාර්ථක භාගය	வெற்றிப்பின்னம்
Fraction	භාගය	பின்னம்
Fractions	භාග	பின்னம்
Greenwich meridian line	ග්‍රිනිච් මධ්‍යාහ්න රේඛාව	கிறின்வீச்கிடைக்கோடு
Greater than	වඩා විශාල	இலும் பெரிய
Perpendicular height (or altitude)	උච්චය	உயரம
Infinite	අපරිමිත	முடிவிலி
International date line	ජාත්‍යන්තර දින රේඛාව	சர்வதேச திகதிக்கோடு

Latitude	අක්ෂාංශ	அகலக்கோடு
Location	පිහිටීම	அமைவு
Longitude	දේශාංශ	நெடுஙே காடு
Maximum value	උපරිම අගය	கூடிய பெறுமானம்
Minimum value	අවම අගය	குறைந்த பெறுமானம
Null set	අභිශූන්‍ය කුලකය	வெறுந்தொடை
Number of elements of a set	කුලකයක අවයව සංඛ්‍යාව	மூலகங்களின் எண்ணிக்கை
Numerator	ලවය	தொகுதி
Ordered pairs	පටිපාටිගත යුගල	வரிசைப்பட்ட சோடி
Percentages	ප්‍රතිශත	சதவீதம்
Polygon	බහු අස්‍රය	பல்கோணி
Likelihood	විය හැකියාව	இயல்தகவு
Probability	සම්භාවිතාව	நிகழ்தகவு
Protractor	කෝණ මානය	பாகைமாணி
Quadrant	වෘත්ත පාදක	காற்பகுதி
Random events	සමහර විට සිදු වන සිද්ධි (අහඹු සිද්ධි)	சிலவேளை நடக்கும்நிகழச்சிகள் (எழுமாறான நிகழ்ச்சிகள)
Range	පරාසය	எண் தொடரி
Ratio	අනුපාතය	விகிதம்
Rectangle	සෘජුකෝණාස්‍රය	செவ்வகம்
Right angled triangle	සෘජුකෝණී ත්‍රිකෝණය	செங்கோண முக்கோணி
Regular tessellation	සවිධි ටෙසලාකරණ	ஒழுங்கான தெசலாக்கம்
Rough sketch	දළ සටහන	பரும்படி படம்
Scale	පරිමාණය	அளவிடை
Sector of a circle	කේන්ද්‍රික ඛණ්ඩය	ஆரைச்சிறை
Segment of a circle	වෘත්ත ඛණ්ඩය	வட்டத்துண்டம்
Semi-regular tessellation	අර්ධ සවිධි සෙලාකරණ	அரைத் தூய தெசலாக்கமி
Set	කුලකය	தொடை
Sides of a triangle	ත්‍රිකෝණයක පාද	முக்கோணியின் பக்கங்கள்
Simple equation	සරල සමීකරණ	எளிய சமன்பாடுகள்
Solution	විසඳුම	தீர்வு
Square	සමචතුරස්‍රය	சதுரம்
Stem and leaf diagram	වෘන්ත පත්‍ර සටහන	தண்டு - இலை வரைபு
Symmetry	සමමිතිය	சமச்சீர்
Tesselation	ටෙසලාකරණ	தெசலாக்கம்
Theoretical probability	සෛද්ධාන්තික සම්භාවිතාව	அறிமுறை நிகழ்தகவு
Time zones	කාල කලාප	காலவலயம்
Triangle	ත්‍රිකෝණය	முக்கோணி
True length	සැබෑ දිග	உண்மை நீளம்
Unknown	අඥාතය	தெரியாக்கணியம்
Volume	පරිමාව	கனவளவு