MATHEMATICS

Grade 9

Part - I

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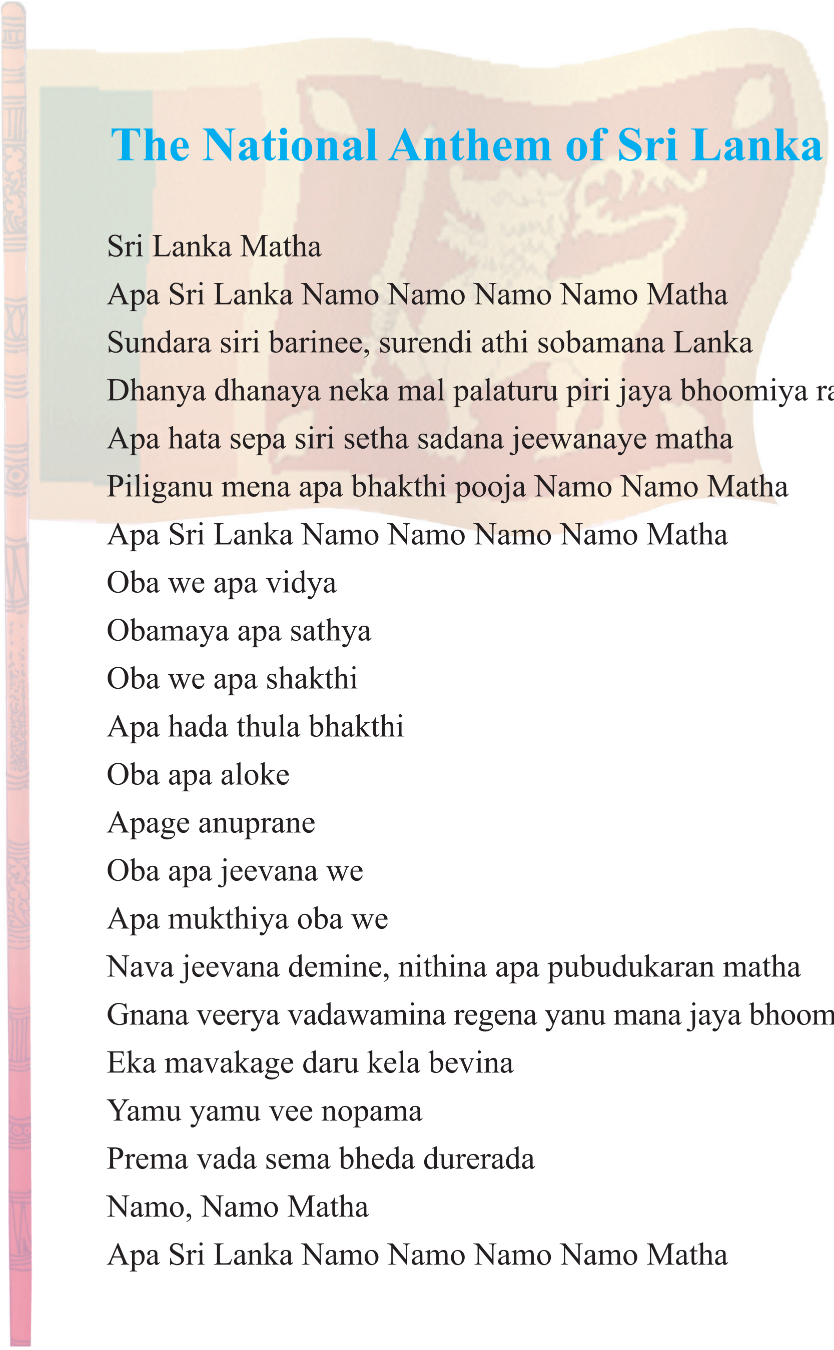

The National Anthem of Sri Lanka

Sri Lanka Matha
Apa Sri Lanka Namo Namo Namo Namo Matha
Sundara siri barinee, surendi athi sobamana Lanka
Dhanya dhanaya neka mal palaturu piri jaya bhoomiya ramya
Apa hata sepa siri setha sadana jeewanaye matha
Piliganu mena apa bhakthi pooja Namo Namo Matha
Apa Sri Lanka Namo Namo Namo Namo Matha
Oba we apa vidya
Obamaya apa sathya
Oba we apa shakthi
Apa hada thula bhakthi
Oba apa aloke
Apage anuprane
Oba apa jeevana we
Apa mukthiya oba we
Nava jeevana demine, nithina apa pubudukaran matha
Gnana veerya vadawamina regena yanu mana jaya bhoomi kara
Eka mavakage daru kela bevina
Yamu yamu vee nopama
Prema vada sema bheda durerada
Namo, Namo Matha
Apa Sri Lanka Namo Namo Namo Namo Matha

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ආනන්ද සමරකෝන්

ஒரு தாய் மக்கள் நாமாவோம்
ஒன்றே நாம் வாழும் இல்லம்
நன்றே உடலில் ஓடும்
ஒன்றே நம் குருதி நிறம்

அதனால் சகோதரர் நாமாவோம்
ஒன்றாய் வாழும் வளரும் நாம்
நன்றாய் இவ் இல்லினிலே
நலமே வாழ்தல் வேண்டுமன்றோ

யாவரும் அன்பு கருணையுடன்
ஒற்றுமை சிறக்க வாழ்ந்திடுதல்
பொன்னும் மணியும் முத்துமல்ல - அதுவே
யான்று மழியாச் செல்வமன்றோ.

ஆனந்த சமரக்கோன்
கவிதையின் பெயர்ப்பு.

Being innovative, changing with right knowledge
Be a light to the country as well as to the world.

Message from the Hon. Minister of Education

The past two decades have been significant in the world history due to changes that took place in technology. The present students face a lot of new challenges along with the rapid development of Information Technology, communication and other related fields. The manner of career opportunities are liable to change specifically in the near future. In such an environment, with a new technological and intellectual society, thousands of innovative career opportunities would be created. To win those challenges, it is the responsibility of the Sri Lankan Government and myself, as the Minister of Education, to empower you all.

This book is a product of free education. Your aim must be to use this book properly and acquire the necessary knowledge out of it. The government in turn is able to provide free textbooks to you, as a result of the commitment and labour of your parents and elders.

Since we have understood that the education is crucial in deciding the future of a country, the government has taken steps to change curriculum to suit the rapid changes of the technological world. Hence, you have to dedicate yourselves to become productive citizens. I believe that the knowledge this book provides will suffice your aim.

It is your duty to give a proper value to the money spent by the government on your education. Also you should understand that education determines your future. Make sure that you reach the optimum social stratum through education.

I congratulate you to enjoy the benefits of free education and bloom as an honoured citizen who takes the name of Sri Lanka to the world.

Akila Viraj Kariyawasam
Minister of Education

Foreword

The educational objectives of the contemporary world are becoming more complex along with the economic, social, cultural and technological development. The learning and teaching process too is changing in relation to human experiences, technological differences, research and new indices. Therefore, it is required to produce the textbook by including subject related information according to the objectives in the syllabus in order to maintain the teaching process by organizing learning experiences that suit to the learner needs. The textbook is not merely a learning tool for the learner. It is a blessing that contributes to obtain a higher education along with a development of conduct and attitudes, to develop values and to obtain learning experiences.

The government in its realization of the concept of free education has offered you all the textbooks from grades 1-11. I would like to remind you that you should make the maximum use of these textbooks and protect them well. I sincerely hope that this textbook would assist you to obtain the expertise to become a virtuous citizen with a complete personality who would be a valuable asset to the country.

I would like to bestow my sincere thanks on the members of the editorial and writer boards as well as on the staff of the Educational Publications Department who have strived to offer this textbook to you.

W. M. Jayantha Wickramanayaka,
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Message of the Board of Compilers

This textbook has been compiled in accordance with the new syllabus to be implemented from 2015.

Textbooks are compiled for students. Therefore, we have made an attempt to compile this textbook in a simple and detailed manner making it possible for you to read and understand it on your own.

We have included descriptions, activities and examples to introduce the subject concepts in an attractive manner and to establish them. Moreover, activities are organized from simple to complex to develop an interest to do them.

We have used the terms related to mathematical concepts in accordance with the glossary of technical terms of mathematics compiled by the Department of Official Languages.

Some subject matter learnt during the earlier grades is necessary to learn the subject content in the grade 9 syllabus. Thus, review exercises are included at the beginning of each chapter to revise previous knowledge. You will be prepared by them for the subject content of grade 9.

You will gain maximum benefit from this textbook by reading the chapters and doing the review exercises of each chapter even before your teacher teaches them in the classroom.

We hope that studying mathematics will be an interesting, joyful and productive experience.

Board of Compilers

Contents

1

Number Patterns

By studying this lesson you will be able to;

- develop the general term of a number pattern with the same difference between adjacent terms,
- develop the number pattern when the general term is known,
- solve problems associated with number patterns.

Introduction to number patterns

Several number patterns are given below.

i. 3, 3, 3, 3, 3, ...
ii. 2, 4, 6, 8, 10, ...
iii. 5, 8, 11, 14, 17, ...
iv. 2, 4, 8, 16, 32, ...
v. 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, ...
vi. 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, ...

The first number pattern is very simple. Every number of this number pattern is 3. While the first number of the second number pattern is 2, all the numbers thereafter are obtained by adding 2 to the previous number.

While the first number of the third number pattern is 5, all the numbers thereafter are obtained by adding 3 to the previous number.

While the first number of the fourth number pattern is 2, all the numbers thereafter are obtained by multiplying the previous number by 2.

The fifth and sixth number patterns have characteristics which are inherent to them.

The numbers of a number pattern are called "terms".

For example, each term of the first number pattern is 3.

The first term of the second number pattern is 2, the second term is 4, the third term is 6, etc. In this pattern, each term which comes after the first term is obtained by adding 2 to the previous term.

The first term of the third number pattern is 5, the second term is 8, the third term is 11, etc. In this pattern, each term which comes after the first term is obtained by adding 3 to the previous term.

In the fourth number pattern, each term which comes after the first term is obtained by multiplying the previous term by 2.

The ways in which the terms of the fifth and sixth number patterns are obtained can also be described as above. However, the descriptions will be more complicated.

Observe that the terms of the number patterns given above are separated by commas and that there are three dots (ellipsis) at the end of each number pattern. This is how number patterns are usually written. The three dots indicate that the number pattern continues.

In mathematics, the word "sequence" is used for the word "pattern".
Accordingly, six "number sequences" (or simply "sequences") are given above.
The order of the terms of a sequence is important.

For example, although the sequence 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 6, 6, ...
and the sequence 1, 2, 1, 2, 3, 4, 3, 4, 5, 6, 5, 6, ... consist of the same numbers, they are two different sequences.

In the above examples of sequences, only a few initial terms are given. However, it is incorrect to presume the pattern of the sequence by considering only a few initial terms.
For example,

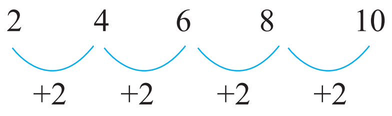
$$1, 2, 3, 4, 5, 1, 2, 3, 4, 5, 1, 2, 3, 4, 5, \ldots$$

is a number pattern; that is, a sequence. If only the first five terms of the sequence are given (that is, 1, 2, 3, 4, 5, …), and a person is asked what the next term is, one may be provided with the incorrect answer 6. Hence, asking for the next term (or next few terms) after giving only the first few terms of a sequence is mathematically incorrect.

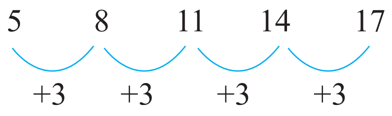
A method of describing a sequence accurately is by providing a rule by which each term of the sequence can be calculated.

The uniqueness (or characteristic) of the second and third sequences of the six sequences given above can be explained as follows.

In the second sequence, every term which comes after the first term is obtained by adding the constant value 2 to the previous term. This can be illustrated as follows.



In the third sequence, every term which comes after the first term is obtained by adding the constant value 3 to the previous term. This can be illustrated as follows.



Here, the meaning of “constant value” is “the value remains unchanged”.

The characteristic which is common to both these patterns can be described as follows.

“The value obtained by subtracting the previous term from any term (except the first term) is a constant (that is, a constant value).”

The value of this constant is 2 for the sequence 2, 4, 6, 8, 10, …
(since $4 - 2 = 6 - 4 = 8 - 6 = 10 - 8 = 2$).

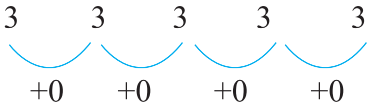
The value of this constant is 3 for the sequence 5, 8, 11, 14, 17, …
(since $8 - 5 = 11 - 8 = 14 - 11 = 17 - 14 = 3$).

Let us study further about sequences of which the difference between every pair of consecutive terms is a constant value.

This constant value is known as the common difference of the sequence. Accordingly,

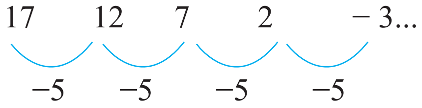
common difference = any term except the first term – the previous term

It can be seen that the first sequence 3, 3, 3, 3, 3, … also has the same characteristic.



Here, the constant value added (that is the common difference) is 0.

Another sequence with the same characteristic is given below.



The first term of this sequence is 17. Every term which comes thereafter is obtained by subtracting 5 from the previous term. That is, by adding −5 to the previous term. Accordingly, the common difference of this sequence is −5.

Common difference = $12 - 17 = 7 - 12 = 2 - 7 = -3 - 2 = -5$.

If the value of the common difference and the first term of a sequence with a common difference are known, the first few terms of the sequence can be written easily. A couple of examples of such sequences are given below.

Example 1

The first three terms of the sequence with first term 4 and common difference 3 are 4, 7 and 10.

Example 2

The first five terms of the sequence with first term 7 and common difference − 4 are 7, 3. −1, −5 and −9.

The first few terms of a sequence with a common difference can be written easily, when the first term and common difference are given. But it is not so easy to find, say the 50th term or the 834th term. The reason is because 50 and 834 are fairly large numbers.

It is important to know the general term of a sequence to be able to find any term of the sequence easily. Now let us see what is meant by the general term.

The general term of a number pattern

First, let us introduce a specific notation to denote the terms of a sequence. For a given sequence, let us denote

the first term by T_1,
the second term by T_2,
the third term by T_3, etc.

For example, with regard to the sequence

$$5, 11, 17, 23, ...$$

we can indicate the terms as follows:

the first term, $T_1 = 5$
the second term, $T_2 = 11$
the third term, $T_3 = 17$
the fourth term, $T_4 = 23$

It is very important to consider the nth term of a sequence, as is usually done in mathematics. Here, n represents any positive integer. The reason for this is that the values n can assume are positive integers such as 1, 2, 3, ... The $\frac{1}{2}$th term, the -4th term, the 3.5th term, etc., have no meaning. When considering a value n, the corresponding nth term is denoted by T_n. This is called the **general term** of the sequence. Accordingly,

the general term (nth term) of a sequence is denoted by T_n.

1.1 Developing the number sequence when the general term is given

In the previous section we learnt the notation that is used to denote the terms of a sequence, in particular, the general term. Now, through a couple of examples, let us consider how to develop the number sequence and how to find a named term of the sequence, when the general term is given.

Example 1

Consider the number sequence with general term $T_n = 2n + 3$.

(i) Write the first three terms of this sequence.
(ii) Find the 20th term.
(iii) Which term is equal to 123?
(iv) Find the $(n + 1)$th term in terms of n.

(i) Since the general term $T_n = 2n + 3$,
when $n = 1$; the first term $T_1 = (2 \times 1) + 3 = 2 + 3 = 5$,
when $n = 2$; the second term $T_2 = (2 \times 2) + 3 = 4 + 3 = 7$,
when $n = 3$; the third term $T_3 = (2 \times 3) + 3 = 6 + 3 = 9$.

Therefore, the first three terms of this number pattern are 5, 7, 9.

(ii) The 20th term is obtained by substituting $n = 20$ in $2n + 3$.

The 20th term, $T_{20} = (2 \times 20) + 3 = 40 + 3$
$= 43$
Therefore, the 20th term is 43.

(iii) Let us assume that the nth term is 123.

$$\begin{aligned} \text{Then, } 2n + 3 &= 123 \\ 2n + 3 - 3 &= 123 - 3 \\ 2n &= 120 \\ n &= \frac{120}{2} \\ &= 60 \end{aligned}$$

Therefore, 123 is the 60th term of the number pattern.

(iv) In order to obtain the $(n + 1)$th term, let us substitute $(n + 1)$ for n.
The $(n + 1)$th term,

$$\begin{aligned} T_{n+1} &= 2\,(n + 1) + 3 \\ &= 2n + 2 + 3 \\ &= 2n + 5 \end{aligned}$$

Therefore, the $(n + 1)$th term, in terms of n, is $2n + 5$.

Example 2

Consider the number pattern with general term $T_n = 56 - 4n$.

(i) Write the first three terms of this number pattern.
(ii) Find the 12th term.
(iii) Show that 0 is a term of this number pattern.
(iv) Show that 18 is not a term of this number pattern.

(i) Since the general term $T_n = 56 - 4n$,

when $n = 1$; the first term $T_1 = 56 - (4 \times 1) = 56 - 4 = 52$
when $n = 2$; the second term $T_2 = 56 - (4 \times 2) = 56 - 8 = 48$
when $n = 3$; the third term $T_3 = 56 - (4 \times 3) = 56 - 12 = 44$
Therefore, the first three terms of the number pattern are 52, 48, 44.

(ii)The 12th term $= 56 - 4 \times 12$

$$= 56 - 48$$
$$= 8$$

(iii) If 0 is a term of the number pattern, then for some integer n, we have

$$56 - 4n = 0.$$

$\therefore\ 56 - 4n + 4n = 4n$ (adding $4n$ to both sides)

$$\frac{56}{4} = \frac{4n}{4}$$
$$14 = n$$
$$n = 14$$

The 14th term of the number pattern is 0.Therefore 0 is a term of this number pattern.

(iv) If 18 is a term of this number pattern, then for some integer n, we have

$$56 - 4n = 18.$$

Then, $56 - 4n + 4n = 18 + 4n$

$$56 - 18 = 18 - 18 + 4n$$
$$38 = 4n$$
$$9\frac{1}{2} = n$$

If 18 is a term of this number pattern, the value of n should be a whole number. Since $n = 9\frac{1}{2}$, 18 is not a term of this number pattern.

Exercise 1.1

1. Complete the table.

The general term of the number pattern	The first term when $n = 1$	The second term when $n = 2$	The third term when $n = 3$	First three terms of the number pattern
$3n + 2$	$(3 \times 1) + 2 = 5$	$(3 \times 2) + 2 = 8$	$(3 \times 3) + 2 = 11$	5, 8, 11
$5n - 1$	$(5 \times 1) - 1 = 4$	....	....	..., ..., ...
$2n + 5$	....	....	....	..., ..., ...
$20 - 2n$	....	....	....	..., ..., ...
$50 - 4n$	....	....	....	..., ..., ...
$35 - n$	....	....	....	..., ..., ...

2. The general term of a number pattern is $4n - 3$.

i. Write the first three terms of this number pattern.
ii. Find the 12th term.
iii. Which term is equal to 97?
iv. Show that 75 is not a term of this number pattern.

3. Consider the number pattern with nth term $7n + 1$.

i. Write the first three terms of this number pattern.
ii. Find the 5th term.
iii. Which term is equal to 36?
iv. Write the $(n+1)$th term, in terms of n.

4. Consider the number pattern with general term $T_n = 50 - 7n$.

i. Write the first three terms of this number pattern.
ii. Find the 10th term.
iii. Write the $(n + 1)$th term, in terms of n.
iv. Show that the terms which come after the 7th term are negative numbers.

1.2 Obtaining an expression for the general term (T_n)

In the previous section an expression was given for the general term T_n. Our objective in this section, is to obtain an expression for T_n in terms of n. Then, any term of the sequence can easily be found by using the obtained expression. Now let us consider how we can develop such an expression, through an example.
Suppose we want to find the 80th term of the sequence 5, 11, 17, 23…, which is a sequence with a common difference. That is, we want to find the value of T_{80}. First, examine the pattern given in the following table.

n	T_n	How T_n can be written in terms of n and the common difference 6.
1	5	$6 \times 1 - 1$ or $5 + 0 \times 6$
2	11	$6 \times 2 - 1$ or $5 + 1 \times 6$
3	17	$6 \times 3 - 1$ or $5 + 2 \times 6$
4	23	$6 \times 4 - 1$ or $5 + 3 \times 6$
5	29	$6 \times 5 - 1$ or $5 + 4 \times 6$

You may be wondering why the expressions $6 \times 1 - 1$, $6 \times 2 - 1$, $6 \times 3 - 1$, etc., given in the 3rd column of the table have been written. Especially, the reason why 1 is subtracted from each term may be unclear to you. This can be explained as follows.

Since the common difference of the given sequence 5, 11, 17, 23, … is 6, let us write the given sequence first and several multiples of 6 below it.

5, 11, 17, 23, 29, ...

6, 12, 18, 24, 30, ...

It is clear that the given sequence can be obtained by subtracting 1 from each multiple of 6.

That is,
the first term of the sequence = the first multiple of 6 – 1
the second term of the sequence = the second multiple of 6 –1
the third term of the sequence = the third multiple of 6 – 1

Accordingly,
the nth term of the sequence = the nth multiple of 6 – 1

$$\therefore\ T_n = 6n - 1$$

Accordingly,

$$T_{80} = 6 \times 80 - 1 = 479.$$

Therefore, the 80th term is 479.

Moreover, an expression for the general term T_n of this sequence was found above as $T_n = 6n - 1$.
We can find any term of the sequence using this expression. For example, in order to find the 24th term of this sequence, n has to be substituted with 24.

$T_{24} = 6 \times 24 - 1 = 143$

Therefore, the 24th term of the sequence is 143.

Let us consider another example.

Example 1

Given that the sequence with first four terms15, 19, 23, 27 has a common difference, let us find an expression for the nth term.

The common difference $= 19 - 15 = 4$.
Let us write the first few terms of the given sequence, and several multiples (positive integer multiples) of 4 below them.

15, 19, 23, 27, ...
4, 8, 12, 16, ...

It is clear that the given number pattern is obtained by adding 11 to each multiple of 4.

Therefore, the expression for the general term T_n is given by $T_n = 4n + 11$.
Let us find the 100th term using this expression.

$T_{100} = 4 \times 100 + 11 = 411$

Now let us consider a sequence with a negative common difference, which therefore consists of terms which are decreasing in value.

Example 2

Let us find an expression for the general term T_n of the sequence with a common difference, of which the first four terms are 10, 7, 4, 1.

The common difference of the sequence 10, 7, 4, 1, … $= 7 - 10 = -3$.

Therefore, let us write the first few terms of the given sequence and a few multiples of –3 (integral), one below the other.

$$10, 7, 4, \ldots$$
$$-3, -6, -9, \ldots$$

It can be seen that the terms of the sequence are obtained by adding 13 to the multiples of −3. Therefore, the general term can be written as

$$T_n = -3n + 13$$

(Or else, it can be written as $T_n = 13 - 3n$ with the positive term first.)

For example, in order to find the 30th term, $n = 30$ should be substituted in the expression for T_n.

$T_{30} = -3 \times 30 + 13 = -77$

Therefore, the 30th term is − 77.

Exercise 1.2

All sequences in this exercise have a common difference.

1. Copy the following table in your exercise book and complete it.

Pattern	The difference between two successive terms	The number, whose multiples are used to develop the pattern
5, 8, 11, 14,...	8 − 5 = 3	3
10, 17, 24, 31, ...		
$2\frac{1}{2}, 4, 5\frac{1}{2}, 7, \ldots$		
20, 17, 14, 11, ...		
50, 45, 40, 35, ...		
0.5, 0.8, 1.1, 1.4, ...		

2. Complete the table in relation to the number pattern 10, 17, 24, 31, ...,

Sequential order of the terms	Term	How the pattern has been developed
1st term	10	$7 \times 1 + ...$
2nd term	17	$7 \times 2 + ...$
3rd term	24	$... + ...$
4th term	31	$... + ...$
*n*th term	....	$... + ... = ...$

3. Find the general term of each of the number patterns given below.

a. 1, 4, 7, 10, ...
b. 1, 7, 13, 19, ...
c. 9, 17, 25, 33, ...
d. 4, 10, 16, 22, ...
e. 22, 19, 16, 13, ...
f. 22, 20, 18, 16, ...

1.3 Solving mathematical problems involving number patterns

We can solve various mathematical problems by developing number patterns using information that is given.

Example 1

A long distance runner trains every day. On the first day he runs 500 m and on each day thereafter he runs 100 m more than on the previous day.

i. Write separately the distances he runs on the first three days.
ii. Find the general term T_n for the distance he runs on the *n*th day, in terms of *n*.
iii. Find the distance he runs on the 20th day.
iv. On which day does he run 3km?

i. The distance run on the first day = 500 m
The distance run on the second day = 500 m + 100 m = 600 m
The distance run on the third day = 500 m + 100 m + 100 m = 700 m
$\therefore$ The first three terms of the number pattern are 500, 600, 700

ii. Let us take the day as n.

The number pattern of the distance run by the athlete is built up by multiples of 100.

Therefore, the general term $T_n = 100n + 400$

iii. It is clear that the distance run on the 20th day is represented by the 20th term.

$$\begin{aligned} \text{The 20th term, } T_{20} &= (100 \times 20) + 400 \\ &= 2000 + 400 \\ &= 2400 \text{ m} \\ &= 2.4 \text{ km} \end{aligned}$$

$\therefore$ The distance run on the 20th day is 2.4 km.

iv. Let us assume that 3000 m are run on the nth day.

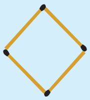
$$\begin{aligned} \text{Then, } 100n + 400 &= 3000 \\ 100n + 400 - 400 &= 3000 - 400 \\ 100\,n &= 2600 \\ \therefore\ n &= \frac{2600}{100} \\ &= 26 \end{aligned}$$

Therefore, 3km are run on the 26th day.

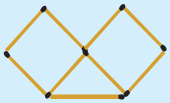
Exercise 1.3

1. A pattern created by using matchsticks is shown below.

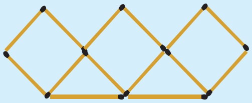
①



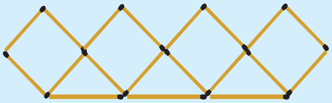
②



③



④



Complete the table in relation to the pattern given above.

Figure Number	1	2	3	4
Total number of matchsticks	....	9	....	....

i. Find the number of matchsticks needed to create the 20th figure.

ii. 219 matchsticks are required to create which figure of this pattern?

iii. Show that one matchstick remains after creating a figure of this pattern by using the maximum number of match sticks from 75 matchsticks.

2. A worker cuts pieces of rods of different lengths from iron rods which are 5 m in length, in order to build a gate by welding pieces together. The length of the shortest piece of iron rod that is cut is 15 cm. All the other pieces are cut such that the difference in length between two successive pieces which are cut is 10 cm.

- **i.** Write the lengths of the shortest three pieces cut by the worker.
- **ii.** Find the length of the 20th piece, when arranged in ascending order of the length, starting from the shortest piece.
- **iii.** Show that a 5 m long rod will not be sufficient to cut the 50th piece, when arranged in ascending order of the length.

3. On the day that their school celebrated "Annual Savings Day", Yesmi and Indunil start saving money by putting Rs 100 each into their respective savings boxes. After that, they put money into their savings boxes once a week, on the same day of the week that they started saving money. Yesmi put Rs 10 and Indunil put Rs 5 each week into their respective boxes.

- **i.** How much does Yesmi have in her savings box in the 5th week?
- **ii.** How much does Indunil have in her savings box in the 10th week?
- **iii.** At the end of 50 weeks, both of them open their savings boxes and check the amount that each has saved. How much more money has Yesmi saved than Indunil in the 50 weeks?

4. The seats in an outdoor stadium are arranged for a drama in 15 rows according to a pattern with a common difference, such that the first row consists of 9 seats, the second row of 12 seats, the third row of 15 seats, etc.

- **i.** How many seats are there in total in the first five rows?
- **ii.** How many seats are there in the 15th row?
- **iii.** Show that the 10th row has 4 times the number of seats in the first row, according to this pattern.
- **iv.** Which row consists of 51 seats?

Miscellaneous Exercise

1. The general terms of a few number patterns are given below.

(a) $3n - 5$ (b) $6n + 5$ (c) $6n - 5$

For each number pattern,

- **i.** write the first three terms.
- **ii.** find the 20th term.
- **iii.** find the $(n - 1)$th term in terms of n.

2. Find the general term of each number pattern given below, given that each has a common difference.

i. $-3, 1, 5, 9, \ldots$ **ii.** $0, 4, 8, 12, \ldots$

iii. $1\frac{1}{2}, 2, 2\frac{1}{2}, \ldots$ **iv.** $-6, -3, 0, 3, \ldots$

3. Show that the general term of the number pattern 42, 36, 30, 24, ... with a common difference is $6(8-n)$.

4. Uditha is employed in a private company. His first monthly salary is Rs 25 000. From the beginning of the second year onwards, he receives an annual salary increment of Rs 2400 per month.

i. How much is his monthly salary at the beginning of the second year?
ii. Write separately, Uditha's monthly salary during the first three years.
iii. Write an expression for his salary in the nth year in terms of n.
iv. Find Uditha's monthly salary in the 5th year, by using the expression obtained in (iii) above.

Summary

Summary

- common difference = any term except the first term – the previous term
- The general term of a sequence is denoted by T_n .
- Any term of a sequence can easily be found by using the general term.

2

Binary Numbers

By studying this lesson, you will be able to;

- identify binary numbers,
- convert a decimal number into a binary number,
- convert a binary number into a decimal number,
- add and subtract binary numbers,
- identify instances where binary numbers are used.

Introduction

Let us recall how numbers are written in the Hindu - Arabic number system, which is the number system we use.

As an example, let us consider the number 3 725. According to what we have learnt in previous grades,

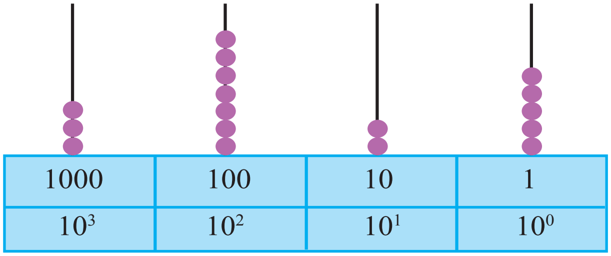
5 denotes the number of 1s (that is, the number of 10^0s),

2 denotes the number of 10s (that is, the number of 10^1s),

7 denotes the number of 100s (that is, the number of 10^2s),

3 denotes the number of 1000s (that is, the number of 10^3s).

The above can be represented on an abacus as shown below.



Observe that the number 3 725 can also be written in terms of powers of 10 as shown below.

$$3\ 725 = 3 \times 1000 + 7 \times 100 + 2 \times 10 + 5 \times 1$$

That is, $3\ 725 = 3 \times 10^3 + 7 \times 10^2 + 2 \times 10^1 + 5 \times 10^0$

If we consider 603 as another example, we can write it as shown below.

$$603 = 6 \times 10^2 + 0 \times 10^1 + 3 \times 10^0$$

In the Hindu - Arabic number system which we use, the place values are powers of ten such as 1, 10, 100 and 1000. Moreover, we use the 10 digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 to write numbers in the Hindu - Arabic number system. The method of writing numbers using these 10 digits and assigning place values which are powers of ten, is called writing the numbers in "base 10". When studying about number bases, these numbers are called "decimal numbers".

Note : $10^0 = 1$. Similarly, any nonzero base raised to the power zero is always equal to one. Accordingly, $2^0 = 1$.

2.1 Expressing numbers in the binary number system

We can use number bases other than base ten to express numbers. For example, we can express numbers in "base two" by using only the digits 0 and 1, and assigning place values which are powers of two. To do this, let us first identify several powers of two.

We can write them as;

$2^0 = 1$ $\quad$ $2^5 = 32$

$2^1 = 2$ $\quad$ $2^6 = 64$

$2^2 = 4$ $\quad$ $2^7 = 128$

$2^3 = 8$ $\quad$ $2^8 = 256$

$2^4 = 16$ $\quad$ $2^9 = 512$

To understand the method of writing numbers in base two, let us first consider the base ten number 13 as an example. Let us see how we can write 13 as a sum of powers of two.

The first few powers of two are;

1, 2, 4 and 8.

Using these numbers which are powers of two, we can write,

$$13 = 8 + 4 + 1$$

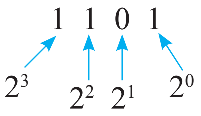
i.e., $13 = 2^3 + 2^2 + 2^0$

i.e., $13 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$.

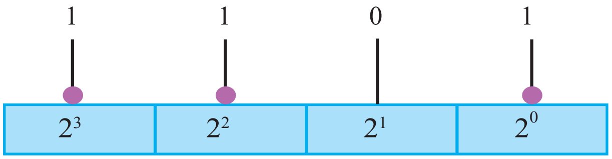
Here, we have written all the non - negative powers of two in descending order, starting from 2^3 and continuing with 2^2, 2^1 and 2^0. Also, since the power 2^3 is included, it is written as 1×2^3 and since the power 2^1 is not included, it is written as 0×2^1. Recall that we use only the digits 1 and 0 when writing base two numbers. Considering the above facts, we can write 13 as a base two number as follows.

1101

The digits 0 and 1 appearing in this base two number can be described as follows.



We can also express it using an abacus as follows.



To indicate that 1101 is a base two number, we usually write "two" as a subscript and express the number as 1101_{two}. In this lesson, whenever necessary, we indicate base ten numbers with the subscript "ten" to differentiate the base two numbers from the base ten numbers. For example, the decimal number 603 is written as 603_{ten}.

Let us consider another example. Let us write the base ten number 20_{ten} as a base two number.

By recalling the powers of two, we can write;

$$
\begin{aligned}
20 &= 16 + 4 \\
&= 2^4 + 2^2 \\
&= 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0.
\end{aligned}
$$

Hence,

$20_{ten} = 10100_{two}$.

There is an important fact to remember here. There is only one way of writing any number as a sum of distinct descending powers of two. For example, $20 = 16 + 4$ can only be written as $2^4 + 2^2$ as a sum of distinct descending powers of two. There is no other way. You can see this for yourself by attempting to find a different way. Moreover, any number can be written as a sum of powers of two. You can verify this too by writing different decimal numbers as a sum of distinct powers of two.

The above method of writing a decimal number as a sum of distinct descending powers of two, cannot be considered as a precise method. The reason for this is because it is difficult to decide what powers of two add up to a number, when the number is large. For example, it is not easy to determine the powers of two that add up to the decimal number 3905_{ten}.

Therefore, let us now consider another method that can be used to convert any decimal number to a binary number fairly easily.

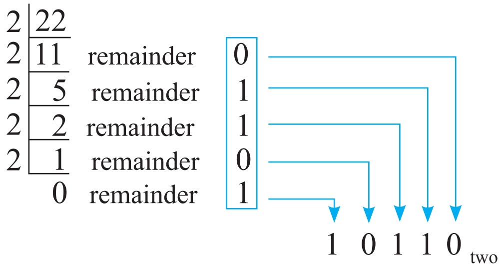
Consider 22_{ten} as an example. To write this as a binary number, we need to first divide 22 by 2 and write the remainder also.

$$\begin{array}{r|l} 2 & \underline{22} \\ & 11 \quad \text{remainder } 0 \end{array}$$

Next we need to divide the quotient 11 which we obtained by dividing 22 by 2, again by 2.

$$\begin{array}{r|l} 2 & \underline{22} \\ 2 & \underline{11} \quad \text{remainder } 0 \\ & \;\;5 \quad \text{remainder } 1 \end{array}$$

We need to continue dividing the quotient by 2 and noting down the remainder, until we get 0 as the quotient and 1 as the remainder. The complete division is shown below.



When the highlighted remainders are written from bottom to top, we obtain the required base two number.

$$22_{ten} = 10110_{two}$$

Let us see whether we can verify this answer using the method we discussed earlier of writing the number as a sum of powers of two.

$$\begin{aligned} 22 = 16 + 4 + 2 = 2^4 + 2^2 + 2^1 &= 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \\ &= 10110_{two} \end{aligned}$$

The answer is verified.

Example 1

Write each decimal number given below as a binary number.

i. 32_{ten}

2	32	
2	16	0
2	8	0
2	4	0
2	2	0
2	1	0
	0	1

$32_{ten} = 100000_{two}$

ii. 154_{ten}

2	154	
2	77	0
2	38	1
2	19	0
2	9	1
2	4	1
2	2	0
2	1	0
	0	1

$154_{ten} = 10011010_{two}$

Exercise 2.1

Convert the decimal numbers (base ten numbers) given below into binary numbers (base two numbers).

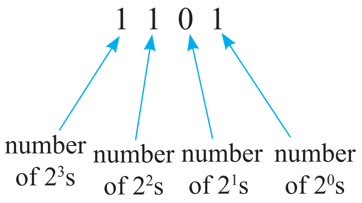
a. 4 **b.** 9 **c.** 16 **d.** 20 **e.** 29

f. 35 **g.** 43 **h.** 52 **i.** 97 **j.** 168

2.2 Converting binary numbers into decimal numbers

Decimal numbers were converted into binary numbers in the above section 2.1. In this section we consider the inverse process; that is, converting binary numbers into decimal numbers. This can be done fairly easily. Let us learn how to do this by considering an example.

In section 2.1, when we wrote the decimal number 13 as a binary number, we obtained 1101_{two}. Let us recall what the digits 1, 1, 0 and 1 represent.



Therefore, by adding all the values of the powers of two in 1101_{two} we get the corresponding decimal representation.

$$\begin{aligned} 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 &= 1 \times 8 + 1 \times 4 + 0 \times 2 + 1 \times 1 \\ &= 8 + 4 + 1 = 13. \end{aligned}$$

By simplifying, we obtain the corresponding decimal number 13.

Example 1

Write 101100_{two} as a decimal number.

First, it should be noted that the place value of the leftmost digit of this base two number is 2^5 and that the other place values are obtained by reducing the index by one (starting from 5) for each move from left to right. Then the required decimal number can be found by multiplying each power of two (place value) by the relevant co-efficient and adding all the terms together.

$$\begin{aligned}101100_{two} &= 1\times 2^5 + 0\times 2^4 + 1\times 2^3 + 1\times 2^2 + 0\times 2^1 + 0\times 2^0\\ &= 2^5 + 2^3 + 2^2 = 32 + 8 + 4\\ &= 44_{ten}\end{aligned}$$

Therefore, when 101100_{two} is written in base 10 we obtain 44_{ten}.

Note: This answer can be verified by converting 44_{ten} back into a binary number.

Exercise 2.2

Convert the binary numbers given below into base ten numbers (decimal numbers).

a. 101_{two} **b.** 1101_{two} **c.** 1011_{two} **d.** 1100_{two} **e.** 11111_{two}

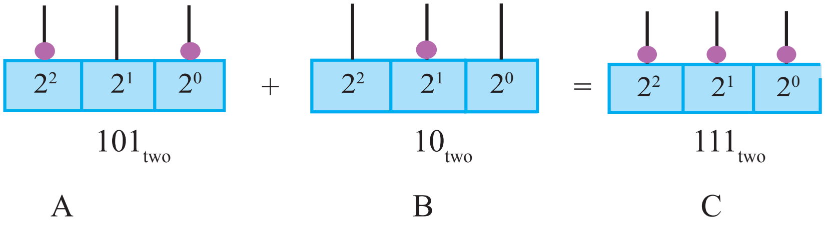
f. 100111_{two} **g.** 110111_{two} **h.** 111000_{two} **i.** 111110_{two} **j.** 110001_{two}

2.3 Adding binary numbers

When representing binary numbers on an abacus, the maximum number of counters that can be placed on a rod is 1. Moreover, instead of placing two counters on a rod, we always place one counter on the rod to the left of it.

Let us learn how to add binary numbers with the aid of two abacuses.

Let us simplify $101_{two} + 10_{two}$.



Let us represent the sum of the numbers represented on the abacuses A and B on the abacus C.
When we consider the two abacuses A and B;

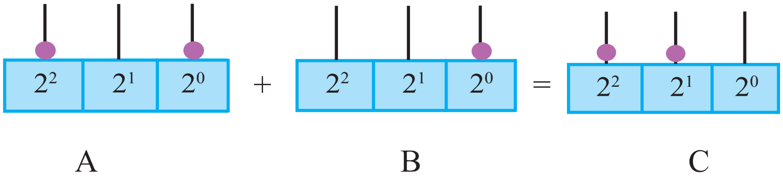
the sum of the counters on the 2^0 rods is 1,

the sum of the counters on the 2^1 rods is 1,

the sum of the counters on the 2^2 rods is 1.

Therefore, $101_{two} + 10_{two} = 111_{two}$

Now, let us obtain the value of $101_{two} + 1_{two}$ using the abacuses.



The counter on the 2^0 rod in A and the counter on the 2^0 rod in B, cannot both be placed on the 2^0 rod in C, because there cannot be two counters on any rod of an abacus used to represent a binary number. Instead, one counter should be placed on the rod to the left of the 2^0 rod. This is shown on the rod 2^1 in C.

Therefore, $101_{two} + 1_{two} = 110_{two}$.

This is clarified further by adding the numbers vertically.

$$\begin{array}{r} 101_{two} \\ +\ \ 1_{two} \\ \hline 110_{two} \\ \hline\hline \end{array}$$

Adding from right to left; first, one 2^0s + one 2^0s = one 2^1s and zero 2^0s.

Then, one 2^1s + zero 2^1s = one 2^1s. Finally, one 2^2s + zero 2^2s = one 2^2s.

Example 1

Find the value.

(i) $11101_{two} + 1101_{two}$ (ii) $1110_{two} + 111_{two}$

(i)
$$\begin{array}{r} {\scriptstyle 11\ \ 1\ \ } \\ 11101_{\text{two}} \\ +\ \ 1101_{\text{two}} \\ \hline 101010_{\text{two}} \\ \hline\hline \end{array}$$

(ii)
$$\begin{array}{r} {\scriptstyle 11\ \ \ \ \ \ } \\ 1110_{\text{two}} \\ +\ \ 111_{\text{two}} \\ \hline 10101_{\text{two}} \\ \hline\hline \end{array}$$

Note: When adding binary numbers observe the relationships given below.

$1_{\text{two}} + 0_{\text{two}} = 1_{\text{two}}$

$1_{\text{two}} + 1_{\text{two}} = 10_{\text{two}}$

$1_{\text{two}} + 1_{\text{two}} + 1_{\text{two}} = 11_{\text{two}}$

Exercise 2.3

1. Find the value.

a.
$$\begin{array}{r} 111_{\text{two}} \\ +101_{\text{two}} \\ \hline \end{array}$$

b.
$$\begin{array}{r} 10111_{\text{two}} \\ +\ \ 1011_{\text{two}} \\ \hline \end{array}$$

c.
$$\begin{array}{r} 1011_{\text{two}} \\ +\ 11101_{\text{two}} \\ \hline \end{array}$$

d. $11101_{\text{two}} + 1110_{\text{two}}$

e. $11011_{\text{two}} + 11_{\text{two}}$

f. $100111_{\text{two}} + 11_{\text{two}} + 1_{\text{two}}$

g. $11_{\text{two}} + 111_{\text{two}} + 1111_{\text{two}}$

h. $11110_{\text{two}} + 1110_{\text{two}} + 110_{\text{two}}$

2. Fill each cage with the suitable digit.

a.
$$\begin{array}{r} 11_{\text{two}} \\ +\ 1\square_{\text{two}} \\ \hline 1\square 1_{\text{two}} \\ \hline\hline \end{array}$$

b.
$$\begin{array}{r} 1\ 10\ \square_{\text{two}} \\ +\ \square 1\ 1_{\text{two}} \\ \hline 1\square 100_{\text{two}} \\ \hline\hline \end{array}$$

c.
$$\begin{array}{r} 10\ 0\ 1_{\text{two}} \\ +\ \ \square 1\ \square_{\text{two}} \\ \hline \square 00\square 0_{\text{two}} \\ \hline\hline \end{array}$$

d.
$$\begin{array}{r} 1\ 1\ 1\ 0_{\text{two}} \\ +\ \ 1\ \square\ \square_{\text{two}} \\ \hline 10\square\ 0\ 1_{\text{two}} \\ \hline\hline \end{array}$$

e.
$$\begin{array}{r} 1\ \square\ 1\ \square_{\text{two}} \\ +\ \ 1\ \square\ 1_{\text{two}} \\ \hline 1\square\ 0\ 0\ 0_{\text{two}} \\ \hline\hline \end{array}$$

f.
$$\begin{array}{r} 1\ 1\ \square\ 1_{\text{two}} \\ +1\ 1\ 1\ 0_{\text{two}} \\ \hline 1\square\ \square\ 1\ \square_{\text{two}} \\ \hline\hline \end{array}$$

2.4 Subtracting binary numbers

When adding binary numbers, we saw that whenever we obtained a sum of 2 in a particular position, we replaced it with 1 in the position left of it.

$$\begin{array}{r} 101_{two} \\ +\quad 1_{two} \\ \hline 110_{two} \\ \hline\hline \end{array} \qquad \text{(right hand column: } 1_{two} + 1_{two} = 10_{two}\text{)}$$

Now let us find the value of $110_{two} - 1_{two}$. According to the above addition, the answer should be 101_{two} . Let us consider how this answer is obtained.

$$\begin{array}{cccc} & 2^2 & 2^1 & 2^0 \\ & 1 & 1 & 0_{two} \\ - & & & 1_{two} \\ \hline & 1 & 0 & 1_{two} \\ \hline\hline \end{array}$$

We cannot subtract 1 from 0 in the rightmost column. Therefore, let us take 1 from the 2^1 column which is to the left of it.The value of this is 2 in the 2^0 column. Now we subtract 1 from this 2 to obtain 1 in the rightmost column. Now there is 0 instead of 1 in the column 2^1.

Therefore, $110_{two} - 1_{two} = 101_{two}$.

Example 1

$$\begin{array}{r} 1101_{two} \\ -\quad 111_{two} \\ \hline 110_{two} \\ \hline\hline \end{array}$$

Let us check the accuracy of the answer by considering $110_{two} + 111_{two}$.

$$110_{two} + 111_{two} = \underline{\underline{1101}}_{\ two}$$

Note: It is very important to develop the habit of checking the accuracy of an answer to a subtraction problem using addition as shown above.

Exercise 2.4

1. Find the value.

a. $\begin{array}{r} 11_{two} \\ -\ \ 1_{two} \\ \hline \end{array}$

b. $\begin{array}{r} 10_{two} \\ -\ \ 1_{two} \\ \hline \end{array}$

c. $\begin{array}{r} 101_{two} \\ -\ \ 1_{two} \\ \hline \end{array}$

d. $\begin{array}{r} 101_{two} \\ -\ \ 11_{two} \\ \hline \end{array}$

e. $111_{two} - 11_{two}$

f. $110_{two} - 11_{two}$

g. $1100_{two} - 111_{two}$

h. $\begin{array}{r} 10001_{two} \\ -\ \ 111_{two} \\ \hline \end{array}$

i. $\begin{array}{r} 100000_{two} \\ -\ 11011_{two} \\ \hline \end{array}$

j. $\begin{array}{r} 100011_{two} \\ -\ 10001_{two} \\ \hline \end{array}$

k. $11000_{two} - 1111_{two}$

l. $101010_{two} - 10101_{two}$

2.5 Applications of binary numbers

The fundamental digits in the binary number system are 0 and 1. Many modern digital instruments are made based on this feature. When designing lighting system circuits, “current off” and “current on” conditions are represented by 0 and 1.

If ⊛ is used to represent the current “on” condition and ○ to represent the current “off” condition, then the combination ⊛ ○ ○ ⊛ is represented by 1001_{two}. The storing of data and computations done in computers and calculators are based on this concept. Any number system can be developed under the same principles used to develop the binary number system. Storing of data can be done using other number systems too.

Note: If a number system is developed using base four, only the fundamental digits 0, 1, 2 and 3 are used.
For example, the decimal number 4 is expressed as 10_{four} in this number system.
In the base five number system, the fundamental digits are 0, 1, 2, 3 and 4, and the decimal number 5 is expressed as 10_{five} in this system.

Miscellaneous Exercise

1. Find the value.

a. $1101_{two} + 111_{two} - 1011_{two}$

b. $11111_{two} - (101_{two} + 11_{two})$

c. $110011_{two} - 1100_{two} - 110_{two}$

2. Write the next number, after adding 1 to each given number. 1_{two} , 11_{two}, 111_{two}, 1111_{two}, 11111_{two}, 111111_{two}

3. Represent the decimal number 4^2 as a binary number.

4. i. Simplify 49_{ten} - 32_{ten} and convert the answer into a binary number.
ii. Convert 49_{ten} and 32_{ten} into binary numbers and find their difference. See whether the answer you obtain is the same as the answer in (1) above.

Summary

Summary

- In the binary number system, the fundamental digits are 0 and 1.
- The place values of the binary number system are; 2^0, 2^1, 2^2, 2^3, 2^4, 2^5, 2^6, etc.

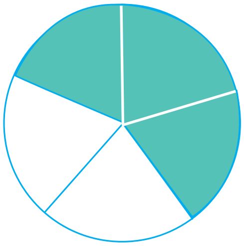
3

Fractions

By studying this lesson, you will be able to;
- simplify expressions of fractions which contain “of”,
- simplify expressions of fractions which contain brackets,
- identify the BODMAS method and solve problems involving fractions.

Fractions

Let us recall the facts that were learnt about fractions in previous grades. The circle shown below is divided into five equal parts of which three are shaded.



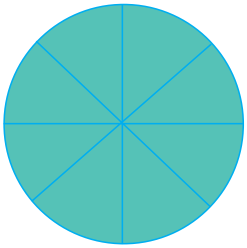
The shaded region can be expressed as $\frac{3}{5}$ of the whole region. We can express this in terms of the area of the circle too. That is, the shaded area is $\frac{3}{5}$ of the area of the whole figure. If the total area of the circle is taken as 1 unit, then the shaded area is $\frac{3}{5}$ units.

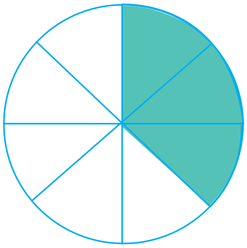
When an object is divided into equal portions, one portion or several portions can be expressed as a fraction. A portion of a collection can also be expressed as a fraction.

For example, if we consider a team consisting of three boys and two girls, then the boys in the team can be considered as $\frac{3}{5}$ of the team. Here, if the whole team is considered as a unit, then the boys in the team can be expressed as $\frac{3}{5}$.

You have learnt that fractions between zero and one such as $\frac{3}{4}$, $\frac{1}{2}$ and $\frac{2}{3}$ are called proper fractions.

Let us now recall the facts that have been learnt previously about mixed numbers and improper fractions.
Two identical circles are given below. One is shaded completely and three parts of the other (which is divided into equal parts) are shaded.





If a circle is considered as one unit, the shaded fraction is $1 + \frac{3}{8}$. This is usually written as $1\frac{3}{8}$, which is called a mixed number ("mixed fractions" are most often called "mixed numbers"). This can also be written as $\frac{11}{8}$, which is called an improper fraction. It is important to remember here that the mixed number and the improper fraction are expressed by taking a circle as one unit.

Some other examples of mixed numbers are $1\frac{1}{2}$, $3\frac{2}{5}$, $2\frac{3}{7}$.

$\frac{3}{2}$, $\frac{8}{5}$, $\frac{11}{4}$ are examples of improper fractions. Fractions such as $\frac{3}{3}$, $\frac{5}{5}$, $\frac{1}{1}$ which are equal to 1 are also considered as improper fractions.

You have learnt to represent mixed numbers as improper fractions and improper fractions as mixed numbers.

Accordingly,

(i) $1\frac{1}{2} = \frac{3}{2}$ and

(ii) $\frac{5}{3} = 1\frac{2}{3}$.

We can obtain a fraction equivalent to a given fraction by multiplying or dividing both the denominator and the numerator by the same number (which is not zero).

For example,

$$\frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10}$$

$$\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$$

The addition and subtraction of fractions can be performed easily if the denominators of the fractions are equal.
For example,

(i) $\frac{1}{5} + \frac{4}{5} - \frac{2}{5}$

$$\frac{1}{5} + \frac{4}{5} - \frac{2}{5} = \frac{1 + 4 - 2}{5}$$
$$= \frac{3}{5}$$

If the denominators of the fractions are unequal, then we convert the fractions into equivalent fractions with equal denominators.

For example,

(ii) $\frac{1}{4} + \frac{2}{3} - \frac{5}{6} = \frac{1 \times 3}{4 \times 3} + \frac{2 \times 4}{3 \times 4} - \frac{5 \times 2}{6 \times 2}$

$$= \frac{3}{12} + \frac{8}{12} - \frac{10}{12}$$
$$= \frac{3 + 8 - 10}{12}$$
$$= \frac{1}{12}$$

- When multiplying two fractions, the numerator of the product is obtained by multiplying the numerators of the two fractions and the denominator is obtained by multiplying the denominators of the two fractions.

Example 1

$\frac{2}{5} \times \frac{1}{3}$

$$\frac{2}{5} \times \frac{1}{3} = \frac{2 \times 1}{5 \times 3} = \underline{\underline{\frac{2}{15}}}$$

Example 2

$1\frac{1}{3} \times 1\frac{3}{4}$

$$\begin{aligned} 1\frac{1}{3} \times 1\frac{3}{4} &= \frac{{}^{1}\cancel{4}}{3} \times \frac{7}{\cancel{4}_{1}} \quad \text{(converting the mixed numbers into improper fractions)} \\ &= \frac{7}{3} \\ &= \underline{\underline{2\frac{1}{3}}} \end{aligned}$$

- If the product of two numbers is 1, then each number is said to be the reciprocal of the other.

Accordingly, since $2 \times \frac{1}{2} = 1$,

2 is the reciprocal of $\frac{1}{2}$ and $\frac{1}{2}$ is the reciprocal of 2.

You have learnt that the reciprocal of a number can be obtained by interchanging the denominator and the numerator.

Hence, the reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$ (In the same way, the reciprocal of $\frac{b}{a}$ is $\frac{a}{b}$.)

- In grade 8 you learnt that dividing a number by another number means multiplying the first number by the reciprocal of the second number.

Let us revise this by considering a couple of examples.

Example 3

$\frac{4}{3} \div 2$

$$\begin{aligned} \frac{4}{3} \div 2 &= \frac{{}^{2}\cancel{4}}{3} \times \frac{1}{\cancel{2}_{1}} \\ &= \underline{\underline{\frac{2}{3}}} \end{aligned}$$

Example 4

$1\frac{2}{7} \div 1\frac{1}{2}$

$$\begin{aligned} 1\frac{2}{7} \div 1\frac{1}{2} &= \frac{9}{7} \div \frac{3}{2} \\ &= \frac{{}^{3}\cancel{9}}{7} \times \frac{2}{\cancel{3}_{1}} \\ &= \underline{\underline{\frac{6}{7}}} \end{aligned}$$

Do the following review exercise to revise what you have learnt thus far about fractions.

Review Exercise

1. For each of the fractions given below, write two equivalent fractions.

i. $\frac{2}{3}$ ii. $\frac{4}{5}$ iii. $\frac{4}{8}$ iv. $\frac{16}{24}$

2. Express each mixed number given below as an improper fraction.

i. $1\frac{1}{2}$ ii. $2\frac{3}{4}$ iii. $3\frac{2}{5}$ iv. $5\frac{7}{10}$

3. Express each improper fraction given below as a mixed number.

i. $\frac{7}{3}$ ii. $\frac{19}{4}$ iii. $\frac{43}{4}$ iv. $\frac{36}{7}$

4. Find the value.

i. $\frac{3}{7}+\frac{2}{7}$ ii. $\frac{5}{6}-\frac{2}{3}$ iii. $\frac{7}{12}+\frac{3}{4}-\frac{2}{3}$

iv. $1\frac{1}{2}+2\frac{1}{4}$ v. $3\frac{5}{6}-1\frac{2}{3}$ vi. $1\frac{1}{2}+2\frac{1}{4}-1\frac{2}{3}$

5. Simplify.

i. $\frac{1}{2}\times\frac{4}{7}$ ii. $\frac{2}{3}\times\frac{5}{8}\times\frac{3}{10}$ iii. $1\frac{3}{5}\times 2\frac{1}{2}$ iv. $3\frac{3}{10}\times 2\frac{1}{3}\times 4\frac{2}{7}$

6. Write the reciprocal of each of the following.

i. $\frac{1}{3}$ ii. $\frac{1}{7}$ iii. $\frac{3}{8}$ iv. 5 v. $2\frac{3}{5}$

7. Simplify.

i. $\frac{6}{7}\div 3$ ii. $8 \div\frac{4}{5}$ iii. $\frac{9}{28}\div\frac{3}{7}$ iv. $5\frac{1}{5}\div\frac{6}{7}$ v. $1\frac{1}{2}\div 2\frac{1}{4}$

3.1 Simplifying expressions of fractions containing “of”

We know that $\frac{1}{2}$ of 100 rupees is 50 rupees.

We also know that this is one half of 100 rupees and that its value can be obtained by dividing 100 by 2.

This can be written as $100 \div 2$.

That is, $100 \times \frac{1}{2}$ (multiplying by the reciprocal).

Accordingly, $\frac{1}{2}$ of 100 $= 100 \times \frac{1}{2} = \frac{1}{\cancel{2}} \times \cancel{100} = 50$.

According to the above facts, $\frac{1}{2}$ of 100 can be written as $\frac{1}{2} \times 100$.

Let us similarly determine how much $\frac{1}{5}$ of 20 kilogrammes is.

This amount can be considered as one part from 5 equal parts into which 20 kilogrammes is divided.

We can write this as $20 \div 5$.

That is, $20 \times \frac{1}{5}$ (multiplying by the reciprocal).

$20 \times \frac{1}{5} = \frac{1}{\cancel{5}} \times \cancel{20}^{4} = 4.$

According to the above facts, $\frac{1}{5}$ of 20 can be written as $\frac{1}{5} \times 20$.

It can be seen from the above instances that we can replace the term “of” by the operation “×”.

$\frac{1}{2}$ of 100 rupees $= \frac{1}{2} \times 100$ rupees

$\frac{1}{5}$ of 20 kilogrammes $= \frac{1}{5} \times 20$ kilogrammes

Now let us find the value of $\frac{1}{2}$ of $\frac{1}{3}$. Let us illustrate this using figures.

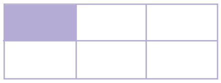
When a unit is divided into three equal parts, one part is $\frac{1}{3}$.

If this figure is taken as one unit, $\frac{1}{3}$ of it is shown below.

$\frac{1}{3}$

Let us separate out $\frac{1}{2}$ of the shaded region.

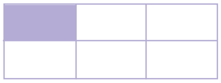
$\frac{1}{2}$



Accordingly,

$\frac{1}{3}$

$\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{6}$



According to the figure, it is clear that $\frac{1}{2}$ of $\frac{1}{3}$ is $\frac{1}{6}$.

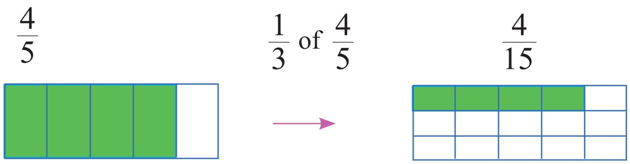
More accurately, if $\frac{1}{3}$ of a unit is taken and then $\frac{1}{2}$ of that $\frac{1}{3}$ is separated out, the portion we get is $\frac{1}{6}$ of the original unit.

Moreover, based on what we have learnt regarding multiplying fractions, we obtain $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

Accordingly, we can express $\frac{1}{2}$ of $\frac{1}{3}$ as $\frac{1}{2} \times \frac{1}{3}$.

Let us consider another example to verify this. Let us simplify $\frac{1}{3}$ of $\frac{4}{5}$.

Let us consider the rectangle given below as one unit.



According to the figure, it is clear that $\frac{1}{3}$ of $\frac{4}{5}$ is $\frac{4}{15}$.

Moreover, $\frac{1}{3} \times \frac{4}{5} = \frac{4}{15}$.

Therefore we can write, $\frac{1}{3}$ of $\frac{4}{5} = \frac{1}{3} \times \frac{4}{5}$.

It is clear that , we can replace “of” by the mathematical operation “multiplication” in the expressions $\frac{1}{2}$ of $\frac{1}{3}$ and $\frac{1}{3}$ of $\frac{4}{5}$.

Example 1

Find the value of $\frac{1}{2}$ of $\frac{2}{3}$.

$\frac{1}{2}$ of $\frac{2}{3} = \frac{1}{2} \times \frac{2}{3}$ (writing × for "of")

$= \underline{\underline{\frac{1}{3}}}$

Example 2

How much is $\frac{2}{3}$ of $1\frac{4}{5}$?

$\frac{2}{3}$ of $1\frac{4}{5} = \frac{2}{3} \times \frac{9}{5}$

$= \frac{6}{5}$

$= \underline{\underline{1\frac{1}{5}}}$

Example 3

How much is $\frac{3}{5}$ of 500 metres?

$\frac{3}{5}$ of $500 = \frac{3}{5} \times 500$

$= \underline{\underline{300 \text{ m}}}$

Exercise 3.1

1. Simplify the following expressions.

i. $\frac{2}{3}$ of $\frac{4}{5}$ **ii.** $\frac{6}{7}$ of $\frac{1}{3}$ **iii.** $\frac{2}{5}$ of $\frac{5}{8}$ **iv.** $\frac{5}{6}$ of $\frac{9}{11}$

v. $\frac{2}{7}$ of $1\frac{3}{4}$ **vi.** $1\frac{1}{3}$ of $2\frac{5}{8}$ **vii.** $1\frac{3}{11}$ of $5\frac{1}{2}$ **viii.** $\frac{5}{9}$ of $1\frac{4}{5}$

2. Find the value.

i. How much is $\frac{3}{4}$ of 64 rupees?

ii. How many grammes is $\frac{2}{5}$ of 400 g?

iii. How many hectares is $\frac{1}{3}$ of 6 ha?

iv. How many metres is $\frac{1}{8}$ of 1 km?

3. A person who owns $\frac{3}{5}$ of a land, gives $\frac{1}{3}$ of it to his daughter. What is the portion received by the daughter as a fraction of the whole land?

4. Nimal's monthly income is 40 000 rupees. He spends $\frac{1}{8}$ of this on travelling. How much does he spend on travelling?

3.2 Simplifying expressions with brackets according to the BODMAS rule

A numerical expression (or algebraic expression) may involve several of the operations addition, subtraction, division, multiplication and raising to the power of. There should be agreement on the order in which these operations should be performed and a set of rules describing it. In previous grades we learnt these rules to some extent. In this section we will discuss the "BODMAS" rule that is used when simplifying fractions.

The acronym "BODMAS" stands for brackets, orders/of, division, multiplication, addition and subtraction. When simplify numerical expressions, priority is given according to the BODMAS order. However, some operations have the same priority. Multiplication and division have equal priority and so do addition and subtraction. Accordingly, expressions should be simplified as follows.

1. First, simplify all expressions within brackets.

2. Then simplify powers and roots (expressions with indices) and the expressions with "of ".

* Simplifying expressions with powers and roots is not included in the syllabus.

3. Next, perform divisions and multiplications. These have equal priority and hence if both these operations are involved, priority is given from left to right.
4. Finally perform addition and subtraction. Since these too have equal priority, precedence is given from left to right, as in 3 above.

The BODMAS rule can be used to simplify expressions with fractions too. In some expressions of fractions the term "of" is used.
For example,

$$\frac{5}{12} \text{ of } \frac{6}{25}$$

As learnt in the previous section, this means $\frac{5}{12} \times \frac{6}{25}$.

A consensus is needed on how a fairly complex expression such as $\frac{2}{3} \div \frac{6}{25} \text{ of } \frac{5}{12} \times \frac{1}{2}$ is to be simplified. Here, precedence is given to "of" over ÷ and ×.

Note: Since "of" and "raising to the power of" have the same priority, the "O" in BODMAS is considered to stand for both "of" and "order". However in this syllabus only "of" is considered.

Now, let us consider how the BODMAS rule is used in simplifying the expression $\frac{1}{4} + \frac{5}{6} \times \frac{1}{2} \div \frac{4}{3} \text{ of } \frac{3}{2}$.

$\frac{1}{4} + \frac{5}{6} \times \frac{1}{2} \div \frac{4}{3} \text{ of } \frac{3}{2} = \frac{1}{4} + \frac{5}{6} \times \frac{1}{2} \div \left(\frac{4}{3} \times \frac{3}{2}\right)$ (inserting brackets after replacing "of" by "×" , to indicate that this operation should be performed first)

$$= \frac{1}{4} + \frac{5}{6} \times \frac{1}{2} \div 2$$

$= \frac{1}{4} + \left(\frac{5}{6} \times \frac{1}{2}\right) \div 2$ (inserting brackets to indicate the mathematical operation which is to be performed next)

$= \frac{1}{4} + \frac{5}{12} \times \frac{1}{2}$ (multiplying by $\frac{1}{2}$ instead of dividing by 2)

$= \frac{1}{4} + \left(\frac{5}{12} \times \frac{1}{2}\right)$ (inserting brackets to indicate the mathematical operation which is to be performed next)

$= \frac{1}{4} + \frac{5}{24}$

$= \frac{6}{24} + \frac{5}{24}$ (writing both fractions with a common denominator)

$= \frac{11}{24}$

Note: The order in which the mathematical operations in an expression should be performed can be indicated very easily using brackets.

Consider the following expression.

$$\frac{5}{4} \times \frac{3}{4} - \frac{1}{3} \text{ of } \frac{1}{5} \div \frac{2}{3} \div \frac{8}{9}$$

How this should be simplified according to the BODMAS rule can be expressed using brackets as follows.

$$\left(\frac{5}{4} \times \frac{3}{4}\right) - \left(\left(\left(\frac{1}{3} \text{ of } \frac{1}{5}\right) \div \frac{2}{3}\right) \div \frac{8}{9}\right)$$

There are disadvantages in using brackets too. When brackets are used, the expression will seem long and complex. Moreover, when simplifying an expression with the aid of a calculator, we have to insert brackets carefully because there is a greater chance of making an error. Therefore, it is important to decide on a convention to simplify expressions without using brackets. Such a convention is necessary especially when writing software for computers and calculators. However, a common convention accepted worldwide has not been agreed upon yet. There are several conventions which are accepted by different countries. Similarly, manufacturers of computers and calculators also have their own conventions.

Now let us consider some examples of expressions with fractions which are simplified using the BODMAS convention.

Example 1

Simplify the expression $\frac{4}{10}$ of $\left(\frac{1}{6}+\frac{1}{4}\right)$ and write the answer in the simplest form.

$$\frac{4}{10} \text{ of } \left(\frac{1}{6}+\frac{1}{4}\right) = \frac{4}{10} \times \left(\frac{2}{12}+\frac{3}{12}\right)$$

$$= \frac{4}{10} \times \frac{5}{12} = \underline{\underline{\frac{1}{6}}}$$

Example 2

Simplify $\left(1\frac{2}{5} \div 2\frac{1}{3}\right)$ of $\left(\frac{2}{3}-\frac{1}{2}\right)$.

$$\left(1\frac{2}{5} \div 2\frac{1}{3}\right) \text{ of } \left(\frac{2}{3}-\frac{1}{2}\right) = \left(\frac{7}{5} \div \frac{7}{3}\right) \text{of} \left(\frac{4}{6}-\frac{3}{6}\right)$$

$$= \left(\frac{7}{5} \times \frac{3}{7}\right) \text{of } \frac{1}{6}$$

$$= \frac{3}{5} \times \frac{1}{6}$$

$$= \underline{\underline{\frac{1}{10}}}$$

Exercise 3.2

1. Simplify and write the answer in the simplest form.

i. $\frac{1}{2}+\frac{2}{3}\times\frac{5}{6}$

ii. $\frac{1}{4}$ of $3\frac{1}{3}\div 2\frac{1}{6}$

iii. $\frac{3}{5}\times\left(\frac{1}{3}+\frac{1}{2}\right)$

iv. $\frac{1}{4}$ of $\left(3\frac{1}{3}\div 2\frac{1}{6}\right)$

v. $3\frac{3}{4}\div\left(2\frac{1}{2}+3\frac{1}{4}\right)$

vi. $\left(1\frac{2}{3}\times\frac{3}{5}\right)+\left(\frac{3}{4}+\frac{1}{2}\right)$

vii. $2\frac{2}{3}\times\left(1\frac{1}{4}-\frac{1}{12}\right)\div 2\frac{1}{3}$

viii. $\frac{5}{6}\div\frac{7}{18}$ of $\frac{2}{3}\times\frac{3}{4}$

2. A person puts aside $\frac{1}{4}$ of his income for food and $\frac{1}{2}$ for his business and saves the remaining amount. What fraction of his income does he save?

3. Kumuduni walked $\frac{1}{8}$ of a journey, travelled $\frac{2}{3}$ of it by train and travelled the remaining distance by bus.

(i). Express the distance she travelled by foot and by train as a fraction of the total distance.

(ii). Express the distance she travelled by bus as a fraction of the total distance.

4. A father gave $\frac{1}{2}$ of his land to his son and $\frac{1}{3}$ to his daughter. The son donated $\frac{1}{5}$ of his portion and the daughter $\frac{2}{5}$ of her portion to a charitable foundation. The foundation decided to construct a building on half the land it received. On what fraction of the total land was the building constructed?

For further knowledge

This is only for your knowledge and will not be checked in exams.
Let us consider the expression

$$8 - 3 \times (4 + 1) + 12 \div 3 \times 3^2 \div 4 \text{ as an example.}$$

How the above expression is simplified using the BODMAS rule is described below.

- First the expression $4 + 1$ which is within brackets is simplified. This is equal to 5. Therefore we obtain

$$8 - 3 \times 5 + 12 \div 3 \times 3^2 \div 4$$

- Next the power 3^2 is simplified. This is equal to 9. Hence we obtain

$$8 - 3 \times 5 + 12 \div 3 \times 9 \div 4$$

- Next the multiplications and divisions are performed from left to right. Therefore, 3×5 is simplified first. This is equal to 15. Therefore we obtain

$$8 - 15 + 12 \div 3 \times 9 \div 4$$

- Next, $12 \div 3$ is simplified. This is equal to 4. Hence we obtain

$$8 - 15 + 4 \times 9 \div 4$$

- Next, 4×9 is simplified. This is equal to 36. Now the expression is

$$8 - 15 + 36 \div 4$$

- Then $36 \div 4$ is simplified. This is equal to 9. Therefore we obtain

$$8 - 15 + 9$$

- Now, since addition and subtraction have equal priority, simplification is done from left to right. Therefore we obtain

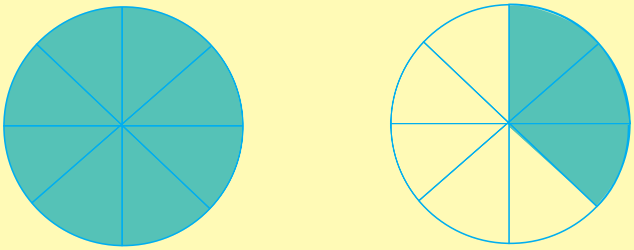
$$-7 + 9$$

- Finally we get $-7 + 9 = 2$ as the answer. According to the BODMAS order,

$$8 - 3 \times (4 + 1) + 12 \div 3 \times 3^2 \div 4 = 2.$$

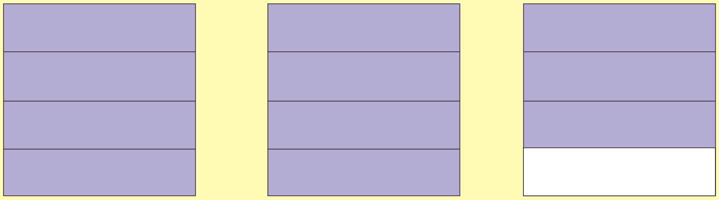
For further knowledge

This is only for your knowledge and will not be checked in exams.
Recall the figure on page 28.



We know that the shaded fraction is $1\frac{3}{8}$, if a circle is considered as one unit. This can be written as $\frac{11}{8}$.

If both these circles are considered as one unit, then the shaded fraction is $\frac{11}{16}$.



In the above diagram, if one square is considered as one unit, the shaded portion is

$2\frac{3}{4}$; That is, $\frac{11}{4}$.

a. What is the shaded fraction if all three squares together are considered as one unit?

b. What is the shaded fraction if half a square is considered as one unit?

Answers

a. $\frac{11}{12}$ **b.** $5\frac{1}{2}$

Summary

Summary

The order in which the mathematical operations are manipulated when simplifying fractions, is as follows:

- B - Brackets
- O - Of
- D - Division
- M - Multiplication
- A - Addition
- S - Subtraction

4

Percentages

By studying this lesson you will be able to;

- calculate the profit earned or loss incurred through a sale,
- calculate the profit percentage or loss percentage,
- identify what commissions and discounts are,
- perform calculations in relation to commissions and discounts.

4.1 Profit and Loss

Most of the items that we use in our day to day lives are bought from supermarkets. People who sell these items are known as sellers whereas people who buy them are known as customers.

The goods sold by sellers are either produced by them or are bought from someone else. In producing or buying goods, a cost is incurred. An item produced or purchased at a cost is generally sold at a price which is greater than the incurred cost. When selling goods at a price which is greater than the cost, it is said that the seller has earned a **profit** from the sale.

A seller will not always be able to sell his goods at a profit. For example, when goods are damaged or about to expire, they may have to be sold at a price which is less than the cost. In such a situation, it is said that the seller has incurred **a loss**. When a seller sells an item at the price at which he bought it, he neither earns a profit nor incurs a loss.

Accordingly, if
the selling price > the cost,
then a profit is earned, and

profit = selling price – cost.

Similarly, if
the cost > the selling price, then a loss is incurred and
loss = cost – selling price.

Example 1

A company which produces shoes incurs a cost of Rs 1000 in manufacturing a pair of shoes. The company sells each pair of shoes at Rs 2600. Find the profit earned by the company in selling one pair of shoes.

The manufacturing cost of a pair of shoes = Rs 1 000
Selling price = Rs 2 600
∴ Profit earned = Rs 2 600 − 1 000
= Rs $\underline{\underline{1\,600}}$

Example 2

A vendor buys a stock of fifty coconuts at the price of Rs 45 per coconut. If the vendor sells all the coconuts at the price of Rs 60 per fruit, calculate his profit.

Method I

The buying price of the stock of coconuts = Rs 45 × 50
= Rs 2 250

Income generated by selling the stock of coconuts = Rs 60 × 50
= Rs 3 000

∴ The profit earned by selling the stock of coconuts = Rs 3 000 – 2 250
= $\underline{\underline{\text{Rs } 750}}$

Method II

The purchase price of a coconut = Rs 45
The selling price of a coconut = Rs 60
The profit earned by selling one coconut = Rs 60 – 45
= Rs 15
The profit earned by selling the whole stock of coconuts = Rs 15 × 50
= $\underline{\underline{\text{Rs } 750}}$

Example 3

A vendor buys a stock of 100 mangoes at the price of Rs 20 each and decides to sell them at the price of Rs 18 each due to the fruits being damaged during transportation. Calculate the loss incurred by the vendor.

Method I

The purchase price of the stock of mangoes = Rs 20 × 100
= Rs 2 000

The amount made by selling the stock of mangoes = Rs 18 × 100
= Rs 1 800

The loss incurred in selling the whole stock of mangoes = Rs 2 000 – 1 800
= $\underline{\underline{\text{Rs 200}}}$

Method II

The purchase price of a mango = Rs 20
The selling price of a mango = Rs 18
The loss incurred in selling a mango = Rs 20 – 18
= Rs 2
The loss incurred in selling the whole stock of mangoes = Rs 2 × 100
= $\underline{\underline{\text{Rs 200}}}$

Example 4

A vendor buys 60 kg of manioc from a farmer at the price of Rs 50 per kilogramme. He initially sells 20 kg at Rs 70 per kilogramme. Of the remaining manioc he sells 15 kg at Rs 60 per kilogramme, 5 kg at Rs 50 per kilogramme and finally 10 kg at Rs 40 per kilogramme. The vendor discards the remaining 10 kg of manioc due to his inability to sell it. Determine whether the vendor earned a profit or incurred a loss from selling the manioc and calculate the profit earned or loss incurred by him.

The cost incurred in buying the manioc = Rs 50 × 60
= Rs 3 000
The amount made by selling the first 20 kg of manioc = Rs 70 × 20
= Rs 1 400

The amount made by selling the next 15 kg of manioc = Rs 60 × 15
= Rs 900
The amount made by selling 5 kg of manioc = Rs 50 × 5
= Rs 250
The amount made by selling 10 kg of manioc = Rs 40 × 10
= Rs 400
The amount made by selling the whole stock of manioc = Rs 1400 + 900 + 250 + 400
= Rs 2950

Since 3000 > 2950, a loss is incurred by the vendor.

The loss incurred by the vendor = Rs 3000 − 2950
= Rs 50

Exercise 4.1

1. Fill in the blanks based on the given information.

Item	Purchase price/ Production cost (Rs)	Selling price (Rs)	Whether it is a profit or a loss	Profit/Loss (Rs)
Wristwatch	500	750	…………………	………………
School Bag	1 200	1 050	…………………	…………………
Calculator	………………………	1 800	Profit	300
Drink Bottle	………………………	750	Loss	175
Water Bottle	350	………………	Loss	50
Box of mathematical instruments	275	………………	Profit	75
Umbrella	………………………	450	Loss	100
Pair of Slippers		700	Profit	150

2. Find the more profitable business of each pair given below.

i. Selling mangoes at Rs 60 per fruit which were bought at Rs 50 per fruit.
Selling oranges at Rs 55 per fruit which were bought at Rs 50 per fruit.

ii. Selling coconuts at Rs 60 per fruit which were bought at Rs 40 per fruit.
Selling jack fruits at Rs 60 per fruit which were bought at Rs 50 per fruit.

iii. Selling pens at Rs 15 each which were bought at Rs 10 each.
Selling books at Rs 28 each which were bought at Rs 25 each.

3. A vendor buys a stock of 100 rambutans at the price of Rs 3 per fruit. He discards 10 fruits which are spoilt and sells the remaining stock at the price of Rs 5 per fruit. Determine whether the vendor earns a profit or incurs a loss and calculate the profit earned or loss incurred by him.

4. A vendor buys a stock of 50 kg of beans at the price of Rs 60 per kilogramme. On the first day he sells 22 kg of beans at the price of Rs 75 per kilogramme and on the second day he sells the remaining stock at the price of Rs 70 per kilogramme.

i. Calculate the profit earned by the vendor on each day and determine on which day he earned a greater profit.

ii. Calculate his total profit.

5. The production cost of a cane chair is Rs 650. A manufacturer produces 20 such chairs. He expects to earn a profit of Rs 7 000 by selling all the chairs. In order to do this, what should be the selling price of a chair?

6. A vendor, who sells apples by the roadside after buying them from a wholesaler, buys 200 apples on a certain day at the price of Rs 25 per fruit. He expects to earn a profit of Rs 1000 by selling the whole stock of apples. In order to do this, determine the price at which he should sell a fruit.

7. A vendor bought a stock of 50 kg of onions at the price of Rs 60 per kilogramme and sold 30 kg of it at the price of Rs 80 per kilogramme. He had to sell the remaining stock of onions at a lesser price because they were close to getting spoilt. Due to this, the vendor neither made a profit nor incurred a loss from selling the whole stock of onions. Find the price at which the vendor sold each kilogramme of the remaining stock of onions.

4.2 Profit percentage/loss percentage

Ramesh and Suresh are two vendors. Ramesh owns a clothing store. He sells a pair of trousers which he bought for Rs 800, at the price of Rs 900. Suresh owns an electrical items store. He sells an electric kettle which he bought for Rs 2500, at the price of Rs 2600.

The items sold by Ramesh and Suresh are not the same, and the buying prices and selling prices of the items are also different. However, the profit earned by them from selling the items is equal.

The profit earned by Ramesh from selling a pair of trousers = Rs 900 – 800
= $\underline{\underline{\text{Rs }100}}$

The profit earned by Suresh from selling an electric kettle = Rs 2600 – 2500
= $\underline{\underline{\text{Rs }100}}$

Can you identify which of these two sellers engaged in the more profitable sale if both had Rs 5000 each?

Even though the profit earned by Ramesh and Suresh is equal, it is clear that the amount of money each person spent in order to earn that profit is not equal. In order to find out which was the more profitable sale, the amount of money spent by each person has to be considered. The below given calculation is performed in order to determine this.

The profit earned by Ramesh after spending Rs 800 = Rs 100

The profit earned by Ramesh as a fraction of the amount he spent = $\frac{100}{800}$

The profit earned by Suresh after spending Rs 2500 = Rs 100

The profit earned by Suresh as a fraction of the amount he spent = $\frac{100}{2500}$

It is easy to compare the fractions $\frac{100}{800}$ and $\frac{100}{2500}$ since the numerators of both fractions are equal. Since $\frac{100}{800} > \frac{100}{2500}$ Ramesh's transaction was more profitable.

Even when the numerators are not equal, the more profitable business is determined using a similar method. Since the comparison of fractions when the denominators are different could be difficult, these fractions are most often converted into percentages to facilitate comparison. Let us calculate these percentages as follows.

Since the profit earned by Ramesh written as a fraction of the cost is $\frac{100}{800}$,

Ramesh’s profit percentage $= \frac{100}{800} \times 100\%$

$= \underline{\underline{12.5\%}}$.

Accordingly, it is clear that the profit earned by Ramesh from spending Rs 100 is Rs 12.50.

Since the profit earned by Suresh written as a fraction of the cost is $\frac{100}{2500}$,

Suresh’s profit percentage $= \frac{100}{2500} \times 100\%$

$= \underline{\underline{4\%}}$.

Accordingly, it is clear that the profit earned by Suresh from spending Rs 100 is Rs 400.

Since 12.5% $>$ 4%, it can be said that Ramesh’s transaction was more profitable.

The meaning of the percentages calculated above can be described as follows.

$\frac{100}{800} \times 100$ is the profit Ramesh earns from spending Rs 100.

$\frac{100}{2500} \times 100$ is the profit Suresh earns from spending Rs 100.

The profit earned/loss incurred by a vendor when the buying price/production cost of the item is Rs 100, is known as the profit/loss percentage. Therefore, by representing the profit/loss as a fraction of the buying price/production cost and multiplying that fraction by 100%, the profit/ loss percentage can be calculated.

$$\text{Profit percentage} = \frac{\text{profit}}{\text{buying price (or production cost)}} \times 100\%$$

$$\text{Loss percentage} = \frac{\text{loss}}{\text{buying price (or production cost)}} \times 100\%$$

Example 1

A vendor buys exercise books at Rs 25 each, and sells them at Rs 30 each. Calculate the profit percentage earned by the vendor from selling one exercise book.

Profit = Rs 30 – 25
= Rs 5

Profit percentage = $\frac{5}{25} \times 100\%$

$= 20\%$

Example 2

A vendor buys a pair of trousers for Rs 500. Due to a damage, he sells it for Rs 450. Determine the loss percentage.

Loss = Rs 500 – 450
= Rs 50

Loss percentage = $\frac{50}{500} \times 100\%$

$= 10\%$

Example 3

A carpenter incurs a cost of Rs 4000 in making a table which he sells at Rs 5600. A blacksmith incurs a cost of Rs 250 in making a knife which he sells at Rs 360. Determine who has engaged in the more profitable sale.

The profit earned by the carpenter as a percentage of the cost incurred $= \frac{1600}{4000} \times 100\% = 40\%$

The profit earned by the blacksmith as a percentage of the cost incurred $= \frac{110}{250} \times 100\% = 44\%$

Therefore, the blacksmith's transaction was more profitable.

Example 4

If a vendor buys an almirah for Rs 30 000 and earns a profit percentage of 15% (of the purchase price) by selling it, calculate the selling price of the almirah.

Method I

Here, what is meant by a profit percentage of 15% is that, if Rs 100 is invested, then a profit of Rs 15 is earned. In other words, if Rs 100 is invested, then the item is sold at the price of Rs 115.

Therefore, the selling price of the item when Rs 30 000 is invested $= \frac{115}{100} \times 30\,000$

$= \underline{\underline{\text{Rs } 34\,500}}$

Method II

As in method I above,
since the profit is Rs 15 when Rs 100 is invested,

the profit earned when Rs 30 000 is invested $= \frac{15}{100} \times 30\,000$

$= \text{Rs } 4\,500$

Therefore, the selling price of the item = cost + profit

$= 30\,000 + 4\,500$

$= \underline{\underline{\text{Rs } 34\,500}}$

Example 5

A vendor buys a pair of shoes for Rs 1500 and sells it at a loss of 2%. What is the selling price of the pair of shoes?

Method I

Since the pair of shoes is sold at a loss of 2%,

the selling price if the item is worth Rs 100 = Rs 98

$\therefore$ The selling price of the item worth Rs 1 500 = Rs $\frac{98}{100} \times 1\,500$

$= \underline{\underline{\text{Rs } 1\,470}}$

Method II

$$\text{The loss incurred} = \text{Rs } 1\ 500 \times \frac{2}{100}$$
$$= \text{Rs } 30$$
$$\therefore \text{ The selling price} = \text{Rs } 1\ 500 - 30$$
$$= \underline{\underline{\text{Rs } 1\ 470}}$$

Example 6

If a vendor earns a profit of 10% by selling a television set at the price of Rs 22 000, find the price at which the vendor bought the set.

Method I

In order to earn a profit of 10% when the purchase price of the item is Rs 100, the item should be sold for Rs 110.

Therefore, the purchase price of an item sold for Rs 110 at a profit of 10% = Rs 100

$\therefore$ The purchase price of an item sold for Rs 22 000 at a profit of 10% $= \text{Rs } \frac{100}{110} \times 22\ 000$

$$= \underline{\underline{\text{Rs } 20\ 000}}$$

Method II

If the purchase price of the item is Rs x, then

$$\text{the profit earned} = \text{Rs } x \times \frac{10}{100}$$
$$= \text{Rs } \frac{x}{10}$$
$$\text{The selling price of the item} = \text{Rs } x + \frac{x}{10}$$
$$\therefore x + \frac{x}{10} = 22\ 000$$
$$\frac{10x + x}{10} = 22\ 000$$
$$\frac{11x}{10} = 22\ 000$$
$$x = 22\ 000 \times \frac{10}{11}$$
$$x = 20\ 000$$

Therefore, the purchase price of the television set is Rs 20 000.

Method III

If the purchase price is Rs x,

$$\text{the selling price} = \text{Rs } x \times \frac{110}{100}$$

$$\therefore\ x \times \frac{110}{100} = 22\ 000$$

$$x = 20\ 000$$

Therefore, the purchase price of the set is Rs 20 000.

Example 7

A vendor had to sell a sports item for Rs 6 800 due to a manufacturing defect, which caused him a loss of 15%. Find the purchase price of the item.

Method I

The selling price of an item which is bought at Rs 100 and sold at a loss of 15%, is Rs 85.

$\therefore$ The purchase price of an item sold at Rs 85 at a loss of 15% = Rs 100

Hence, the purchase price of an item sold at Rs 6 800 at a loss of 15% $= \text{Rs } \frac{100}{85} \times 6\ 800$

$$= \underline{\underline{\text{Rs } 8\ 000}}$$

Method II

If the purchase price of the item is Rs x,

$$\text{the loss incurred} = \text{Rs } x \times \frac{15}{100}$$

$$= \text{Rs } \frac{3x}{20}$$

$\therefore$ The selling price of the item

$$= \text{Rs } x - \frac{3x}{20}$$

$$\text{Then, } x - \frac{3x}{20} = 6\ 800$$

$$\frac{20x - 3x}{20} = 6\ 800$$

$$\frac{17x}{20} = 6\ 800$$

$$x = 6\ 800 \times \frac{20}{17}$$

$$\underline{\underline{x = 8\ 000}}$$

$\therefore$ The purchase price of the item is Rs 8000.

Exercise 4.2

1. Fill in the blanks in the table based on the information that is given.

	Purchase price (Rs)	Selling price (Rs)	Whether it is a profit or a loss	Profit/ Loss (Rs)	Profit/ Loss percentage
i.	400	440	Profit	40	10%
ii.	600	720	....	....	....
iii.	1500	1200	....	....	....
iv.	60	....	Profit	....	60%
v.	180	....	Profit	....	30%
vi.	150	75	Loss	....	....
vii.	200	....	Loss	....	10%

2. If a vendor buys a pair of trousers for Rs 500 and sells it at Rs 650, determine

i. his profit,

ii. the profit percentage.

3. If an electric iron which is worth Rs 2500 is sold at the price of Rs 2300, determine

i. the loss,

ii. the loss percentage.

4. A vendor buys a stock of 100 mangoes at the price of Rs 18 each. He discards 20 mangoes due to them being spoilt and sells the rest of the stock at the price of Rs 30 per fruit. Determine whether he has earned a profit or incurred a loss and calculate,

i. the profit earned/ loss incurred,

ii. profit/ loss percentage.

5. The production costs of several types of clothing produced and sold by a certain tailor, together with their selling prices are given in the table below.

The types of clothing	**Production cost (Rs) per item**	**Selling price (Rs) per item**
Shirts	300	350
Pairs of trousers	400	450
Frocks	500	575
Raincoats	1000	1150

  - **i.** For each of the above items, find the profit and the profit percentage earned by the tailor.
  - **ii.** Giving reasons, write the most profitable item that is produced by the tailor.

6. If a bookseller earns a profit of 25% by selling a novel worth Rs 300, calculate the selling price of the novel.

7. If a bicycle worth Rs 12 000 is sold at a loss of 10%, calculate the selling price of the bicycle.

8. A carpenter spends Rs 1800 in producing a chair. He sells the chair to a vendor at a profit of 20%. The vendor then sells the chair to a customer at a profit of 20%.
  - **i.** How much does the vendor spend to buy the chair?
  - **ii.** How much does the customer spend to buy the chair?
  - **iii.** Write with reasons whether the carpenter or the vendor earns a greater profit.

9. If a vendor earns a profit of 10% by selling a refrigerator for Rs 33 000, calculate its purchase price.

10. If a vendor incurs a loss of 5% by selling an electric stove for Rs 28 500, calculate its purchase price.

11. The profit/loss percentages of several items sold by a vendor and their selling prices are given in the table below. Calculate the purchase price of each item.

Item	**Selling price**	**Profit percentage**	**Loss percentage**
Clock	3 240	8%	-
Electric stove	7 500	25%	-
Camera	12 048	-	4%

4.3 Discounts and Commissions

Discounts

A discount of 20% is given on every book

The price at which an item is expected to be sold is called the marked price. According to the Consumer Affairs Authority Act, the price of an item that is for sale needs to be marked on the item.

A notice displayed in a bookshop is given in the picture shown above. What is mentioned in the notice is that a discount of 20% is given when a book is bought. This means that, when the book is purchased, 20% will be reduced from the price mentioned on the book. The amount that is reduced is called a "**discount**". Most often, a discount is indicated as a percentage of the marked price.

Since customers usually tend to buy goods from shops which offer discounts, there is an increase in sales in these shops. Due to this, the profits of the shop also increase. Discounts result in direct benefits for customers while the shop owners too gain long term benefits.

Example 1

Kaveesha buys books which are worth Rs 1500 from a bookshop which offers a discount of 20%. Calculate the discount that Kaveesha receives.

$$\text{Discount} = \text{Rs } 1\ 500 \times \frac{20}{100}$$

$$= \text{Rs } 300$$

Example 2

The production cost of a mobile phone is Rs 9000. The price of the phone has been marked keeping a profit of Rs 3000. If the phone is sold at a discount of 10% on the marked price, find the selling price.

Method I

The marked price = Rs 9000 + 3000
= Rs 12 000

$$\text{Discount} = \text{Rs } 12\,000 \times \frac{10}{100}$$
$$= \text{Rs } 1\,200$$

$$\therefore \text{ Selling price} = \text{Rs } 12\,000 - 1\,200$$
$$= \underline{\underline{\text{Rs } 10\,800}}$$

Method II

Since an item of marked price Rs 100 is sold for Rs 90 when the discount is 10%,

the selling price of an item of marked price Rs 100, sold at a discount of 10% = Rs 90

$\therefore$ The selling price of an item of marked price Rs 12 000, sold at a discount of 10% $= \text{Rs } \frac{90}{100} \times 12\,000$

$$= \underline{\underline{\text{Rs } 10\,800}}$$

Note: In the above example, two methods of solving the problem have been given. Since the second method presented is shorter than the first, it is important to practice this method.

Example 3

A discount of Rs 250 is given on the marked price of Rs 2000 when a certain wristwatch is bought. Find the discount percentage offered.

$$\text{The discount percentage} = \frac{250}{2000} \times 100\%$$
$$= \underline{\underline{12.5\%}}$$

Example 4

If a storybook is sold for Rs 460 at a discount of 8%, what is the marked price?

$$\text{The marked price} = \text{Rs } 460 \times \frac{100}{92}$$
$$= \underline{\underline{\text{Rs } 500}}$$

Commissions

A notice issued by a company which facilitates the sale of properties, vehicles and houses is shown in the above picture. While such companies find customers for these kinds of sales, once the deal is over, a certain percentage of the value of the transaction is charged by the company. Such companies are known as brokerages. The amount that is charged by such companies for facilitating the sale is known as the commission. This is usually a percentage of the value of the transaction.

Example 5

What is the commission charged by a company for facilitating the sale of a motorcar worth Rs 3 000 000, if a commission of 5% is charged?

$$\text{The commission charged} = \text{Rs } 3\,000\,000 \times \frac{5}{100}$$
$$= \underline{\underline{\text{Rs } 150\,000}}$$

Example 6

A real estate company charges a fee of Rs 36,000 to facilitate the sale of a land worth Rs 1 200 000. Calculate the commission percentage charged by the company.

$$\text{The commission percentage} = \frac{36\,000}{1\,200\,000} \times 100\%$$
$$= \underline{\underline{3\%}}$$

Exercise 4.3

1. A discount of 5% is offered when a television set of marked price Rs 25 000 is purchased.

(i) How much is offered as the discount (in rupees) ?
(ii) Find the selling price of the television set.

2. Nimithee buys a pair of trousers worth Rs 1 500 and a shirt worth Rs 1 200 from a shop which offers a discount of 5%. How much does Nimithee have to pay for both the items?

3. Two notices displayed during the festive season in two shoe shops which sell the same types of shoes are given below.

Shop A	**Shop B**
A discount of 8% on all purchases	**A reduction of Rs 100 on all purchases of value greater than Rs 1000**

i. How much needs to be paid when purchasing a pair of shoes of marked price Rs 1 500 from shop A?

ii. How much needs to be paid when purchasing a pair of shoes of marked price Rs 1 500 from shop B?

iii. What is the discount percentage offered by shop B for this pair of shoes?
iv. Is it more beneficial for the customer to buy the pair of shoes from shop A or from shop B?

4. A seller of bicycles buys a bicycle for Rs 8 000 and marks its selling price so that he earns a profit of 25%. When selling the bicycle, if the payment is done outright, a discount of 10% is offered to the customer.

i. Find the marked price of the bicycle.
ii. Find the price of the bicycle when the discount is given.
iii. If the seller marks the selling price so that he earns a profit of 20% on the amount he paid for the bicycle, then find its selling price.

5. A vendor marks the price of an item such that he earns a profit of 10%. He intends to offer a discount of 10% on the marked price when the item is sold. Describe the profit earned or loss incurred by the vendor at the sale of the item.

6. A company charges a commission of 3% on the sale of a land. When selling a land worth Rs 5 000 000,

i. how much needs to be paid as the commission?
ii. how much does the land owner receive after paying the commission?

7. If a broker charged Rs 25 000 for selling a generator which was worth Rs 300 000, calculate the commission percentage that he charged.

8. A person who sells his vehicle is left with Rs 570 000 after paying Rs 30 000 to the broker.

i. What is the selling price of the vehicle?
ii. What is the commission percentage charged by the broker?

9. A person paid a commission of 3% when he purchased a house. If he paid Rs 54 000 as commission, find the amount he paid for the house.

Miscellaneous Exercise

1. Kasun decides to sell 10 perches of a land he owns at the price of Rs 300 000 per perch. He promises a commission of 3% on the sale of the land to a broker. If he gives a discount of 1% on the original price to the buyer, find his income from the sale of the land, after paying the commission to the broker.

2. Amal who is a car dealer purchases a car for Rs 5 000 000. He intends selling the car for Rs 6 000 000. However, he gives a discount of 3% on this price to the buyer and a commission of 2% to a broker. Determine Amal's profit.

Summary

Summary

- profit = selling price – cost
- loss = cost – selling price
- Profit percentage $= \dfrac{\text{profit}}{\text{buying price (or production cost)}} \times 100\%$
- Loss percentage $= \dfrac{\text{loss}}{\text{buying price (or production cost)}} \times 100\%$

5 Algebraic Expressions

By studying this lesson, you will be able to;

- find the value of simple algebraic expressions by substituting directed numbers,
- expand the product of two binomial expressions of the form $(x \pm a)\ (x \pm b)$,
- verify the expansion of the product of two binomial expressions by considering areas.

Algebraic expressions

Do the following exercise to review what you have learnt in grade 8, related to algebraic expressions.

Review Exercise

1. Expand the following expressions.

a. $5\ (x + 2)$ **b.** $3\ (y + 1)$ **c.** $4\ (2m + 3)$
d. $3\ (x - 1)$ **e.** $4\ (3 - y)$ **f.** $2\ (3x - 2y)$
g. $-2\ (y + 3)$ **h.** $-3\ (2 + x)$ **i.** $-5\ (2a + 3b)$
j. $-4\ (m - 2)$ **k.** $-(5 - y)$ **l.** $-10\ (-3b - 2c)$

2. Expand the following expressions.

a. $x\ (a + 2)$ **b.** $y\ (2b - 3)$ **c.** $a\ (2x + 3y)$
d. $2a\ (x + 5)$ **e.** $2b(y - 2)$ **f.** $3p\ (2x - y)$
g. $(-3q)\ (p + 8)$ **h.** $(-2x)\ (3 - 2y)$ **i.** $(-5m)\ (x - 2y)$

3. Find the value of each of the following expressions when $x = 3$ and $y = -2$.

a. $x + y$ **b.** $x - y$ **c.** $3x - 2y$
d. $-2x + y$ **e.** $2(x + y)$ **f.** $3\ (2x - y)$

4. Expand and simplify each of the following expressions.

a. $3\ (x + y) + 2\ (x - y)$ **b.** $5(a + b) + 4\ (a + c)$
c. $4\ (a + b) + 3\ (2a - b)$ **d.** $2(a - b) + (2a - b)$
e. $5\ (m + n) + 2\ (m + n)$ **f.** $3(m + n) - (m - n)$
g. $5\ (x - y) - 3\ (2x + y)$ **h.** $2(3p - q) - 3\ (p - q)$
i. $-4\ (m + n) + 2\ (m + 2)$ **j.** $-4\ (a - b) - 2\ (a - b)$

5.1 Substitution

In grade 8, you learnt to find the value of an algebraic expression by substituting integers for the unknown terms. Let us now find out how to obtain the value of an algebraic expression by substituting directed numbers.

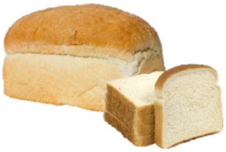
◆ 20 adults and 16 children went on a trip. Each adult was given x amount of bread and each child was given y amount of bread for breakfast.

Let us write the total amount of bread that was distributed as an algebraic expression.

Amount of bread given to the 20 adults $= 20x$
Amount of bread given to the 16 children $= 16y$
Total amount of bread that was distributed $= 20x + 16y$

Let us find out the total amount of bread that was distributed, if an adult was given half a loaf of bread and a child was given a quarter loaf of bread.

Then $x = \frac{1}{2}$ and $y = \frac{1}{4}$. To find out the total amount of bread that was distributed, $x = \frac{1}{2}$ and $y = \frac{1}{4}$ should be substituted in the expression $20x + 16y$.



Accordingly, the total number of loaves of bread that were distributed

$$= 20 \times \frac{1}{2} + 16 \times \frac{1}{4}$$
$$= 10 + 4$$
$$= 14$$

Example 1

Find the value of each of the following algebraic expressions when $a = \frac{1}{2}$.

i. $2a + 3$

$$\begin{aligned} 2a + 3 &= 2 \times \frac{1}{2} + 3 \\ &= 1 + 3 \\ &= \underline{\underline{4}} \end{aligned}$$

ii. $6 - 4a$

$$\begin{aligned} 6 - 4a &= 6 - 4 \times \frac{1}{2} \\ &= 6 - 2 \\ &= \underline{\underline{4}} \end{aligned}$$

iii. $3a - 1$

$$\begin{aligned} 3a - 1 &= 3 \times \frac{1}{2} - 1 \\ &= \frac{3}{2} - 1 \\ &= \frac{3-2}{2} \\ &= \underline{\underline{\frac{1}{2}}} \end{aligned}$$

Example 2

Find the value of each of the following algebraic expressions when $b = -\frac{2}{3}$.

i. $3b + 5$

$$\begin{aligned} &3b + 5 \\ &= 3 \times \frac{-2}{3} + 5 \\ &= (-2) + 5 \\ &= \underline{\underline{3}} \end{aligned}$$

ii. $5 - 6b$

$$\begin{aligned} &5 - 6b \\ &= 5 - 6 \times (-\frac{2}{3}) \\ &= 5 + (-6) \times (-\frac{2}{3}) \\ &= 5 + 4 \\ &= \underline{\underline{9}} \end{aligned}$$

iii. $2b + \frac{1}{3}$

$$\begin{aligned} &2b + \frac{1}{3} \\ &= 2 \times (\frac{-2}{3}) + \frac{1}{3} \\ &= \frac{-4}{3} + \frac{1}{3} \\ &= \frac{-3}{3} \\ &= \underline{\underline{-1}} \end{aligned}$$

Example 3

Find the value of each of the following algebraic expressions when $x = \frac{1}{2}$ and $y = -\frac{1}{4}$.

i. $2x + 4y$

$$\begin{aligned} 2x + 4y &= 2 \times \frac{1}{2} + 4 \times \left(\frac{-1}{4}\right) \\ &= 1 - 1 \\ &= \underline{\underline{0}} \end{aligned}$$

ii. $2x - 2y$

$$\begin{aligned} 2x - 2y &= 2 \times \frac{1}{2} - 2 \times \left(\frac{-1}{4}\right) \\ &= 1 + \frac{1}{2} \\ &= \underline{\underline{1\frac{1}{2}}} \end{aligned}$$

iii. $4xy$

$$\begin{aligned} 4xy &= 4 \times \frac{1}{2} \times \left(\frac{-1}{4}\right) \\ &= \underline{\underline{\frac{-1}{2}}} \end{aligned}$$

iv. $-2xy$

$$\begin{aligned} -2xy &= -2 \times \left(\frac{1}{2}\right) \times \left(\frac{-1}{4}\right) \\ &= \underline{\underline{\frac{1}{4}}} \end{aligned}$$

Exercise 5.1

1. Find the value of each of the following algebraic expressions when $x = \frac{1}{4}$.

i. $4x$ **ii.** $2x$ **iii.** $3x$ **iv.** $-8x$

2. Find the value of each of the following algebraic expressions when $y = \frac{-1}{3}$.

i. $3y$ **ii.** $2y$ **iii.** $-6y$ **iv.** $-4y$

3. Find the value of each of the following algebraic expressions when $a = -2$ and $b = \frac{1}{2}$.

i. $a + 2b$ **ii.** $4b - a$ **iii.** $3a + b$

4. Find the value of each of the following algebraic expressions when $x = \frac{2}{3}$ and $y = \frac{3}{4}$.

i. $3x + 4y$ **ii.** $3x - 2y$ **iii.** $8y - 6x$

5. Find the value of each of the following algebraic expressions when $p = -\frac{1}{2}$ and $q = -3$.

i. $2p + q$ **ii.** $4p - q$ **iii.** $6pq - 2$

5.2 The product of two binomial expressions

Let us first recall what is meant by algebraic symbols, algebraic terms, algebraic expressions and binomial expressions. The letters $x, y, z, a, b, c, \ldots$ are considered as algebraic symbols.

Algebraic symbols such as x, y and z are also considered as algebraic terms.

When an algebraic symbol is multiplied or divided by a number, as for example, $2x$, $5y$, $-2a$ and $\frac{x}{3}$, it too is considered as an algebraic term.

Similarly, when an algebraic symbol is multiplied or divided by another algebraic symbol, as for example, xy, ay and $\frac{b}{z}$, it is also called an **algebraic term**. The products and quotients of algebraic symbols and numbers such as $2xy$, $-3zab$ and $\frac{2}{5}xy$ are also called **algebraic terms**.

Algebraic terms can also be considered as algebraic expressions (expressions with one term).

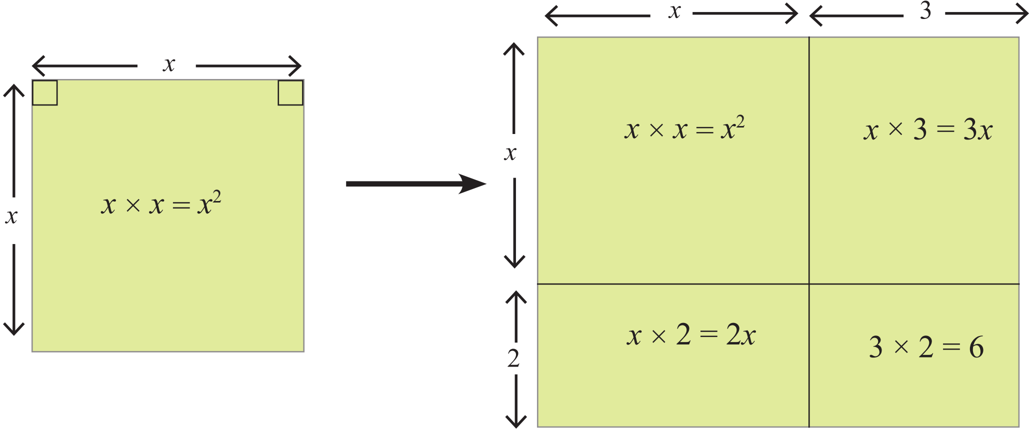
A sum or a difference of algebraic terms is called an **algebraic expression**. For example, $x + y$, $2a + xyz$, $4xy^2 - yz$ and $-2x + 3xy$ are algebraic expressions. Similarly, when a number is added to or subtracted from an algebraic term, it is also called an algebraic expression. For example, $4 + x$ and $1 - 3ab$ are **algebraic expressions**.

All the algebraic expressions we have considered thus far have consisted of two terms. A "binomial algebraic expression" (or simply a "binomial expression) is an expression which is a sum or difference of two terms.

However, there can be any number of terms in an algebraic expression.
$3 + ax - 2xyz + xy$ is an algebraic expression with four terms. It has three algebraic terms and a number (constant term).

In this lesson we will be studying binomial expressions. Now let us consider the product of two binomial expressions.

Let us take the length of a side of the square shaped flower bed shown in the figure below as x units. If a larger rectangular flower bed is made by increasing the length of one side by 3 units and the length of the adjacent side by 2 units, let us consider how an algebraic expression can be constructed in terms of x, for the area of the larger flower bed.



The length of the larger flower bed $= x + 3$ units
The breadth of the larger flower bed $= x + 2$ units

According to the figure,
the area of the larger flower bed = length × breadth= $(x + 3)(x + 2)$ square units ——(1)

Observe that $(x + 3)\ (x + 2)$ is a product of two binomial expressions.

The area of the larger flower bed can also be found by using a different method, that is, by adding the areas of the four smaller sections of which it is composed. The four sections are, the initial square shaped section and the three smaller rectangular sections in the figure.

Accordingly,
the area of the larger flower bed = the sum of the areas of the four smaller sections

$$= x^2 + 2x + 3x + 6 \text{ square units}$$
$$= x^2 + 5x + 6 \text{ square units} \text{ ——————} (2)$$

Irrespective of the method used to find the area, the expressions obtained for the area should be equal to each other. Therefore, from (1) and (2) the following equality is established.

$$(x + 3)\ (x + 2) = x^2 + 5x + 6$$

Let us now consider how this equality can be obtained without the aid of a figure.

Let us multiply the terms within the second pair of brackets by the two terms within the first pair of brackets.

$$(x + 3)\ (x + 2) = (x + 3)\ (x + 2)$$
$$= x\ (x + 2) + 3\ (x + 2)$$
$$= x^2 + 2x + 3x + 6$$
$$= x^2 + 5x + 6$$

Accordingly, the product of two binomial expressions can be obtained in the above manner without the aid of a figure.

Let us consider another activity similar to the one above.

Activity 1

Fill in the blanks using the given information.

A square shaped metal sheet of side length x centimetres is shown in Figure I. Figure II illustrates how two strips of width 2 centimetres and 3 centimetres respectively have been cut off from the two sides of the sheet.

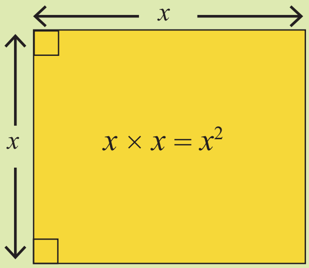


Figure I

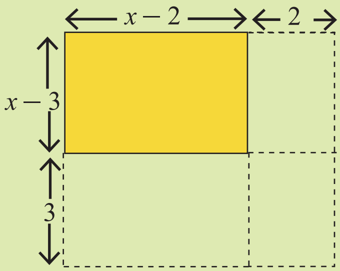


Figure II

The area of the remaining rectangular sheet $= (x-2)\,(x-3)$

According to Figure II,

the area of the remaining rectangular sheet = the area of the square − the area of the three rectangular parts—②

$$= x^2 - 2\,(\ldots\ldots\ldots) - \ldots\,(x-2) - 2 \times 3$$

Accordingly, $(x-2)\,(x-3) = x^2 - 2(\ldots\ldots\ldots) - \ldots(x-2) - 2 \times 3$

$= \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$= \ldots\ldots\ldots\ldots\ldots\ldots$

Let us consider a few examples to develop a better understanding of how the product of two binomial expressions is obtained.

Example 1

$(x+5)\,(x+3)$

$$\begin{aligned}(x+5)\,(x+3) &= x\,(x+3) + 5\,(x+3)\\ &= x^2 + 3x + 5x + 15\\ &= \underline{\underline{x^2 + 8x + 15}}\end{aligned}$$

Example 2

$(x+5)\,(x-3)$

$$\begin{aligned}(x+5)\,(x-3) &= x\,(x-3) + 5\,(x-3)\\ &= x^2 - 3x + 5x - 15\\ &= \underline{\underline{x^2 + 2x - 15}}\end{aligned}$$

Example 3

$(x-5)\,(x+3)$

$$\begin{aligned}(x-5)\,(x+3) &= x\,(x+3) - 5\,(x+3)\\ &= x^2 + 3x - 5x - 15\\ &= \underline{\underline{x^2 - 2x - 15}}\end{aligned}$$

Example 4

$(x-5)\,(x-3)$

$$\begin{aligned}(x-5)\,(x-3) &= x\,(x-3) - 5\,(x-3)\\ &= x^2 - 3x - 5x + 15\\ &= \underline{\underline{x^2 - 8x + 15}}\end{aligned}$$

Example 5

Show that $(x + 8)(x - 3) = x^2 + 5x - 24$ when $x = 5$.

L.H.S. $= (x + 8)(x - 3)$
When $x = 5$
L.H.S. $= (5 + 8)(5 - 3)$
$= 13 \times 2$
$= 26$

R. H. S. $= x^2 + 5x - 24$
When $x = 5$
R. H. S. $= 25 + 25 - 24$
$= 26$

L.H.S. = R. H. S.

$\therefore (x + 8)(x - 3) = x^2 + 5x - 24$

Exercise 5.2

1. Expand and simplify each of the following products of binomial expressions.

a. $(x + 2)(x + 4)$ **b.** $(x + 1)(x + 3)$ **c.** $(a + 3)(a + 2)$
d. $(m + 3)(m + 5)$ **e.** $(p - 4)(p - 3)$ **f.** $(k - 3)(k - 3)$

2. Draw relevant rectangles for each product of binomial expressions in **a.**, **b.** and **e.** of **1.** above and verify the answers obtained in **1.** by calculating their areas.

3. Expand and simplify each of the following products of binomial expressions.

a. $(x + 2)(x - 5)$ **b.** $(x + 3)(x - 7)$ **c.** $(m + 6)(m - 1)$
d. $(x - 2)(x + 3)$ **e.** $(x - 5)(x + 5)$ **f.** $(m - 1)(m + 8)$
g. $(x - 3)(x - 4)$ **h.** $(y - 2)(y - 5)$ **i.** $(m - 8)(m - 2)$
j. $(x - 3)(2 - x)$ **k.** $(5 - x)(x - 4)$ **l.** $(2 - x)(3 - x)$

4. Join each of the expressions in column A, with the corresponding simplified expression in column B.

A	**B**
$(x + 2)(x + 1)$	$x^2 + 3x - 10$
$(x + 3)(x - 4)$	$x^2 - 25$
$(x + 5)(x - 2)$	$x^2 - 6x + 9$
$(x - 3)(x - 3)$	$x^2 + 3x + 2$
$(x - 5)(x + 5)$	$x^2 - x - 12$

5. Verify that $(x + 5)(x + 6) = x^2 + 11x + 30$ for each instance given below.

i. $x = 3$ **ii.** $x = -2$

6. Verify that $(x - 2)(x + 3) = x^2 + x - 6$, for each instance given below.

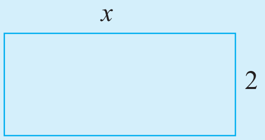
i. $x = 1$ **ii.** $x = 4$ **iii.** $x = 0$

7. Verify that $(2 - x)(4 - x) = x^2 - 6x + 8$, for each instance given below.

i. $x = 2$ **ii.** $x = 3$ **iii.** $x = -2$

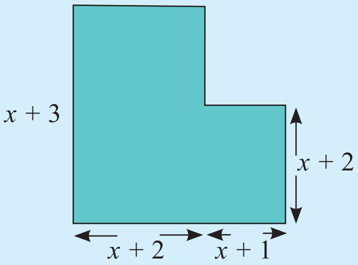
8. The length and breadth of a rectangular piece of decorative paper are 15 cm and 8 cm respectively. Two strips of breadth x cm each are cut off from the length and the breadth of this paper. Using a figure, obtain an expression for the area of the remaining portion. (Consider $x < 8$ cm).

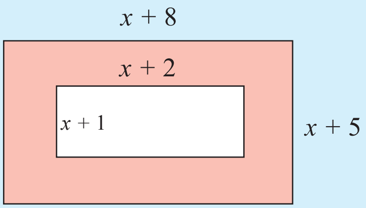
9. A rectangular flower bed of length x metres and breadth 2 metres is shown in the figure. Two metres are reduced from its length and x meters are added to its breadth. Construct an expression in terms of x for the area of the new flower bed by using a figure. (Consider that $x > 2$ m).



Miscellaneous Exercise

1. Write an expression for the shaded area in the given figure and simplify it.





2. If $(x + a)(x + 4) = x^2 + bx + 12$, find the values of a and b.

6

Factors of Algebraic Expressions

By studying this lesson, you will be able to;

- factorize algebraic expressions with four terms when the factors are binomial expressions,
- factorize trinomial quadratic expressions of the form $x^2 + bx + c$,
- factorize algebraic expressions written as a difference of two squares.

Factors of algebraic expressions

The meanings of many algebraic terms were explained in the previous lesson. In this lesson we will consider what is meant by the factors of an algebraic expression (or an algebraic term).

Consider the term $2xy$. It is formed by the product of 2, x and y. Therefore, 2, x and y are all factors of $2xy$.

$2x + 2y$ is a binomial expression. It is the sum of two algebraic terms. 2 and x are factors of $2x$. Similarly, 2 and y are factors of $2y$. Accordingly, 2 is a factor of both the terms $2x$ and $2y$. You have learnt in grade 8 that the above binomial expression can be written as $2\,(x + y)$ by factoring out the common factor 2. Hence;

$$2x + 2y = 2\,(x + y)$$

What is important here is that the algebraic expression $2x + 2y$, which is the sum of $2x$ and $2y$, is expressed as a product of 2 and $x + y$. We say that 2 and $x + y$ are factors of $2x + 2y$. That is, the algebraic expression $2x + 2y$ can be expressed as a product of its factors 2 and $x + y$.

One factor of the above algebraic expression $2x + 2y$ is the number 2 and another factor is the algebraic expression $x + y$. However, an algebraic expression could also be expressed as a product of algebraic terms or algebraic expressions. For example, since the expression $xy + 5xz$ can be written as $x(y + 5z)$, x and $y + 5z$ are factors of it.

According to the facts learnt in lesson 5, when the algebraic expression $x(y + 5z)$ which is a product is expanded, we obtain the algebraic expression $xy + 5xz$, which is a sum of algebraic terms. In this lesson we will study the inverse of the process that was learnt in lesson 5. That is, we will learn how to write a given algebraic expression as a product of factors.

Observe how each algebraic expression given below has been written as a product of factors as learnt in grade 8.

- $3x + 12 = 3\ (x + 4)$
- $6a + 12b - 18 = 6\ (a + 2b - 3)$
- $-2x - 6y = -2\ (x + 3y)$
- $3x - 6xy = 3x\ (1 - 2y)$

In the second example above, the common factor of the terms of the expression $6a + 12b - 18$ is 6. Observe that this is the highest common factor of 6, 12 and 18. When a number is a common factor, we should always consider the highest common factor. Furthermore, when factorizing algebraic expressions, the numbers need not be factorized further. For example, $6x + 6y$ is written as $6(x + y)$ and not as $2 \times 3\ (x + y)$.

Do the following review exercise to establish these facts further.

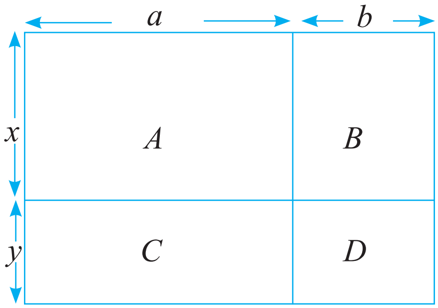
Review Exercise

Write each of the following algebraic expressions as a product of factors.

a. $8x + 12y$
b. $9a + 18y$
c. $3m + 6$
d. $20a - 30b$
e. $4p - 20q$
f. $12 - 4k$
g. $3a + 15b - 12$
h. $12a - 8b + 4$
i. $9 - 3b - 6c$
j. $-12x + 4y$
k. $-8a - 4b$
l. $-6 + 3m$
m. $ab + ac$
n. $p - pq$
o. $ab + ac - ad$
p. $3x + 6xy$
q. $6ab - 9bc$
r. $4ap + 4bp - 4p$
s. $x^3 + 2x$
t. $3m - 2nm^2$
u. $6s - 12s^2t$

6.1 Factors of algebraic expressions with four terms

The figure of a large rectangle which is composed of the four rectangular sections A, B, C and D is given below.



Let us find the area of each rectangle in terms of the given algebraic symbols x, y, a and b.

The area of section $A = a \times x = ax$

The area of section $B = b \times x = bx$

The area of section $C = a \times y = ay$

The area of section $D = b \times y = by$

Now let us find the area of the large rectangle.

The length of the large rectangle = $a + b$
The breadth of the large rectangle = $x + y$
Hence, the area of the large rectangle = $(a + b)(x + y)$

Now, since the total area of the 4 small rectangles = the area of the large rectangle, $ax + ay + bx + by = (a + b)(x + y)$.

We can verify the total above equality by expanding the product $(a + b)(x + y)$ by using the method learnt in the previous lesson.

Let us expand it as follows.

$$\begin{aligned}(a + b)(x + y) &= a(x + y) + b(x + y)\\ &= ax + ay + bx + by\end{aligned}$$

The validity of the equality is verified.

In this lesson we expect to learn how to write an expression of the form $ax + bx + ay + by$ as a product of two factors as $(a + b)(x + y)$. First, we need to observe that the four terms ax, ay, bx and by have no common factors. Therefore, the factoring out of a common factor cannot be done directly. However, if we consider the four terms pairwise, the expression can be factored as follows.

$$ax + bx + ay + by = (ax + bx) + (ay + by)$$
$$= x(a + b) + y(a + b)$$

The final expression is the sum of the two expressions $x(a + b)$ and $y(a + b)$. Now observe that the two expressions $x(a + b)$ and $y(a + b)$ have a common factor $(a + b)$. Therefore, by factoring out this expression, we can rewrite the given expression as a product of two factors as $(a + b)(x + y)$.

$$ax + bx + ay + by = x(a + b) + y(a + b)$$
$$= (a + b)(x + y)$$

Example 1

Factorize $3x + 6y + kx + 2ky$.

$$3x + 6y + kx + 2ky = 3(x + 2y) + k(x + 2y)$$
$$= \underline{\underline{(x + 2y)(3 + k)}}$$

Example 2

Factorize $a^2 - 3a + ab - 3b$.

$$a^2 - 3a + ab - 3b = a(a - 3) + b(a - 3)$$
$$= \underline{\underline{(a - 3)(a + b)}}$$

Example 3

Factorize $x^2 + xy - x - y$.

$$x^2 + xy - x - y = x^2 + xy - 1(x + y)$$
$$= x(x + y) - 1(x + y)$$
$$= \underline{\underline{(x + y)(x - 1)}}$$

Exercise 6.1

Factorize each of the following algebraic expressions.

a. $ax + ay + 3x + 3y$
b. $ax - 8a + 3x - 24$
c. $mp - mq - np + nq$
d. $ak + al - bk - bl$
e. $x^2 + 4x - 3x - 12$
f. $y^2 - 7y - 2y + 14$
g. $a^2 - 8a + 2a - 16$
h. $b^2 + 5b - 2b - 10$
i. $5 + 5x - y - xy$
j. $ax - a - x + 1$

6.2 Factors of trinomial quadratic expressions of the form of $x^2 + bx + c$

Recall how we obtained the product of the two algebraic expressions $(x + 3)$ and $(x + 4)$.

$$\begin{aligned}(x + 3)(x + 4) &= x(x + 4) + 3(x + 4)\\ &= x^2 + 4x + 3x + 12\\ &= x^2 + 7x + 12\end{aligned}$$

Since we have obtained $x^2 + 7x + 12$ as the product of $(x + 3)$ and $(x + 4)$, the expressions $(x + 3)$ and $(x + 4)$ are factors of $x^2 + 7x + 12$. Expressions of the form $x^2 + 7x + 12$ consisting of three terms of which one is a quadratic term are called trinomial quadratic expressions.

Note:

The trinomial quadratic expressions we consider here can in general be written in the form $x^2 + bx + c$. Here b and c are numerical values. For example, $x^2 + 7x + 12$ is the trinomial quadratic expression that is obtained when $b = 7$ and $c = 12$. In general, bx is called the middle term and c is called the constant term. The expression $x^2 + 7x + 12$ can be written as a product of two factors as $(x + 3)(x + 4)$. However, there are some trinomial quadratic expressions which cannot be written as a product of two such factors, as for example, the expression $x^2 + 3x + 4$.

Here, we only consider how to find the factors of those trinomial quadratic expressions that can be written as a product of two factors.

To find out how to write a trinomial quadratic expression as a product of two binomial terms, let us analyze the steps we carried out in obtaining the product of two binomial expressions, in the opposite direction.

- In the trinomial quadratic expression $x^2 + 7x + 12$, the middle term $7x$ has been written as a sum of two terms as $3x + 4x$.

  There are many ways of writing $7x$ as a sum of two terms. For example, $7x = 5x + 2x$ and $7x = 8x + (-x)$. The importance of $3x$ and $4x$ can be explained as follows.

- The product of $3x$ and $4x = 3x \times 4x = 12x^2$.

- Moreover, the product of the first and last terms of the trinomial quadratic expression $x^2 + 7x + 12$ is also $x^2 \times 12 = 12x^2$.

The observations from the above analysis can be used to factorize trinomial quadratic expressions. The middle term should be written as a sum of two terms. Their product should be equal to the product of the first and last terms of the expression to be factorized.

Let us factorize $x^2 + 6x + 8$. The middle term is $6x$. It should be written as a sum of two terms, and their product should be equal to $x^2 \times 8 = 8x^2$.

Based on the above facts, we have to find a pair of linear terms of which the product is $8x^2$ and the sum is $6x$. The table below shows the possible ways of writing $8x^2$ as a product of two linear terms.

Pair of linear terms	**Product**	**Sum**
x, $8x$	$x \times 8x = 8x^2$	$x + 8x = 9x$
$2x$, $4x$	$2x \times 4x = 8x^2$	$2x + 4x = 6x$

According to the table, it is clear that the middle term $6x$ is obtained from $2x + 4x$. Let us factorize the expression $x^2 + 6x + 8$.

$$\begin{aligned} x^2 + 6x + 8 &= x^2 + 2x + 4x + 8 \\ &= x\,(x + 2) + 4\,(x + 2) \\ &= (x + 2)\,(x + 4) \end{aligned}$$

$\therefore$ $x + 2$ and $x + 4$ are factors of $x^2 + 6x + 8$.

Instead of writing the middle term as $2x + 4x$, let us write it as $4x + 2x$ and factorize to see whether we obtain different factors.

$$\begin{aligned} x^2 + 6x + 8 &= x^2 + 4x + 2x + 8 \\ &= x\,(x + 4) + 2\,(x + 4) \\ &= (x + 4)\,(x + 2) \end{aligned}$$

The same pair of factors are obtained. Therefore, we see that the order in which the selected pair is written does not affect the final answer.

Example 1

Factorize $x^2 + 5x + 6$.
In this expression,
the product of the first and last terms $= x^2 \times 6 = 6x^2$
The middle term $= 5x$

We can factorize this expression as below, because $2x + 3x = 5x$ and $(2x)\,(3x) = 6x^2$

$$\begin{aligned} x^2 + 5x + 6 &= x^2 + 2x + 3x + 6 \\ &= x\,(x + 2) + 3\,(x + 2) \\ &= \underline{\underline{(x + 2)\,(x + 3)}} \end{aligned}$$

Example 2

Factorize $x^2 - 8x + 12$.

The product of the first and last terms of the expression is $x^2 \times 12 = 12x^2$ and the middle term is $(-8x)$. Here we have a negative term. The table given below shows the various ways in which two terms in x can be selected such that their product is $12x^2$.

x,	$12x$
$2x$,	$6x$
$3x$,	$4x$
$-2x$,	$-6x$
$-3x$,	$-4x$
$-x$,	$-12x$

According to the table, if we write $-8x = (-2x) + (-6x)$, then we obtain $(-2x)(-6x) = 12x^2$.

Hence,
$$\begin{aligned} x^2 - 8x + 12 &= x^2 - 2x - 6x + 12 \\ &= x\,(x - 2) - 6\,(x - 2) \\ &= \underline{\underline{(x - 2)\,(x - 6)}} \end{aligned}$$

Example 3

Factorize $y^2 + 2y - 15$.

The product of the first and last terms of the expression $= y^2 \times -15 = -15y^2$

The middle term $= 2y$

By writing $-15y^2 = (5y)\,(-3y)$, the middle term is obtained as $(5y) + (-3y) = 2y$

Therefore,

$$\begin{aligned} y^2 + 2y - 15 &= y^2 - 3y + 5y - 15 \\ &= y\,(y - 3) + 5\,(y - 3) \\ &= \underline{\underline{(y - 3)\,(y + 5)}} \end{aligned}$$

Example 4

Factorize $a^2 - a - 20$.
The product of the first and last terms of the expression is $= a^2 \times (-20) = -20a^2$ and the middle term is $(-a)$.

Since $-20a^2 = (-5a)\,(4a)$ and $(-5a) + (4a) = -a$, the expression can be factored as follows.

$$\begin{aligned} a^2 - a - 20 &= a^2 + 4a - 5a - 20 \\ &= a\,(a+4) - 5\,(a+4) \\ &= (a+4)\,(a-5) \end{aligned}$$

Exercise 6.2

Factorize the quadratic expressions given below.

a. $x^2 + 9x + 18$
b. $y^2 + 11y + 30$
c. $a^2 + 10a + 24$
d. $b^2 - 8b + 15$
e. $x^2 - 5x + 6$
f. $m^2 - 12m + 20$
g. $a^2 + a - 12$
h. $p^2 + 5p - 24$
i. $p^2 + 6p - 16$
j. $x^2 - x - 12$
k. $a^2 - 3a - 40$
l. $r^2 - 3r - 10$
m. $y^2 + 6y + 9$
n. $k^2 - 10k + 25$
o. $4 + 4x + x^2$
p. $36 + 15x + x^2$
q. $30 - 11a + a^2$
r. $54 - 15y + y^2$

Note:
When factorizing trinomial quadratic expressions, writing the middle term as a sum of two suitable terms is an important step. Although a specific method has been given above to find the two terms, an easier method is to write the middle term as a sum of two terms and check whether their product is equal to the product of the first and last terms of the given expression. This skill can be mastered with practice. However, once the two terms have been written, we have to be careful when simplifying the expression. In example 4 above, when the common factor -5 is factored out from the expression $-5a - 20$, we obtain $-5(a+4)$ This is often mistakenly written as $-5(a-4)$.

6.3 Factors of an expression written as a difference of two squares

Consider the product of the two binomial expressions $(x - y)$ and $(x + y)$.

$$\begin{aligned} (x-y)\,(x+y) &= x\,(x+y) - y\,(x+y) \\ &= x^2 + xy - xy - y^2 \\ &= x^2 - y^2 \end{aligned}$$

Accordingly, $(x+y)(x-y)$ is equal to the expression x^2-y^2. The expression x^2-y^2 is said to be a difference of two squares.

The fact that $(x+y)(x-y)=x^2-y^2$ means that $x+y$ and $x-y$ are factors of x^2-y^2 .

Let us see whether we can find the factors of x^2-y^2 by considering it as a quadratic expression in x. We can rewrite it as a trinomial quadratic expression in x by writing the middle term as 0. We then obtain the expression x^2+0-y^2. Now consider its factorization.

The product of the first and last terms of the expression is $= x^2 \times (-y^2) = -x^2y^2$ and the middle term is 0.
Now,

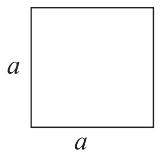
$$-x^2y^2 = (-xy) \times (xy) \text{ and } -xy + xy = 0$$

Therefore, $x^2+0-y^2 = x^2-xy+xy-y^2$
$= x(x-y)+y(x-y)$
$= (x-y)(x+y)$

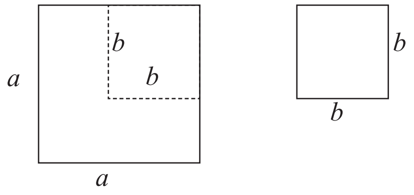
Again we obtain $x^2-y^2=(x-y)(x+y)$.

Now let us consider how to find the factors of the difference of two squares by considering areas.

Consider a square of side length a units.



From this square, cut out a square of side length b units.



The area of the remaining portion is a^2-b^2 square units.

Let us rearrange the remaining portion as follows.

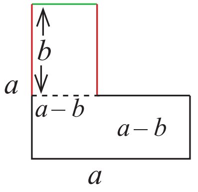


Figure I

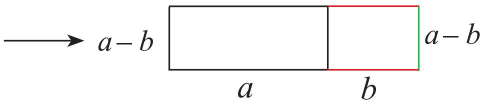


Figure II

The area of the remaining portion according to figure II is $(a-b)(a+b)$.
Accordingly, $a^2-b^2=(a-b)(a+b)$.

Now let us consider some examples of the factorization of expressions which are the difference of two squares.

Example 1

Factorize x^2-25.

$$x^2-25=x^2-5^2$$
$$=\underline{\underline{(x-5)(x+5)}}$$

Example 2

Factorize $9-y^2$.

$$9-y^2=3^2-y^2$$
$$=\underline{\underline{(3-y)(3+y)}}$$

Example 3

Factorize $4a^2-49$.

$$4a^2-49=2^2a^2-7^2$$
$$=\underline{\underline{(2a-7)(2a+7)}}$$

Example 4

Factorize $1-4b^2$.

$$1-4b^2=1^2-2^2b^2$$
$$=\underline{\underline{(1-2b)(1+2b)}}$$

Example 5

Factorize $2x^2-72$.

$$2x^2-72=2(x^2-36)$$
$$=2(x^2-6^2)$$
$$=\underline{\underline{2(x-6)(x+6)}}$$

Example 6

Find the value 33^2-17^2.

$$33^2-17^2=(33+17)(33-17)$$
$$=50\times 16$$
$$=\underline{\underline{800}}$$

Example 7

Factorize $\dfrac{x^2}{4} - \dfrac{1}{9}$.

$$\frac{x^2}{4} - \frac{1}{9} = \frac{x^2}{2^2} - \frac{1}{3^2}$$

$$= \left(\frac{x}{2} + \frac{1}{3}\right)\left(\frac{x}{2} - \frac{1}{3}\right)$$

Example 8

Factorize $1 - \dfrac{9x^2}{16}$.

$$1 - \frac{9x^2}{16} = 1^2 - \left(\frac{3x}{4}\right)^2$$

$$= \left(1 - \frac{3x}{4}\right)\left(1 + \frac{3x}{4}\right)$$

Exercise 6.3

Factorize the expressions given below.

a. $x^2 - 100$ **b.** $m^2 - 36$ **c.** $p^2 - 81$

d. $4 - b^2$ **e.** $16 - a^2$ **f.** $64 - y^2$

g. $x^2 - 4y^2$ **h.** $9a^2 - 16b^2$ **i.** $100x^2 - 1$

j. $25m^2 - n^2$ **k.** $49 - 81p^2$ **l.** $25a^2b^2 - 9c^2$

Miscellaneous Exercise

1. Factorize the following algebraic expressions by changing the order in which the terms appear as required.

i. $ax + by - ay - bx$ **ii.** $9p - 2q - 6q + 3p$

iii. $x - 12 + x^2$ **iv.** $4 - k^2 - 3k$

2. Factorize the following algebraic expressions.

i. $8x^2 - 50$ **ii.** $3x^2 - 243$

iii. $a^3 b^3 - ab$ **iv.** $3 - 12q^2$

3. Find the value.

i. $23^2 - 3^2$ **ii.** $45^2 - 5^2$ **iii.** $102^2 - 2^2$

4. Join each algebraic expression in column A with the product of its factors in column B.

A	**B**
$x^2 - x - 6$	$\left(\frac{x}{5} - 1\right)\left(\frac{x}{5} + 1\right)$
$x^2 + 5x - 3x - 15$	$2x\,(x - 2)\,(x + 2)$
$2x^3 - 8x$	$(x - 3)\,(x + 5)$
$4x^2 - 9m^2$	$(x - 3)\,(x + 2)$
$\frac{x^2}{25} - 1$	$(2x - 3m)\,(2x + 3m)$

7

Axioms

By studying this lesson, you will be able to;

- identify five fundamental axioms of mathematics,
- develop geometrical relationships and solve problems involving calculations using the five fundamental axioms.

Axioms

Statements which are considered to be self-evident and are accepted without proof are called axioms. In mathematics, axioms are used to explain facts logically, develop relationships and reach conclusions.

Euclid, who is considered to be the father of geometry lived in Greece around 300 B.C. He introduced certain axioms related to mathematics in his book "Elements". Some of them are unique to geometry. Others are common axioms which can be used in other areas including algebra.

We consider five common axioms in this lesson. They can be summarized as given below.

1. Quantities which are equal to the same quantity, are equal.
2. Quantities which are obtained by adding equal quantities to equal quantities, are equal.
3. Quantities which are obtained by subtracting equal quantities from equal quantities, are equal.
4. Products which are equal quantities multiplied by equal quantities, are equal.
5. Quotients which are equal quantities divided by nonzero equal quantities, are equal.

By "quantities" we usually mean lengths, areas, volumes, masses, speeds, magnitudes of angles, etc.

These five axioms are very important because we can derive many results related to algebra and geometry by using them. Let us study these axioms in detail.

Axiom 1

Quantities which are equal to the same quantity, are equal.

We can write this axiom briefly as given below.

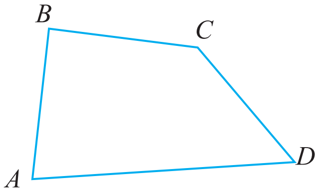
If $b = a$ and $c = a$, then $b = c$.

According to this axiom,

'If Hasith's age is the same as Kasun's and Harsha's age is also the same as Kasun's, then Hasith's age is the same as Harsha's.'

How Axiom 1 is used to obtain geometrical results is seen in the simple example given below.

In the quadrilateral $ABCD$ shown below $BC = AB$ and $CD = AB$.

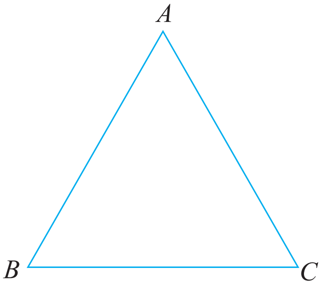


According to the above axiom,

$BC = CD$.

Example 1

In the triangle ABC, $AB = AC$ and $AB = BC$ If $AC = 5$ cm then determine the perimeter of the triangle ABC.

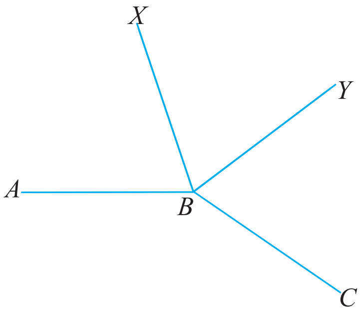


Since $AC = 5$ cm and $AC = AB$, according to Axiom 1, $AB = 5$ cm.
Since $AB = 5$ cm and $AB = BC$, according to Axiom 1, $BC = 5$ cm.
The perimeter of the triangle $ABC = AC + BC + AB$

$$= 5 \text{ cm} + 5 \text{ cm} + 5 \text{ cm}$$
$$= 15 \text{ cm}$$

Example 2

In the figure given below, $X\hat{B}Y = A\hat{B}X$ and $X\hat{B}Y = C\hat{B}Y$. Find the relationship between $A\hat{B}X$ and $C\hat{B}Y$.



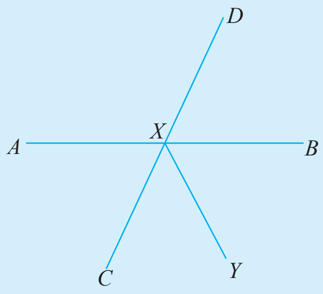
$$X\hat{B}Y = A\hat{B}X \text{ (given)}$$

$$X\hat{B}Y = C\hat{B}Y \text{ (given)}$$

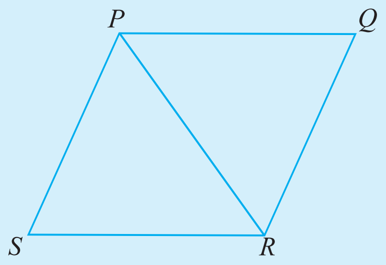
$\therefore$ According to Axiom 1, $A\hat{B}X = C\hat{B}Y$

Exercise 7.1

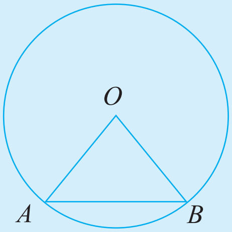
1. The straight lines AB and CD intersect at X. In the figure, $D\hat{X}B = B\hat{X}Y$. If $A\hat{X}C = 70^{\circ}$, find the magnitude of $B\hat{X}Y$.



2. In the parallelogram $PQRS$, $PQ = PR$ and $PQ = PS$. Based on its sides, mention what type of triangle PSR is.



3. The points A and B are located on the circle with centre O, such that $OA = AB$. Based on its sides, mention what type of triangle ABO is.



Axiom 2

Quantities which are obtained by adding equal quantities to equal quantities, are equal.

We can write this briefly as given below.

If $a = b$, then $a + c = b + c$.

This axiom can be written as given below too.

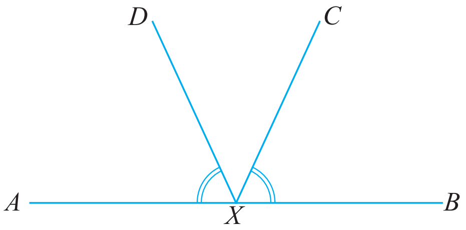
If $x = y$ and $p = q$, then $x + p = y + q$.

According to this axiom,

"If the cost incurred in purchasing vegetables is equal to the cost incurred in purchasing milk and the cost incurred in purchasing fruits is equal to the cost incurred in purchasing eggs, then the cost incurred in purchasing vegetables and fruits is equal to the cost incurred in purchasing milk and eggs."

Let us consider a simple geometrical result that can be derived using the above axiom.

In the figure given below, the point X is located on the straight line AB. Also, $A\hat{X}D = B\hat{X}C$.



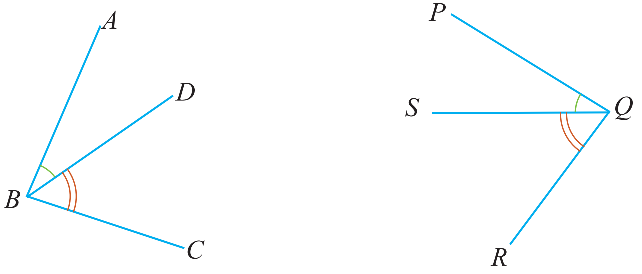
$$A\hat{X}D = B\hat{X}C \text{ (given)}$$

$\therefore$ According to Axiom 2, $\underbrace{A\hat{X}D + C\hat{X}D}_{} = \underbrace{B\hat{X}C + C\hat{X}D}_{}$

$$\therefore\ A\hat{X}C = B\hat{X}D$$

Example 1

In the figures given below, $A\hat{B}D = P\hat{Q}S$ and $C\hat{B}D = R\hat{Q}S$. Show that $A\hat{B}C = P\hat{Q}R$.



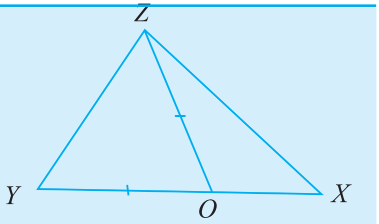
$A\hat{B}D = P\hat{Q}S$, $C\hat{B}D = R\hat{Q}S$.

$\therefore$ According to this axiom, $A\hat{B}D + C\hat{B}D = P\hat{Q}S + R\hat{Q}S$

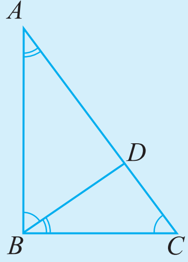
$\therefore$ $A\hat{B}C = P\hat{Q}R$

Exercise 7.2

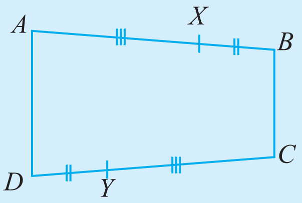
1. The point O is located on the side XY of the triangle XYZ such that $OZ = OY$. Show that $XY = OZ + OX$.



2. The point D is located on the side AC of the triangle ABC. If $A\hat{B}D = B\hat{C}D$ and $C\hat{B}D = B\hat{A}D$, show that $B\hat{A}D + B\hat{C}D = A\hat{B}C$.



3. The points X and Y are located on the sides AB and CD respectively of the quadrilateral $ABCD$, such that $AX = CY$ and $BX = DY$. Show that $AB = CD$.



Axiom 3

Quantities which are obtained by subtracting equal quantities from equal quantities, are equal.

We can write this briefly as given below.

If $a = b$, then $a - c = b - c$.

This axiom can be written as given below too.

If $a = b$ and $c = d$, then $a - c = b - d$.

A simple result in geometry that can be obtained by using the above axiom is given below.

In the figure given below, $AD = CB$.

$$AD = CB$$

According to Axiom 3, $AD - CD = CB - CD$

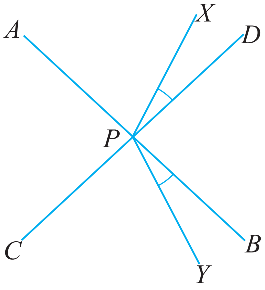
$$\therefore\ AC = DB$$

Example 1

The straight line segments AB and CD intersect at P. $X\hat{P}D = B\hat{P}Y$

(i) Show that $A\hat{P}X = C\hat{P}Y$.

(ii) If $A\hat{P}D = 95^{\circ}$ and $X\hat{P}D = 20^{\circ}$, find the magnitude of $C\hat{P}Y$.



(i) $A\hat{P}D = B\hat{P}C$ (vertically opposite angles)

$X\hat{P}D = B\hat{P}Y$ (given)

$\therefore$ According to this axiom, $\underbrace{A\hat{P}D - X\hat{P}D} = \underbrace{B\hat{P}C - B\hat{P}Y}$

$$\therefore\ A\hat{P}X = C\hat{P}Y$$

(ii) $A\hat{P}X = A\hat{P}D - X\hat{P}D$

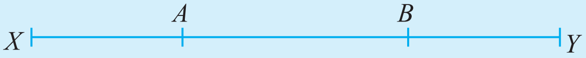
$A\hat{P}X = 95^{\circ} - 20^{\circ}$

$A\hat{P}X = 75^{\circ}$

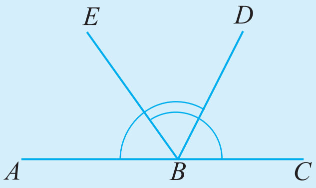
$\therefore C\hat{P}Y = 75^{\circ}$

Exercise 7.3

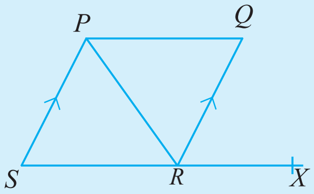
1. The points A and B are located on the line XY such that $XB = AY$. If $XY = 16$ cm and $BY = 6$ cm, find the length of AB.



2. The point B is located on the line AC. If $A\hat{B}D = C\hat{B}E$, show that $A\hat{B}E = C\hat{B}D$.



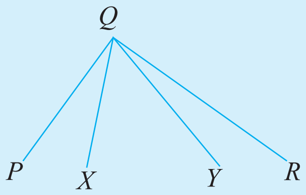
3. In the quadrilateral $PQRS$ in the figure, $S\hat{P}R = P\hat{R}Q$. If $Q\hat{P}S = P\hat{R}X$ and $S\hat{P}R = Q\hat{R}X$, show that $Q\hat{P}R = Q\hat{R}X$.



4. In the figure given here, $P\hat{Q}Y = X\hat{Q}R$. If $P\hat{Q}R = 110^{\circ}$ and $P\hat{Q}X = 35^{\circ}$,

(i) find the magnitude of $R\hat{Q}Y$.

(ii) find the magnitude of $X\hat{Q}Y$.



Axiom 4

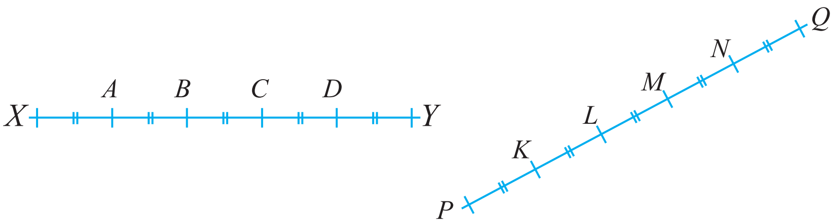
Products which are equal quantities multiplied by equals, are equal.

We can write this briefly as given below.

If $a = b$, then $ca = cb$.

Let us first consider an application of this axiom in geometry.

- As indicated in the figure given below, the points A, B, C and D are located on the straight line XY, such that $XA = AB = BC = CD = DY$. The points K, L, M and N are located on the straight line PQ, such that $PK = KL = LM = MN = NQ$. It is also given that $XA = PK$.



Let us show that $XY = PQ$.

Since $XA = AB = BC = CD = DY$, it is clear that $5\,XA = XY$.
Similarly, since $PK = KL = LM = MN = NQ$, we obtain that $5\,PK = PQ$.
Since $XA = PK$, by applying Axiom 4 we obtain $5\,XA = 5\,PK$.
$\therefore\ XY = PQ$.

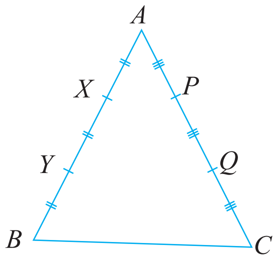
Although it is important to understand how results are derived by using axioms, in practice, when deriving geometrical results, the relevant axioms are not mentioned. This is because, as implied by the word "axiom", the validity of the derivation is obvious.

Now, let us consider how this axiom is applied in algebra.

If $x = 5$ and $y = 2x$, let us find the value of y.
Since $x = 5$, applying the above axiom and multiplying both sides by 2 we obtain $2x = 2 \times 5$.
But $2 \times 5 = 10$.
Hence by Axiom 1 we obtain
$y = 10$.

Example 1

The points X and Y are located on the side AB of the triangle ABC such that $AX = XY = YB$. The points P and Q are located on the side AC such that $AP = PQ = QC$. If $AX = AP$, find the relationship between AB and AC.



$$
\begin{aligned}
AX &= XY = YB \text{ (given)}\\
\therefore AB &= 3AX\\
AP &= PQ = QC \text{ (given)}\\
\therefore AC &= 3AP\\
AX &= AP \text{ (given)}
\end{aligned}
$$

According to Axiom 4;

$$
\begin{aligned}
3AX &= 3AP\\
\therefore AB &= AC
\end{aligned}
$$

Axiom 5

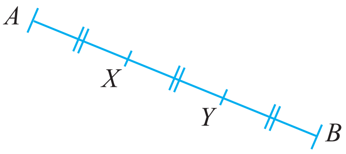
Quotients which are equal quantities divided by nonzero equals, are equal.

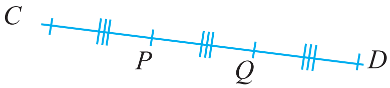
We can write this briefly as given below.

If $a = b$ then $\frac{a}{c} = \frac{b}{c}$.

Here, c is a nonzero number since it is meaningless to divide by zero.

- The line segments AB and CD in the figure are equal in length (that is, $AB = CD$). The points X and Y are located on AB such that $AX = XY = YB$. The points P and Q are located on CD such that $CP = PQ = QD$.





Let us show that $AX = CP$.

Since $AX = XY = YB$, we obtain that $\frac{AB}{3} = AX$.

Since $CP = PQ = QD$, we obtain that $\frac{CD}{3} = CP$.

Since $AB = CD$, according to Axiom 5,

$$\frac{AB}{3} = \frac{CD}{3}$$

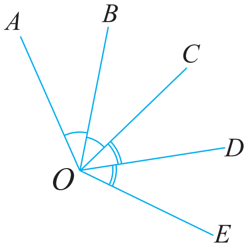
$\therefore AX = CP.$

Example 1

In the figure given below, $A\hat{O}B = B\hat{O}C$ and $C\hat{O}D = D\hat{O}E$. If $A\hat{O}C = C\hat{O}E$,

(i) find the relationship between $A\hat{O}B$ and $D\hat{O}E$.

(ii) If $B\hat{O}C = 35^{o}$, find the magnitude of $D\hat{O}E$.



(i) $A\hat{O}B = B\hat{O}C$ (given)

$\therefore A\hat{O}B = \dfrac{A\hat{O}C}{2}$

$C\hat{O}D = D\hat{O}E$ (given)

$\therefore D\hat{O}E = \dfrac{C\hat{O}E}{2}$

$A\hat{O}C = C\hat{O}E$ (given)

$\therefore$ According to Axiom 5, $\dfrac{A\hat{O}C}{2} = \dfrac{C\hat{O}E}{2}$

$\therefore \quad A\hat{O}B = D\hat{O}E$

(ii) $A\hat{O}B = B\hat{O}C$ (given) $\qquad B\hat{O}C = 35^{o}$ (given)

$\therefore A\hat{O}B = 35^{o}$ (by axiom 1)

$A\hat{O}B = D\hat{O}E$ (Proved above)

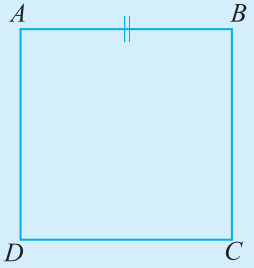
$\therefore D\hat{O}E = 35^{o}$ (by axiom 1)

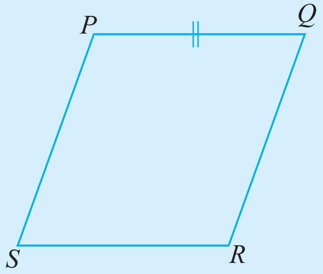
Exercise 7.4

1. The square $ABCD$ and the rhombus $PQRS$ are such that $AB = PQ$.

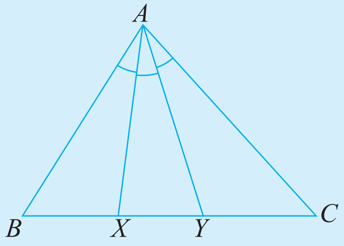
(i) Using Axiom 4, show that the perimeter of the square $ABCD$ is equal to the perimeter of the rhombus $PQRS$.

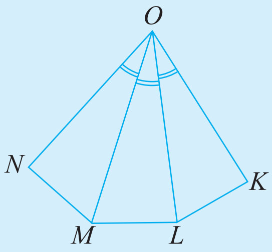
(ii) If $AB = 7$cm, find the perimeter of the rhombus $PQRS$.





2.





In the triangle ABC, $B\hat{A}X = X\hat{A}Y = C\hat{A}Y$. In the pentagon $KLMNO$ $M\hat{O}N = L\hat{O}M = K\hat{O}L$. If $B\hat{A}C = K\hat{O}N$,

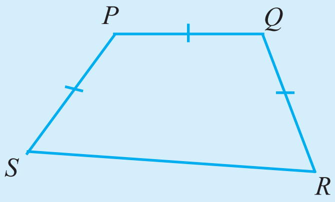
(i) show that $X\hat{A}Y = M\hat{O}L$.

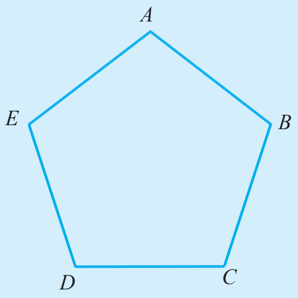
(ii) If $X\hat{A}Y = 30^{o}$, determine the magnitude of $K\hat{O}N$.

3. In the quadrilateral $PQRS$, $PQ = QR = SP$ and $2PQ = RS$. The perimeter of the regular pentagon $ABCDE$ is equal to that of the quadrilateral $PQRS$.

(i) Find the relationship between PQ and AB.

(ii) If $AB = 8$ cm, find the perimeter of the quadrilateral $PQRS$.





Further applications of the axioms

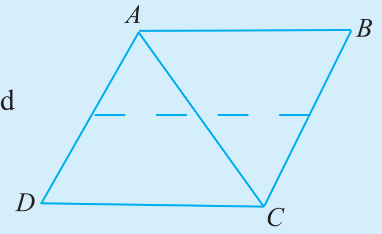
Example 1

Solve the equation given below using the axioms.
$2x + 5 = 13$

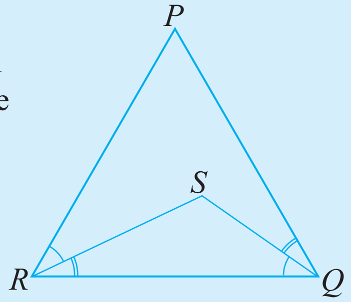
Here, solving the equation means finding the value of x. Since the quantity $2x + 5$ is equal to the quantity 13, according to Axiom 3, the quantities obtained by subtracting 5 from these two quantities are also equal.
$\therefore\ 2x + 5 - 5 = 13 - 5.$
Simplifying this we obtain,
$2x = 8.$
Since the quantity $2x$ is equal to the quantity 8, the quantities that are obtained by dividing these two quantities by 2 are also equal. Therefore, $\frac{2x}{2} = \frac{8}{2}$.

When we simplify this we obtain $x = 4$. That is, the solution of the equation is 4.

Miscellaneous Exercise



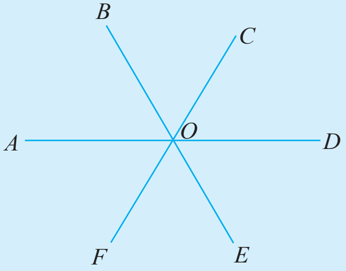
1. In the quadrilateral $ABCD$, $AD = AC$, $BC = AC$, $AB = BC$ and $AD = CD$. Show that $ABCD$ is a rhombus.



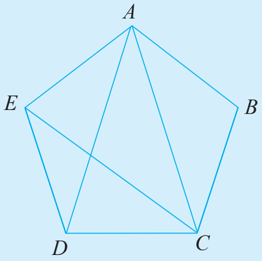
2. As indicated in the figure, the point S is located such that $P\hat{R}S = S\hat{Q}R$ and $Q\hat{R}S = P\hat{Q}S$. By applying the axioms,

(i) show that $P\hat{R}Q = P\hat{Q}R$,
(ii) show that if $R\hat{P}Q = P\hat{R}Q$, then all the angles of the triangle PQR are equal in magnitude.

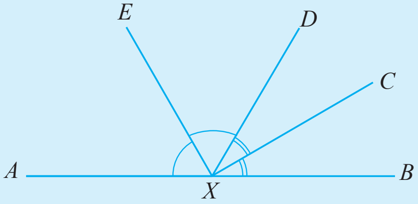
3. As indicated in the figure, the straight lines AD, BE and CF intersect at the point O. If $D\hat{O}E = A\hat{O}F$, show that $B\hat{O}D = D\hat{O}F$.



4. In the regular pentagon $ABCDE$,
$E\hat{A}D = D\hat{A}C = B\hat{A}C$ and $B\hat{C}A = A\hat{C}E = D\hat{C}E$.
(i) Show that $B\hat{C}A = B\hat{A}C$.
(ii) If $B\hat{A}C = 36^{\circ}$ find the magnitude of $C\hat{D}E$.



5. The point X lies on the straight line AB. Also, $A\hat{X}E = E\hat{X}D$ and $B\hat{X}C = C\hat{X}D$. Determine the magnitude of $C\hat{X}E$.



Summary

Summary

- Quantities which are equal to the same quantity, are equal.
- Quantities which are obtained by adding equal quantities to equal quantities, are equal.
- Quantities which are obtained by subtracting equal quantities from equal quantities, are equal.
- Products which are equal quantities multiplied by equal quantities, are equal.
- Quotients which are equal quantities divided by nonzero equal quantities, are equal.

8

Angles related to straight lines and parallel lines

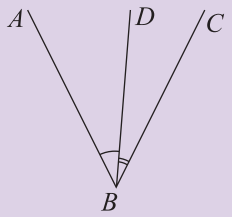
By studying this lesson, you will be able to;

- identify and verify the theorems related to the adjacent angles/vertically opposite angles formed by one straight line meeting or intersecting another straight line, and use them to solve problems,
- identify the angles formed when a transversal intersects two straight lines,
- identify and verify the theorems related to the angles formed when a transversal intersects two straight lines, and use them to solve problems.

Introduction

Let us first recall the basic geometrical facts we learnt in previous grades.

Adjacent angles



The angles $A\hat{B}D$ and $D\hat{B}C$ in the above figure have a common vertex. This common vertex is B. They also have a common arm BD. The pair of angles $A\hat{B}D$ and $D\hat{B}C$ lie on opposite sides of the common arm BD. Such a pair of angles is known as a pair of adjacent angles.

$A\hat{B}D$ and $D\hat{B}C$ are a pair of adjacent angles.

However, $A\hat{B}D$ and $A\hat{B}C$ are not a pair of adjacent angles. This is because, these two angles are not on opposite sides of the common arm AB.

Complementary angles

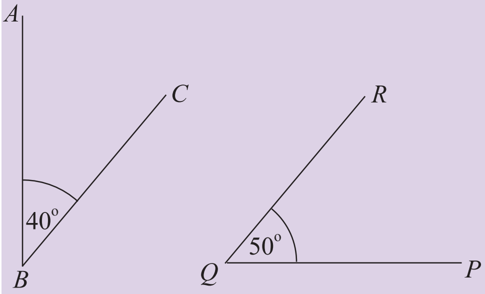


Figure I

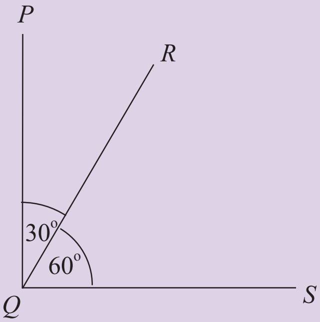


Figure II

In figure I, since $A\hat{B}C + P\hat{Q}R = 40^\circ + 50^\circ = 90^\circ$, $A\hat{B}C$ and $P\hat{Q}R$ are a pair of complementary angles.

In figure II, $P\hat{Q}R$ and $R\hat{Q}S$ are a pair of adjacent angles. Furthermore, since $P\hat{Q}R + R\hat{Q}S = 90^\circ$, they are a pair of complementary angles too. Therefore, $P\hat{Q}R$ and $R\hat{Q}S$ are a pair of complementary adjacent angles.

Supplementary angles

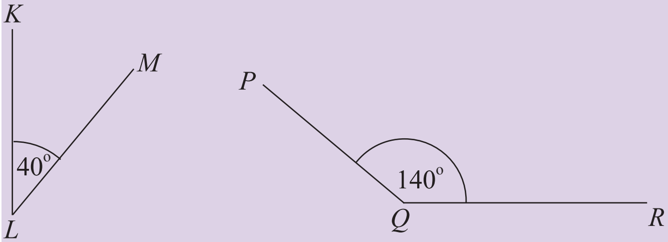


Figure I

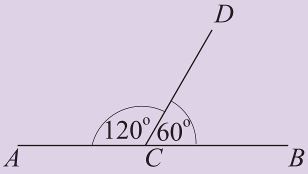
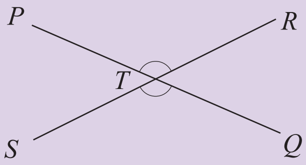


Figure II

In figure I, since $K\hat{L}M + P\hat{Q}R = 180^\circ$, $K\hat{L}M$ and $P\hat{Q}R$ are a pair of supplementary angles. In figure II, $A\hat{C}D$ and $B\hat{C}D$ are a pair of adjacent angles. Furthermore, since $A\hat{C}D + B\hat{C}D = 180^\circ$, they are a pair of supplementary angles too. Therefore, $A\hat{C}D$ and $B\hat{C}D$ are a pair of supplementary adjacent angles.

Vertically opposite angles

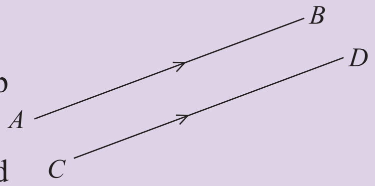


The pair of angles $P\hat{T}R$ and $S\hat{T}Q$, formed by the intersection of the straight lines PQ and RS at the point T, are vertically opposite angles.

Similarly, $P\hat{T}S$ and $R\hat{T}Q$ are another pair of vertically opposite angles.
Vertically opposite angles are equal in magnitude.

$$\text{Therefore, } P\hat{T}R = S\hat{T}Q \text{ and } P\hat{T}S = R\hat{T}Q.$$

Parallel lines



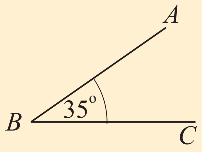
Two straight lines in a plane which do not intersect each other are called parallel straight lines. The gap between two parallel straight lines is a constant.

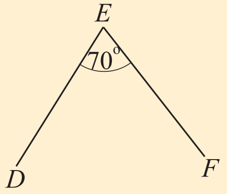
As shown in the figure, parallel lines are indicated using arrow. We use the notation $AB//CD$ to indicate that AB and CD are Parallel.

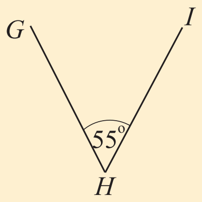
Do the following exercise, to strengthen your understanding of the above facts.

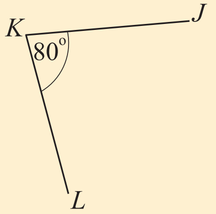
Review Exercise

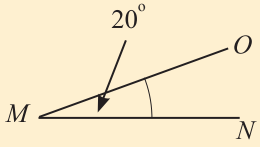
1. From the angles given below, select and write the pairs which are complementary.

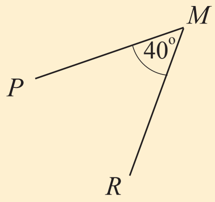


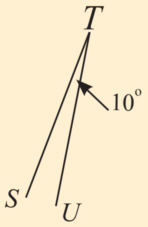






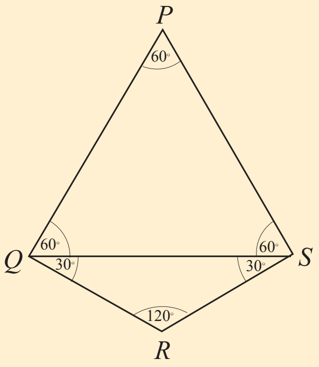






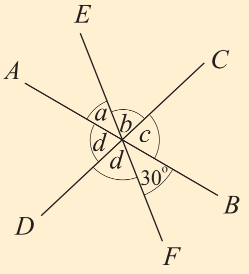
2. Based on the magnitudes of the angles shown in the figure, write

i. four pairs of complementary angles,
ii. two pairs of complementary adjacent angles,
iii. two pairs of supplementary angles.

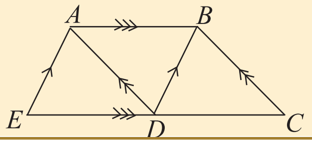


3. The straight line segments *AB, CD* and *EF* shown in the figure intersect at a point. According to the information given in the figure,

i. find the value denoted by *a*.
ii. give reasons why $b = d$.
iii. find the value denoted by *d*.
iv. find the values denoted by *b* and *c*.

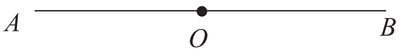


4. Name three pairs of parallel straight lines.

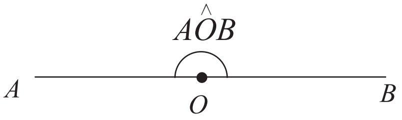


8.1 Angles related to straight lines

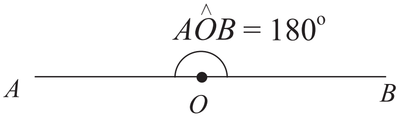
Let us assume that the point *O* is located on the straight line *AB*.



Then $A\hat{O}B$ can be considered as an angle between the arms *AO* and *OB*. Such an angle is known as a straight angle.

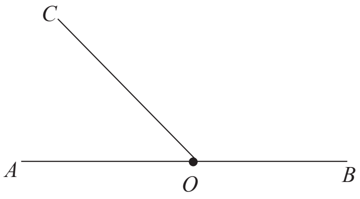


The unit "degree" which is used to measure angles has been selected such that a straight angle is equal in magnitude to 180°. Therefore, we can write $A\hat{O}B = 180^\circ$.



Accordingly, the magnitude of a straight angle is180°.

In the following figure, two angles have been drawn at the point *O* which is located on the straight line *AB*.



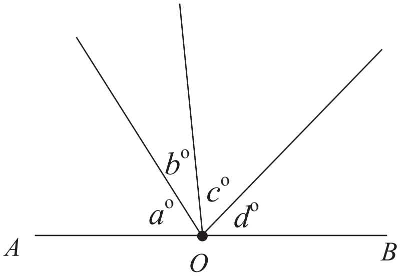
Here, $A\hat{O}C$ and $B\hat{O}C$ are a pair of adjacent angles. In a situation such as this, we say that the adjacent angles $A\hat{O}C$ and $B\hat{O}C$ are on the straight line *AB*. Furthermore, since $A\hat{O}B = 180^\circ$, it is clear that,

$$A\hat{O}C + B\hat{O}C = 180^\circ.$$

Hence, the two angles $A\hat{O}C$ and $B\hat{O}C$ are a pair of supplementary adjacent angles. The facts we have discussed can be stated as a theorem as follows.

Theorem:
The sum of the adjacent angles formed by a straight line meeting another straight line is two right angles.

The facts discussed above can be generalized further. As an example, in the figure given below, four angles have been drawn at the point *O* on the straight line *AB*.



The values of these angles in degrees have been denoted by *a, b, c* and *d*. In a situation such as this too, we say that the angles are on the straight line *AB*. Furthermore, since $A\hat{O}B = 180^o$, it is clear that,

$$a + b + c + d = 180.$$

It is also clear that this relationship holds true for any number of angles on a straight line. Therefore,

"The sum of the magnitudes of the angles on a straight line is 180°."

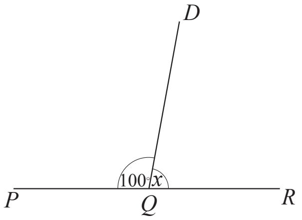
Now, by considering some examples, let us learn how to solve problems using these theorems.

Example 1

In each figure given below, if *PQR* is a straight line, then find the value denoted by *x*.

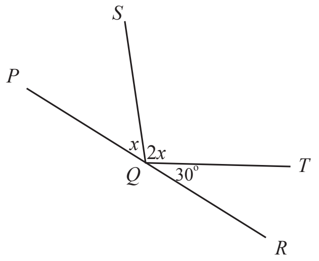
$P\hat{Q}D + D\hat{Q}R = 180^o$ (Angles on the straight line *PQR*)

$$100^o + x = 180^o$$
$$x = 180^o - 100^o$$
$$= \underline{\underline{80^o}}$$



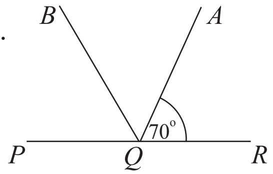
$P\hat{Q}S + S\hat{Q}T + T\hat{Q}R = 180^o$ (Angles on the straight line *PQR*)

$$x + 2x + 30^o = 180^o$$
$$3x + 30^o = 180^o$$
$$3x = 180^o - 30^o$$
$$3x = 150^o$$
$$x = \underline{\underline{50^o}}$$



Example 2

In the given figure, $A\hat{Q}R = 70^\circ$ and the bisector of $P\hat{Q}A$ is QB. If PQR is a straight line, then find the magnitude of $A\hat{Q}B$.



Since PQR is a straight line,

$$\begin{aligned} P\hat{Q}A + A\hat{Q}R &= 180^\circ \quad \text{(angles on the straight line } PQR\text{)} \\ P\hat{Q}A + 70^\circ &= 180^\circ \\ \therefore \quad P\hat{Q}A &= 180^\circ - 70^\circ \\ &= 110^\circ \end{aligned}$$

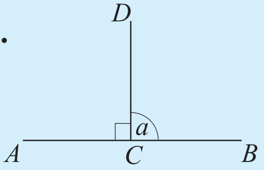
Since BQ is the bisector of $P\hat{Q}A$,

$$\begin{aligned} P\hat{Q}B = A\hat{Q}B &= \frac{1}{2} P\hat{Q}A \\ \therefore \quad A\hat{Q}B &= \frac{110^\circ}{2} \\ &= \underline{\underline{55^\circ}} \end{aligned}$$

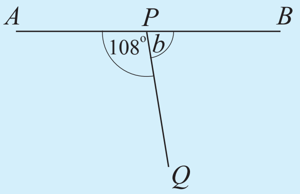
Exercise 8.1

1. In each of the following figures, AB is a straight line. Based on the information in each figure, find the value of the angle denoted by the lower case letter.

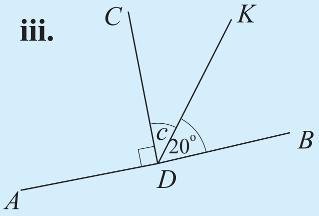
i.



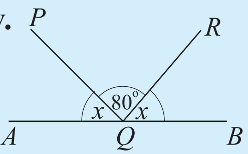
ii.



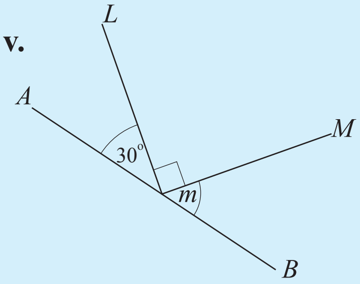
iii.



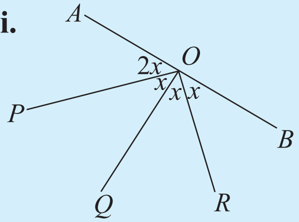
iv.



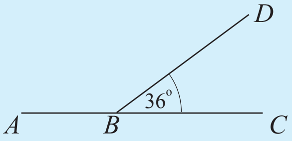
v.



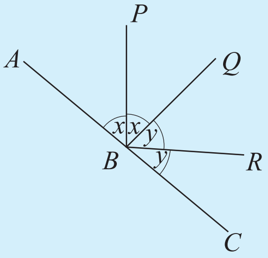
vi.



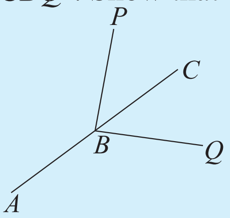
2. In the figure, ABC is a straight line. If $D\hat{B}C = 36^{\circ}$, show that the magnitude of $A\hat{B}D$ is four times the magnitude of $D\hat{B}C$.



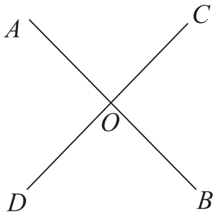
3. In the figure, ABC is a straight line. Based on the information given in the figure, show that $P\hat{B}R$ is a right angle.



4. In the figure, ABC is a straight line. Moreover, $P\hat{B}C = C\hat{B}Q$. Show that $A\hat{B}P = A\hat{B}Q$.



8.2 Vertically opposite angles



In the figure, the straight lines AB and CD intersect each other at O.

The vertex O is common to both the angles $A\hat{O}C$ and $D\hat{O}B$. Furthermore, they are on opposite sides of O.

The pair of angles $A\hat{O}C$ and $D\hat{O}B$ are known as a pair of vertically opposite angles.

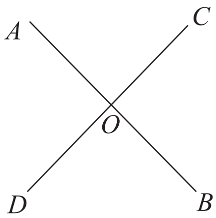
Similarly, $A\hat{O}D$ and $B\hat{O}C$ lie on opposite sides of O, which is the common vertex of these two angles.

Therefore $A\hat{O}D$ and $B\hat{O}C$ are also a pair of vertically opposite angles.

Accordingly, it is clear that two pairs of vertically opposite angles are formed by the intersection of two straight lines.
Let us now consider a theorem related to vertically opposite angles.

Theorem:
The vertically opposite angles formed by the intersection of two straight lines are equal.

By considering the figure, it can clearly be seen that ‘vertically opposite angles are equal’. However, by using the fact we learnt earlier, that ‘the sum of the angles on a straight line is 180’, and also the axioms learnt in the previous lesson, this theorem can be proved as follows.



Data: The straight lines *AB* and *CD* intersect each other at *O*.

To Prove: $A\hat{O}C = B\hat{O}D$ and

$$A\hat{O}D = B\hat{O}C$$

Proof:

Since *AB* is a straight line,

$$A\hat{O}C + B\hat{O}C = 180^{\circ} \text{ (angles on the straight line } AB\text{)}$$

Similarly, since *CD* is a straight line,

$$B\hat{O}C + B\hat{O}D = 180^{\circ} \text{ (angles on the straight line } CD\text{)}$$

$$\therefore \; A\hat{O}C + B\hat{O}C = B\hat{O}C + B\hat{O}D \text{ (axiom)}$$

Subtracting $B\hat{O}C$ from both sides of the equation,

$$A\hat{O}C + \cancel{B\hat{O}C} - \cancel{B\hat{O}C} = \cancel{B\hat{O}C} - \cancel{B\hat{O}C} + B\hat{O}D$$

$$\therefore \; A\hat{O}C = B\hat{O}D$$

Similarly, $A\hat{O}D + A\hat{O}C = 180^{\circ}$ (angles on the straight line *CD*)

$\hat{AOC} + \hat{BOC} = 180^\circ$ (since AB is a straight line)

$\therefore \quad \hat{AOD} + \hat{AOC} = \hat{AOC} + \hat{BOC}$ (axiom)

Subtracting $\hat{AOC}$ from both sides of the equation we obtain,

$\hat{AOD} = \hat{BOC}$.

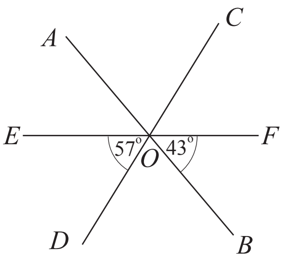
Let us consider the following examples to learn how this theorem is used to solve problems.

Example 1

Based on the information given in the figure, giving reasons, determine the following.

(i) The magnitude of $\hat{DOB}$

(ii) The magnitude of $\hat{AOC}$.



(i) Since EOF is a straight line,

$\hat{EOD} + \hat{DOB} + \hat{BOF} = 180^\circ$ (sum of the magnitudes of the angles on a straight line)

$57^\circ + \hat{DOB} + 43^\circ = 180^\circ$

$\hat{DOB} = 180^\circ - (57^\circ + 43^\circ)$

$\therefore \quad \hat{DOB} = 80^\circ$

(ii) $\hat{AOC} = \hat{DOB}$ (vertically opposite angles)

$\hat{DOB} = 80^\circ$ (proved above)

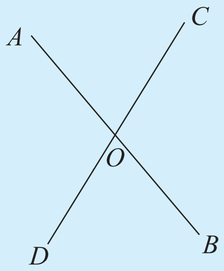
$\therefore \quad \hat{AOC} = \underline{\underline{80^\circ}}$

Exercise 8.2

1. In the figure, the straight lines AB and CD intersect each other at O.

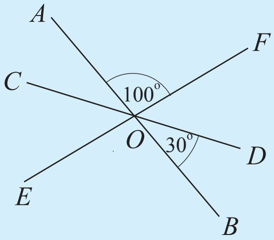
i. If $\hat{AOC} = 80^\circ$, find the magnitude of $\hat{BOD}$.

ii. Name an angle which is equal in magnitude to $\hat{AOD}$.



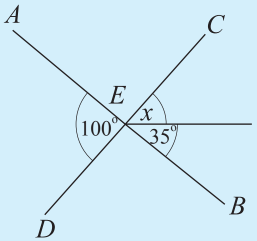
2. In the figure, the straight lines AB, CD and EF intersect at O. Based on the information provided in the figure, find the magnitude of each of the following angles.

i. $A\hat{O}C$
ii. $B\hat{O}E$
iii. $C\hat{O}E$



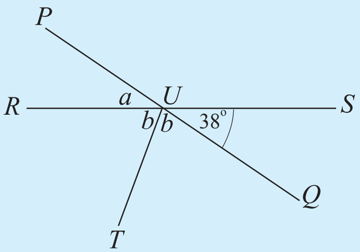
3. Based on the information given in each of the figures shown below, find the value of each English letter representing an angle.

i.



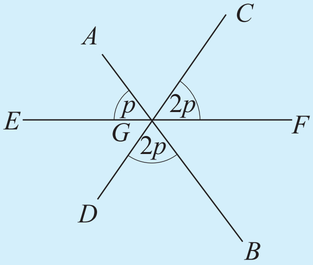
In the figure AB and CD are straight lines.

ii.



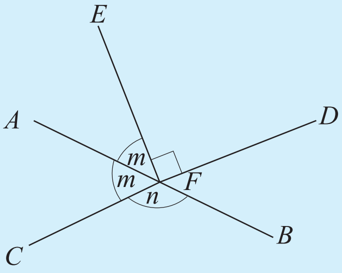
In the figure, RS and PQ are straight lines.

iii.



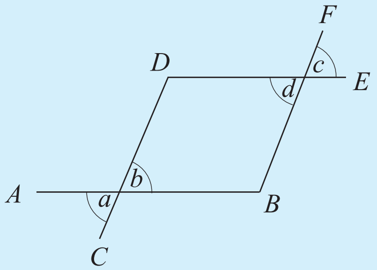
In the figure, the straight lines AB, CD and EF intersect at G.

iv.



In the figure, AB and CD are straight lines.

4. In the figure, *AB, CD, DE* and *BF* are straight lines. Moreover, $a = d$. Fill in the blanks given below to prove that $b = c$.



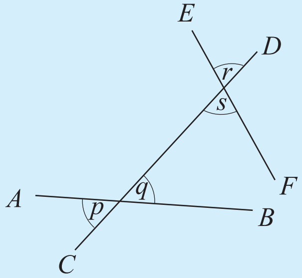
$a = b$ (................)

$d =$ ..... (................)

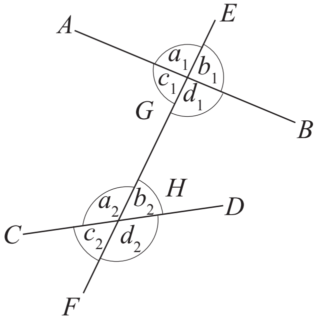
But, ... = ... (data)

$\therefore$ $b = c$

5. In the figure, *AB, CD* and *EF* are straight lines. Moreover $p = r$. Prove that $s = q$.



8.3 Corresponding angles, alternate angles and allied angles



In the above figure, the two straight line *AB* and *CD* are intersected by the straight line *EF* at the points *G* and *H* respectively. The line *EF* is known as a transversal.

A line intersecting two or more straight lines is known as a transversal.

In the above figure, there are four angles around the point *G* and four angles around the point *H*. According to where these angles are located, they are given special names in pairs.

Corresponding angles

Consider the four pairs of angles given below.

(i) a_1 and a_2 (ii) b_1 and b_2 (iii) c_1 and c_2 (iv) d_1 and d_2

Each of these pairs of angles is a pair of corresponding angles.

To be a pair of corresponding angles, the following characteristics should be there in the two angles.

1. Both angles should lie on the same side of the transversal.

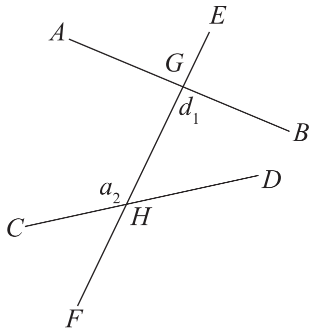
According to the above figure, both the angles a_1 and a_2 are on the left side of the transversal. Similarly, both the angles b_1 and b_2 are on the right side of the transversal. Also, the two angles c_1 and c_2 are on the left side of the transversal and the two angles d_1 and d_2 are on the right side of the transversal.

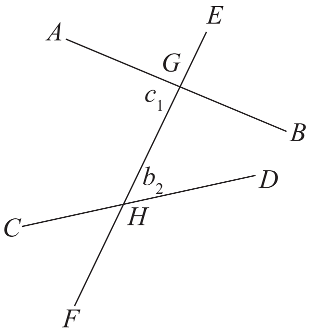
2. Both angles should be in the same direction with respect to the two straight lines.

According to the given figure, the angles a_1 and a_2 lie above the lines AB and CD respectively. The angles b_1 and b_2 also lie above the lines AB and CD respectively. Similarly, the angles c_1 and c_2 lie below the lines AB and CD respectively, and the angles d_1 and d_2 also lie below the lines AB and CD respectively.

In the given figure, the pairs of angles $A\hat{G}E$ and $C\hat{H}G$, $B\hat{G}E$ and $D\hat{H}G$, $A\hat{G}H$ and $C\hat{H}F$, $B\hat{G}H$ and $D\hat{H}F$ are pairs of corresponding angles.

Alternate angles





In the figures, the two pairs of angles given below are pairs of alternate angles.

(i) a_2 and d_1

(ii) c_1 and b_2

The characteristics common to these pairs of angles that can be used to identify pairs of alternate angles are the following.

1. The two angles should be on opposite sides of the transversal.

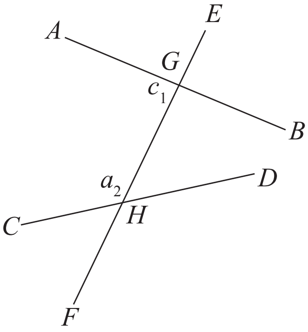
According to the above figure, the angles a_2 and d_1 lie on opposite sides of the transversal. Similarly, c_1 and b_2 also lie on opposite sides of the transversal.

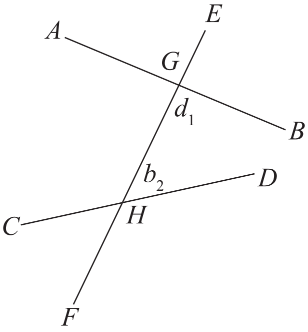
2. The line segment of the transversal, which lies between the two straight lines, should be a common arm of the two angles.

According to the given figure, the line segment GH is a common arm of the angles a_2 and d_1 and also of the angles c_1 and b_2.

In the given figure, the pair of angles $B\hat{G}H$ and $G\hat{H}C$ and the pair of angles $A\hat{G}H$ and $G\hat{H}D$ are pairs of alternate angles.

Allied Angles





In the figures, the two pairs of angles given below are pairs of allied angles.

(i) a_2 and c_1
(ii) d_1 and b_2

In the figure, the two straight lines are intersected by a transversal. The pairs of angles on the same side of the transversal segment GH, between the straight lines AB and CD are

(i) the pair $A\hat{G}H$ and $C\hat{H}G$
(ii) the pair $B\hat{G}H$ and $D\hat{H}G$

For all four of these angles, GH is a common arm. A pair of angles on the same side of the common arm GH and between the straight lines AB and CD is called a pair of allied angles, Accordingly,

> while the pair of angles $A\hat{G}H$ and $C\hat{H}G$ is a pair of allied angles, the pair of angles $B\hat{G}H$ and $D\hat{H}G$ is also a pair of allied angles.

Exercise 8.3

1. Consider the figures given below of two straight lines intersected by a transversal.

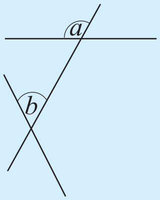


Figure 1

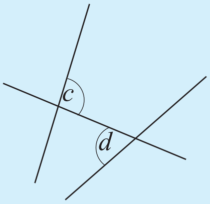


Figure 2

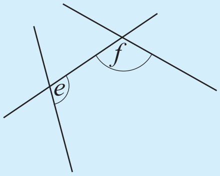
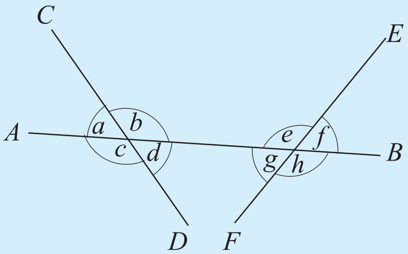


Figure 3

By considering the angles represented by the lower case English letters in the given figures, fill in the blanks.

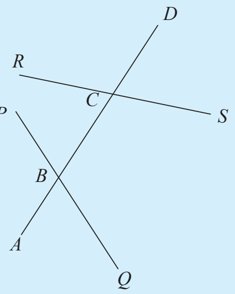
(i) In figure 1, a and b form a pair of .... angles.
(ii) In figure 2, c and d form a pair of .... angles.
(iii) In figure 3, e and f form a pair of .... angles.

2. Consider the figure given below of two straight lines intersected by a transversal. Its angles are indicated by lowercase English letters.



i. Name the line which can be considered as the transversal.
ii. Name the two straight lines which are intersected by the transversal.
iii. One pair of corresponding angles is the pair of angles a and e. Write the other three pairs of corresponding angles in a similar manner.
iv. Write the two pairs of allied angles in terms of the lower case English letters.
v. Write the two pairs of alternate angles in terms of the lower case English letters.

3. Answer the questions given below in relation to the given figure of two straight lines intersected by a transversal.



(i) Name the angle which together with $A\hat{B}P$ forms a pair of corresponding angles.

(ii) Name the angle which together with $B\hat{C}S$ forms the following.

a) Pair of allied angles

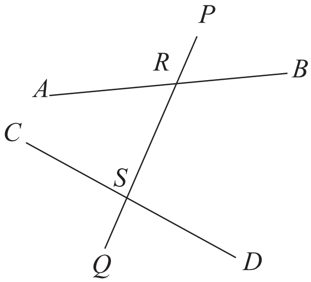
b) Pair of alternate angles

c) Pair of corresponding angles.

(iii) What type of angles is the pair $R\hat{C}D$ and $P\hat{B}C$?

(iv) What type of angles is the pair $P\hat{B}C$ and $B\hat{C}R$?

8.4 Angles related to parallel lines



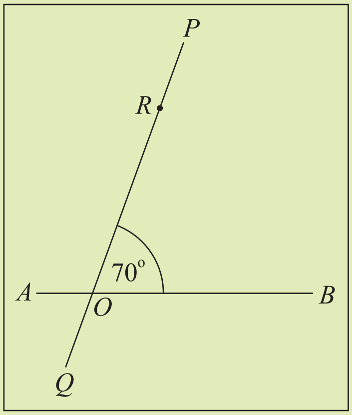
As indicated in the figure, the transversal PQ intersects the two straight lines AB and CD at R and S respectively. Now let us consider how the two lines AB and CD are positioned in each of the following cases.

- ★ When a pair of corresponding angles are equal
- ★ When a pair of alternate angles are equal
- ★ When the sum of a pair of allied angles is 180°

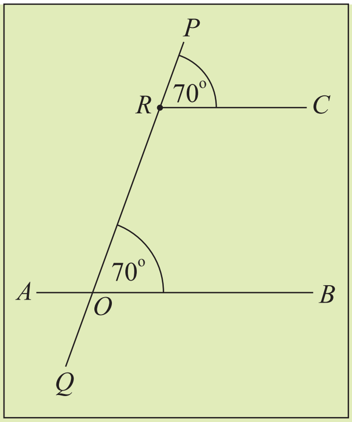
Do the following activity to identify this.

Activity 1

Step 1: On a sheet of A4 paper, draw two straight lines AB and PQ such that they intersect at O and such that $P\hat{O}B = 70^{\circ}$, as shown in the figure. Mark a point R on OP.



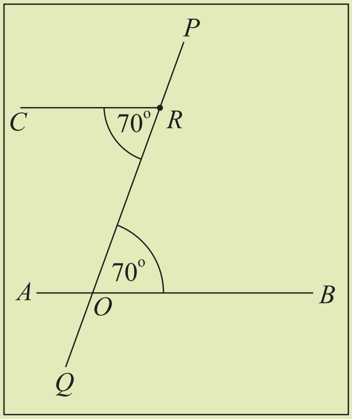
Step 2: Using the protractor, as shown in the figure, draw $P\hat{R}C$ at R such that its magnitude is 70°. Observe that $P\hat{O}B$ and $P\hat{R}C$ are a pair of corresponding angles (considering PQ as a transversal which intersects the straight lines RC and AB)



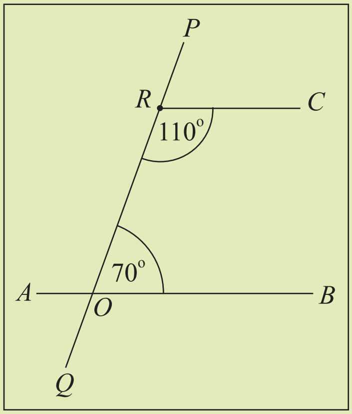
Step 3: Using a set square and a straight edge, examine whether the lines AB and RC are parallel.

Step 4: Selecting different values for $P\hat{O}B$, repeat the above three steps and in each case examine whether the lines AB and RC are parallel.

Step 5: Carry out steps 1 to 3 which were performed for corresponding angles, for alternate angles too. When completing these steps you will obtain a figure similar to the one shown here.



Step 6: Carry out the steps which were performed for corresponding angles, for allied angles too. In this case, the line drawn in step 2 above should be drawn as shown in the figure, such that $C\hat{R}O = 180^{\circ} - 70^{\circ} = 110^{\circ}$.



In doing the above activity you would have observed that, when

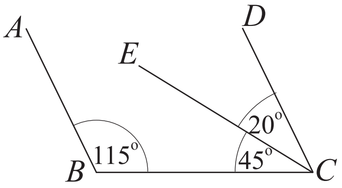
(i) a pair of corresponding angles are equal or
(ii) a pair of alternate angles are equal or
(iii) the sum of a pair of allied angles is equal to 180°,

then the straight lines *AB* and *RC* are parallel.

This result which is true in general, can be expressed as a theorem as follows.

Theorem : When two straight lines are intersected by a transversal, if
(i) a pair of corresponding angles are equal or
(ii) a pair of alternate angles are equal or
(iii) the sum of a pair of allied angles is 180°, then the two straight lines are parallel to each other.

Example 1



Based on the information given in the figure, show that *AB* and *DC* are parallel. The two angles $A\hat{B}C$ and $B\hat{C}D$ formed by the transversal *BC* meeting the two straight lines *AB* and *DC,* are a pair of allied angles.

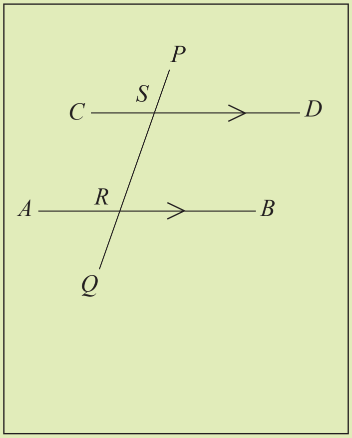
$$A\hat{B}C = 115^\circ$$
$$B\hat{C}D = B\hat{C}E + E\hat{C}D = 45^\circ + 20^\circ = 65^\circ$$
$$\therefore \quad A\hat{B}C + B\hat{C}D = 115^\circ + 65^\circ = 180^\circ$$

Since the sum of the pair of allied angles $A\hat{B}C$ and $B\hat{C}D$ is 180° , *AB* and *DC* are parallel.

Now let us consider another theorem which is related to parallel lines.

Activity 2

Step 1: On a sheet of A4 paper, draw two straight lines AB and CD parallel to each other (the parallel lines can be drawn using a set square and a straight edge), and a transversal PQ such that it intersects the lines AB and CD at R and S respectively, as shown in the figure.



Step 2: Use a protractor to measure the magnitudes of the below given angles.

(i) Measure the magnitudes of the pair of corresponding angles $S\hat{R}B$ and $P\hat{S}D$ and check whether they are equal. Similarly, measure the magnitudes of the other pairs of corresponding angles and check whether they too are equal.

(ii) Measure the magnitudes of the pair of alternate angles $C\hat{S}R$ and $S\hat{R}B$ and check whether they are equal. Similarly, measure the magnitudes of the other pair of alternate angles and check whether they too are equal.

(iii) Measure the magnitudes of the pair of allied angles $D\hat{S}R$ and $S\hat{R}B$ and check whether they are supplementary. Similarly, measure the magnitudes of the other pair of allied angles and check whether they too are supplementary.

Step 3: Change the inclination of the transversal PQ and repeat the above two steps.

You would have observed in the above activity that when two parallel lines are intersected by a transversal,

(i) each pair of corresponding angles is equal,
(ii) each pair of alternate angles is equal,
(iii) each pair of allied angles is supplementary.

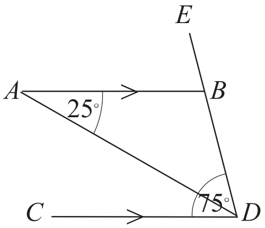
This result is true in general and can be expressed as a theorem as follows.

Theorem: When a transversal intersects a pair of parallel lines,

- **i.** the corresponding angles formed are equal,
- **ii.** the alternate angles formed are equal,
- **iii.** the sum of each pair of allied angles formed equals two right angles.

Observe that the above theorem is the converse of the theorem learnt earlier.

Example 2



In the above figure, the straight lines AB and CD are parallel (this is denoted by $AB//CD$). Moreover, $B\hat{D}C = 75^\circ$ and $B\hat{A}D = 25^\circ$.

(i) Giving reasons, determine the magnitude of $A\hat{B}E$.

(ii) Giving reasons, determine the magnitude of $A\hat{D}B$.

(i) $B\hat{D}C = 75^\circ$ (data)

$B\hat{D}C = A\hat{B}E$ (corresponding angles, $AB//CD$)

$\therefore A\hat{B}E = \underline{\underline{75^\circ}}$

(ii) $B\hat{A}D = 25^\circ$ (data)

$B\hat{A}D = A\hat{D}C$ (alternate angles, $AB//CD$)

$\therefore A\hat{D}C = 25^\circ$

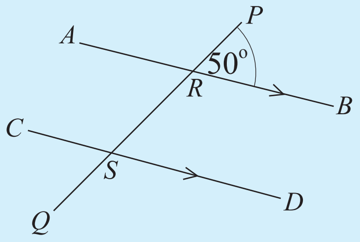
But $A\hat{D}B = B\hat{D}C - A\hat{D}C$

$= 75^\circ - 25^\circ$

$= \underline{\underline{50^\circ}}$

Exercise 8.4

1.

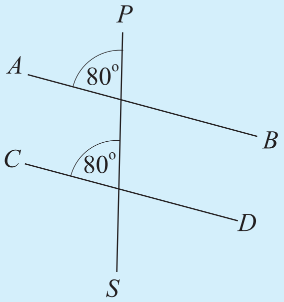


In the figure, $AB//CD$. If $P\hat{R}B = 50^\circ$, find the magnitude of each of the following angles.

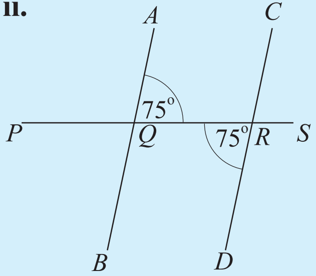
i. $R\hat{S}D$ (ii) $A\hat{R}S$ (iii) $C\hat{S}Q$ (iv) $Q\hat{S}D$

2. Based on the information in each of the following figures, giving reasons state whether AB and CD are parallel.

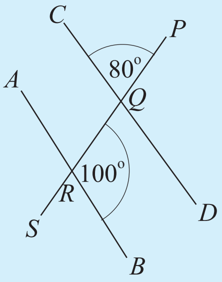
i.



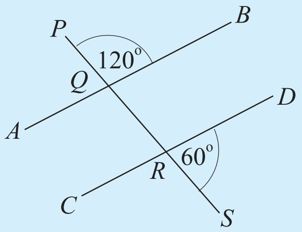
ii.



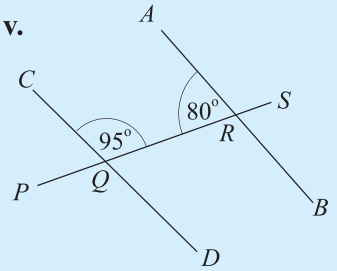
iii.



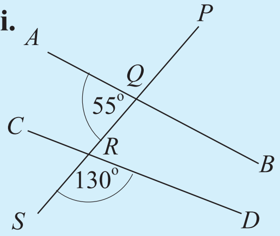
iv.



v.

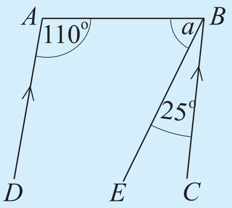


vi.

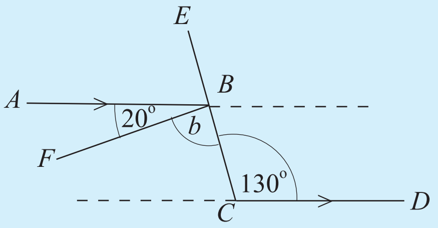


3. Find the value of each angle denoted by a lowercase English letter in the figures given below.

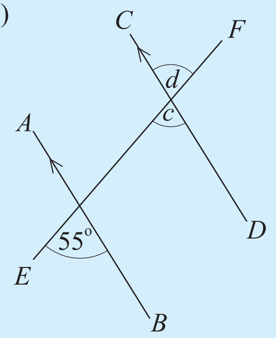
i.



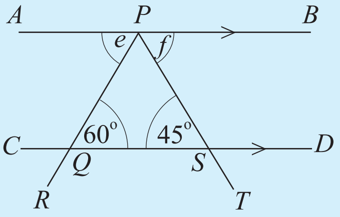
ii.



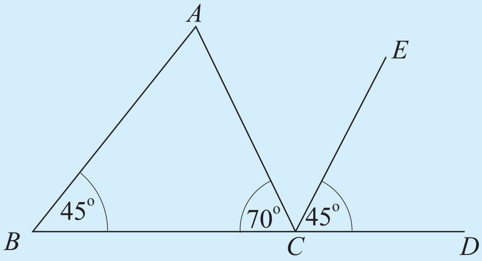
(iii)



(iv)



4.

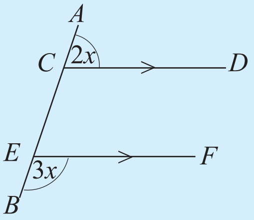


Based on the information given in the figure, show that *AB*//*EC*.

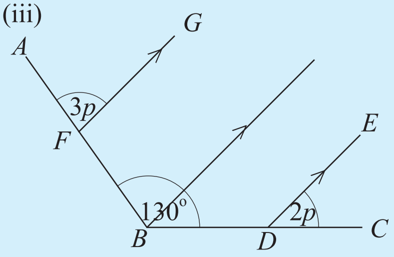
Miscellaneous Exercise

1. Find the magnitude of each of the angles denoted using lowercase English letters in the following figures.

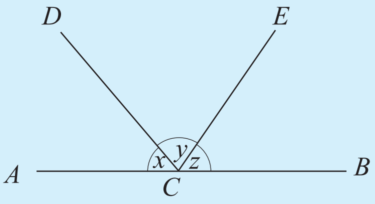
(i)



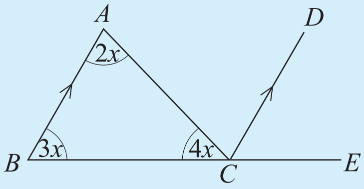
(iii)



2.



In the figure, x, y and z denote the magnitude of the relevant angle. If $x + z = y$, find the value of y.

3. 

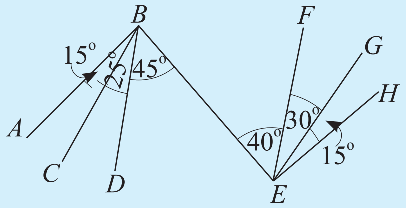
Based on the information in the figure,

(i) write the values of $D\hat{C}E$ and $A\hat{C}D$ in terms of x.

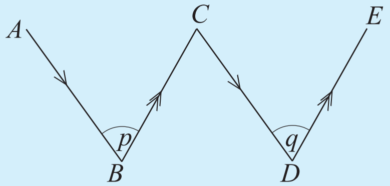
(ii) find the value of x,

(iii) find the magnitude of each angle in the triangle.

4. Write all the pairs of parallel lines in the given figure, indicating the reasons for your selections.



5. In the figure, $A\hat{B}C = p$ and $C\hat{D}E = q$. Show that $p = q$.



Summary

Summary

- The sum of the adjacent angles formed by a straight line meeting another straight line is two right angles.
- The vertically opposite angles formed by the intersection of two straight lines are equal.
- When two straight lines are intersected by a transversal, if
  - **i.** a pair of corresponding angles are equal or
  - **ii.** a pair of alternate angles are equal or
  - **iii.** the sum of a pair of allied angles is 180°, then the two straight lines are parallel to each other.
- When a transversal intersects a pair of parallel lines,
  - **i.** the corresponding angles formed are equal,
  - **ii.** the alternate angles formed are equal,
  - **iii.** the sum of each pair of allied angles formed equals two right angles.

9

Liquid Measurements

By studying this lesson, you will be able to;

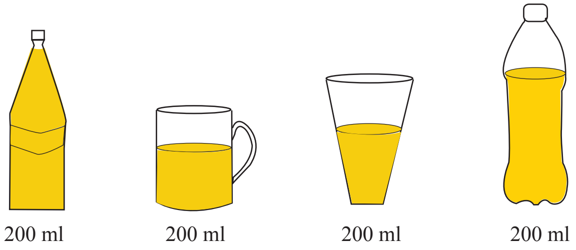
- determine the relationship between
  millilitres (ml) and cubic centimetres (cm^3),
  litres (*l*) and cubic centimetres (cm^3),
  litres(*l*) and cubic metres (m^3), as units which are used to measure liquid volumes, and
- solve problems involving units which are used to measure liquid volumes.

Volume and capacity

We know that the amount of space occupied by a solid or a liquid is known as its volume.

A solid has a definite shape and a definite volume. Although a liquid has a definite volume, it does not have a definite shape. A liquid always takes the shape of its container.

The below given pictures show 200 ml of drink in different shaped containers



When that quantity of drink is poured into the different shaped containers, even though the liquid takes the shape of the container, its volume of 200 ml remains unchanged. In the first picture, the container is completely filled with the 200 ml of drink. Therefore, the capacity of that container is 200 ml. The **capacity** of a container is the maximum volume that it can hold.

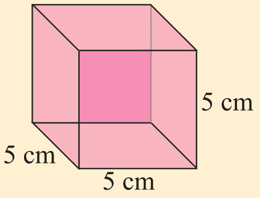
Do the following review exercise to recall the facts that you have learnt earlier in relation to volume and capacity.

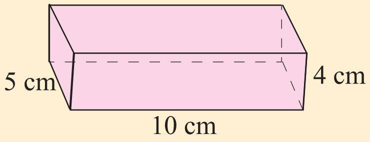
Review Exercise

1. Complete the table given below using the fact that 1 *l* =1000 ml.

ml	*l* and ml		*l* (in decimal form)
	l	ml	
2500	2	500	2.5
....	3	000	
3500	3		
....	4	500	4.5
....	0	500	
200			
50			
....			3.25
....	0	25	
....			0.005

2. Complete the two tables given below based on the way the volumes of the cube and the cuboid in the figure have been calculated.





Volume = 5 cm × 5 cm × 5 cm = 125 cm^3

Volume = 10 cm × 5 cm × 4 cm = 200 cm^3

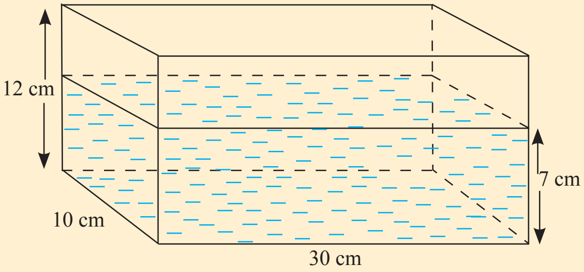
(i) Cube

The length of a side (cm)	**Volume (cm^3)**
2	…. × …. × …. = ….
4	
6	
7	
8	
10	
12	

(ii) Cuboid

Length (cm)	**Width (cm)**	**Height (cm)**	**Volume (cm^3)**
3	2	2	... × ... × ... = ...
5	3	4	
8	6	5	
10	5	10	
10	5	6	
12	10	8	
12	6	5	
15	8	10	
20	7	8	

3. The internal length, width and height of the container in the figure are 30 cm, 10 cm and 12 cm respectively. This container has been filled with water up to a height of 7 cm.



Determine the following.

i. The capacity of the container.
ii. The volume of water required to fill the whole container.
iii. The volume of water in the container, if the container is filled with water only up to a height of 7 cm.
iv. When the water level is 7 cm, if due to a leak it decreases to 5 cm within an hour, the volume of water that has leaked out during that hour.

9.1 The relationship between a cubic centimetre and a millilitre

A syringe used by doctors is given in the above figure. The amount of liquid medicine injected into a patient can be identified using the scale indicated on the syringe.

The units of measurement are indicated as cc/ ml.
cc means "**cubic centimetre**". It consists of the initial letters of these two words. A cubic centimetre is the volume of a cube of side length 1 cm.

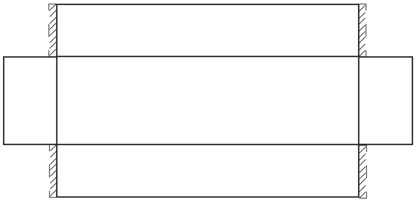
The back slash (/) means "or". It indicates that the amount of medicine can be expressed in terms of either cc or ml. The question which arises immediately is whether 1 cc is equal to 1 ml. In the metric system, 1 ml is defined such that it is equal to1 cc. Accordingly,

1 cubic centimetre = 1 millilitre

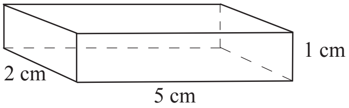
$$1 \text{ cm}^3 = 1 \text{ ml}$$

Do the following activity in order to study this fact further.

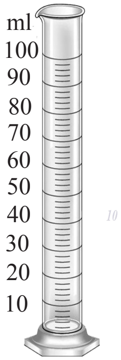
Activity



The net to construct the container



The cuboid shaped container of volume 10 cm^3



Measuring cylinder

- Construct a container of dimensions 5 cm × 2 cm × 1 cm using a net prepared from a thin plastic sheet as shown in the figure (paste the edges properly using sellotape or a suitable glue so that there is no water leakage).
- Obtain a measuring cylinder of capacity 100 ml from the laboratory.
- Draw the below given table in your exercise book.

Number of times water is poured from the cuboid shaped container into the measuring cylinder	The volume of water poured into the measuring cylinder	
	In cm^3 according to the cuboid shaped container	In ml according to the measuring cylinder
	10	
	20	
	30	
	40	
	50	

- Fill the cuboid shaped container completely with water and pour that water into the measuring cylinder.
- After pouring the water into the measuring cylinder, note down its reading.
- Repeat this process several times. Note down the reading on each occasion.
- Determine a relationship between the units of the volume of the container (cm^3) and the units (ml) marked on the measuring cylinder.

Based on the activity the following equalities are obtained.

$$10\text{ cm}^3 = 10\text{ ml}$$
$$20\text{ cm}^3 = 20\text{ ml}$$

Accordingly, it is clear that $1\text{ cm}^3 = 1\text{ ml}$.

This relationship can be used when solving problems related to liquid volumes in containers.

Example 1

A cuboid shaped glass container of length 20 cm, width 15 cm and height 10 cm is filled with a liquid medicine.

i. Find the volume of the container in cubic centimetres.
ii. What is the capacity of the container in litres?
iii. If the liquid in the container is to be stored in phials of capacity 50 ml each, find the number of phials required to store all the liquid in the container.

i. The volume of the container $= 20\text{ cm} \times 15\text{ cm} \times 10\text{ cm}$
$= 3000\text{ cm}^3$

ii. The capacity of the container $= 3000\text{ ml}$
$= 3\ l$

iii. The total amount of liquid $= 3000\text{ ml}$

The number of phials of capacity 50 ml each that are required $= 3000 \div 50$
$= 60$

Example 2

800 *l* of water is in a cuboid shaped concrete tank consisting of a base of length 2 m and width 1 m. Find the height to which the water is filled in the tank.

Let us construct an equation, assuming that the tank is filled up to a height of x cm, and by solving it find the height to which it is filled.

Let us first convert all the measurements into centimetres.

The length of the tank = 2 m = 200 cm
The width of the tank = 1 m = 100 cm

The volume of water in the tank if the water level is x cm $= 200 \text{ cm} \times 100 \text{ cm} \times x \text{ cm}$

$= 20\,000x \text{ cm}^3$

It is given that the volume of water in the tank is 800 *l*.

$\therefore$ Volume of water in the tank = 800 *l*

= 800 000 ml

$= 800\,000 \text{ cm}^3$

Since the volume of water represented above in two ways is equal,

$$20\,000 \times x = 800\,000$$

$$x = \frac{800\,000}{20\,000}$$

$$= 40$$

$\therefore$ The height of the water in the tank is 40 cm.

Exercise 9.1

1. Join each of the volumes in box *A* with the volume in box *B* which is equal to it.

A	*B*
1000 cm^3	25 ml
10 cm^3	25 *l*
3000 cm^3	1 *l*
1500 cm^3	10 ml
25000 cm^3	1.5 *l*
25 cm^3	3 *l*

2. The dimensions of several cuboid shaped containers are given in the following table. Complete this table.

Length (cm)	Width (cm)	Height (cm)	Capacity		
			cm^3	ml	*l*
20	10	5			
40	20	10			
35	12	10			
50	35	12			
40	35	25			
25	20	18			

3. A cuboid shaped tank of base area 240 cm^2 is filled up to a height of 12 cm with water. Find the volume of water in the tank in,

i. cubic centimetres **ii.** millilitres **iii.** litres

4. A cuboid shaped container has a square base of area 225 cm^2. An amount of 3.6 *l* of water has been filled into this container.

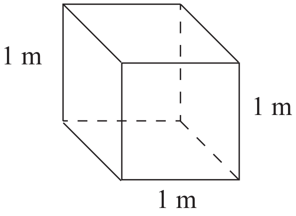
- **i.** Find the height of the water in the container.
- **ii.** If the height of the container is 24 cm, show that the water is filled to $\frac{2}{3}$ of its capacity.

5. Show that a barrel of capacity 15 *l* can be filled completely by pouring water 15 times using a completely filled cube shaped container of side length 10 cm.

9.2 The relationship between a litre and a cubic metre

The need for a unit which is larger than ml or *l* arises when it is necessary to measure large volumes of liquid such as the quantity of liquid in an oil tank or a swimming pool. In such instances a large unit called cubic metre is used.

In order to identify a cubic metre, let us calculate the capacity of a cube shaped container of side length 1 m.



The capacity of the container shown in the figure $= 1 \text{ m} \times 1 \text{ m} \times 1 \text{ m} = 1 \text{ m}^3$

However, since 1 m = 100 cm,

the capacity of the container, $1 \text{ m}^3 = 100 \text{ cm} \times 100 \text{ cm} \times 100 \text{ cm}$

$$= 1\ 000\ 000 \text{ cm}^3$$

$$= 1\ 000\ 000 \text{ ml (Since } 1 \text{ cm}^3 = 1 \text{ ml)}$$

$$= \frac{1000\ 000}{1000}\ l \text{ (Since } 1000 \text{ ml} = 1\ l)$$

$$= 1\ 000\ l$$

Accordingly,

one cubic metre is equal to 1000 *l*.

$1 \text{ m}^3 = 1000\ l$

Example 3

The internal length, width and height of a cuboid shaped tank in which water is stored for the daily use of a household are 1.5 m, 1 m and 1 m respectively.

(i) What is the capacity of the tank in litres?
(ii) If the residents use 300 *l* per day, for how many days will a completely filled tank be sufficient?

(i) The capacity of the tank $= 1.5 \text{ m} \times 1 \text{ m} \times 1 \text{ m}$
$= 1.5 \text{ m}^3$
$= 1\,500\ l$ (Since $1 \text{ m}^3 = 1000\ l$)

(ii) The volume of water used per day $= 300\ l$
The volume of water in the tank $= 1\,500\ l$

$\therefore$ The number of days for which the water is sufficient $= \dfrac{1\,500}{300}$

= five days

Exercise 9.2

1. Complete the table.

The inner dimensions of the cuboid shaped tank			The capacity of the tank	
Length (m)	Width (m)	Height (m)	m^3	*l*
2	2	1	.....	.....
2	1.5	1	.....	.....
1	1	0.5	.....	.....
4	1	.....	8	.....
.....	1.5	3.0	.....	9000
1	.....	1	1.5	.....

2. The length, width and depth of a swimming pool are 50 m, 25 m and 3 m respectively.

- **i.** Find the capacity of the swimming pool
- **ii.** If the swimming pool is filled with water up to a height of 1.2 m, what is the volume of water in the swimming pool in litres?
- **iii.** How much more water is required to fill the swimming pool completely?

3. A bowser of capacity 6.5 m^3 is filled completely with oil. This bowser is supposed to distribute 850 *l* of oil each to 8 filling stations. Is the oil in the bowser sufficient for all 8 filling stations? Give reasons for your answer.

4. A person requires on average 150 *l* of water daily. If a cuboid shaped tank of length $1\frac{1}{2}$ m, width 1 m and height 1 m is completely filled with water, for how many people in total will this quantity of water be sufficient for a day?

5. The length of an interior side of a cube shaped tank is 1 m. The tank is completely filled with water. When a tap from which the water in the tank is discharged is opened, water flows out from the tank at a constant rate of 50 *l* per minute. Determine how long after the tap is opened the tank becomes empty, if the water flows out at this constant rate.

Miscellaneous Exercise

1. The capacity of a large drink bottle is 1.5 *l*. It is expected to serve a quantity of 150 ml of this drink in small glasses to each of the guests at a party. If there are 225 guests at the party, find the minimum number of large drink bottles needed to serve all the guests.

2. Household storage tanks of capacity 500 *l*, 1000 *l* and 2000 *l* are available for sale. The head of a family of 5 members intends to buy one of these tanks to store water for their household use. If each family member requires a maximum of 150 *l* per day and 200 *l* of water is required each day for other household chores. Determine which tank best suits his requirements, if the head of the family intends to fill the tank only once a day,

Summary

Summary

- 1 cm^3 = 1 ml
- 1 m^3 = 1000 *l*

Revision Exercise – First term.

Part – I

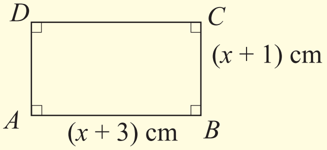
1. Write the general term of the number pattern 5, 8, 11, 14, …, with a common difference.

2. Fill in the blank: $10011_{\text{two}} - \ldots\ldots_{\text{two}} = 0011_{\text{two}}$.

3. The value of $\frac{1}{3}$ of a certain amount of money is Rs 800. What is the value of $\frac{3}{4}$ of that amount of money?

4. If a profit of Rs 300 is earned by selling an item for Rs 1500, what is the profit percentage?

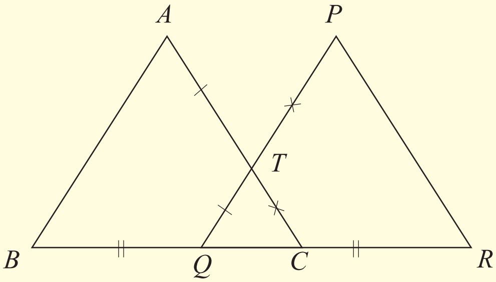
5.



Express the area of the rectangle *ABCD* in terms of *x*.

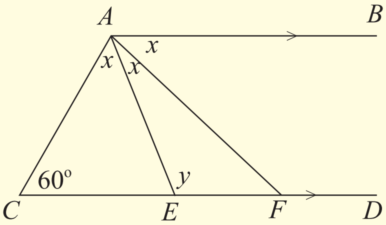
6. Factorize $2x^2 - x - 6$

7.



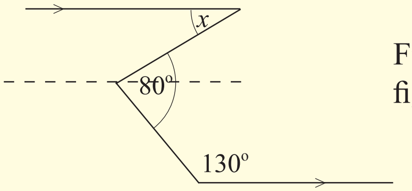
Show using axioms and the information in the figure that,
(i) $AC = PQ$ and
(ii) $BC = QR$.

8.

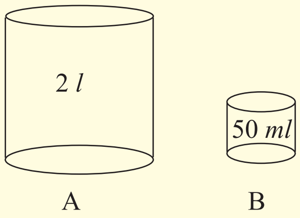


Given that the lines *AB* and *CD* are parallel, find the value of *y*.

9. Find the values of *b* and *c* if $(x + 4)(x - 3) = x^2 + bx + c$.

10. 

Find the value of x using the information in the figure.

11. 

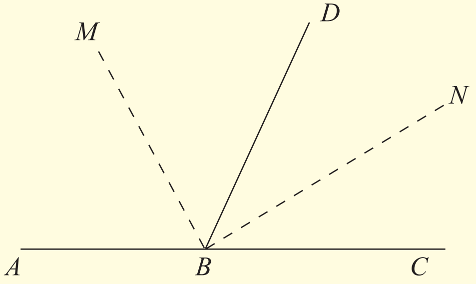
How many times should water be poured into the container *A* of capacity 2*l* from the completely filled container *B* of capacity 50 ml to fill $\frac{3}{4}$.

12. brokerage fee of 3% is charged when a land is sold. If the land owner received 27 lakhs in rupees after the brokerage fee was paid, find the price at which the land was sold.

13. What is the fraction by which $1\frac{3}{4}$ has to be multiplied to obtain $3\frac{3}{4}$?

14.
$$\begin{array}{r} 1101_{two} \\ + \quad 1111_{two} \\ \hline \ldots\ldots\ldots\ldots \\ - \quad 101_{two} \\ \hline \ldots\ldots\ldots\ldots \\ \hline\hline \end{array}$$

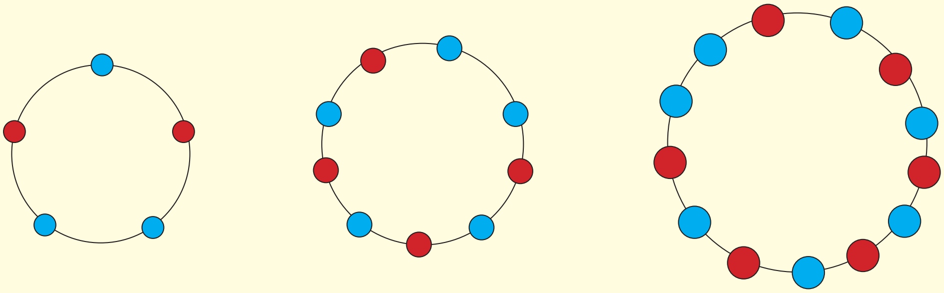
Fill in the blanks.

15. 

The bisectors of $A\hat{B}D$ and $D\hat{B}C$ are BM and BN respectively. Find the value $A\hat{B}M + C\hat{B}N$, if $A\hat{B}C$ is a straight line.

Part II

1.



1. Adecoration is madeby preparing circles of various sizes and placing red and blue bulbs in a pattern with a common difference such that the first three arrangements contain 3, 5, 7 blue bulbs and 2, 4, 6 red bulbs respectively as shown in the figure.

   (i) Write the number of blue bulbs and the number of red bulbs in the 4th and 5th arrangements.

   (ii) Identify the patterns of the number of blue bulbs and the number of red bulbs in the arrangements and construct two expressions in n for the number of bulbs of each colour in the nth arrangement.

   (iii) Find an expression for the total number of bulbs inthe nth arrangement, using the expressions in (ii) above.

   (iv) Find the number of blue bulbs and the number of red bulbs in the 10th arrangement using the expressions in (ii) above.

   (v) Which arrangement is prepared using a total of 61 bulbs? Find the number of blue bulbs in that arrangement.

2. (a) Simplify.

i. $$\frac{2\frac{1}{5}+\frac{1}{2}}{\frac{3}{10}}$$

ii. $(1\frac{1}{8} \text{ of } 1\frac{1}{3}) \div 2\frac{1}{2}$

(b) **i.** $\frac{1}{4}$ th of a certain land contains mango trees. What fraction of the total land is the remaining portion of land?

ii. If $\frac{1}{3}$ of the remaining land contains banana trees, express the portion of land in which banana trees are grown as a fraction of the whole land.

iii. In what fraction of the total land are the mango trees and banana trees grown?

iv. If the area of the portion in which these trees are not grown is 3 hectares, what is the total area of the land?

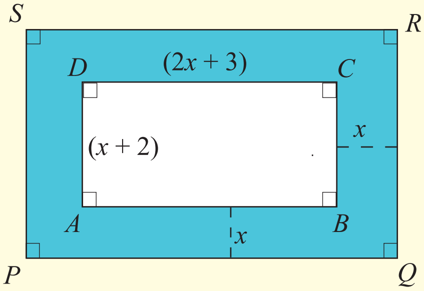
3. (a) The selling price of an item which was bought for Rs 8000 was marked keeping aprofit of 25%. A discount of 10% was given when the item was purchased outright. Find the profit percentage earned by the seller.

(b) A person marks the price of an item to earn a profit of 15%. If its price had been marked to earn a profit of 20%, an extra Rs 200 could have been earned. Find the purchase price and the marked price of the item.

4. (a) Find the value of each of the following expressions when $a = -2$ and $b = 3$.

i. $2a + 3b$ **ii.** $b - 2a$ **iii.** $\frac{a}{3} - \frac{b}{2}$

(b)



a The rectangle $ABCD$ in the figure represents picture. Its length is $(2x+3)$ cm and its breadth is $(x + 2)$ cm.

i. Obtain an expression in terms of x for the area of $ABCD$.

ii. he shaded part in the figure represents a band of breadth x cm which is pasted bordering $ABCD$. Find an expression for the area of the rectangle $PQRS$ and express the area of the shaded part in terms of x using the expression found in (i) above too.

iii. Calculate the area of the shaded part if $x = 3$ cm.

(c) Factorize the following expressions.

i. $5x^2 + 12y^2 - 4xy - 15xy$

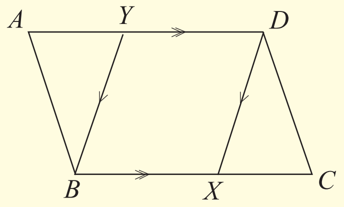
ii. $6(x - 1) + 3x - 3$

iii. $t^2 - 8t + 15$

iv. $3k^2 - 12k$

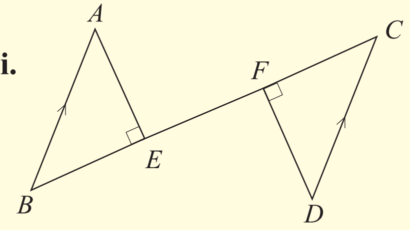
5. (a) Obtain the following results using axioms and the information in the figure.

i.



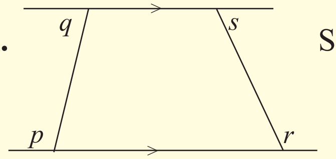
Show that $A\hat{Y}B = D\hat{X}C$.

ii.



Show that $B\hat{A}E = F\hat{D}C$.

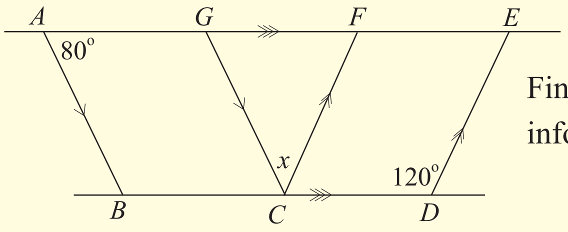
iii.



Show that $\hat{p} - \hat{s} = \hat{r} - \hat{q}$.

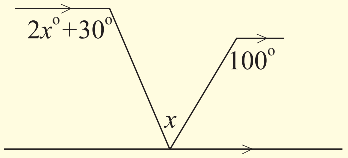
(b)

i.



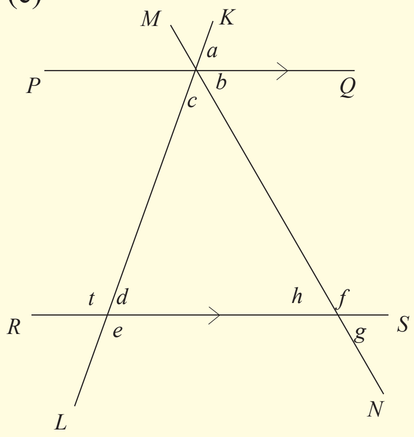
Find the value of x using the information in the figure.

ii.



Find the value of x using the information in the figure.

(c)



The parallel lines *PQ* and *RS* are intersected by the transversals *MN and KL.* Answer the following questions using the information in the figure.

i. Write all the instances where the sum of the given angles is 180^o.

ii. Write the pairs of allied angles in the figure.

iii. Providing reasons, write all the angles which are equal to each other.

iv. Is $\hat{a} + \hat{e} = 180^o$? Explain your answer.

v. Using axioms, show that $t - f = h - d$.

vi. Find the values of all the angles indicated by lowercase letters if $e = 140^o$, $f = 110^o$.

6. The length, breadth and height of a water tank are 2m, 1.5m and 1m respectively.

i. Express the capacity of the tank in liters.

ii. If the daily water requirement of a person is 150 *l*, how much water is required daily for 4 people?

iii. For how many days will the water in this tank be sufficient for4 people if it is full?

iv. If water is supplied to the tank at a rate of 100 *l* per minute, how much time is needed to fill thetank if it is empty?

v. On a day when the tank is filled to its capacity, 900 *l* of water leaks out due to a fault in the delivery pipeline. Find the height of the remaining water.

Glossary

A

Addition	එකතු කිරීම	கூட்டல்
Allied angles	මිත්‍ර කෝණ	நேயக்கோணங்கள்
Algebraic expressions	වීජීය ප්‍රකාශන	அட்சரகணிதக் கோவைகள்
Algebraic term	වීජීය පදය	அட்சரகணித உறுப்பு
Alternate angles	ඒකාන්තර කෝණ	ஒன்றுவிட்டகோணங்கள்

B

Base	පාදය	அடி
Binary numbers	ද්වීමය සංඛ්‍යා	துவித எண்கள்
Brackets	වරහන්	அடைப்பு
Broker	තැරැව්කරුවා	தரகர்

C

Capacity	ධාරිතාව	கொள்ளளவு
Commission	කොමිස්	தரகு (கமிஷன்)
Common factors	පොදු සාධක	பொதுக்காரணிகள்
Converse	විලෝමය	மறுதலை
Conversion	පරිවර්තනය	மாற்றல்
Corresponding angles	අනුරූප කෝණ	ஒத்தகோணங்கள்

D

Difference of terms	පද අතර වෙනස	உறுப்புக்களுக்கிடையேயானவித்தியாசம்
Discount	වට්ටම	கழிவு

F

First term	පළමුවන පදය	முதலாம் உறுப்பு
Fractions	භාග	பின்னங்கள்

G

General term	සාධාරණ පදය	பொது உறுப்பு

I

integers	නිඛිල	நிறைவெண்கள்

L

Loss	අලාභය	நட்டம்

M

Marked Price	ලකුණු කළ මිල	குறித்த விலை

N

Number sequence	සංඛ්‍යා අනුක්‍රම	எண் தொடரி

P

Place Value	ස්ථානීය අගය	இடப்பெறுமானம்
Power	බලය	வலு
Profit	ලාභය	இலாபம்

S

Sciencetific notation	විද්‍යාත්මක අංකනය	விஞ்ஞானமுறைக் குறிப்பீடு
Selling Price	විකුණුම් මිල	விற்றவிலை
Subtraction	අඩු කිරීම	கழித்தல்

T

Theorem	ප්‍රමේයය	தேற்றம்

V

Vertically opposite angles	ප්‍රතිමුඛ කෝණ	குத்தெதிர்க்கோணங்கள்
Volume	පරිමාව	கனவளவு

Lesson Sequence

Contents	Numbers of Periods
First term	
1. Number Patterns	03
2. Binary Numbers	03
3. Fractions	05
4. Percentages	06
5. Algebraic Expressions	05
6. Factors of the Algebraic Expressions	05
7. Axioms	04
8. Angles related to straight lines and parallel lines	07
9. Liquid Measurements	03
Second term	
10. Direct proportion	06
11. Calculator	02
12. Indices	03
13. Round off and Scientific Notation	05
14. Loci and Constructions	09
15. Equations	06
16. Angles of a triangle	09
17. Formulae	02
18. Circumference of circle	05
19. Pythagoras relation	04
20. Graphs	04
Third term	
21. Inequalities	03
21. Sets	07
23. Area	05
24. Probability	05
25. Angles of polygons	05
26. Algebraic fractions	03
27. Scale drawings	08
28. Data representation and prediction	10

MATHEMATICS

Grade 9

Part - II

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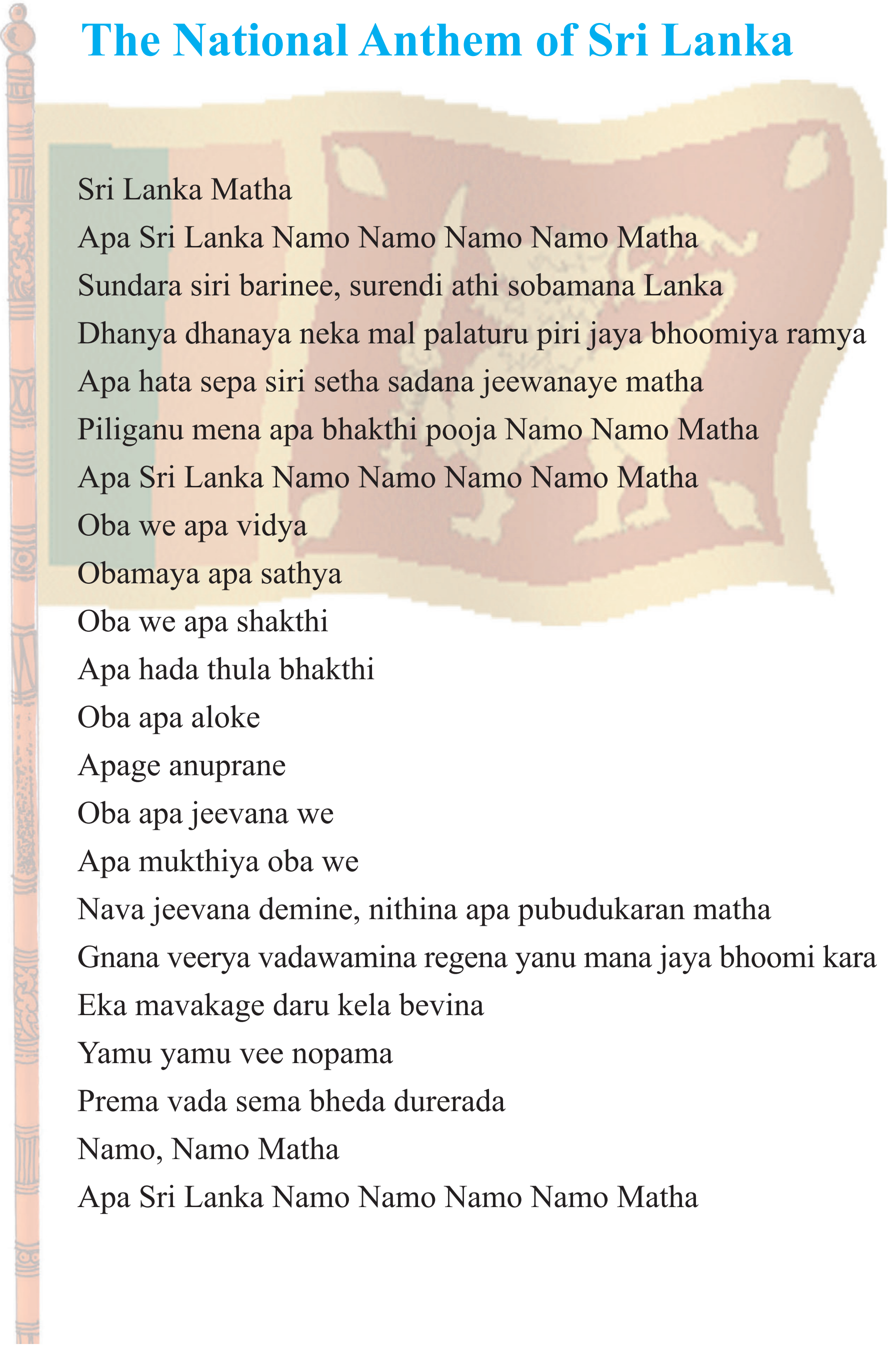
The National Anthem of Sri Lanka



Sri Lanka Matha
Apa Sri Lanka Namo Namo Namo Namo Matha
Sundara siri barinee, surendi athi sobamana Lanka
Dhanya dhanaya neka mal palaturu piri jaya bhoomiya ramya
Apa hata sepa siri setha sadana jeewanaye matha
Piliganu mena apa bhakthi pooja Namo Namo Matha
Apa Sri Lanka Namo Namo Namo Namo Matha
Oba we apa vidya
Obamaya apa sathya
Oba we apa shakthi
Apa hada thula bhakthi
Oba apa aloke
Apage anuprane
Oba apa jeevana we
Apa mukthiya oba we
Nava jeevana demine, nithina apa pubudukaran matha
Gnana veerya vadawamina regena yanu mana jaya bhoomi kara
Eka mavakage daru kela bevina
Yamu yamu vee nopama
Prema vada sema bheda durerada
Namo, Namo Matha
Apa Sri Lanka Namo Namo Namo Namo Matha

අපි වෙමු එක මවකගෙ දරුවෝ
එක නිවසෙහි වෙසෙනා
එක පාටැති එක රුධිරය වේ
අප කය තුළ දුවනා

එබැවිනි අපි වෙමු සොයුරු සොයුරියෝ
එක ලෙස එහි වැඩෙනා
ජීවත් වන අප මෙම නිවසේ
සොඳින සිටිය යුතු වේ

සැමට ම මෙත් කරුණා ගුණෙනී
වෙළී සමගි දමිනී
රන් මිණි මුතු නො ව එය ම ය සැපතා
කිසි කල නොම දිරනා

ආනන්ද සමරකෝන්

ஒரு தாய் மக்கள் நாமாவோம்
ஒன்றே நாம் வாழும் இல்லம்
நன்றே உடலில் ஓடும்
ஒன்றே நம் குருதி நிறம்

அதனால் சகோதரர் நாமாவோம்
ஒன்றாய் வாழும் வளரும் நாம்
நன்றாய் இவ் இல்லினிலே
நலமே வாழ்தல் வேண்டுமன்றோ

யாவரும் அன்பு கருணையுடன்
ஒற்றுமை சிறக்க வாழ்ந்திடுதல்
பொன்னும் மணியும் முத்துமல்ல - அதுவே
யான்று மழியாச் செல்வமன்றோ.

ஆனந்த சமரக்கோன்
கவிதையின் பெயர்ப்பு.

Being innovative, changing with right knowledge
Be a light to the country as well as to the world.

Message from the Hon. Minister of Education

The past two decades have been significant in the world history due to changes that took place in technology. The present students face a lot of new challenges along with the rapid development of Information Technology, communication and other related fields. The manner of career opportunities are liable to change specifically in the near future. In such an environment, with a new technological and intellectual society, thousands of innovative career opportunities would be created. To win those challenges, it is the responsibility of the Sri Lankan Government and myself, as the Minister of Education, to empower you all.

This book is a product of free education. Your aim must be to use this book properly and acquire the necessary knowledge out of it. The government in turn is able to provide free textbooks to you, as a result of the commitment and labour of your parents and elders.

Since we have understood that the education is crucial in deciding the future of a country, the government has taken steps to change curriculum to suit the rapid changes of the technological world. Hence, you have to dedicate yourselves to become productive citizens. I believe that the knowledge this book provides will suffice your aim.

It is your duty to give a proper value to the money spent by the government on your education. Also you should understand that education determines your future. Make sure that you reach the optimum social stratum through education.

I congratulate you to enjoy the benefits of free education and bloom as an honoured citizen who takes the name of Sri Lanka to the world.

Akila Viraj Kariyawasam
Minister of Education

Foreword

The educational objectives of the contemporary world are becoming more complex along with the economic, social, cultural and technological development. The learning and teaching process too is changing in relation to human experiences, technological differences, research and new indices. Therefore, it is required to produce the textbook by including subject related information according to the objectives in the syllabus in order to maintain the teaching process by organizing learning experiences that suit to the learner needs. The textbook is not merely a learning tool for the learner. It is a blessing that contributes to obtain a higher education along with a development of conduct and attitudes, to develop values and to obtain learning experiences.

The government in its realization of the concept of free education has offered you all the textbooks from grades 1-11. I would like to remind you that you should make the maximum use of these textbooks and protect them well. I sincerely hope that this textbook would assist you to obtain the expertise to become a virtuous citizen with a complete personality who would be a valuable asset to the country.

I would like to bestow my sincere thanks on the members of the editorial and writer boards as well as on the staff of the Educational Publications Department who have strived to offer this textbook to you.

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Commissioner General of Educational Publications,
Educational Publications Department,
Isurupaya,
Battaramulla.
2019.04.10

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Message of the Board of Compilers

This textbook has been compiled in accordance with the new syllabus to be implemented from 2015.

Textbooks are compiled for students. Therefore, we have made an attempt to compile this textbook in a simple and detailed manner making it possible for you to read and understand it on your own.

We have included descriptions, activities and examples to introduce the subject concepts in an attractive manner and to establish them. Moreover, activities are organized from simple to complex to develop an interest to do them.

We have used the terms related to mathematical concepts in accordance with the glossary of technical terms of mathematics compiled by the Department of Official Languages.

Some subject matter learnt during the earlier grades is necessary to learn the subject content in the grade 9 syllabus. Thus, review exercises are included at the beginning of each chapter to revise previous knowledge. You will be prepared by them for the subject content of grade 9.

You will gain maximum benefit from this textbook by reading the chapters and doing the review exercises of each chapter even before your teacher teaches them in the classroom.

We hope that studying mathematics will be an interesting, joyful and productive experience.

Board of Compilers

Contents

10

Direct Proportion

By studying this lesson you will be able to;

- identify direct proportions,
- solve problems related to direct proportion using the unitary method,
- solve problems related to direct proportion using the definition,
- write the relationship between two directly proportional quantities in the form $y = kx$,
- solve problems related to the conversion of foreign currencies using the knowledge on direct proportions.

10.1 Introduction to direct proportion

The way the price of a certain type of pen varies depending on the quantity of pens is given in the following table.

Number of pens	Price (Rs)
1	15
2	30
3	45
4	60
5	75
6	90

It is clear from the above table that the price increases as the number of pens increases. Let us consider the number of pens and the price as two quantities.

Based on the above example, a few ratios of different amount of pens and the ratios of the corresponding prices are shown in the following table. Observe that these ratios are equal.

Ratio of two amounts of pens	Ratio of the corresponding prices
1 : 2	15 : 30 = 1 : 2
1 : 3	15 : 45 = 1 : 3
2 : 3	30 : 45 = 2 : 3
3 : 5	45 : 75 = 3 : 5
2 : 5	30 : 75 = 2 : 5

Two distinct quantities are said to be in direct proportion if they increase or decrease in the same ratio.

Therefore, if two quantities are in direct proportion, then when one quantity increases, the other quantity will also increase in the same ratio.
Similarly, if two quantities are in direct proportion and one quantity decreases, then the other quantity will also decrease in the same ratio.

Exercise 10.1

1. For each of the cases given below, write whether the two quantities are directly proportional or not.

a. The number of books and the price

b. The distance travelled by an object moving at a constant speed and the time taken for the journey.

c. The speed of a vehicle and the time taken to travel a certain distance

d. The length of a side of a square and its perimeter

e. The length of a side of a square and its area

f. The number of people needed to finish a task and the number of days taken for it

g. The number of units of electricity consumed by a household and the monthly bill

10. 2 Solving problems related to direct proportion using the unitary method

Suppose we want to find the price of 5 cakes of a certain type of soap, given that the price of 3 cakes of soap of that type is Rs 120.

As you have learnt in previous grades, we can first find the price of one cake of soap and thereby easily find the price of 5 cakes of soap.

Price of 3 cakes of soap = Rs 120
Price of 1 cake of soap = Rs 120 ÷ 3
= Rs 40
Price of 5 cakes of soap = Rs 40 × 5
= Rs 200

This method of calculation can also be explained as follows.

There are two quantities. They are the number of cakes of soap and the price. Initially the price of one cake of soap is found. It is Rs. 40. To find the price of five cakes of soap, the price of one cake of soap is multiplied by 5. Here the price of one cake of soap is clearly the constant value of the following fraction.

$$\frac{\text{price of 3 cakes of soap}}{\text{number of cakes of soap}}$$

The method of solving a problem based on the value of a unit is called the unitary method.

Let us learn how to solve problems related to direct proportion using the unitary method by considering a few examples.

Example 1

If a person walking at a constant speed takes 5 minutes to walk 800 m, calculate the distance he walks in 12 minutes.

Distance walked in 5 minutes in metres = 800
Distance walked in 1 minute in metres = 800 ÷ 5
= 160
Distance walked in 12 minutes in metres = 160 × 12
= 1 920

∴ The distance walked in 12 minutes is 1920 m.

Example 2

If the mass of 10 identical balls used in a cricket match is 3 kg, what is the mass of 3 such balls?

Mass of 10 balls in kilogrammes = 3

Mass of 1 ball in grammes = $3000 \div 10$

= 300

Mass of 3 balls in grammes = 300×3

= 900

$\therefore$ The mass of 3 balls is 900g.

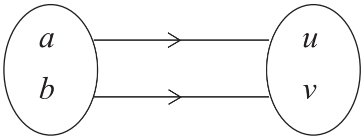
Do the following exercise using the unitary method.

Exercise 10.2

1. If the price of 8 oranges is Rs 320, find the price of 5 such oranges.
2. If the price of 5 m of a certain fabric is Rs 750, find the price of 12 m of that fabric.
3. If the mass of a parcel containing 15 apples is 3.6 kg, find the mass of a parcel containing 8 such apples.
4. If a printing machine makes 240 copies in 5 minutes, determine the number of copies it makes in 12 minutes.
5. If a motor vehicle moving at a constant speed travels 12 km in 15 minutes, calculate the distance it travels in 40 minutes.
6. If a motorbike can travel 90 km on 2 *l* of petrol, find the distance it can travel on 5 *l* of petrol.
7. If the time taken for a tank of capacity 1000 litres to be filled using a pump that releases water at a constant rate is 5 minutes, find the time taken in seconds to fill a tank of capacity 1600 litres.

10.3 Solving problems related to direct proportion using the definition

In the first section of this lesson it was explained that if two quantities are directly proportional, then the ratio of any two values of the first quantity is equal to the ratio of the corresponding values of the second quantity. This can be shown algebraically as below. Let us assume that the price of an amount a of a certain item is Rs u and the price of an amount b of the same item is Rs v.



Then we can write $a : b = u : v$.

This can be expressed in terms of fractions as $\frac{a}{b} = \frac{u}{v}$ (or $\frac{b}{a} = \frac{v}{u}$).
By cross multiplication, $a \times v = b \times u$
Let us learn how to solve problems related to direct proportion using this feature by considering the following examples.

Example 1

If the price of 5 mangoes is Rs 75, find the price of 8 mangoes.

Let us take the price of 8 mangoes as x. Then we can illustrate this information using an arrow diagram as shown below.

Number of mangoes Price (Rs)

$$\begin{tikzcd} 5 \arrow[r] & 75 \\ 8 \arrow[r] & x \end{tikzcd}$$

Using this as the base, let us write an algebraic equation as shown below and by solving it, find the value of x; that is, the price of 8 mangoes.

$$5 : 8 = 75 : x$$

Therefore, $$\frac{5}{8} = \frac{75}{x}$$

$$5x = 75 \times 8$$

$$x = \frac{75 \times 8}{5}$$

$$x = 120$$

Accordingly, the price of 8 mangoes is Rs 120.

Example 2

Find the price at which an item bought for Rs 500 should be sold to earn a profit of 15%.

Let us write the information in this problem as follows, so that we can use direct proportions. "If the selling price of an item bought for Rs 100 is Rs 115 (since the profit is 15%), find the selling price of an item bought for Rs 500."

Let us assume that the selling price of an item bought for Rs 500 is Rs x.

$$\begin{tikzcd}
\text{Purchase price (Rs)} & \text{Selling price (Rs)} \\
100 \arrow[r] & 115 \\
500 \arrow[r] & x
\end{tikzcd}$$

$100 : 500 = 115 : x$

$$\frac{100}{500} = \frac{115}{x}$$

$$100\,x = 115 \times 500$$

$$x = \frac{115 \times 500}{100}$$

$$x = 575$$

Accordingly, the selling price should be Rs 575.

Exercise 10.3

1. For each of the proportions given below, write the suitable value in the blank space.

a. 2 : 5 = 8 : ......

b. 3 : 4 = ... : 20

c. 5 : 3 = 40 : ......

d. 4 : 1 = ... : 8

e. 8 : ...... = 24 : 15

f. ...... : 6 = 35 : 30

2. Solve each problem given below using proportions, by first drawing an arrow diagram and then writing an algebraic equation.

a. If the price of 10 kg of rice is Rs 850, find the price of 7 kg of rice.

b. If the mass of 9 cm^3 of a certain type of metal is 108 g, find the mass of 12 cm^3 of this metal.

c. If the distance travelled in 4 hours by a motorbike moving at a constant speed is 240 km, find the distance travelled by it in 3 hours.

d. Find the amount needed to buy an item worth Rs 800 from a shop which offers a discount of 3%.

e. If a commission of 12% is given when an item is sold, what is the commission given for an item worth Rs 15 000?

f. If the price of 4 pencils is Rs 48, find the number of pencils that can be bought for Rs 132.

g. If the price of 12 bottles is Rs 4800, find the number of bottles that can be bought for Rs 6000.

10.4 Solving problems related to direct proportion algebraically

If the price of 1 pen is Rs 15, then

- the price of 2 pens is Rs 30.
- the price of 3 pens is Rs 45.
- the price of 4 pens is Rs 60.

If we consider the above four instances, it can be observed that if the amount of money spent is divided by the number of pens, the value that is obtained is a constant.

That is, $\dfrac{\text{money spent}}{\text{number of pens}} = \text{constant value.}$

This constant value is the price of one pen. Accordingly, if the money spent for x pens is y,

we can write $\dfrac{y}{x} = k$; here k is a constant.

This equation can also be written as $y = kx$.

Let us learn how to solve problems related to direct proportions using the above algebraic equation, by considering the following examples.

Example 1

If the price of 3 exercise books is Rs 75, find the price of 5 such exercise books. Let us take the number of books as x and the price as y.

Then we can write $y = kx$; where k is a constant. The value of k can be found using the information given in the problem.

Since the price of 3 exercise books is Rs 75, when $x = 3$, $y = 75$.

By substituting these values in the equation we obtain, $75 = k \times 3$.
By solving this we obtain $k = 25$.

By substituting this value of k in the first equation, we obtain the relationship between x and y as $y = 25x$.

Now, using this equation, for any value of x the corresponding value of y and for any value of y the corresponding value of x can be found.
In this problem, since we need the price of 5 exercise books, y needs to be found when $x = 5$.

By substituting $x = 5$ in the equation $y = 25x$ we get,

$$\begin{aligned} y &= 25 \times 5 \\ &= 125 \end{aligned}$$

Accordingly, the price of 5 exercise books is Rs 125.

Example 2

If a vendor sells an item he bought for Rs 500 such that he earns a profit of 20%, determine the selling price of the item.

Taking the purchase price of the item as x and selling price as y we can write $\frac{y}{x} = k$.

Since the selling price is Rs 120 when the purchase price is Rs 100, we obtain $\frac{120}{100} = k$.

Let us assume that the selling price of an item bought for Rs 500 is y. Then we obtain the equation $\frac{y}{500} = k$.

Since k is a constant, we can write, $\frac{y}{500} = \frac{120}{100}$.

Therefore, $y = \frac{120 \times 500}{100}$.

$y = 600.$

$\therefore$ The selling price of the item is Rs 600.

Exercise 10.4

Do the problems in this exercise, using the algebraic equation method.

1. If the price of 3 shirts is Rs 1200, find the price of 5 shirts.

2. If the daily wage of 8 labourers who are paid equal wages is Rs 7200, find the daily wage of 3 labourers.

3. If a distance of 25 m is represented by 5 cm on a map drawn to scale, find the actual distance represented by 8 cm on this map.

4. If a machine in a factory produces 7500 drink bottles in 5 hours, find the number of drink bottles it produces in 7 hours.

5. A bookstore offers a discount of 8% on every book that is purchased. Find the amount a person has to pay if he purchases books worth Rs 1 200.

10.5 Foreign currency

We know that every country has its own currency unit and that the rate of conversion of the currency of one country to that of another country varies depending on the countries. The rate at which one country exchanges its currency with that of another country is called the **exchange rate**. This rate is not a constant value; it increases and decreases daily due to various reasons.

The currency units used by certain countries and their exchange rates with respect to the Sri Lankan rupee on a particular day, is given below.

Here the exchange rate given is the value of one foreign currency unit in Sri Lankan rupees.

Country/Union	**Foreign currency unit**	**Exchange rate (Rs)**
United States of America	American Dollar	151.20
England	Sterling Pound	185.90
European Union	Euro	160.60
Japan	Yen	1.33
India	Indian Rupee	2.26
Saudi Arabia	Saudi Riyal	40.32
Singapore	Singapore Dollar	107.30

(From the internet on 2017-03-05)

Now let us consider how to solve problems related to exchange rates using proportions.

Example 1

On a day that the exchange rate is Rs 151 for an American dollar, how many Sri Lankan rupees will a person who converts 50 American dollars receive?

Value of 1 American dollar = Rs 151
Value of 50 American dollars = Rs 151 × 50
= Rs $\underline{\underline{7550}}$

Therefore, the person will receive Rs 7550.

Example 2

A person visiting England, converted Rs 74 000 into sterling pounds on a day when the exchange rate was Rs 185 for a sterling pound. How many sterling pounds did he receive?

The value of 185 Sri Lankan rupees = 1 sterling pound

The value of 1 Sri Lankan rupee = $\frac{1}{185}$ sterling pounds

The value of 74 000 Sri Lankan rupees = $\frac{1}{185}$ × 74 000 sterling pounds

= 400 sterling pounds

(It is easy to simplify this if we keep $\frac{1}{185}$ as a fraction without converting it into a decimal number). Therefore, the amount of sterling pounds he received is 400.

Exercise 10.5

Do the following exercise by using the exchange rate table given earlier.

1. If the monthly salary of a person working in a foreign country is 1500 American dollars, what is his salary in Sri Lankan rupees?

2. If the price of a television set imported from Japan is 12 500 yen, what is its value in Sri Lankan rupees?

3. A monthly allowance of 2500 sterling pounds is given to a scholarship student engaged in further studies in Great Britain. How much is this amount in Sri Lankan rupees?

4. A sports equipment in a duty free shop is worth 750 euros. How many Sri Lankan rupees have to be paid to purchase it?

5. A pilgrim who travels to India, converts 56 000 Sri Lankan rupees into Indian rupees. How many Indian rupees does he receive?

6. How many Singapore dollars are received when readymade garments worth Rs 600 880 are exported from Sri Lanka to Singapore?

11

The Calculator

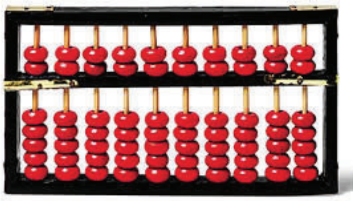
By studying this lesson you will be able to,

- identify and use the keys of =, %, x^2 and $\sqrt{x}$ in the scientific calculator.

The Calculator

Since ancient times, humans have used various devices to perform calculations. During the period when animal husbandry commenced, men used pebbles to count the number of animals they owned. There is evidence to show that counting was done by drawing lines on clay tablets during a later period. The Egyptians started using a device known as the abacus for calculations in 1000 B.C. The modern day abacus was invented by the Chinese in the 15th century. John Napier who lived in the 17th century invented “Napier’s bones” which is a device that was used to calculate products and quotients of numbers.

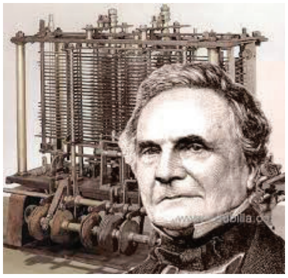
Egyptian abacus



Modern abacus

The first mechanical calculator was invented by the French mathematician Blaise Pascal(1623-1662). In the year 1833, the Englishman Charles Babbage (1791-1871) introduced a more advanced calculator. Based on this, the electrically operated computer was invented. The production of the modern compact calculators commenced with the development of electronics.

Blaise Pascal



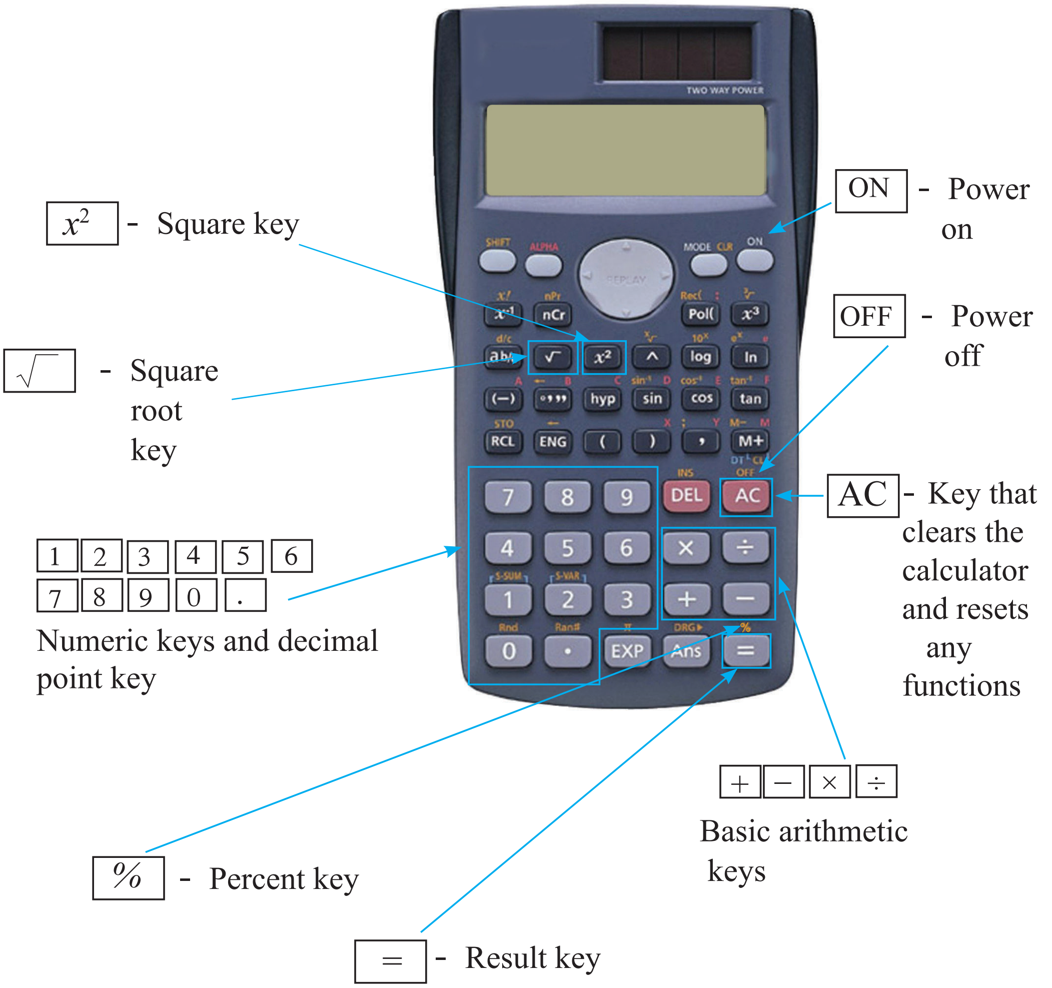
Charles Babbage

Nowadays, two types of calculators are manufactured. They are the ordinary calculators and the scientific calculators. Ordinary calculators can perform only normal mathematical operations such as addition, subtraction, multiplication, division and square roots. However, by using a scientific calculator, operations such as x^2, x^3 $\sqrt[x]{y}$, 10^x can be performed.

The Scientific Calculator

The scientific calculator, like the ordinary calculator, consists of a key pad to enter data and a screen to display results. However, the number of keys on the keypad and the number of digits that can be displayed on the screen of a scientific calculator are greater than those of an ordinary calculator.

Let us identify the keys on the keypad of a scientific calculator.



11.1 Performing calculations using a calculator

When performing calculations using a calculator, the keys need to be pressed in a specific order.

Example 1

The order in which the keys need to be pressed to obtain the value of 27 + 35 is the following.

$\boxed{\text{ON}} \to \boxed{2} \to \boxed{7} \to \boxed{+} \to \boxed{3} \to \boxed{5} \to \boxed{=}\ \boxed{62}$

Example 2

The order in which the keys need to be pressed to obtain the value of 208 − 159 is the following.

$\boxed{\text{ON}} \to \boxed{2} \to \boxed{0} \to \boxed{8} \to \boxed{-} \to \boxed{1} \to \boxed{5} \to \boxed{9} \to \boxed{=}\ \boxed{49}$

Example 3

The order in which the keys need to be pressed to obtain the value of 5.25×35.4 is the following.

$\boxed{\text{ON}} \to \boxed{5} \to \boxed{.} \to \boxed{2} \to \boxed{5} \to \boxed{\times} \to \boxed{3} \to \boxed{5} \to \boxed{.} \to \boxed{4} \to \boxed{=}\ \boxed{185.85}$

Example 4

The order in which the keys need to be pressed to obtain the value of 5.52 ÷ 6 is the following.

$\boxed{\text{ON}} \to \boxed{5} \to \boxed{.} \to \boxed{5} \to \boxed{2} \to \boxed{\div} \to \boxed{6} \to \boxed{=}\ \boxed{0.92}$

To switch the calculator off after a calculation is done, press the $\boxed{\text{OFF}}$ key. If you wish to start another calculation without switching the calculator off, use the $\boxed{AC}$ key to clear the screen and reset the functions.

Example 5

Write the order in which the keys need to be pressed to do the following simplifications.

(i) 53 + 42 − 25 (ii) 35 × 45 ÷ 21

$\boxed{\text{ON}} \to \boxed{5} \to \boxed{3} \to \boxed{+} \to \boxed{4} \to \boxed{2} \to \boxed{-} \to \boxed{2} \to \boxed{5} \to \boxed{=}\ \boxed{70}$

$\boxed{\text{AC}} \to \boxed{3} \to \boxed{5} \to \boxed{\times} \to \boxed{4} \to \boxed{5} \to \boxed{\div} \to \boxed{2} \to \boxed{1} \to \boxed{=}\ \boxed{75}$

Exercise 11.1

Simplify each of the following using a calculator. Indicate the order in which the keys need to be pressed to obtain the correct answer.

a. 45 + 205
b. 350 − 74
c. 824 × 95
d. 3780 ÷ 35
e. 3.52 + 27.7
f. 43.5 − 1.45
g. 7.35 × 6.2
h. 134.784 ÷ 31.2
i. 12.5 ÷ 50 × 4.63
j. 15.84 − 6.75 × 3.52
k. 120.82 ÷ 0.0021 × 5
l. 0.006 ÷ 0.33 × 0.12

Performing calculations using an ordinary calculator or a scientific calculator

Let us consider how simplifications are done using a calculator when more than one operation is involved.

In simplifying 75 + 6 ÷ 3 using an ordinary calculator, when the keys are pressed in the order

$\boxed{\text{ON}} \rightarrow \boxed{7} \rightarrow \boxed{5} \rightarrow \boxed{+} \rightarrow \boxed{6} \rightarrow \boxed{\div} \rightarrow \boxed{3} \rightarrow \boxed{=}$, the operations are performed in the order that they have been entered and 27 is obtained as the answer. That is, a wrong answer is obtained by 75 + 6 ÷ 3 = 81 ÷ 3 = 27.
(This is wrong according to the “BODMAS Rule”.)

When we enter the above data in a scientific calculator in the same order, the operations are performed according to the accepted order of performing mathematical operations and the answer 77 is obtain by performing the operations as follows: 75 + 6 ÷ 3 = 75 + 2 = 77.

Note: When performing calculations using an ordinary calculator, the order in which data is entered should be chosen with care. However, the correct answer can be obtained when data is entered into a scientific calculator in the order in which they appear. Most manufacturers of calculators follow the BODMAS rule when programming their products. However, there are some calculators that do not perform the operations according to this rule. The order in which data should be entered into these calculators is indicated in the instruction booklet that is provided. If such a booklet is not available, an understanding of how the calculator operates can be gained by performing a few simple calculations. Otherwise, the operations that need to be performed initially have to be enclosed within brackets. For example, if the data in the expression 1 − 5 + 12 ÷ 3 × 2 is entered in the given order, some calculators may perform the multiplication before the division. However, according to the BODMAS rule, since multiplication and division have the same priority, moving from left to right, the division should be peformed first.

11.2 Using the $\boxed{\%}$ key in the scientific calculator

The % key is used to calculate percentages. Both the symbols "=" and "%" appear on the same key in most calculators. To activate the % key, press the $\boxed{\text{Shift}}$ key and then the $\boxed{=}$ key.

Example 1

The keys of the calculator need to be pressed in the following order to find 25% of 480.

$\boxed{\text{ON}}$ or $\boxed{\text{AC}} \rightarrow \boxed{4} \rightarrow \boxed{8} \rightarrow \boxed{0} \rightarrow \boxed{\times} \rightarrow \boxed{2} \rightarrow \boxed{5} \rightarrow \boxed{\text{SHIFT}} \rightarrow \overset{\%}{\boxed{=}} \rightarrow \boxed{=} \quad \boxed{120}$

Example 2

Let us express $\frac{2}{8}$ as a percentage. To do this, the keys of the calculator need to be pressed in the following order.

$\boxed{\text{ON}} \rightarrow \boxed{2} \rightarrow \boxed{\div} \rightarrow \boxed{8} \rightarrow \boxed{\text{SHIFT}} \rightarrow \overset{\%}{\boxed{=}} \rightarrow \boxed{=} \quad \boxed{25}$

Example 3

The keys of the calculator need to be pressed in the following order to find 35% of Rs 2500.

$\boxed{\text{ON}} \rightarrow \boxed{2} \rightarrow \boxed{5} \rightarrow \boxed{0} \rightarrow \boxed{0} \rightarrow \boxed{\times} \rightarrow \boxed{3} \rightarrow \boxed{5} \rightarrow \boxed{\text{SHIFT}} \rightarrow \overset{\%}{\boxed{=}} \rightarrow \boxed{=} \quad \boxed{875}$

Example 4

The population of a certain village is 550. Of this number, 66 are school children. To find what percentage of the population are school children, the keys of the calculator need to be pressed in the following order.

$\boxed{\text{ON}} \rightarrow \boxed{6} \rightarrow \boxed{6} \rightarrow \boxed{\div} \rightarrow \boxed{5} \rightarrow \boxed{5} \rightarrow \boxed{0} \rightarrow \boxed{\text{SHIFT}} \rightarrow \overset{\%}{\boxed{=}} \rightarrow \boxed{=} \quad \boxed{12}$

Exercise 11.2

1. Simplify each of the following using a calculator. Indicate the order in which the keys of the calculator need to be pressed to get the correct answer.

a. 350 × 3% **b.** 7520 × 60% **c.** 75.3 × 5%

2. Using a calculator, express each of the following as a percentage.

a. $\frac{1}{5}$ **b.** $\frac{12}{25}$ **c.** $\frac{7}{20}$

Use a calculator to find the solution to each of the following problems.

3. A chair which was produced at a cost of Rs 450 was sold at a profit of 22%. What was the profit?

4. Of the 750 pupils in a certain school, 20% travel to school by bus. How many pupils travel to school by bus?

5. Nimal's monthly salary is Rs 35 000. Of this amount, he deposits Rs 7000 in a savings account. What percentage of his salary does he save?

6. Of the 650 students in a certain school, 143 learn music. Express the number of students who learn music as a percentage of the total number of students in the school.

7. It was stated that the amount of unfilled grain in a stock of paddy is less than 2%. There were 6 kg of unfilled grain in 350 kg of that stock. Was the statement that was made true?

11.3 Performing calculations using the $\boxed{x^2}$ key

To find the value of powers with index two such as 2^2, 5^2 and 3.21^2, we use the $\boxed{x^2}$ key.

Example 1

The order in which the keys need to be pressed to obtain the value of 3^2 is the following.

$\boxed{\text{ON}} \rightarrow \boxed{3} \rightarrow \boxed{x^2} \rightarrow \boxed{=} \rightarrow \boxed{9}$

Example 2

The order in which the keys need to be pressed to obtain the value of 4.1^2 is the following.

$\boxed{\text{AC}} \rightarrow \boxed{4} \rightarrow \boxed{\cdot} \rightarrow \boxed{1} \rightarrow \boxed{x^2} \rightarrow \boxed{=} \boxed{16.81}$

Example 3

The order in which the keys need to be pressed to obtain the value of $5^2 \times 12^2$ is the following.

$\boxed{\text{AC}} \rightarrow \boxed{5} \rightarrow \boxed{x^2} \rightarrow \boxed{\times} \rightarrow \boxed{1} \rightarrow \boxed{2} \rightarrow \boxed{x^2} \rightarrow \boxed{=} \boxed{3600}$

Example 4

The order in which the keys need to be pressed to find the area of a square of side length 6 cm is the following.
Since the area of the square $= 6 \times 6$ cm^2

$\boxed{\text{ON}} \rightarrow \boxed{6} \rightarrow \boxed{x^2} \rightarrow \boxed{=} \rightarrow 36$

$\therefore$ The area of the square is 36 cm^2.

Exercise 11.3

1. Find the value of each of the following powers using a scientific calculator. Indicate the order in which the keys of the calculator need to be pressed to obtain the correct answer.

a. 2^2 **b.** 8^2 **c.** 127^2
d. 3532^2 **e.** 3.5^2 **f.** 6.03^2

2. Find the value of each of the following using a scientific calculator. Indicate the order in which the keys of the calculator need to be pressed to obtain the correct answer.

a. 3×5^2 **b.** $3^2 \times 4^2$ **c.** $(3.5)^2$
d. $4^2 + 3^2$ **e.** $10^2 - 6^2$ **f.** $10^2 - 3^2 \times 5$

11.4 Performing calculations using the $\boxed{\sqrt{\ }}$ key in a scientific calculator

To find the value of the square root of a number we use the $\boxed{\sqrt{\ }}$ key.

Example 1

The order in which the keys of a scientific calculator need to be pressed to find the value of $\sqrt{25}$ is the following.

$\boxed{\text{ON}} \rightarrow \boxed{\sqrt{\ }} \rightarrow \boxed{2} \rightarrow \boxed{5} \rightarrow \boxed{=} \rightarrow 5$

Example 2

The order in which the keys of a scientific calculator need to be pressed to find the value of $\sqrt{44\ 521}$ is the following.

$\boxed{\text{ON}} \rightarrow \boxed{\sqrt{\ }} \rightarrow \boxed{4} \rightarrow \boxed{4} \rightarrow \boxed{5} \rightarrow \boxed{2} \rightarrow \boxed{1} \rightarrow \boxed{=} \rightarrow 211$

Example 3

The order in which the keys of a scientific calculator need to be pressed to find the value of $\sqrt{5.29}$ is the following.

$\boxed{\text{ON}} \rightarrow \boxed{\sqrt{\ }} \rightarrow \boxed{5} \rightarrow \boxed{.} \rightarrow \boxed{2} \rightarrow \boxed{9} \rightarrow \boxed{=}$ 2.3

Exercise 11.4

1. Find the square root of each of the following numbers using a scientific calculator. Indicate the order in which the keys of the calculator need to be pressed to obtain the correct answer.

a. 64 **b.** 81 **c.** 2704
d. 3356 **e.** 3500 **f.** 362404

2. Find the value of each of the following using a scientific calculator. Indicate the order in which the keys of the calculator need to be pressed to obtain the correct answer.

a. $\sqrt{49}$ **b.** $\sqrt{121}$ **c.** $\sqrt{625}$
d. $\sqrt{20.25}$ **e.** $\sqrt{5.76}$ **f.** $\sqrt{0.1225}$

For further knowledge

The order in which the keys of a calculator need to be pressed to find the value of $\sqrt{4^2 + 3^2}$ is the following.

$\boxed{\text{ON}} \rightarrow \boxed{\sqrt{\ }} \rightarrow \boxed{(} \rightarrow \boxed{4} \rightarrow \boxed{x^2} \rightarrow \boxed{+} \rightarrow \boxed{3} \rightarrow \boxed{x^2} \rightarrow \boxed{)} \rightarrow \boxed{=}$ 5

Miscellaneous Exercise

1. Simplify using a scientific calculator. Indicate the order in which the keys need to be pressed to obtain the correct answer.

a. $5 + 6 \div 2 + 4 \times 5$ **b.** $6225 + 37 \times 0.25$ **c.** $42.48 \div 5.31$
d. $428 + 627 \times 5\ \%$ **e.** $5.3^2 \div 6.01$ **f.** $\frac{7}{130} \times 2\ \% + 560$

2. Of the 35 seeds that Saman planted, 21 germinated. Using a scientific calculator, find what percentage of the seeds that were planted germinated.

3. Nimal received a salary increment of 12%. If his salary before the increment was Rs 45 200, how much was his salary after the increment?

4. Find the value of a if $a = 1.33^2$.

5. Find the value of p if $p = \sqrt{18.49 - 2}$.

12

Indices

By studying this lesson you will be able to;

- identify the laws of indices on the product of powers, the quotient of powers and the power of a power,
- simplify algebraic expressions using the above mentioned laws of indices,
- identify the zero index and negative indices and simplify algebraic expressions containing these.

Indices

You have learnt about powers of numbers such as 2^1, 2^2 and 2^3 in previous grades. The values of these powers can be obtained as follows.

$$\begin{aligned} 2^1 &= 2 \\ 2^2 &= 2 \times 2 = 4 \\ 2^3 &= 2 \times 2 \times 2 = 8 \\ &\vdots \qquad \vdots \end{aligned}$$

You have also learnt about powers of algebraic symbols such as x^1, x^2 and x^3 .These can be expanded and written as follows.

$$\begin{aligned} x^1 &= x \\ x^2 &= x \times x \\ x^3 &= x \times x \times x \\ &\vdots \qquad \vdots \end{aligned}$$

Moreover, you have learnt how to write the expanded form of a product of powers of algebraic symbols and numerical values. For example, $5^2a^3b^2$ is written in expanded form as follows.

$$5^2a^3b^2 = 5 \times 5 \times a \times a \times a \times b \times b.$$

You have also learnt that a power of a product such as $(xy)^2$ can be expressed as a product of powers as x^2y^2 and a power of a quotient such as $\left(\frac{x}{y}\right)^2$ can be expressed as a quotient of powers as $\frac{x^2}{y^2}$.

Do the following review exercise to recall what you have learnt in previous grades regarding indices.

Review Exercise

1. Evaluate the following.

i. 2^5 **ii.** $(-3)^2$ **iii.** $(-4)^2$

iv. $\left(\frac{2}{3}\right)^2$ **v.** $(-3)^3$ **vi.** $(-4)^3$

2. Fill in the blanks.

i.
$$\begin{aligned}(xy)^2 &= (xy) \times ......\\ &= ...... \times ...... \times x \times y\\ &= x \times x \times ...... \times ......\\ &= \underline{\underline{x^2 \times y^2}}\end{aligned}$$

ii.
$$\begin{aligned}(pq)^3 &= ...... \times ...... \times ......\\ &= p \times q \times ...... \times ......\\ &= p \times p \times p \times ..... \times ..... \times .....\\ &= \underline{\underline{p^3 \times q^3}}\end{aligned}$$

iii.
$$\begin{aligned}(2ab)^2 &= ...... \times ......\\ &= ...... \times ...... \times b \times ...... \times ...... \times b\\ &= 2 \times 2 \times ...... \times ...... \times ...... \times ......\\ &= \underline{\underline{4a^2b^2}}\end{aligned}$$

iv.
$$\begin{aligned}9p^2q^2 &= ......^2 \times p^2 \times q^2\\ &= .... \times .... \times p \times p \times .... \times ....\\ &= (3 \times p \times q) \times (..... \times ..... \times .....)\\ &= \underline{\underline{(3pq)^2}}\end{aligned}$$

3. Expand and write each of the following expressions as a product.

i. $2a^2$ **ii.** $3x^2y^2$ **iii.** $-5p^2q$

iv. $(-3)^5$ **v.** $(ab)^3$ **vi.** $x^4 \times y^4$

12.1 Products of powers with the same base

2^3 and 2^5 are two powers with the same base.
They can be expanded and written as follows.

$2^3 = 2 \times 2 \times 2$ and
$2^5 = 2 \times 2 \times 2 \times 2 \times 2$

Let us obtain the product of these two powers.

$$\begin{aligned}2^3 \times 2^5 &= (2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2 \times 2)\\ &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2\\ &= 2^8\end{aligned}$$

2 is repeatedly multiplied three times in 2^3. Furthermore, 2 is repeatedly multiplied five times in 2^5.

Therefore, when multiplying 2^3 by 2^5, 2 is repeatedly multiplied $3 + 5 = 8$ times.

This can be expressed as $2^3 \times 2^5 = 2^{3+5} = 2^8$.

It is important to remember that when two powers are multiplied, their indices can be added only if they have the same base. Then the base of the power that is obtained is also this common base.

Let us obtain the product $x^3 \times x^5$ accordingly. Since x^3 and x^5 have the same base, the two indices are added to get the product.

That is, $x^3 \times x^5 = x^{3+5}$

$= x^8$

This can be expressed as a law of indices as follows.

$$a^m \times a^n = a^{m+n}$$

This law can be extended to any number of powers.

For example,

$$a^m \times a^n \times a^p = a^{m+n+p}$$

Let us understand how this law is used to simplify expressions by considering some examples.

Example 1

Simplify the following.

i. $x^2 \times x^5 \times x$ ii. $a^2 \times b^2 \times a^2 \times b^3$ iii. $2x^2 \times 3x^5$

i.

$$\begin{aligned} x^2 \times x^5 \times x &= x^{2+5+1} \text{ (since } x = x^1) \\ &= \underline{\underline{x^8}} \end{aligned}$$

ii.

$$\begin{aligned} a^2 \times b^2 \times a^2 \times b^3 &= a^2 \times a^2 \times b^2 \times b^3 \\ &= a^{2+2} \times b^{2+3} \\ &= a^4 \times b^5 \\ &= \underline{\underline{a^4 b^5}} \end{aligned}$$

iii.

$$\begin{aligned} 2x^2 \times 3x^5 &= 2 \times x^2 \times 3 \times x^5 \\ &= 2 \times 3 \times x^2 \times x^5 \\ &= 6x^{2+5} \\ &= \underline{\underline{6x^7}} \end{aligned}$$

Do the following exercise using the law of indices on the product of powers.

Exercise 12.1

1. Fill in the blanks.

i. $2^5 \times 2^2$

$2^5 \times 2^2 = 2^{\dots + \dots}$

$= \underline{\underline{2^{\dots}}}$

ii. $x^4 \times x^2$

$x^4 \times x^2 = x^{\dots + \dots}$

$= \underline{\underline{x^{\dots}}}$

iii. $a^3 \times a^4 \times a$

$a^3 \times a^4 \times a = a^{\dots + \dots + \dots}$

$= \underline{\underline{a^{\dots}}}$

iv. $5p^3 \times 3p$

$= 5 \times \dots \times 3 \times \dots$

$= 15p^{\dots + \dots}$

$=15\dots$

v. $x^2 \times y^3 \times x^5 \times y^5$

$= x^{\dots} \times x^{\dots} \times y^{\dots} \times y^{\dots}$

$= x^{\dots + \dots} \times y^{\dots + \dots}$

$= \dots^{\dots} \times \dots^{\dots}$

2. Join each expression in column A with the expression in column B which is equal to it.

A	B
$x^3 \times x^7$	x^7
$x^5 \times x^2 \times x$	x^8
$x^7 \times x$	x^9
$x^2 \times x^2 \times x^6$	x^{10}
$x^2 \times x^3 \times x^2 \times x$	

3. Simplify and find the value.

i. $3^5 \times 3^5$

ii. $7^2 \times 7^3 \times 7$

4. Simplify.

i. $x^3 \times x^6$

ii. $x^2 \times x^2 \times x^2$

iii. $a^3 \times a^2 \times a^4$

iv. $2x^3 \times x^5$

v. $5p^2 \times 2p^3$

vi. $4x^2 \times 2x \times 3x^5$

vii. $m^2 \times 2n^2 \times m \times n$

viii. $2a^2 \times 3b^2 \times 5a \times 2b^3$

5. A pair of positive integral values that m and n can take so that the equation $x^m \times x^n = x^8$ holds true is 3 and 5. Write all such pairs of positive integral values.

6. Write a value of a for which the equation $a^2 + a^3 = a^5$ holds if the statement is true and a value of a for which it is false.

12.2 Quotients of powers with the same base

Let us see whether there is a law of indices for the quotients of powers with the same base, similar to the one obtained above for the product.

$x^5 \div x^2$ can also be expressed as $\dfrac{x^5}{x^2}$.

$$\text{Now, } \frac{x^5}{x^2} = \frac{x \times x \times x \times x \times x}{x \times x}$$

$$= x \times x \times x$$

$$= \underline{\underline{x^3}}$$

$\therefore \ \dfrac{x^5}{x^2} = x^3$. When the index of the power in the numerator is 5 and the index of the power in the denominator is 2, then the index of the quotient is $5 - 2 = 3$. The base of the quotient is x, which is the common base of the original two powers.

Therefore, $x^5 \div x^2$ can be simplified easily by subtracting the indices as follows.

$$x^5 \div x^2 = x^{5-2} = x^3$$

When powers with the same base are divided, the index of the divisor is subtracted from the index of the dividend. The base remains the same.

$$a^m \div a^n = a^{m-n}$$

It is important to remember this law of indices too.
Let us understand how this law is used to simplify expressions by considering the following examples.

Example 1

Simplify the following expressions.

a. $x^5 \times x^2 \div x^3$

$$(x^5 \times x^2) \div x^3 = x^{5+2} \div x^3$$

$$= x^{7-3}$$

$$= \underline{\underline{x^4}}$$

b. $4x^8 \div 2x^2$

$$4x^8 \div 2x^2 = \frac{\overset{2}{\cancel{4}}x^8}{\underset{1}{\cancel{2}}x^2}$$

$$= 2x^{8-2}$$

$$= \underline{\underline{2x^6}}$$

c. $\dfrac{a^3 \times a^2}{a}$

$$\frac{a^3 \times a^2}{a} = a^{3+2-1}$$

$$= \underline{\underline{a^4}}$$

Now do the following exercise.

Exercise 12.2

1. Simplify using the laws of indices.

i. $a^5 \div a^3$ **ii.** $\frac{x^7}{x^2}$ **iii.** $2x^8 \div x^3$ **iv.** $4p^6 \div 2p^3$

v. $\frac{10m^5}{2m^2}$ **vi.** $\frac{x^2 \times x^4}{x^3}$ **vii.** $n^5 \div (n^2 \times n)$ **viii.** $\frac{2x^3 \times 2x}{4x}$

ix. $\frac{x^5 \times x^2 \times 2x^6}{x^7 \times x^2}$ **x.** $\frac{a^5 \times b^3}{a^2 \times b^2}$ **xi.** $\frac{2p^4 \times 2q^3}{p \times q}$

2. Write five pairs of positive integral values for *m* and *n* which satisfy the equation $a^m \div a^n = a^8$

3. For each of the algebraic expressions in column A, select the algebraic expression in column B which is equal to it and combine the two expressions using the " = " sign.

A

$2a^5 \div 2a^2$

$a^6 \div a^4$

$\frac{a^7 \times a^2}{a^6}$

$\frac{a^3}{a}$

$\frac{4a^5 \times a}{4a^3}$

B

a

a^2

a^3

12.3 Negative indices

In the previous section we identified that $x^5 \div x^2$ is x^3. We know that this can be obtained by expanding $\frac{x^5}{x^2}$ and simplifying it as follows.

$$\frac{\overset{1}{\cancel{x}} \times \overset{1}{\cancel{x}} \times x \times x \times x}{\underset{1}{\cancel{x}} \times \cancel{x}_1} = x^3$$

Let us simplify $x^2 \div x^5$ in a similar manner.

i. When expanded,

$$\frac{x^2}{x^5} = \frac{\overset{1}{\cancel{x}} \times \overset{1}{\cancel{x}}}{\overset{1}{\cancel{x}} \times \overset{1}{\cancel{x}} \times x \times x \times x}$$

$$= \underline{\underline{\frac{1}{x^3}}}$$

ii. Using laws of indices,

$$\frac{x^2}{x^5} = x^{2-5}$$

$$= \underline{\underline{x^{-3}}}$$

The two simplifications of $x^2 \div x^5$ obtained in (i) and (ii) above must be equal. Therefore, $\frac{1}{x^3} = x^{-3}$. Observe here that the index of the power in the denominator changes signs when it is brought to the numerator. This is an important feature related to indices which can be used when we need to change a negative index into a positive index. We can similarly write $x^3 = \frac{1}{x^{-3}}$.

This law can be expressed as follows.

$$x^n = \frac{1}{x^{-n}}$$

Accordingly, $a^{-m} = \frac{1}{a^m}$, $a^m = \frac{1}{a^{-m}}$, $\frac{a^{-m}}{a^{-n}} = \frac{a^n}{a^m}$ (By applying the above feature to both powers simultaneously)

We can use this law of indices to simplify algebraic expressions as shown in the following examples.

Example 1

Evaluate the following.

(i) 2^{-5} (ii) $\frac{1}{5^{-2}}$

i. $2^{-5} = \frac{1}{2^5}$

$$= \frac{1}{2 \times 2 \times 2 \times 2 \times 2}$$

$$= \underline{\underline{\frac{1}{32}}}$$

ii. $\frac{1}{5^{-2}} = 5^2$

$$= \underline{\underline{25}}$$

Example 2

Simplify: $\dfrac{2x^{-2} \times 2x^{3}}{2x^{-4}}$

$$\begin{aligned}\frac{2x^{-2} \times 2x^{3}}{2x^{-4}} &= \frac{2 \times x^{-2} \times 2 \times x^{3}}{2 \times x^{-4}}\\ &= \frac{\overset{1}{\cancel{2}} \times x^{4} \times 2 \times x^{3}}{\underset{1}{\cancel{2}} \times x^{2}} \qquad \left(\text{since } x^{-2} = \frac{1}{x^{2}} \text{ and } \frac{1}{x^{-4}} = x^{4}\right)\\ &= \frac{2x^{7}}{x^{2}}\\ &= 2x^{7-2}\\ &= \underline{\underline{2x^{5}}}\end{aligned}$$

Exercise 12.3

1. Write each of the following with positive indices.

i. 3^{-4} **ii.** x^{-5} **iii.** $2x^{-1}$ **iv.** $5a^{-2}$ **v.** $5p^{2}q^{-2}$

vi. $\dfrac{1}{x^{-5}}$ **vii.** $\dfrac{3}{a^{-2}}$ **viii.** $\dfrac{2x}{x^{-4}}$ **ix.** $\dfrac{a}{2b^{-3}}$ **x.** $\dfrac{m}{(2n)^{-2}}$

xi. $\dfrac{t^{-2}}{m}$ **xii.** $\dfrac{p}{q^{-2}}$ **xiii.** $\dfrac{x^{-2}}{2y^{-2}}$ **xiv.** $\left(\dfrac{2x}{3y}\right)^{-2}$

2. Evaluate the following.

i. 2^{-2} **ii.** $\dfrac{1}{4^{-2}}$ **iii.** 2^{-7} **iv.** $(-4)^{-3}$ **v.** 3^{-2}

vi. $\dfrac{5}{5^{-2}}$ **vii.** 10^{-3} **viii.** $\dfrac{3^{-2}}{4^{-2}}$

3. Simplify and write the answers with positive indices.

i. $a^{-2} \times a^{-3}$ **ii.** $a^{2} \times a^{-3}$ **iii.** $\dfrac{a^{2}}{a^{-5}} \times a^{-8}$ **iv.** $2a^{-4} \times 3a^{2}$ **v.** $3x^{-2} \times 4x^{-2}$

vi. $\dfrac{10x^{-5}}{5x^{2}}$ **vii.** $\dfrac{4x^{-3} \times x^{-5}}{2x^{2}}$ **viii.** $\dfrac{(2p)^{-2} \times (2p)^{3}}{(2p)^{4}}$

12.4 Zero index

A power of which the index is zero, is known as a power with zero index. 2^{0} is an example of a power with zero index.

When $x^5 \div x^5$ is simplified using the laws of indices we obtain,

$$x^5 \div x^5 = x^{5-5} = x^0$$

When it is expanded and simplified we obtain, $x^5 \div x^5 = \frac{x \times x \times x \times x \times x}{x \times x \times x \times x \times x}$

$$= 1$$

Since the answers obtained when $x^5 \div x^5$ is simplified by the two methods should be the same, we obtain $x^0 = 1$.

$x^0 = 1$ where, x is any number except 0.

This result is also used when simplifying algebraic expressions.

Note:

Example 1

Simplify.

i. $\frac{x^0 \times x^7}{x^2}$

$$\frac{x^0 \times x^7}{x^2} = 1 \times x^7 \div x^2$$
$$= 1 \times x^{7-2}$$
$$= \underline{\underline{x^5}}$$

ii. $\left(\frac{x^5 \times x^2}{a}\right)^0$

$$\left(\frac{x^5 \times x^2}{a}\right)^0 = \underline{\underline{1}}$$

(Since the whole term within brackets is the base, and 0 is the index, its value is 1.)

Let us improve our skills in simplifying expressions which contain powers with zero index by doing the following exercise.

Exercise 12.4

1. Simplify the following expressions.

i. $x^8 \div x^8$ **ii.** $(2p)^4 \times (2p)^{-4}$ **iii.** $\frac{a^2 \times a^3}{a \times a^4}$

iv. $\frac{y^4 \times y^2}{y^6}$ **v.** $\frac{p^3 \times p^5 \times p}{p^6 \times p^3}$ **vi.** $\frac{x^{-2} \times x^{-4} \times x^6}{y^{-2} \times y^8 \times y^{-6}}$

2. Evaluate the following.

i. $2^0 \times 3$ **ii.** $(-4)^0$ **iii.** $\left(\frac{x}{y}\right)^0 + 1$ **iv.** $\left(\frac{x^2}{y^2}\right)^0$ **v.** $5^0 + 1$

vi. $\left(\frac{2}{3}\right)^0$ **vii.** $(2ab)^0 - 2^0$ **viii.** $(abc)^0$

12.5 Power of a power

$(x^2)^3$ is the third power of x^2. A power of this type is known as a power of a power. We can simplify this as follows.

$$\begin{aligned}(x^2)^3 &= x^2 \times x^2 \times x^2\\ (x^2)^3 &= (x \times x) \times (x \times x) \times (x \times x)\\ &= x \times x \times x \times x \times x \times x\\ &= x^6\end{aligned}$$

Therefore, $(x^2)^3 = x^6$.
Observe that the index 6 is 3 twos; that is, 2 × 3. Therefore, we can write $(x^2)^3 = x^{2\times 3} = x^6$.
Hence, when simplifying an algebraic expression of a power of a power, the indices are multiplied.
This is also a law of indices which can be expressed as follows.

$$(a^m)^n = a^{m\times n} = a^{mn}$$

Example 1

Simplify the following.

i. $(a^5)^2 \times a$ **ii.** $(p^3)^4 \times (x^2)^0$ **iii.** $(2x^2y^3)^2$

i. $$\begin{aligned}(a^5)^2 \times a &= a^{5\times 2} \times a\\ &= a^{10} \times a^1\\ &= a^{10+1}\\ &= a^{11}\end{aligned}$$

ii. $$\begin{aligned}(p^3)^4 \times (x^2)^0 &= p^{3\times 4} \times x^{2\times 0}\\ &= p^{12} \times x^0\\ &= p^{12} \times 1\\ &= p^{12}\end{aligned}$$

iii. $$\begin{aligned}(2x^2y^3)^2 &= (2 \times x^2 \times y^3)^2\\ &= 2^2 \times x^4 \times y^6\\ &= 4\,x^4\,y^6\end{aligned}$$

Let us improve our skills in simplifying expressions which contain the power of a power by doing the following exercise.

Exercise 12.5

1. Evaluate the following.

i. $(2^4)^2$ **ii.** $(3^2)^{-1}$ **iii.** $(2^3)^2 + 2^0$

iv. $(5^2)^{-1} + \frac{1}{5}$ **v.** $(4^0)^2 \times 1$ **vi.** $(10^2)^2$

2. Simplify the express using positive indices.

i. $(x^3)^4$

ii. $(p^{-2})^2$

iii. $(a^2 b^2)^2$

iv. $(2x^2)^3$

v. $\left(\frac{x^5}{x^2}\right)^3$

vi. $\left(\frac{a^3}{b^2}\right)^2$

vii. $\left(\frac{m^3}{n^2}\right)^{-2}$

viii. $(p^{-2})^{-4}$

ix. $(a^0)^2 \times a$

Miscellaneous Exercise

1. Evaluate the following.

i. $5^3 \times 5^2$

ii. $5^3 \div 5^2$

iii. $5^0 \times 5 \times 5^2$

iv. $(5^{-1})^2$

v. $\{(5^2)^0\}^4$

vi. $\frac{5^3 \times 5^{-1}}{(5^2)^2}$

vii. $5^2 \div 10^2$

viii. $5^2 \times 10^3 \times 5^{-1} \times 10^{-2}$

2. Simplify the following.

i. $(2x^5)^2$

ii. $(2ab^2)^3$

iii. $2x \times (3x^2)^2$

iv. $\frac{(4p^2)^3}{(2p^2q)^2}$

v. $\frac{(2p^2)^3}{3pq}$

vi. $\frac{(2a^2)^2}{5b^3} \times \frac{(3b^2)^2}{2a}$

Summary

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $x^n = \frac{1}{x^{-n}}$
- $(a^m)^n = a^{m \times n} = a^{mn}$
- $x^0 = 1$ where $x \neq 0$.

13

Rounding off and Scientific Notation

By studying this lesson you will be able to;

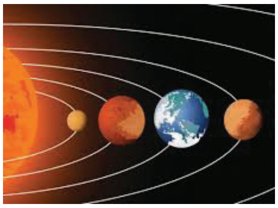
- identify the scientific notation and write numbers up to the millions period in scientific notation,
- convert numbers expressed in scientific notation to normal form,
- identify the rules related to rounding off numbers,
- round off a given number to the nearest ten, nearest hundred, nearest thousand and nearest decimal place,
- solve problems related to rounding off.

Introduction

∢ It is the opinion of scientists that dinosaurs are a species of animals that lived on earth 140 000 000 years ago.

∢ The atomic radius of the Hydrogen atom is 0.000 000 000 053 m.

∢ The distance from the sun to the earth is 149 600 000 000 m.



∢ The speed of light is 299 790 000 meters per second.

The above are four instances where numbers have been used to provide information. Using the information in the last two statements, let us find the time taken for a light ray from the sun to approach earth.

Time =149 600 000 000 ÷ 299 790 000 seconds.

Since there are many digits in each of these numbers, they are lengthy. Therefore more space is required to write them and computations such as the above become difficult. Since a calculator can display only a limited number of characters, it is difficult to do such calculations even with an ordinary calculator. Therefore the need arises to represent such numbers in a more concise way to facilitate calculations.

In this lesson we will learn a method of writing these numbers in a concise way so that it is easy to manipulate them. Let us first do the below given review exercise to recall the facts that have been learnt in previous grades which are relevant to this lesson.

Review Exercise

1. Complete the following table.

Number	As a power of 10
1	1 $= 10^0$
10	10 $= 10^1$
100	10 × 10 $= 10^{\cdots}$
1000	.... × .... ×......... $= 10^{\cdots}$
10000	.... $= 10^{\cdots}$
100000	.... = ...
....	.... $= 10^6$
....	10 × 10 × 10 × 10 × 10 × 10 × 10 = ...

2. Fill in the table given below with the following numbers according to the instructions given in the table.

5.37, 87.5, 0.75, 4.02, 1.01, 10.1, 4575, 0.07, 9, 12.3, 2.7, 9.9

Numbers that are between 1 and 10	
Numbers that are not between 1 and 10	

13.1 Scientific Notation

> The number of students sitting for the G.C.E (O/L) examination this year exceeds 700 000.
>
> \- A news item

Several ways in which the six digit number mentioned in the above news item can be expressed are given below.

i. $700 \times 1000 \longrightarrow 700 \times 10^3$

ii. $70 \times 10\ 000 \longrightarrow 70 \times 10^4$

iii. $\boxed{7 \times 100\ 000 \longrightarrow 7 \times 10^5}$

From the above, the last form is used often as it can be easily written and is the most concise form. It is a product of two parts. The first part is 1 or a number between 1 and 10 while the second part is a power of 10.

$$\underset{\substack{\uparrow \\ \text{Number between} \\ \text{1 and 10 or 1}}}{7} \times \underset{\substack{\nwarrow \\ \text{Power of 10}}}{10^5}$$

Writing a number with many digits in this manner as a product of two numbers, where one is between 1 and 10 or 1 and the other is a power of 10, is known as the scientific notation.

> If A is a number between 1 and 10 or 1 and n is an integer, then $A \times 10^n$ is a number written in scientific notation (Here $1 \leq A < 10$).

Let us write 280 000 in scientific notation.

Taking the first couple of digits in 280 000 and writing it as a number between 1 and 10 we get 2.8.

$$\begin{aligned} \therefore\ 280\ 000 &= 2\,\widetilde{80000} \\ &= 2.8 \times 100\ 000 \\ &= 2.8 \times 10^5 \end{aligned}$$

Therefore 280 000 expressed in scientific notation is 2.8×10^5.

Example 1

Write the following numbers in scientific notation.

a. 20 000 **b.** 4240 **c.** 1 million **d.** 3.47 **e.** 34.7
f. 6 **g.** 289.325 **h.** 2491.32

a. $20\,000 = 2.0 \times 10\,000$
$= \underline{\underline{2 \times 10^4}}$

b. $4240 = 4.24 \times 1000$
$= \underline{\underline{4.24 \times 10^3}}$

c. 1 million $= 1000\,000$
$= \underline{\underline{1 \times 10^6}}$

d. $3.47 = 3.47 \times 1$
$= \underline{\underline{3.47 \times 10^0}}$ (since$1 = 10^0$)

e. $34.7 = 3.47 \times 10$
$= \underline{\underline{3.47 \times 10^1}}$

f. $6 = 6 \times 1$
$= \underline{\underline{6 \times 10^0}}$

g. $289.325 = 2.89325\times 100$
$= \underline{\underline{2.89325 \times 10^2}}$

h. 2491.32

$2491.32 = 2.49132 \times 10^3$

By shifting the decimal point 3 places to the left, we obtain 2.49132×10^3.

Exercise 13.1

1. Complete the following table according to the given examples.

	Number	**1 or a number between 1 and 10 × a power of 10**	**Scientific notation**
	48	4.8×10	4.8×10^1
a.	8		
b.	99		
c.	78		
	548	5.48×100	5.48×10^2
d.	999		
e.	401		
f.	111		
	34 700	3.47×10000	3.47×10^4
g.	54 200		
h.	49 40000		
i.	10 00000		

2. Write each of the following numbers in scientific notation.

a. 200
b. 254
c. 1010
d. 5290
e. 74300
f. 340000
g. 6581200
h. 7.34
i. 18.5
j. 715.8

3. A few important facts about Sri Lanka are given below. Write the numbers which are related to these facts in scientific notation.

The height of Piduruthalagala mountain is 2524 m.
The area of Sinharaja forest is 9300 hectares.
The length of Mahaweli river is 335 km.
The total area of Sri Lanka is 65 610 km^2.

13.2 Writing a number between 0 and 1 in scientific notation

Consider the pattern given below.

$$10\,000 = 10^4$$
$$1000 = 10^3$$
$$100 = 10^2$$
$$10 = 10^1$$
$$1 = 10^0$$
$$0.1 = \frac{1}{10} = \frac{1}{10^1} = 10^{-1}$$
$$0.01 = \frac{1}{100} = \frac{1}{10^2} = 10^{-2}$$
$$0.001 = \frac{1}{1000} = \frac{1}{10^3} = 10^{-3}$$

It is clear that,

when writing 0.1 as a power of 10 the index is –1

when writing 0.01 as a power of 10 the index is −2

when writing 0.001 as a power of 10 the index is –3.

0.75 is a number which is less than 1. When it is written in terms of a number between 1 and 10, it should be written as 7.5 divide by 10. The way this is done mathematically can be expressed as follows.

Since $0.75 \times 10 = 7.5$,

$$\begin{aligned} 0.75 &= \frac{7.5}{10} \\ &= \frac{7.5}{10^1} \quad (\text{Since } 10 = 10^1) \\ &= \underline{\underline{7.5 \times 10^{-1}}} \quad \left(\text{Since } \frac{1}{10^1} = 10^{-1}\right) \end{aligned}$$

Accordingly, the number 0.75 has been expressed as the product of a number between 1 and 10 and a power of 10.

$\therefore$ 0.75 expressed in scientific notation is 7.5×10^{-1} .

In the same manner, let us write 0.0034 in scientific notation.

Since $0.0034 \times 1000 = 3.4$,

$$\begin{aligned} 0.0034 &= \frac{3.4}{1000} \\ &= \frac{3.4}{10^3} \\ &= \underline{\underline{3.4 \times 10^{-3}}} \end{aligned}$$

Note: When a number between 0 and 1 is written in scientific notation, the index of the power of 10 is a negative integer.

Example 1

Express each of the following numbers in scientific notation.

a. 0.8453 **b.** 0.047 **c.** 0.000017

a. $$\begin{aligned} 0.8453 &= 8.453 \div 10 \\ &= \frac{8.453}{10} \\ &= \frac{8.453}{10^1} \\ &= \underline{\underline{8.453 \times 10^{-1}}} \end{aligned}$$

b. $$\begin{aligned} 0.047 &= 4.7 \div 100 \\ &= \frac{4.7}{100} \\ &= \frac{4.7}{10^2} \\ &= \underline{\underline{4.7 \times 10^{-2}}} \end{aligned}$$

c. $$\begin{aligned} 0.000017 & \\ &= 1.7 \div 100000 \\ &= \frac{1.7}{10^5} \\ &= \underline{\underline{1.7 \times 10^{-5}}} \end{aligned}$$

Exercise 13.2

1. Copy the following table and complete it.

Number less than 1	Expressed in terms of a number between 1 and 10	Scientific notation
a. 0.041	$\frac{4.1}{100} = \frac{4.1}{10^2}$	4.1×10^{-2}
b. 0.059		
c. 0.0049		
d. 0.000 135	$\frac{1.35}{10000} = \frac{1.35}{10^4}$	$\ldots\ldots\ldots \times 10^{-4}$
e. 0.000 005		
f. 0.000 003 9		
g. 0.111345		

2. Write each of the following numbers in scientific notation.

a. 0.08

b. 0.543

c. 0.0004

d. 0.0019

e. 0.00095

f. 0.000 000 054

3. Express each of the following numbers in scientific notation.

The radius of an atom is 0.000 0000 01 cm.
The mass of one cubic centimetre of air is 0.00129 g.
The mass of one cubic centimetre of hydrogen is 0.000 088 9 g.

13.3 Converting numbers expressed in scientific notation to general form

As an example, let us convert the number 5.43×10^4 written in scientific notation to general form.

Method I

$5.43 \times 10^4 = 5.43 \times 10000$
$= 54\,300$
$\therefore\ 5.43 \times 10^4 = 54300$

Method II

Since it is multiplied by 10^4, (that is 10 000) shifting the decimal point 4 places to the right, we obtain 54300.

54 300
54 300

Another example is given below. This is an instance where the index of the power of 10 is a negative number.

Method I

$$5.43 \times 10^{-4} = 5.43 \times \frac{1}{10^4}$$
$$= 5.43 \div 10000$$
$$= 0.000543$$

Method II

Since it is divided by 10^4, shifting the decimal point 4 places to the left we obtain 0.000543.

0.000543

Example 1

Convert the following numbers to general form.

(i) 8.9×10^3

(i) $8.9 \times 10^3 = 8.9 \times 1000$

$= \underline{\underline{8900}}$

8900.

(ii) 8.9×10^{-3}

(ii) $8.9 \times 10^{-3} = 8.9 \times \frac{1}{10^3}$

$= \underline{\underline{0.0089}}$

0.0089

Here for example, 8.9×10^3 can be directly written as 8 900. When multiplying, if the index of the power of 10 is a positive integer, then the decimal point should be shifted to the right, the same number of positions as the index (adding zeros if necessary). When multiplying, if the index of the power of 10 is a nagative integer, then the decimal point should be shifted to the left, the same number of positions as the index.

Exercise 13.3

1. Fill in the given blanks to convert each of the following numbers expressed in scientific notation to general form.

i. $5.43 \times 10^3 = 5.43 \times$ ....
$= \underline{\underline{\text{............}}}$

ii. $7.25 \times 10^5 =$ .... × ....
$= \underline{\underline{\text{............}}}$

iii. $6.02 \times 10^1 =$ .... × ....
$= \underline{\underline{\text{............}}}$

iv. $5.99 \times 10^{-2} = 5.99 \times \frac{1}{10^{\cdots}}$
$= \frac{5.99}{\text{.......}}$
$= \underline{\underline{0.0599}}$

v. $1.06 \times 10^{-6} = 1.06 \times$ ....
$= \frac{1.06}{\text{.......}}$
$= \underline{\underline{\text{............}}}$

2. Convert the following numbers to general form.

a.	8.9×10^2	**f.**	7.2×10^{-1}
b.	1.05×10^4	**g.**	8.34×10^{-3}
c.	7.994×10^5	**h.**	5.97×10^{-4}
d.	8.02×10^3	**i.**	9.12×10^{-5}
e.	9.99×10^7	**j.**	5.00×10^{-6}

3. Select the larger number from each of the number pairs given below.

a.	$2.1 \times 10^4, 3.7 \times 10^4$	**d.**	$2.1 \times 10^4, 2.1 \times 10^{-4}$
b.	$2.1 \times 10^4, 3.7 \times 10^3$	**e.**	$2.1 \times 10^4, 3.7 \times 10^{-3}$
c.	$2.1 \times 10^4, 3.7 \times 10^5$	**f.**	$2.1 \times 10^{-4}, 3.7 \times 10^{-3}$

4. Write the following numbers in general form.

The area of the earth covered by land is 1.488×10^8 km^2.
The area of the earth covered by the oceans is 3.613×10^8 km^2.
The total surface area of the earth is 5.101×10^8 km^2.

Rounding off numbers

> It is reported that 2 500 people attended the book exhibition held at Sarasvathi hall over the weekend.
>
> - A news item

The number of tickets that were sold over the weekend to those who attended the exhibition mentioned in the news item was 2 483. Accordingly, the actual number of people who attended the exhibition is 2 483. The number 2 500 which is mentioned in the news item is a number which is close to 2 483, easy to remember and has a special feature. Moreover it is sufficient to communicate an idea of the number that attended the exhibition.

Rounding off a number means representing the value of the number by a value which is close to it, which is simple and, easy to remember and communicate. There are many ways of rounding off numbers. Let us consider a few of them.

13.4 Rounding off to the nearest 10

Representing a number by the multiple of 10 which is nearest to it is known as "rounding off to the nearest 10".

Let us round off 2 483, which is the number of people who attended the exhibition, to the nearest 10. The number 2 483 lies between the two multiples of ten, 2 480 and 2 490. However it is closer to 2 480 than to 2 490. Accordingly, when 2 483 is rounded off to the nearest 10, we obtain 2 480.

We can describe this more generally as follows.

When rounding off 2 481, 2 482, 2 483 and 2 484 to the nearest 10 we obtain 2 480. This is because the multiple of 10 which all these numbers are closest to is 2 480. Similarly, if we round off 2 486, 2 487, 2 488 and 2 489 to the nearest 10 we obtain 2 490. The reason for this is also the same as the above. Even though the remaining number 2 485 is at an equal distance from the two multiples of ten 2 480 and 2 490, when rounding it off to the nearest 10, the convention is to round it off to the nearest 10 which is greater than it, that is, to 2 490. Finally, it is clear that when 2 480 is rounded off to the nearest 10 we obtain 2 480 itself and when 2 490 is rounded off to the nearest 10 we obtain 2 490 itself.

Example 1

Round off to the nearest 10.

i. 273 **ii.** 1428 **iii.** 7196.

i. 270

ii. 1430

iii. 7200

Exercise 13.4

1. Round off each of the following numbers to the nearest 10.

a. 33 **b.** 247 **c.** 3 008
d. 59 **e.** 306 **f.** 4 010
g. 85 **h.** 1514 **i.** 1 895
j. 12 345 **k.** 234 532 **f.** 997 287

2. The height of the mountain Piduruthalagala is 2 524 m. Round off this number to the nearest 10.

3. Write every whole number which when rounded off to the nearest 10 is equal to 140.

4. Write every whole number which when rounded off to the nearest 10 is equal to 80.

What is the smallest whole number which when rounded off to the nearest 10 is 80?
What is the largest whole number which when rounded off to the nearest 10 is 80?

5. When a certain number is rounded off to the nearest 10, the number 260 is obtained. Find separately the least and the greatest value that this number can take.

13.5 Rounding off to the nearest 100 or 1000

"Rounding off to the nearest 100" or "to the nearest 1000" is defined in the same way that "rounding off to the nearest 10" was defined.

For example, the number 7 346 is between the two multiples of hundred, 7 300 and 7 400 and is closer to 7 300 than to 7 400. Therefore when 7 346 is rounded off to the nearest 100, we obtain 7 300. Similarly, if we round off 7 675 to the nearest 100 we obtain 7 700. In general, if we round off a number from 7 300 to 7 349 (both included) to the nearest 100 we obtain 7 300, and if we round off a number from 7 350 to 7 400 (both included) to the nearest 100 we obtain 7 400.

Now, let us consider how to round off numbers to the nearest 1000. For example, when 41 873 is rounded off to the nearest 1000 we obtain 42 000. The reason for this is because 41 873 is closer to 42 000 than to 41 000.

It must be clear to you by this time, what occurs when we round off numbers. Now let us consider a method that can be used to round off numbers easily.

- Let us round off 2 425 to the nearest 100.

2425

The two multiples of 100 between which 2425 lies are 2400 and 2500. The value of 2425 is less than the value of 2450 which is exactly at the centre between these two multiples of 100. Therefore, 2425 is closer to 2400 than to 2500.

Accordingly, when 2425 is rounded off to the nearest 100 we obtain 2400.

- Let us round off 2485 to the nearest 100.

2485

The two multiples of 100 between which 2485 lies are 2400 and 2500. The value of 2485 is greater than the value of 2450 which is exactly at the centre between these two multiples of 100. Therefore, 2485 is closer to 2500 than to 2400.

Accordingly, when 2485 is rounded off to the nearest 100 we obtain 2500.

- Let us round off 2450 to the nearest 100.

2450

The two multiples of 100 between which 2450 lies are 2400 and 2500. The number 2450 is exactly at the centre between these two multiples of 100. According to the convention, the number which is at the centre is rounded off to the nearest multiple of 100 greater than that number.

Accordingly, when 2450 is rounded off to the nearest 100 we obtain 2500.

- Let us round off 2485 to the nearest 1000.

2485

The two multiples of 1000 between which 2485 lies are 2000 and 3000. The value of 2485 is less than the value of 2500 which is exactly at the centre between these two multiples of 1000. Therefore, 2485 is closer to 2000 than to 3000.

Accordingly, when 2485 is rounded off to the nearest 1000 we obtain 2000.

- Let us round off 2754 to the nearest 1000.

2754

The two multiples of 1000 between which 2754 lies are 2000 and 3000. The value of 2754 is greater than the value of 2500 which is exactly at the centre between these two multiples of 1000. Therefore, 2754 is closer to 3000 than to 2000.

Accordingly, when 2754 is rounded off to the nearest 1000 we obtain 3000.

- Let us round off 12 500 to the nearest 1000.

12500

The two multiples of 1000 between which 12 500 lies are 12 000 and 13 000. The number 12 500 is exactly at the centre between these two multiples of 1000. According to the convention, the number which is at the centre is rounded off to the nearest multiple of 1000 greater than that number.

Accordingly, when 12 500 is rounded off to the nearest 1000 we obtain 13 000.

Exercise 13.5

1. Round off each of the following numbers to the nearest 100.

a. 54 **b.** 195 **c.** 1009 **d.** 2985 **e.** 72324 **f.** 7550

2. Round off each of the following numbers to the nearest 1000.

a. 1927 **b.** 2433 **c.** 19999 **d.** 45874 **e.** 38000 **f.** 90500

3. The number of students in a school is 2 059. Round off this number to the,
i. nearest 10
ii. nearest 100
iii. nearest 1000.

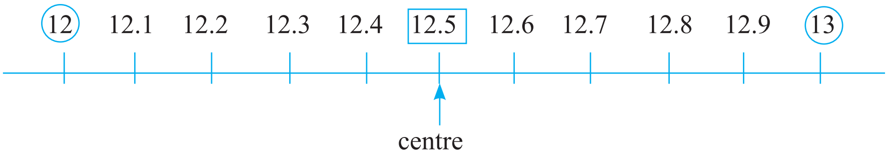
4. When a number is rounded off to the nearest 100, the number 4 500 is obtained.
i. What is the smallest whole number it could be?
ii. What is the largest whole number it could be?

Rounding off decimal numbers

When the mass of a 5 year old child was measured, it was 12.824 kg. If we write this in grammes, it is 12 824 g. This value was obtained because the scale used for this purpose gives the mass to the nearest gramme. However, for practical purposes the mass is usually required to the nearest kilogramme, to the nearest 10th of a kilogramme or to the nearest 100th of a kilogramme.

It is useful to know how to round off any given decimal number to the nearest whole number, nearest first decimal place, nearest second decimal place, etc. In this lesson we will learn how to round off decimal numbers.

Initially, let us consider how to round off a number with one decimal digit to a whole number.
Let us round off 12.7 to the nearest whole number.



The whole numbers on either side of 12.7 are 12 and 13.

Since the numbers 12.1, 12.2, 12.3 and 12.4 are closer to the whole number 12 than to the whole number 13, when these numbers are rounded off to the nearest whole number we obtain 12. Similarly, since the numbers 12.6, 12.7, 12.8 and 12.9 are closer to 13 than to 12, when these numbers are rounded off to the nearest whole number we obtain 13. Furthermore, as in the above sections, 12.5 rounded off to the nearest whole number is accepted by convention to be 13. Accordingly, when 12.7 is rounded off to the nearest whole number we obtain 13.

Similarly,
12.3 rounded off to the nearest whole number is 12 and
12.5 rounded off to the nearest whole number is 13.

Rounding off to a given decimal place

Round off 3.74 to the nearest first decimal place.

In 3.74, the digit in the first decimal place is 7 and the digit in the second decimal place is 4. When rounding off to the first decimal place, the digit in the second decimal place is considered and the digit in the first decimal place is adjusted accordingly if necessary.
The rule used here for rounding off is similar to that used in the previous sections. Since the number with one decimal digit which is closest to the numbers 3.71, 3.72, 3.73 and 3.74 is 3.7, when these numbers are rounded off to the first decimal place we obtain 3.7. Similarly when the numbers 3.75, 3.76, 3.77, 3.78 and 3.79 are rounded off to the first decimal place we obtain 3.8. Accordingly, 3.74 rounded off to the first decimal place is 3.7.

The rule for rounding off numbers to other decimal places is also the same. Let us consider the following example.

Example 2

Round off

i. 3.784 **ii.** 3.796

to the nearest second decimal place.

When rounding off to the nearest second decimal place, the digit in the third decimal place needs to be considered.

i. 3.784 lies between 3.78 and 3.79. Since 3.784 is closer to 3.78 than to 3.79, when it is rounded off to the nearest second decimal place we obtain 3.78.

ii. 3.796 lies between 3.79 and 3.80. Since 3.796 is closer to 3.80 than to 3.79, when it is rounded off to the nearest second decimal place we obtain 3.80.

Exercise 13.6

1. Round off each of the following numbers to the nearest whole number and to the nearest first decimal place.

i. 5.86 **ii.** 12.75 **iii.** 10.43 **iv.** 123.79
v. 8.04 **vi.** 13.99 **vii.** 101.98 **viii.** 100.51

2. The value of π is 3.14159… . Round off this value to

i. the nearest whole number
ii. the nearest first decimal place
iii. the nearest second decimal place.

3. The diameter of a sphere is 3.741 cm. Round off this value to

i. the nearest first decimal place
ii. the nearest second decimal place.

4. According to a survey plan, the area of a plot of land is 0.785 ha. Round off this value to
i. the nearest first decimal place
ii. the nearest second decimal place.

5. In an animal farm, the mean amount of milk obtained from a healthy cow per day is 5.25 l. If there are 45 such animals, round off the amount of milk obtained per a day

i. to the nearest litre

ii. to the nearest first decimal place.

Miscellaneous Exercise

1. Write each of the following groups of numbers in ascending order.

i. 3.10×10^{2} , 3.10×10^{-4} , 3.10×10^{0} , 3.10×10^{5}

ii. 4.78×10^{-2} , 1.43×10^{4} , 9.99×10^{-3} , 2.32×10^{1}

iii. 7.85×10^{0} , 7.85×10^{-4} , 7.85×10^{2} , 7.85×10^{-2}

2. There are 250 labourers working in a factory which pays Rs 1 230 per day as wages to a labourer.

i. Find the amount of money required per day to pay the wages of all these labourers.

ii. Write 1 230 and 250 in scientific notation.

iii. Using the numbers written in (ii) above in scientific notation, find the amount of money required per day for wages.

iv. Compare the values obtained in (i)and (iii) above.

3. The volume of tea produced in a day at a certain tea factory is 1 500 kg. If the factory operates for 30 days during a certain month, show that the volume of tea produced that month is 4.5×10^{4} kg.

4. Fill in the tables given below.

(a)

Expression	The expression obtained when the numbers in the given expression are rounded off to the nearest whole number	The value obtained for the expression by taking the product after rounding off the numbers
59.2 × 9.97	60 × 10	600
8.4 × 5.7	8 × 6	48
12.3 × 11.95	.... ×........	....
10.15 × 127.6	.... ×........	....
459.7 × 3.51	.... ×........	....
109.5 × 4.49	.... ×........	....

(b)

Expression	Product without rounding off the numbers	Value obtained by rounding off the product to the nearest whole number
59.2 × 9.97	590.224	590
8.4 × 5.7		
12.3 × 11.95		
10.15 × 127.6		
459.7 × 3.51		
109.5 × 4.49		

Summary

- Scientific notation is a method of expressing a number concisely to facilitate calculations.
- If $1 \leq A < 10$ and $n \in \mathbb{Z}$ then $A \times 10^n$ is a number expressed in scientific notation.

14

Loci and Constructions

By studying this lesson you will be able to,

- identify four basic loci,
- construct a line perpendicular to a given line,
- construct the perpendicular bisector of a straight line segment,
- construct and copy angles,
- solve problems related to loci and constructions.

Loci

A few motions that you can observe in the environment are given below. Let us consider the path of each motion.

1. Cotton floating in the air
2. A bird flying
3. A ball hit by a bat
4. A fruit falling from a tree
5. The tip of a hand of a working watch
6. A child riding a see-saw

You may observe that even though the motions of 1 and 2 are complex and unpredictable, the motions of 3 to 6 have a definite path. It is important to learn about loci in geometry to develop a proper understanding of the paths of the objects undergoing these motions.

A set of points satisfying one or more conditions is known as a locus.

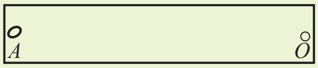
14.1 Basic Loci

Now let us consider four basic loci.

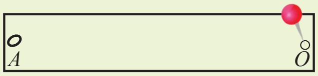
1. The locus of points which are at a constant distance from a fixed point.

Activity 1

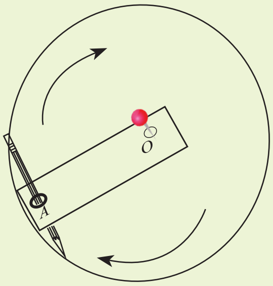
Step 1: Take a strip of cardboard of length 5 cm, make two small holes near the two ends and name them O and A.



Step 2: Keep the above strip of cardboard on a piece of paper, place a pin through the hole O and keep it firm on the piece of paper.



Step 3: Place a pencil point through the hole A and while holding the pin tightly so that it doesn't move, move the pencil and mark the path it takes.



Step 4: At the end of the activity, identify the locus that is obtained.

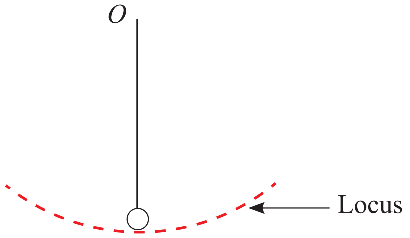
In the above activity, you would have obtained a circular path. Accordingly,

The locus of points on a plane which are at a constant distance from a fixed point is a circle.

Example 1

Draw a sketch of the locus of the bottommost point of the bob of a pendulum in a working pendulum clock.

The locus relevant to this motion is a part of a circle with centre the fixed point of the rod/string to which the bob is connected and radius the distance from this fixed point to the bottommost point of the bob.

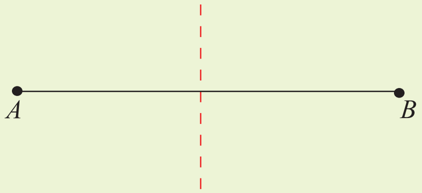


2. The locus of points which are equidistant from two fixed points.

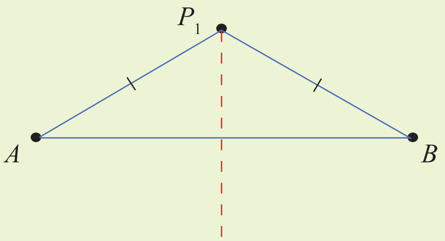
Activity 2

Step 1: Draw a straight line segment of length 10 cm on an oil paper/tissue paper and name it AB.

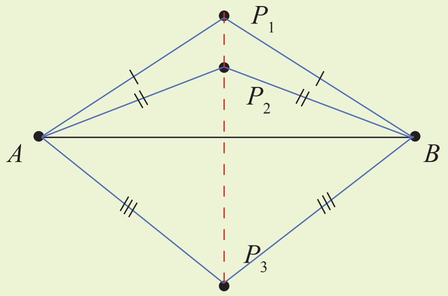
Step 2: Identify the axis of symmetry of the straight line AB by folding the tissue paper such that the two points A and B coincide and mark it with a dashed line.



Step 3: Mark a point P_1 on the dashed line, draw the straight lines P_1A and P_1B, and measure and write their lengths.



Step 4: Mark several more points on the dashed line and measure and write the distances from the points A and B to each of these points.



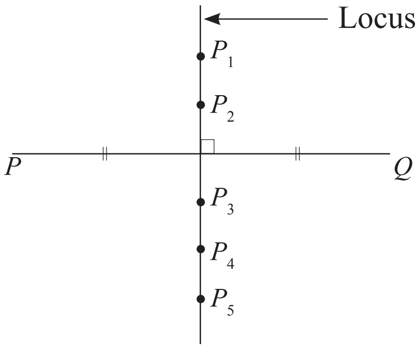
Step 5: Check whether the distances from the points A and B to each of these points on the dashed line are equal and write your conclusion.

When folding the paper as above, such that A and B coincide, observe that the fold line obtained is perpendicular to AB and that it passes through the midpoint of AB. This line is called the perpendicular bisector of the line segment AB. Observe further that the distances from the points A and B to any point you chose on the perpendicular bisector are equal.

The locus of points which are equidistant from two given points is the perpendicular bisector of the line joining the two points.

Example 2

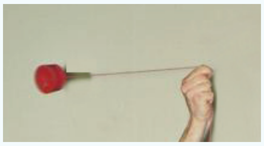
Draw a rough sketch of the locus of points which are equidistant from the two given points P and Q. Name five points on the locus as P_1, P_2, P_3, P_4 and P_5.



Exercise 14.1

1. By a rough sketch, show the locus relevant to each motion given below.

a. The path of a rubber bushing which is tied to one end of a rope of length 50 cm and rotated, by holding the stretched rope at the other end as shown in the figure



b. The path of the tip of a hand of a working clock

c. The figure shows two houses located 50 m from each other on horizontal ground. It is required to build a wall exactly halfway between the two houses (Points A and B). Indicate by a rough sketch where the wall should be built.

c. The path of the fire of a torch held by a fire torch rotating dancer in a perahera (while the fire torch rotator is stationery)

d. The path of a person riding a Ferris wheel

e. The paths of the two children riding a see-saw while sitting at the two ends of the see-saw

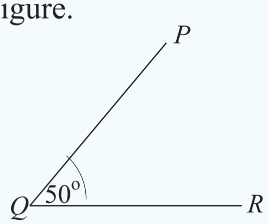
2. In the given figure, P and Q are two trees planted on flat ground, at a horizontal distance of 25 m from each other.

i. It is required to fix a tap at a distance of 15 m from each tree. With the knowledge on loci, draw a rough sketch to indicate how the points where the tap can be fixed are found.

ii. It is required to cut a drain equidistant from the trees. Draw a rough sketch to indicate the location of the drain.

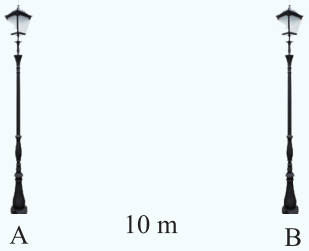
3. Draw an angle of 50° and name it $P\hat{Q}R$ as shown in the figure.

With the knowledge on loci, draw a rough sketch to indicate how the point which is equidistant from the points Q and R and lying on the arm PQ is found, and name it S.



4. A and B are two lamp posts located at a distance of 10 m from each other.

i. It is required to fix another lamp post C at a distance of 6 m from A and 8 m from B. Mark the location of the lamp post C on a suitable rough sketch.



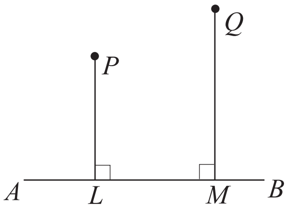
ii. It is required to fix a lamp post D equidistant form the posts A and B. Mark the location of the lamp post D on a suitable rough sketch.

14.2 More on basic loci

3. The locus of points which are at a constant distance from a fixed line

The distance from a point to a line is the length of the perpendicular line drawn from the point to the line.
Accordingly,

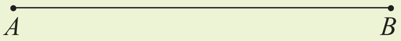
the distance from P to the line AB is the length of PL and the distance from Q to the line AB is the length of QM.



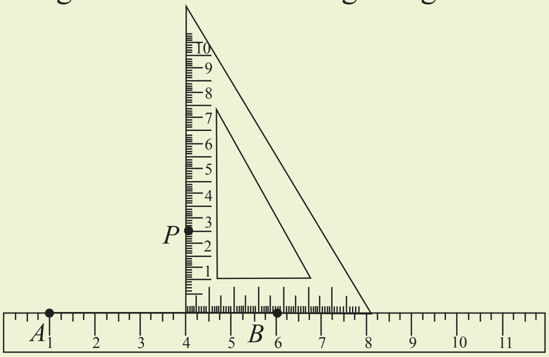
Now let us do the following activity to determine the locus of points which are at a constant distance from a fixed line.

Activity 1

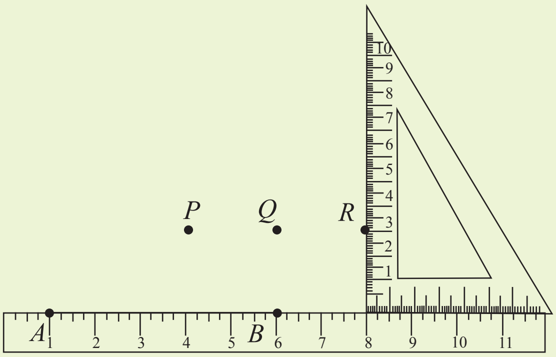
Step 1: Draw a straight line segment in your exercise book and name it *AB*.



Step 2: Place a straight edge on the line *AB* and a set square touching the straight edge as shown in the figure. Mark a point 3 cm from *AB* using the scale on the straight edge and name it *P*.



Step 3: By changing the position of the set square, mark a couple more points at a distance of 3 cm from *AB* and name them *Q* and *R*.



Step 4: Using a straight edge, join the above marked points *P*, *Q* and *R*.

Step 5: Describe the locus of points which are at a distance of 3 cm from the line *AB*. Observe that a similar locus can be drawn on the other side of *AB* too.

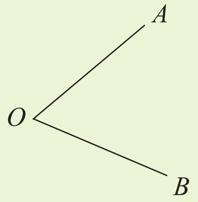
It is clear from the above activity that, the locus of a point at a constant distance of 3 cm from the line *AB* is a straight line parallel to the line *AB* and at a distance of 3 cm from it. Moreover, two loci can be drawn on either side of *AB*.

The locus of points which are at a constant distance from a straight line are the two straight lines parallel to it and at the given constant distance from it, on either side of it.

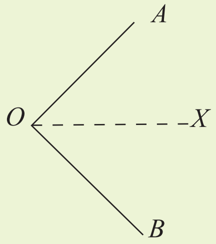
4. The locus of points equidistant from two intersecting straight lines.

Activity 2

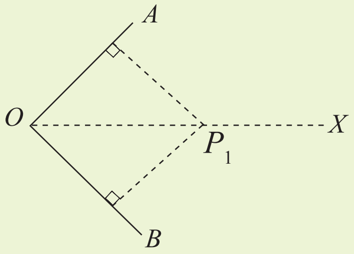
Step 1: On a transparent paper (like oil paper) draw a pair of straight lines as shown in the figure and name them *OA* and *OB*.



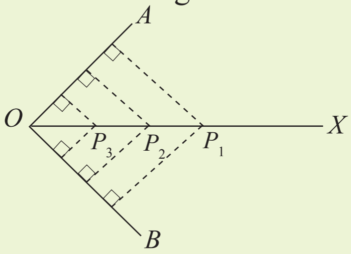
Step 2: Fold the transparent paper such that *OA* and *OB* coincide, and mark the fold line with a dotted line. Name it *OX*.



Step 3: Mark a point on the dotted line and name it P_1. Using a set square, draw two lines from P_1 perpendicular to *OA* and *OB* respectively, measure their lengths and write them down.



Step 4: Mark more points on line OX as shown in the figure and name them P_2, P_3, etc. From each of these points, draw perpendicular lines to OA and OB, measure their lengths and write them down.



Step 5: Measure $A\hat{O}X$ and $B\hat{O}X$ and write what can be concluded about the line OX.

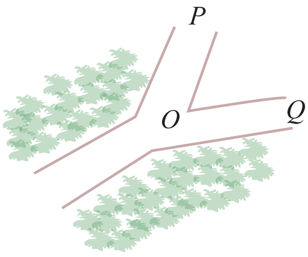
From the above activity it is clear that OX is the line that divides the angle $A\hat{O}B$ into two equal angles and that the distances from any point on the line OX to the lines OA and OB are equal.

Furthermore, since the paper was folded such that OA and OB coincide, the angles $A\hat{O}X$ and $B\hat{O}X$ are equal to each other.

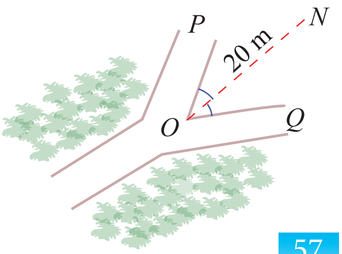
OX is known as the angle bisector of $A\hat{O}B$.

The locus of points equidistant from two intersecting straight lines is the angle bisector of the angles formed by the intersection of the two lines.

Example 1



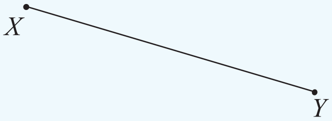
OP and OQ are two roads which diverge at the junction O. It is required to fix a notice board at a point which is 20 m from the junction O and at an equal distance from both these roads. Using the knowledge on loci, indicate by a rough sketch how you would find the place where the notice board should be fixed.



The required position (say N) should be a point on the angle bisector of $Q\hat{O}P$. Since $ON = 20$ m, N should be the point on the angle bisector a distance of 20 m from O.

Exercise 14.2

1. Draw a straight line segment and name it XY. Illustrate by a rough sketch, the locus of points which are 4 cm away from it.



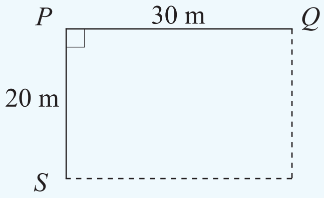
2. A student walks on a straight road rotating a wheel of diameter 20 cm which is fixed to a handle. Illustrate by a rough sketch the locus of the centre of the wheel.

3. The figure shows the positions of the hour hand and the minute hand of a clock at a certain instant. At this moment, the second hand is located at an equal distance from these two hands. Indicate the position of the second hand by a separate rough sketchs.

4. *A* drain PQ of length 50 m which is located in a certain plot of land is shown in the figure. A tap needs to be fixed at a distance of 10 m from PQ and at an equal distance from both the ends P and Q. Illustrate by a rough sketch the position/positions where the water tap can be fixed.

5. A piece of cake cut from a round (circular) cake is shown in the figure. It is required to divide this piece of cake into two equal pieces. Using the knowledge on loci, indicate by a sketch how this piece should be cut.

6. PQ and PS are two boundaries of a rectangular plot of land. A tree needs to be planted in this plot of land such that it is 8 m from the boundary PQ and 5 m from the boundary PS. Illustrate by a rough sketch where the tree should be planted and name it T.



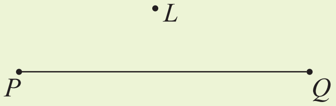
14.3 Constructing lines perpendicular to a given straight line

Let us explain two phrases that are commonly used in constructions. When drawing a circle using a pair of compasses, phrases such as "taking a certain point as the centre" and "taking a certain length as the radius" are often used. For example, "Taking point *A* as the centre" means the circle or arc should be drawn with the point of the pair of compasses kept at the point *A*; and "Taking *AB* as the radius" means that the distance between the point of the pair of compasses and the pencil point should be equal to the length of *AB*.

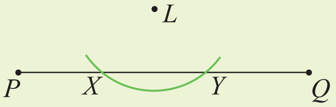
1. Constructing a line perpendicular to a given line from an external point

Activity 1

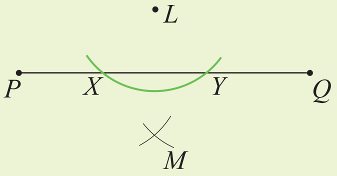
Step 1: Draw a straight line segment in your exercise book and name it *PQ*. Mark a point external to *PQ* and name it *L*.



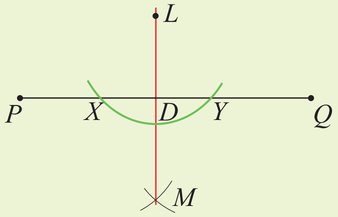
Step 2: Taking a length which is more than the distance from *L* to *PQ* as the radius and *L* as the centre, drawn arc such that it intersects the line *PQ*. Name the points of intersection *X* and *Y*.



Step 3: Taking each of the points *X* and *Y* as the centre and using the same radius, draw two arcs such that they intersect each other as shown in the figure. Name the point of intersection *M*.



Step 4: Join the points L and M and name the point at which LM intersects PQ as D. Measure and write the magnitude of $L\hat{D}P$.

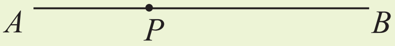


At the end of the above construction, you would have obtained that $L\hat{D}P = 90^{o}$. That is, LD is the perpendicular line drawn from the point L to the line PQ.

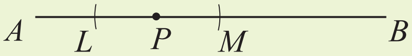
2. Constructing a line perpendicular to a given line through a point on the line

Activity 2

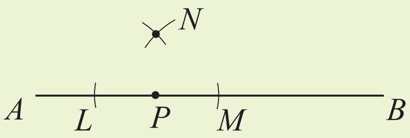
Step 1: Draw a straight line and name it AB. Mark a point on it and name it P.



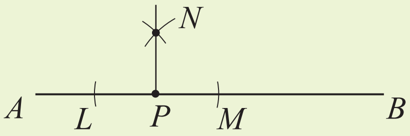
Step 2: Taking a length less than the length of PA as the radius, and taking P as the centre, draw two arcs using the pair of compasses such that they intersect the line segments PA and PB. Name the two points of intersection L and M.



Step 3: Taking a length greater than the one taken in step 2 as the radius, and taking L and M as the centres, draw two arcs such that they intersect each other as shown in the figure. Name the point of intersection N.



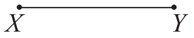
Step 4: Join *NP*, measure the magnitude of the angle $N\hat{P}A$ and write its value.



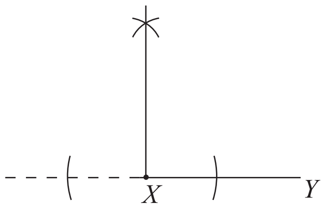
At the end of the above construction you would have obtained that $N\hat{P}A = 90^\circ$. That is, the line drawn perpendicular to *AB* through the point *P* is *PN*.

3. Constructing a line perpendicular to a given straight line segment through an end point

Let us assume that we need to draw a line perpendicular to the line segment *XY* through the point *X*.



Produce the line *YX* and do this construction using the method identified above.

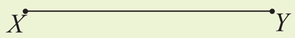


4. Constructing the perpendicular bisector of a straight line segment

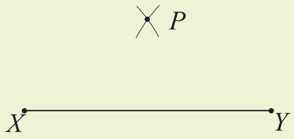
The straight line which is perpendicular to a given line segment and which passes through the midpoint of that line segment, was identified earlier as the perpendicular bisector of that line segment.

Draw a straight line segment and name it *XY*. Let us do the activity given below to construct the perpendicular bisector of this line segment.

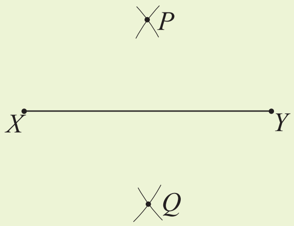
Activity 3



Step 1: Taking a length greater than half the length of XY as the radius, and without changing it, draw two arcs with X and Y as the centres, such that they intersect each other. Name the point of intersection P.

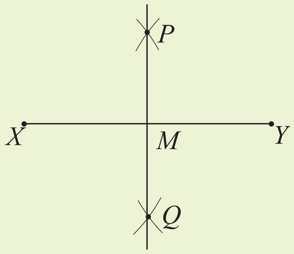


Step 2: As done above, taking X and Y as the centres, draw two other arcs such that they intersect each other on the side of XY opposite to the side on which P is located. Name the point of intersection Q.



Note: It is not necessary to use the same radius in the above two steps.

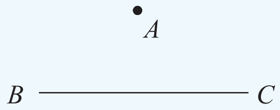
Step 3: Join PQ and name the point at which PQ intersects XY as M. Measure XM and MY and the magnitude of $X\hat{M}P$. What can be concluded regarding the line PQ?



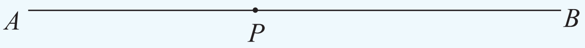
You would have identified in the above activity that $XM = MY$ and $X\hat{M}P = 90^{\circ}$. Accordingly, PQ bisects the line segment XY perpendicularly. Therefore, PQ is the perpendicular bisector of XY.

Exercise 14.3

1. Draw a straight line as shown in the figure and name it *BC*. Construct a perpendicular line from the point *A* to the line *BC*.



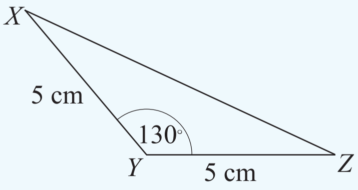
2. Draw the line *AB* such that *AB* = 7 cm. Mark the point *P* on *AB* such that *AP* = 3 cm and construct a line perpendicular to *AB* through *P*.



3. Draw any acute angled triangle and name it *PQR*.

- **i.** Construct a line perpendicular to *QR* from *P*.
- **ii.** Construct a line perpendicular to *PR* from *Q*.
- **iii.** Construct a line perpendicular to *PQ* from *R*.

4. i. Using a protractor, draw an angle of 130° and as shown in the figure mark 5 cm on each arm and complete the triangle *XYZ*.



- **ii.** Construct a perpendicular line from *Y* to the line *XZ* and name the point at which it meets *XZ* as *D*.
- **iii.** Measure and write the lengths of *XD* and *ZD*.

5. Construct a rectangle of length 6 cm and breadth 4 cm.

6. **a.** Draw a straight line segment *PQ* such that *PQ* = 10 cm.
- **b.** Mark the point *B* on the line *PQ* such that *PB* = 2 cm.
- **c.** Construct a line perpendicular to *PQ* through *B*.
- **d.** Mark a point *A* on the perpendicular line such that *BA* = 6 cm and complete the triangle *ABQ*.

e. Construct the perpendicular bisector of the line segment *BQ* and name the point it intersects *AQ* as *O*.

f. Construct a circle with *O* as the centre and *OA* as the radius.

14.4 Constructions related to angles

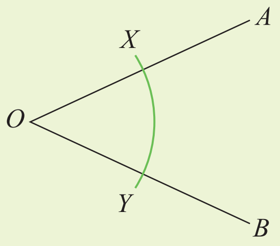
Constructing the angle bisector

The line drawn through a given angle such that it divides the angle into two equal angles, is known as the angle bisector of the given angle.

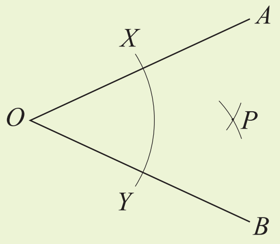
Activity 1

Draw any angle and name it $A\hat{O}B$. Perform the following steps to construct the bisector of this angle.

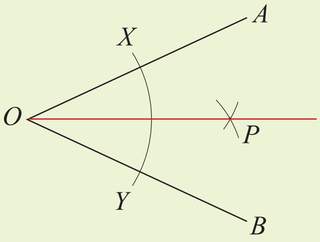
Step 1: Draw an arc with *O* as the centre such that it intersects the arms *OA* and *OB*. Name the points of intersection *X* and *Y*.



Step 2 : Using a pair of compasses and taking a suitable radius, construct two arcs with *X* and *Y* as the centres such that they intersect each other as shown in the figure. Name the point of intersection *P*.



Step 3 : Join *OP*. Measure $A\hat{O}P$ and $B\hat{O}P$ and check whether they are equal.

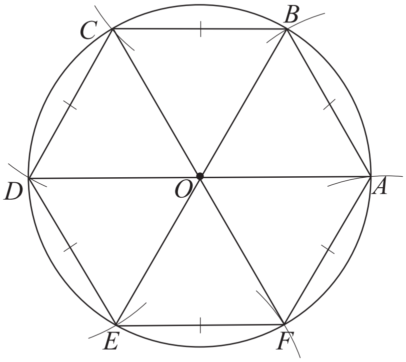


It must have been clear to you at the end of the above activity that $A\hat{O}P = B\hat{O}P$. That is, *OP* is the angle bisector of $A\hat{O}B$.

14.5 Construction of angles

By now we have learnt to draw angles using the protractor. However we can construct a few special angles by using a straight edge and a pair of compasses only. Let us recall how we constructed a regular hexagon in grade 8 by using a pair of compasses.

Here, taking the length of a side of the regular hexagon which needs to be drawn as the radius, a circle is drawn, and with the same radius, arcs are marked on it.The points at which the arcs intersect the circle are joined to each other and to the centre as shown in the figure.



Then every angle of each equilateral triangle that is formed is 60°.

Therefore, $A\hat{O}B = 60^{\circ}$ and $A\hat{O}C = 120^{\circ}$.

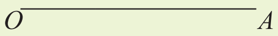
Let us use the principles that were used in this construction to construct certain special angles.

1. Constructing an angle of 60°

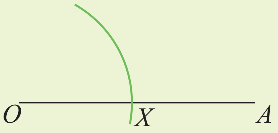
Activity 1

Suppose we need to construct an angle of 60°at O with OA as an arm.

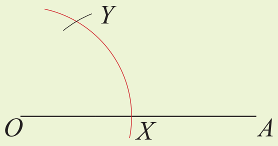
Step 1: Draw a straight line segment in your exercise book and name it OA.



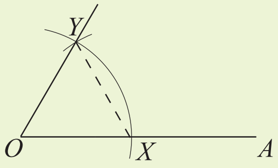
Step 2: Taking O as the centre, construct an arc such that it intersects OA as shown in the figure. Name the point of intersection X.



Step 3: Without changing the length of the radius, and taking X as the centre, draw another arc using the pair of compasses, such that it intersects the first arc. Name this point of intersection Y.



Step 4: Join the points O and Y and produce it as required. Measure $A\hat{O}Y$ and check whether it is 60°.



The triangle OXY in the above figure is an equilateral triangle. The reason for this can be explained as follows.

Since OX and OY are radii of the circle with centre O, $OX = OY$.
Similarly, since XO and XY are radii of the circle with centre X, $XO = XY$.
Accordingly, $OX = XY = OY$.
Therefore, OXY is an equilateral triangle.
Therefore, every angle of it is 60°.

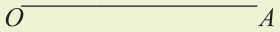
Therefore $X\hat{O}Y = 60^{o}$.

Suppose we need to construct an angle of 120° at O with OA as an arm.

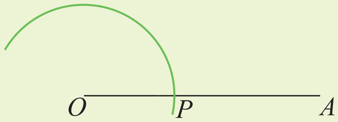
2. Constructing an angle of 120°

Activity 2

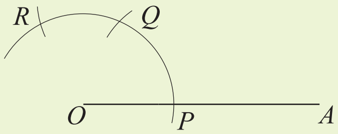
Step 1: Construct a straight line segment and name it OA.



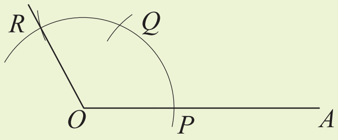
Step 2: Taking O as the centre, construct an arc such that it intersects OA as shown in the figure. Name the point of intersection P.



Step 3: Without changing the length of the radius, and taking P as the centre, draw a small arc using the pair of compasses, such that it intersects the first arc as shown in the figure, and name that point of intersection Q. Now, without changing the radius, take Q as the centre and draw another small arc such that it too intersects the first arc and name that point of intersection R.



Step 4: Join OR and produce it as required. Measure and check the magnitude of $A\hat{O}R$.



The reason why $A\hat{O}R = 120^{o}$ is the following. As discussed above, $P\hat{O}Q = 60^{o}$. Furthermore, QOR is also an equilateral triangle. Therefore, $Q\hat{O}R = 60^{o}$. Accordingly,

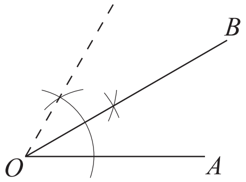
$$\begin{aligned} P\hat{O}R &= P\hat{O}Q + Q\hat{O}R \\ &= 60^{o} + 60^{o} \\ &= 120^{o} \end{aligned}$$

3. Constructing angles of 30°, 90° and 45°

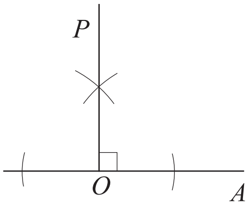
By constructing suitable angle bisectors we can construct the angles 30°, 90° and 45°. By considering the information and figures given below construct the given angles.

Angle of 30°

Construct an angle of 60° and construct its angle bisector. Then $A\hat{O}B = 30^{o}$.

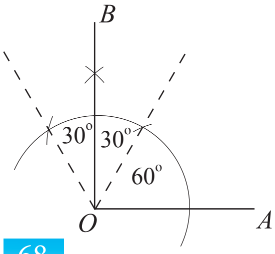


Angle of 90°



Method I

At O, construct a line perpendicular to the line segment AO. Then $A\hat{O}P = 90^{o}$.



Method II

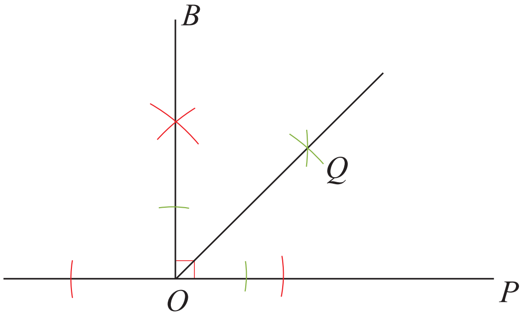
Construct an angle of 120°and bisect one 60° angle.

Then $A\hat{O}B = 90^{o}$.

Angle of 45°

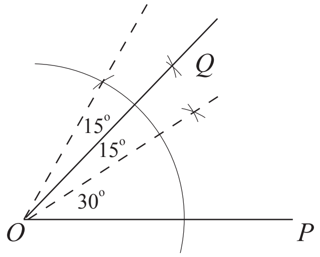
Method I

Construct an angle of 90° and bisect it. Then $P\hat{O}Q = 45^{\circ}$.



Method II

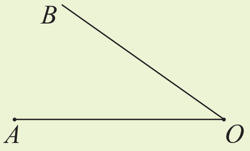
Construct an angle of 60° and bisect it. Again bisect one of the resulting 30° angles. Then, $P\hat{O}Q = 30^{\circ} + 15^{\circ} = 45^{\circ}$.

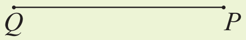


Copying a given angle

Let us suppose that we need to construct an angle equal to a given angle $A\hat{O}B$ at a point P, with PQ as an arm. For this, let us do the following activity.

Activity 3

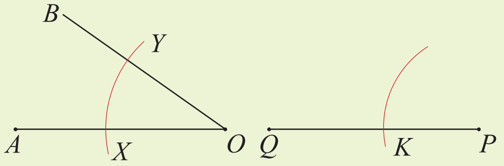




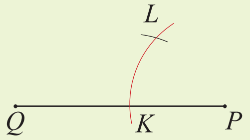
Step 1: Draw any angle and name it $A\hat{O}B$. Draw the arm PQ on which $A\hat{O}B$ needs to be copied.

Step 2: Taking O as the centre, draw an arc as shown in the figure such that it intersects the arms OA and OB, and name the points of intersection X and Y. Using the same radius and taking P as the centre, draw an arc longer than the previous arc such that it intersects PQ.

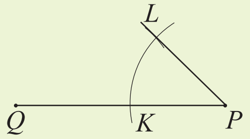
Name the point at which the arc intersects PQ as K.



Step 3: Taking XY as the length of the radius and K as the centre, using the pair of compasses, construct a small arc such that it intersects the initial arc and name the point of intersection L.



Step 4: Join PL and produce it as required. Using a protractor (or any other method), check whether $A\hat{O}B$ and $Q\hat{P}L$ are equal.



Exercise 14.4

1. **i.** Draw a straight line segment of length 8 cm and name it PQ.
   ii. Construct an angle of $60°$ at P such that PQ is an arm.
   iii. Construct an angle of $60°$ at Q such that QP is an arm.

2. **i.** Draw a straight line segment of length 6.5 cm and name it AB.
   ii. Construct an angle of $90°$ at A such that AB is an arm.

iii. Construct an angle of 30°at B such that BA is an arm.

iv. Produce the constructed lines so that they intersect. Name their point of intersection as C and form the triangle ABC.

3. Construct angles of magnitude 15° and 75°.

4. To construct the triangle shown in the figure below, do the following constructions.

i. Draw a straight line segment of length 7 cm and name it PQ.

ii. Construct an angle of 30° at P such that PQ is an arm.

iii. Construct an angle of 45° at Q such that QP is an arm.

iv. Complete the triangle PQR and measure and write the magnitude of $P\hat{R}Q$.

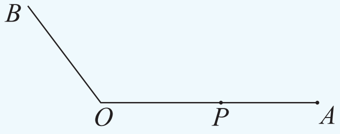
5.

i. Draw a straight line segment OA of length 10 cm.

ii. Draw an arm BO such that $A\hat{O}B$ is an obtuse angle.

iii. Mark the point P on OA such that $OP = 7$ cm.

iv. Construct a line segment PC such that C is on the same side of OA as B and such that $A\hat{P}C = A\hat{O}B$.



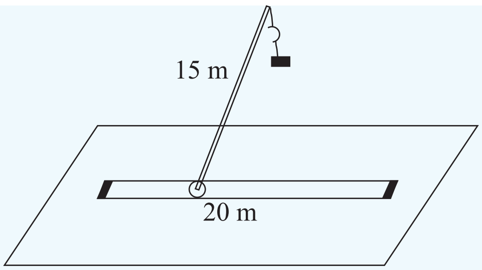
6. **i.** Draw any acute angle and name it $K\hat{L}M$.

ii. Copy the angle $\hat{L}$ at M such that $K\hat{L}M = L\hat{M}N$, where N is on the same side of LM as K.

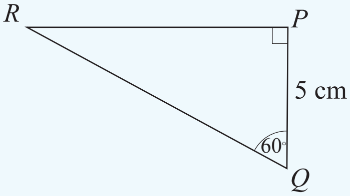
iii. Name the point of intersection of the lines LK and MN as P (produce the lines if necessary) and measure and write the lengths of PL and PM.

Miscellaneous Exercise

1. In a factory, a 15 m long arm of a crane is fixed to a groove of length 20 m. It can be moved along the groove and also rotated in a horizontal plane about the end points of the groove. Draw a rough sketch, and indicate with measurements the path on the horizontal plane where the crane can exchange goods.

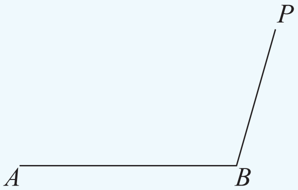


2. To construct the triangle shown in the figure, carry out the steps given below.



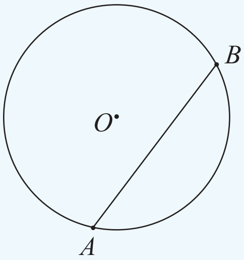
i. Draw a straight line segment PQ where $PQ = 5$ cm.
ii. Construct an angle of 90° at P.
iii. Construct an angle of 60° at Q.
iv. Complete the triangle PQR and measure and write the magnitude of $\hat{R}$.

3. i. As shown in the figure, draw an obtuse angle $A\hat{B}P$.



ii. Locate a point K such that $A\hat{B}P = B\hat{P}K$ and such that the two angles form a pair of alternate angles. Join PK.

4. **i.** Draw a circle of radius 4 cm and name its centre O.
ii. Mark two points A and B on the circle 6 cm apart from each other, and draw the line AB.
iii. Construct a perpendicular line from O to AB and name the point at which it meets AB as N.
iv. Measure and write the lengths of AN and BN.



Summary

A set of points satisfying one or more conditions is known as a locus.

Basic Loci

- **The locus of points on a plane which are at a constant distance from a fixed point is a circle.**
- **The locus of points which are equidistant from two given points is the perpendicular bisector of the line joining the two points.**
- **The locus of points which are at a constant distance from a straight line are the two straight lines parallel to it and at the given constant distance from it, on either side of it.**
- **The locus of points equidistant from two intersecting straight lines is the angle bisector of the angles formed by the intersection of the two lines.**

15

Equations

By studying this lesson you will be able to;

- solve linear equations containing brackets,
- solve linear equations containing fractions,
- solve simultaneous linear equations when the coefficient of one unknown is equal in both equations.

Linear equations

Do the following exercise to recall the facts that you have learnt in previous grades on solving linear equations.

Review Exercise

Solve the following linear equations.

a. $x + 12 = 20$

b. $x - 7 = 2$

c. $5 + m = 8$

d. $2x = 16$

e. $-3x = 6$

f. $2p + 1 = 5$

g. $3b - 7 = 2$

h. $\frac{x}{2} = 3$

i. $\frac{2p}{2} = 6$

j. $\frac{m}{5} - 1 = 8$

k. $2(x + 3) = 11$

l. $3(1 - x) = 9$

15.1 Solving linear equations with two types of brackets

You may have observed that there are some equations with brackets in the review exercise. In this lesson we expect to learn how to solve linear equations with two types of brackets. Let us first consider how to construct a linear equation with several brackets and find its solution.

Note: There are several types of brackets that we use.

( ) → Parentheses

{ } → Curly Brackets

[ ] → Square Brackets

When applying brackets, the usual practice is to first use parentheses, then curly brackets and finally square brackets.

“The result of adding three to a certain number and subtracting one from twice this value, and finally multiplying the resulting value by five and adding two is equal to 47”.

Let us consider how to construct an equation using the above information and then solve it.

If the number is x, when 3 is added, we obtain $x + 3$.

Twice this expression can be written as $2(x + 3)$ using parentheses.

The expression that is obtained when 1 is subtracted from this is $2(x+3) - 1$.

Using curly brackets to write five times this expression we obtain,

$$5\{2\ (x + 3) - 1\}$$

It is given that when 2 is added to this expression it is equal to 47. Therefore,

$$5\{2\ (x + 3) - 1\} + 2 = 47$$

Now, by solving this equation, let us find the value of the number (x).

First, by simplifying the expression with parentheses we obtain

$$5\{2x + 6 - 1\} + 2 = 47.$$

When we simplify the expression within curly brackets we obtain,

$$5\{2x + 5\} + 2 = 47.$$

Now, simplifying the expression with curly brackets we obtain

$$10x + 25 + 2 = 47.$$
$$10x + 27 \quad = 47$$

Subtracting 27 from both sides we obtain,

$$10x + 27 - 27 = 47 - 27.$$

That is, $10x = 20$.

Dividing both sides by10 we obtain,

$$\frac{10x}{10} = \frac{20}{10}$$
$$x = 2$$

Therefore, the number is 2.

Let us consider a few more examples of equations with brackets to improve our skills of solving such equations.

Example 1

Solve $2\{3\,(2x-1)+4\}=38$.

$\frac{2\{3\,(2x-1)+4\}}{2}=\frac{38}{2}$ (dividing both sides by 2)

$3(2x-1)+4=19$

$6x-3+4\ =19$ (simplifying the expression with parentheses)

$6x+1=19$

$6x+1-1=19-1$ (subtracting 1 from both sides)

$6x=18$

$\frac{6x}{6}=\frac{18}{6}$ (dividing both sides by 6)

$\underline{\underline{x=3}}$

Example 2

Solve $5\{4\,(x+3)-2\,(x-1)\}=72$.

$5\{4\,(x+3)-2\,(x-1)\}=72$

$5\{4x+12-2x+2\}=72$ (simplifying the expression with parentheses)

$5\{2x+14\}=72$

$10x+70=72$ (simplifying the expression with curly brackets)

$10x+70-70=72-70$ (subtracting 70 from both sides)

$\frac{10x}{10}=\frac{2}{10}$ (dividing both sides by 10)

$\underline{\underline{x=\frac{1}{5}}}$

Exercise 15.1

Solve the following equations.

a. $2\{2\,(x-1)+2\}=18$

b. $5\{3\,(x+2)-2\,(x-1)\}=60$

c. $6+2\,\{x+3\,(x+2)\}=58$

d. $5\{2+3\,(x+2)\}=10$

e. $2\,\{3\,(y-1)-2y\}=2$

f. $7x+5\,\{4-(x+1)\}=17$

15.2 Solving linear equations containing fractions

Now, let us consider how to construct a linear equation with fractions and find its solution.

A vendor bought a stock of mangoes to sell. He discarded 10 fruits which were rotten. The rest he divided into 12 equal piles of 5 mangoes each.

Let us construct an equation using the above data.

If the vender bought x mangoes to sell,
when 10 mangoes are discarded, the remaining amount is $x - 10$.
The number of piles that can be made when the remaining mangoes are divided into groups of 5 is $\frac{x-10}{5}$. It is given that the number of piles is 12.

Therefore, $\frac{x-10}{5} = 12$

Now, let us solve this equation and find x.

$$\frac{x-10}{5} = 12$$

Multiplying both sides of the equation by 5,

$$5 \times \frac{x-10}{5} = 12 \times 5$$

$$x - 10 = 60$$

Adding 10 to both sides,

$$x - 10 + 10 = 60 + 10$$

$$x = 70$$

Therefore, the vendor bought 70 mangoes to sell.

Let us study the following examples to learn more on solving linear equations with fractions.

Example 1

Solve $\frac{x+3}{2} = 15.$

$$\frac{x+3}{2} = 15$$

$$2 \times \frac{x+3}{2} = 15 \times 2 \text{ (multiplying both sides by 2)}$$

$$x + 3 = 30$$

$$x + 3 - 3 = 30 - 3 \text{ (subtracting 6 from both sides)}$$
$$\underline{\underline{x = 27}}$$

Example 2

Solve $\frac{y}{2} - \frac{y}{3} = 9.$

$$\frac{y}{2} - \frac{y}{3} = 9$$
$$6 \times \frac{y}{2} - 6 \times \frac{y}{3} = 9 \times 6$$ (multiplying both sides by 6, the L.C.M. of the denominators 2 and 3)
$$3y - 2y = 54$$
$$\underline{\underline{y = 54}}$$

Example 3

Solve $2\left(\frac{m}{3} - 1\right) = 10.$

$$2\left(\frac{m}{3} - 1\right) = 10$$
$$\frac{2}{2}\left(\frac{m}{3} - 1\right) = \frac{10}{2}$$ (dividing both sides by 2)
$$\frac{m}{3} - 1 = 5$$
$$\frac{m}{3} - 1 + 1 = 5 + 1$$ (adding 1 to both sides)
$$\frac{m}{3} = 6$$
$$3 \times \frac{m}{3} = 6 \times 3$$ (multiplying both sides by 3)
$$\underline{\underline{m = 18}}$$

Note: When solving equations, it is not necessary to write the reason for each simplification.

Exercise 15.2

Solve each of the following equations.

a. $\frac{x-2}{5} = 4$

b. $\frac{y+8}{3} = 5$

c. $\frac{2a}{3} + 1 = 7$

d. $\frac{5b}{2} - 3 = 2$

e. $\frac{2p+3}{4} = 5$

f. $\frac{3m-2}{7} = 4$

g. $\frac{3x}{2} + \frac{x}{4} = 7$ **h.** $\frac{2m}{3} - \frac{3m}{5} = 1$ **i.** $4\left(\frac{3x}{2} - 1\right) = 12$

j. $\frac{1}{3}\left(\frac{2a}{3} - 3\right) = 2$ **k.** $\frac{m-3}{2} + 1 = 4$ **l.** $\frac{x+1}{2} + \frac{x}{3} = 8$

m. $\frac{y+1}{2} + \frac{y-3}{4} = \frac{1}{2}$ **n.** $\frac{x+3}{2} - \frac{x+1}{3} = 2$

15.3 Solving simultaneous equations

You have learnt in previous grades and in the earlier section of this lesson how to find the value of the unknown by solving a linear equation.
In this section we will learn how to solve linear equations with two unknowns.

Suppose it is given that the sum of two numbers is 6.

If we take the two numbers as x and y, then we can construct the equation $x + y = 6$, based on the given statement.
Here, x and y are not unique. The following table shows several different pairs of values of x and y which satisfy the above equation.

x	y	$x + y$
-1	7	6
0	6	6
1	5	6
2	4	6
3	3	6
4	2	6
5	1	6
6	0	6

Table 1

By observing the above table, we can conclude that there are infinitely many pairs of values of x and y which satisfy the equation $x + y = 6$.
If there is another relationship between x and y, we can construct another equation and by solving both equations simultaneously we can find the values of x and y that satisfy both equations.

Suppose it is given that the difference of the two numbers is 2. If we take the larger number as x, we can construct the equation $x - y = 2$, based on the given statement.

There are infinitely many pairs of values of x and y which satisfy this equation too as can be concluded from observing the following table.

x	y	$x - y$
6	4	2
5	3	2
4	2	2
3	1	2
2	0	2
1	–1	2

Table 2

By observing Tables 1 and 2, you can see that there is only one pair of values of x and y which satisfies both $x + y = 6$ and $x - y = 2$. This pair is $x = 4$ and $y = 2$. Therefore, the solution of the above two equations is $x = 4$ and $y = 2$.

A pair of equations of this type with two unknowns is known as a pair of simultaneous equations. "Simultaneous" means "occurring at the same time".
Let us learn how to solve pairs of simultaneous equations using several other methods which are shorter, by considering the following examples.

Example 1

Solve the pair of simultaneous equations $x + y = 6$ and $x - y = 2$.

To facilitate finding the solution, let us label the two equations as ① and ②.

$x + y = 6$ _______ ①
$x - y = 2$ _______ ②

Method I

We can name this method "the method of substitution".

By making x the subject of equation ②, we can write it as

$$x = 2 + y.$$

By substituting this expression for x in equation ① we obtain,

$$2 + y + y = 6.$$
$$2 + 2y = 6$$

This is a linear equation in one unknown.

Let us find the value of y by solving it.

$$2 - 2 + 2y = 6 - 2$$
$$2y = 4$$
$$\frac{2y}{2} = \frac{4}{2}$$
$$\underline{\underline{y = 2}}$$

We can now find the value of x by substituing $y = 2$ in $x = 2 + y$.

$$x = 2 + 2$$
$$\underline{\underline{x = 4}}$$

Method II

This method can be named "the method of elimination".

$x + y = 6$ ______ ①

$x - y = 2$ ______ ②

First, observe that $+ y$ occurs in equation (1) and $- y$ occurs in equation ②.

By adding both equations we get

$x + y + x - y = 6 + 2$

Here we have used the axiom "Quantities which are obtained by adding equal quantities to equal quantities, are equal".

Now we obtain a linear equation in x, since $+ y$ and $-y$ cancel off.

Let us solve it and find the value of x.

$$2x = 8$$
$$\frac{2x}{2} = \frac{8}{2}$$
$$\underline{\underline{x = 4}}$$

To find the value of y, let us substitute $x = 4$ in equation ①,

$$4 + y = 6$$
$$4 - 4 + y = 6 - 4 \qquad x = 4$$
$$\underline{\underline{y = 2}} \qquad y = 2$$

Note that in the above pair of simultaneous equations, the coefficient of y was 1 in one equation and -1 in the other. That is, the numerical values of these coefficients are equal (when the signs are ignored).

Let us consider a few more examples. We will use the 2nd method to solve them.

Example 2

Solve $2m + n = 10$

$m - n = 2$.

$2m + n = 10 \longrightarrow$ ①

$m - n = 2 \longrightarrow$ ②

Adding ① and ②, $2m + n + m - n = 10 + 2$

$$3m = 12$$

$$\frac{3m}{3} = \frac{12}{3}$$

$$\underline{\underline{m = 4}}$$

By substituting $m = 4$ in ①,

$$2 \times 4 + n = 10$$

$$8 + n = 10$$

$$n = 10 - 8$$

$$\underline{\underline{n = 2}}$$

$m = 4$

$n = 2$

Example 3

Solve $2a + b = 7$

$a + b = 4.$

$2a + b = 7 \longrightarrow$ ①

$a + b = 4 \longrightarrow$ ②

In these equations, the coefficient of b is equal. Therefore, to eliminate b, we must subtract one equation from the other.

① – ②, $2a + b - (a + b) = 7 - 4$ (As there is a subtraction, it is essential to use brackets and write $(a + b)$)

$$2a + b - a - b = 3$$

$$\underline{\underline{a = 3}}$$

By substituting $a = 3$ in ②,

$$3 + b = 4$$

$$b = 4 - 3$$

$$\underline{\underline{b = 1}}$$

Example 4

Solve
$$\begin{aligned} x + 2y &= 11 \\ x - 4y &= 5\,. \end{aligned}$$

$$\begin{aligned} x + 2y &= 11 \longrightarrow ① \\ x - 4y &= 5 \longrightarrow ② \end{aligned}$$

Here the coefficients of x are equal. Therefore, let us subtract one equation from the other to eliminate x.

$$① - ②,\ x + 2y - (x - 4y) = 11 - 5$$
$$x + 2y - x + 4y = 6$$
$$6y = 6$$
$$\frac{6y}{6} = \frac{6}{6}$$
$$\underline{\underline{y = 1}}$$

By substituting $y = 1$ in ① ,

$$x + 2 \times 1 = 11$$
$$x + 2 = 11$$
$$x + 2 - 2 = 11 - 2$$
$$\underline{\underline{x = 9}}$$

Exercise 15.3

1. Solve each of the following pairs of simultaneous equations.

a. $a + b = 5$, $a - b = 1$

b. $x + y = 8$, $2x + y = 2$

c. $m + 2n = 7$, $m - n = 1$

d. $4c - b = 7$, $4c - 2b = 2$

e. $2a + 3b = 16$, $4a + 3b = 26$

f. $3k + 4l = 4$, $3k - 2l = 16$

g. $x + 3y = 12$, $-x + y = 8$

h. $3m - 2n = 10$, $-3m + n = -14$

2. The sum of two numbers is 10 and their difference is 2. Taking the two numbers as x and y, construct a pair of simultaneous equations and solve them.

3. Two pens and a pencil cost Rs 32. A pen and a pencil cost Rs 20. Taking the price of a pen as Rs p and the price of a pencil as Rs q,construct a pair of simultaneous equations and by solving the pair find the price of a pen and the price of a pencil.

16

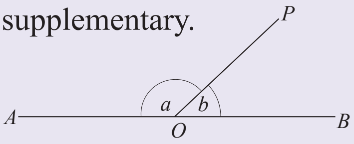
Angles of a Triangle

By studying this lesson you will be able to;

- solve simple problems using the theorem "The sum of the interior angles of a triangle is 180^{o}",
- solve simple problems using the theorem "The exterior angle of a triangle is equal to the sum of the interior opposite angles".

Let us recall several results in geometry that you have learnt earlier related to straight lines.

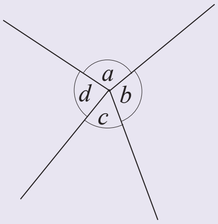
- A pair of adjacent angles on a straight line are supplementary.



AOB is a straight line.

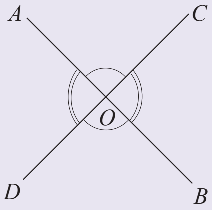
$\therefore\ a + b = 180^{o}.$

- The sum of the angles around a point is 360^{o}.



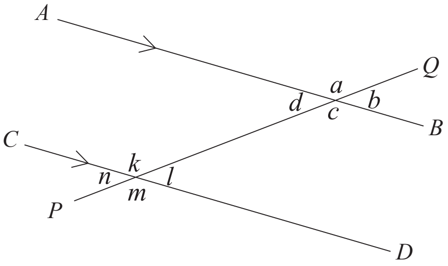
$$a + b + c + d = 360^{o}$$

- The vertically opposite angles formed by the intersection of two straight lines are equal.



AB and *CD* are straight lines. $A\hat{O}C = B\hat{O}D$ and $A\hat{O}D = C\hat{O}B$.

- Angles related to parallel lines

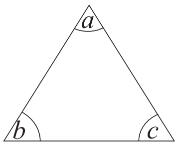


$AB // CD$.

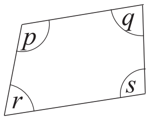
- $c = k$ and $d = l$ (alternate angles)
- $a = k,\ b = l,\ d = n,\ c = m$ (corresponding angles)
- $d + k = 180^{o}$ and $c + l = 180^{o}$ (allied angles)

In the lesson on triangles and quadrilaterals learnt in grade 8, we identified that;

- the sum of the interior angles of a triangle is 180^{o} and the sum of the interior angles of a quadrilateral is also 360^{o}.

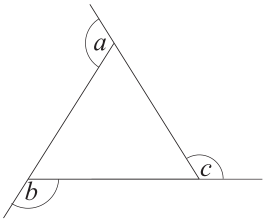


$a + b + c = 180^{o}$

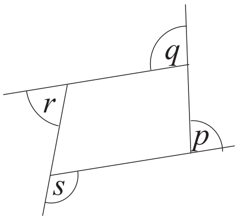


$p + q + r + s = 360^{o}$

- the sum of the exterior angles of a triangle is 180^{o} and the sum of the exterior angles of a quadrilateral is also 360^{o}.



$a + b + c = 360^{o}$



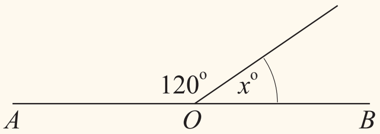
$p + q + r + s = 360^{o}$

Do the following review exercise to further establish the above facts.

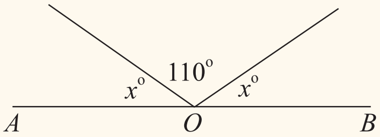
Review Exercise

a. AOB is a straight line. Find the value of x.

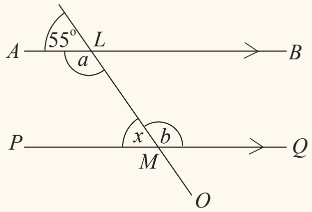
i.



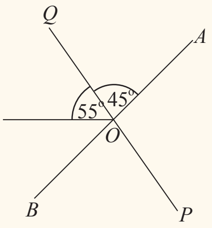
ii.



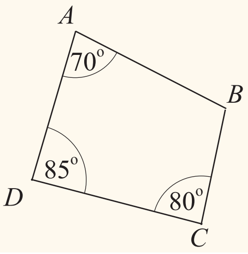
b. Find the magnitude of each of the angles a, b and x, using the information in the figure.



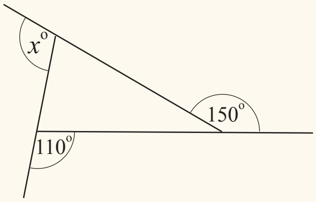
c. AOB and POQ are straight lines. Find the magnitudes of $P\hat{O}B$, $Q\hat{O}B$ and $A\hat{O}P$.



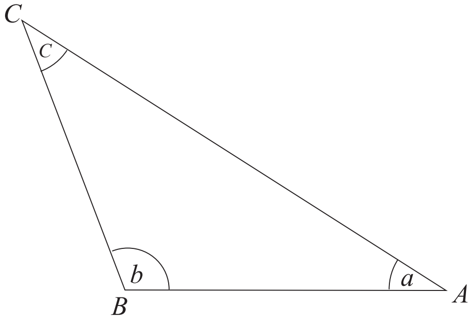
d. Find the magnitude of $A\hat{B}C$ using the information in the figure.



e. Find the value of x, using the information in the figure.



16.1 Interior angles of a triangle



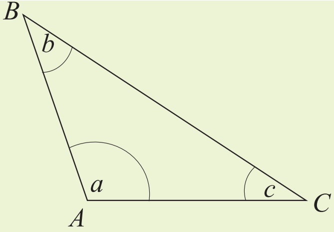
a, b and c are the interior angles of the triangle ABC in the above figure. As discussed earlier, the sum of the interior angles of a triangle is 180^{o}. Hence,

$$A\hat{B}C + B\hat{C}A + C\hat{A}B = 180^{o}.$$

Let us do the following activity to verify the above relationship.

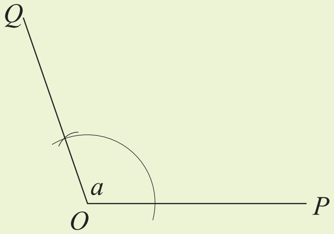
Activity 1

Step 1: Draw a triangle in your exercise book and name it ABC. (The interior angles are given as a, b, c).

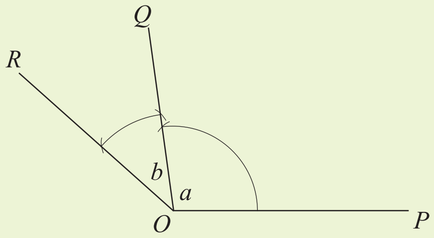


Step 2: Draw a straight line segment in your exercise book and name it OP.

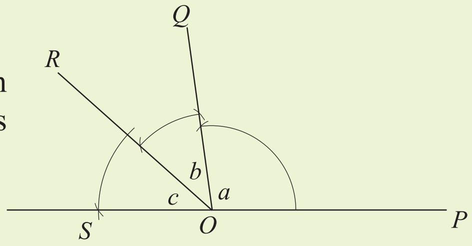
Step 3: Using a straight edge and a pair of compasses, copy the angle $C\hat{A}B$ (a) at O, such that O is the vertex and OP is an arm. (This angle is indicated as $P\hat{O}Q$ in the figure).



Step 4: As done above, copy the angle $A\hat{B}C$ at O, such that O is the vertex and OQ is an arm. (This angle is indicated as $Q\hat{O}R$ in the figure).



Step 5: Copy the angle $A\hat{C}B$ at O, such that O is the vertex and OR is an arm. (This angle is indicated as $R\hat{O}S$ in the figure).



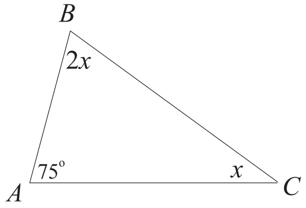
Examine whether POS is a straight line by using a straight edge or a protractor;

It can be concluded that the sum of the interior angles of the triangle ABC is 180^{o}. This can be stated as a theorem as follows.

Theorem: The sum of the three interior angles of a triangle is 180^{o}.

Now let us consider a few examples to see how this theorem can be used to solve problems.

Example 1



Determine the magnitudes of $A\hat{C}B$ and $A\hat{B}C$ of the triangle ABC, using the information given in the figure.

$$\begin{aligned} 75^\circ + 2x + x &= 180^\circ \\ 3x &= 180^\circ - 75^\circ \\ 3x &= 105^\circ \\ x &= \frac{105^\circ}{3} \\ &= 35^\circ \end{aligned}$$

$$\therefore\ A\hat{C}B = x = 35^\circ$$

$$A\hat{B}C = 2x = 2 \times 35^\circ = 70^\circ$$

Example 2

The magnitudes of the interior angles of a triangle are in the ratio 2:3:4. Determine the magnitudes of these angles and giving reasons mention what type of triangle it is.

The ratio of the magnitudes of the angles $= 2:3:4$

$\therefore$ The fractions related to the angles $= \frac{2}{9}, \frac{3}{9}, \frac{4}{9}$

The sum of the three angles $= 180^\circ$

$\therefore$ The smallest angle $= \overset{20}{\cancel{180}}{}^\circ \times \frac{2}{\underset{1}{\cancel{9}}} = 40^\circ$

The medium angle $= 180^\circ \times \frac{3}{9} = 60^\circ$

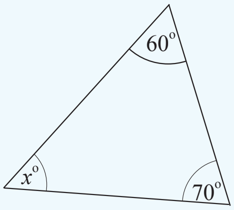
The largest angle $= 180^\circ \times \frac{4}{9} = 80^\circ$

Hence, the interior angles of the triangle are of magnitudes 40°, 60° and 80°. This is an acute triangle since every angle is less than 90°.

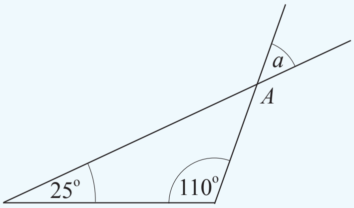
Exercise 16.1

1. Find the magnitude of each angle indicated by a lowercase letter in the following figures, using the information provided in the figures.

i.



ii.



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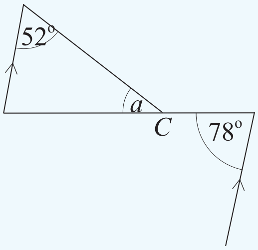

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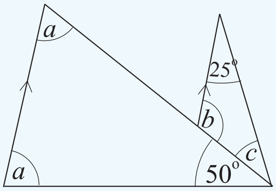

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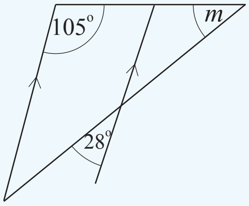
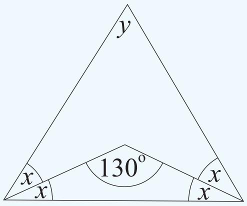
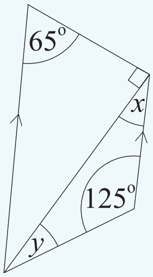


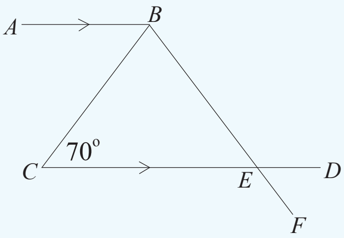
vi.



vii.

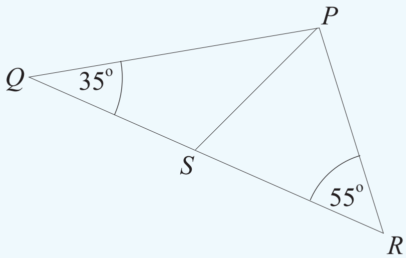


2.



In the given figure, $A\hat{B}C = C\hat{B}E$. $B\hat{C}E = 70^\circ$. Find the magnitude of $D\hat{E}F$.

3.



In the triangle PQR, the point S is located on QR such that $Q\hat{P}S = R\hat{P}S$. Moreover, $P\hat{Q}S = 35^{\circ}$ and $P\hat{R}S = 55^{\circ}$.

(i) Find the magnitude of $Q\hat{P}R$.

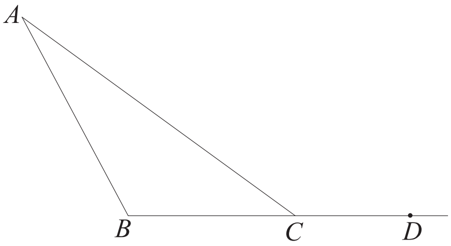
(ii) Find the magnitude of $P\hat{S}R$.

4. In the triangle XYZ, $\hat{X} + \hat{Y} = 115^{\circ}$ and $\hat{Y} + \hat{Z} = 100^{\circ}$. Find the magnitudes of $\hat{X}$, $\hat{Y}$ and $\hat{Z}$.

5. The ratio of the magnitudes of the interior angles of a triangle is 1: 2 : 3. Find the magnitude of each angle separately and with reasons mention what type of a triangle it is.

6. An interior angle of a triangle is 75°. The ratio of the magnitudes of the remaining two angles is 1 : 2. Find the magnitude of each of these angles.

16.2 Exterior angles of a triangle

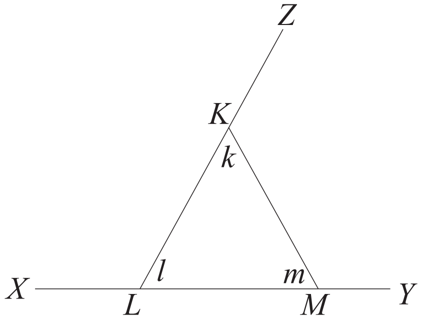


The side BC of the triangle ABC shown in the figure is produced and the point D is marked on BC produced. The angle $A\hat{C}D$ which is formed outside the triangle is called an exterior angle of the triangle.

The interior angle of the triangle, which is adjacent to the exterior angle $A\hat{C}D$ is $A\hat{C}B$. The other two interior angles which are not adjacent to the exterior angle are called the interior opposite angles.

Accordingly, in this figure, the interior opposite angles relevant to the exterior angle $A\hat{C}D$ are $C\hat{A}B$ and $A\hat{B}C$.

Now, let us consider another instance.



In the triangle *KLM* in the above figure, *k*, *l* and *m* are the interior angles. Three exterior angles have been created by producing the sides of the triangle.

The interior opposite angles relevant to $K\hat{M}Y$ are *k* and *l*.

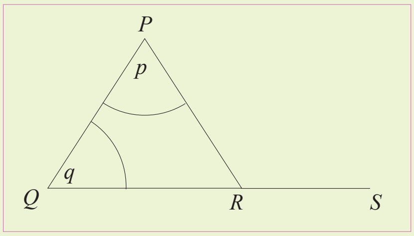
The interior opposite angles relevant to $M\hat{K}Z$ are *l* and *m*.

The interior opposite angles relevant to $X\hat{L}K$ are *k* and *m*.

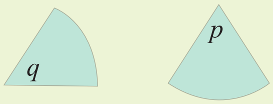
Now let us develop a relationship between an exterior angle and the interior opposite angles of a triangle.

Activity 1

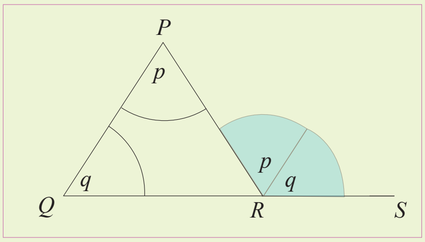
Step 1: Draw a triangle on a piece of Bristol board or on a thick sheet of paper as shown in the figure. Produce a side to create an exterior angle. Mark and shade the interior opposite angles relevant to it (Indicated by *p* and *q* in the figure).



Step 2: Using a blade, cut and separate out the interior opposite angles you marked as laminas.



Step 3: Place the two laminas of the interior opposite angles such that they coincide with the exterior angle as shown in the figure, and paste them

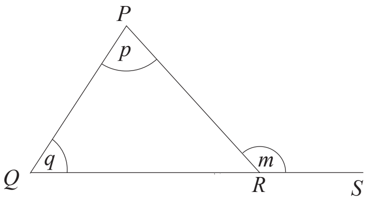


Compare your completed work with those of your friends. Write the conclusion that can be arrived at through this activity.

From the above activity, we can say that the exterior angle of a triangle is equal to the sum of the interior opposite angles.

Draw an acute triangle, a right triangle and an obtuse triangle in your exercise book and in each triangle, mark an exterior angle and the relevant interior opposite angles. Measure them using a protractor and verify the above relationship for the three triangles by obtaining the sum of the interior opposite angles.

The above result can be expressed as follows.



$m = p + q$.

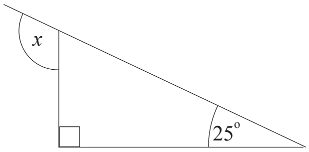
That is, $P\hat{R}S = R\hat{P}Q + P\hat{Q}R$.

This can be expressed as a theorem.

Theorem: **If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles.**

Now, let us consider a few examples to see how this result can be used to solve problems.

Example 1



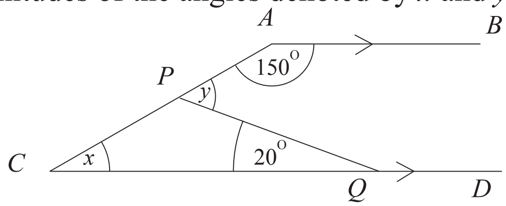
Find the magnitude of the angle indicated by x in the figure.

$x = 90^\circ + 25^\circ$

$= \underline{\underline{115^\circ}}$

Example 2

Find the magnitudes of the angles denoted by x and y in the figure.



$x + 150^\circ = 180^\circ$ (since AB // CD and allied angles are supplementary)

$x = 180^\circ - 150^\circ = 30^\circ$

$y = x + 20^\circ$ (the exterior angle of triangle PCQ = the sum of the interior opposite angles)

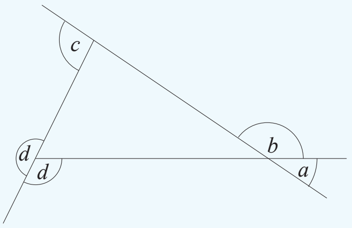
$y = 30^\circ + 20^\circ$

$= \underline{\underline{50^\circ}}$

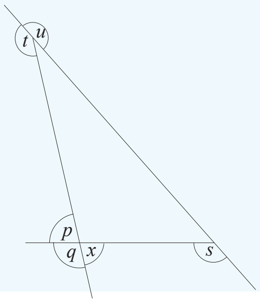
Exercise 16.2

1. Select and write the letters corresponding to the angles which are exterior angles of the given triangles.

i.

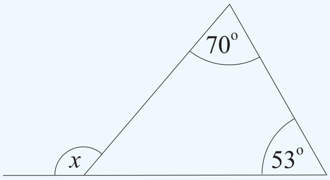


ii.



2. Find the magnitude of each angle denoted by a lowercase letter in the following figures.

i.



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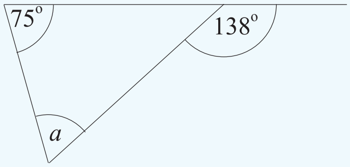

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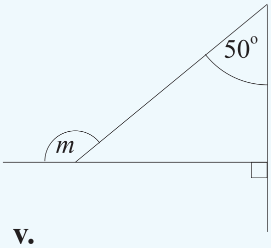

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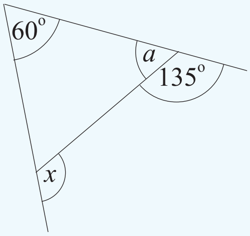
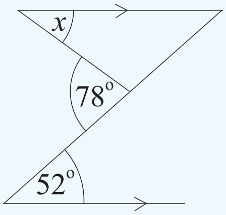


v.



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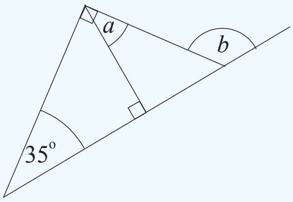
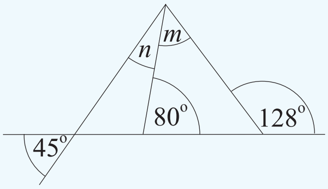
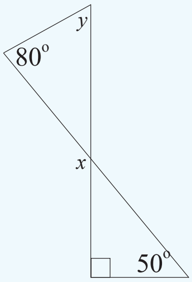


vii.

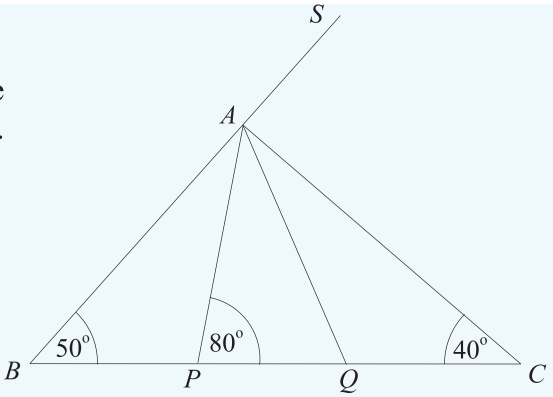


viii.

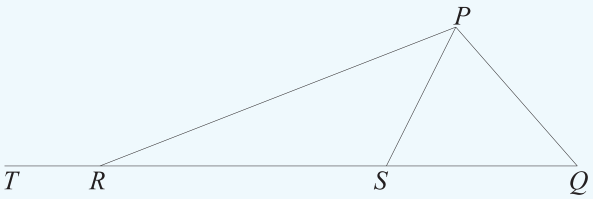


3. In the triangle ABC in the figure, the points P and Q are located on the side BC such that $B\hat{A}P = C\hat{A}Q$. The side BA is produced to S.

i. Find the magnitude of $B\hat{A}P$.
ii. Find the magnitude of $A\hat{Q}P$.
iii. Find the magnitude of $S\hat{A}Q$.

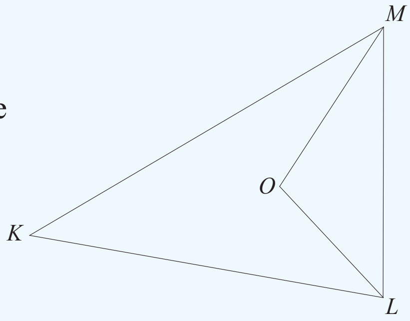


4. In the triangle PQR shown in the figure, the bisector of $\hat{P}$ meets QR at S. Moreover, $S\hat{P}Q = S\hat{Q}P$. If $S\hat{Q}P = a^{\circ}$, then find $P\hat{R}T$ in terms of a.

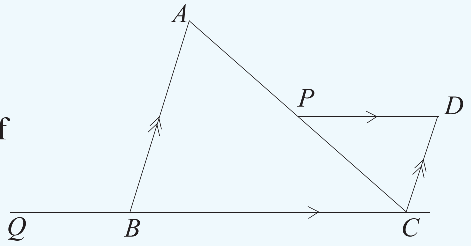


Miscellaneous Exercise

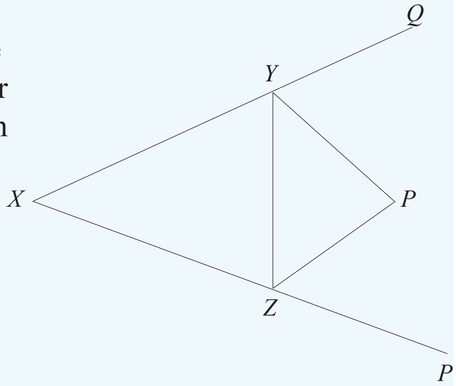
1. In the triangle KLM, the angle bisectors of $\hat{M}$ and $\hat{L}$ meet at O. Moreover, $\hat{K} = 70^{\circ}$. Find the magnitude of $L\hat{O}M$.



2. In the given figure, $A\hat{P}D = 140^{\circ}$ and $P\hat{D}C = 85^{\circ}$. Find the magnitude of $A\hat{B}Q$.



3. The sides XY and XZ of the triangle XYZ have been produced. The bisectors of the exterior angles at Y and Z intersect at P. Find $Y\hat{P}Z$ in terms of $\hat{X}$.



Summary

- **The sum of the three interior angles of a triangle is 180^{o}.**
- **If a side of a triangle is produced the exterior angle so formed is equal to the sum of the two interior opposite angles.**

17

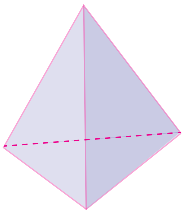
Formulae

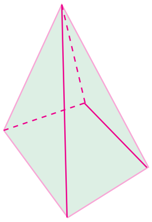
By studying this lesson you will be able to;

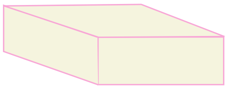
- change the subject of a formula,
- find the value of a variable term in a formula when the values of the other variables are given.

Introducing formulae

In grade 8 you learnt Euler's relationship which is an equation expressing the relationship between the number of edges, number of vertices and number of faces in a solid.







This relationship is the following.

Number of edges = Number of vertices + Number of faces – 2

By taking the number of edges as E, the number of vertices as V and the number of faces as F, this relationship can be expressed as follows.

$$E = V + F - 2$$

A relationship between two or more quantities expressed as an equation, is known as a "formula".

The quantities in a formula are known as variables. In general, a formula is written with just one variable which is called the "subject of the formula" on one side of the equal sign (usually the left hand side), and the remaining variables on the other side. For example, in the formula $E = V + F - 2$ stated above, E is the subject.

Let us consider another formula. Temperature can be expressed in degrees Celsius or degrees Fahrenheit. The relationship between these two units is given below.

$$F = \frac{9}{5}C + 32$$

Here, F denotes the temperature in Fahrenheit and C denotes the temperature in Celsius. The subject of this formula is F.

Some formulae that are frequently used in Science and Mathematics are given below.

$p = 2\,(a + b)$

$v = u + at$

$s = \frac{n}{2}\,(a + l)$

$y = mx + c$

$C = 2\,\pi r$

$A = \pi r^2$

17.1 Changing the subject of a formula

E is the subject of the formula $E = V + F - 2$. If required, we can make either V or F the subject of this formula. This can be done in a manner similar to solving equations by using axioms.

As an example, let us make V the subject of the formula $E = V + F - 2$.
V is on the right hand side of this equation. F and -2 are also on the same side of the equation as V. To remove the terms F and -2 from the right hand side, let us add $-F$ and $+2$ to both sides of the equation.

We then obtain, $E + (-F) + 2 = V + F - 2 + (-F) + 2$.

Now, by simplifying both sides we obtain,
$E - F + 2 = V$ (since $F + (-F) = 0$ and $-2 + 2 = 0$)

The subject V appears on the right hand side.
Since the subject is usually written on the left hand side, we re-write the above equation as follows with V on the left hand side.
$V = E - F + 2$

The following examples show how the subject of formulae of various forms are changed.

Example 1

Make a the subject of the formula $v = u + at$.

Here the variable a is multiplied by the variable t. Therefore, we need to first make the term at the subject.

Subtracting u from both sides of $v = u + at$ we obtain

$$v - u = u + at - u$$
$$v - u = at$$

Now by dividing both sides by t to make a the subject we obtain,

$$\frac{v-u}{t} = \frac{at}{t}$$

By simplifying this we get the formula $a = \dfrac{v-u}{t}$ with a as the subject.

Example 2

Make n the subject of the formula $S = \dfrac{n}{2}(a + l)$.

$$S = \frac{n}{2}(a + l).$$

Here, the variable n which is to be made the subject is divided by 2 and the result is multiplied by $(a + l)$. Therefore both sides of the formula need to be multiplied by 2 and divided by $(a + l)$ to make n the subject.

By multiplying both sides by 2 we obtain,

$$2S = \cancel{2}^{1} \times \frac{n}{\cancel{2}_{1}} \times (a + l)$$
$$2S = n(a + l)$$

Now, by dividing both sides by $(a + l)$ we obtain

$$\frac{2S}{a+l} = \frac{n\,\cancel{(a+l)}^{1}}{\cancel{(a+l)}_{1}}$$

$$\frac{2S}{a+l} = n$$

$$n = \frac{2S}{a+l}$$

Example 3

Make n the subject of the formulal $l = a + (n-1)d$.

$l = a + (n-1)d$

Let us consider the variable n which is to be made the subject. Observe that the right hand side of the formula is formed by subtracting 1 from n to obtain $(n-1)$, then multiplying $(n-1)$ by d to obtain $(n-1)d$ and finally adding a to $(n-1)d$.

To make n the subject, we need to perform the inverse operations corresponding to the arithmetic operations performed in the above three steps (i.e., the inverse operation “addition” of the operation “subtraction, the inverse operation “division” of the operation “multiplication”, etc.), starting from the last step and moving up.

Expressed in another way, this means that we make n the subject of the formula by using the relevant axioms.

Therefore, let us first subtract a from both sides of the equation and simplify.

$$l = a + (n-1)d$$
$$l - a = a + (n-1)d - a$$
$$l - a = (n-1)d$$

Now let us divide both sides by d and simplify.

$$\frac{l-a}{d} = \frac{(n-1)\cancel{d}^{1}}{\cancel{d}_{1}}$$
$$\therefore \quad \frac{l-a}{d} = n-1$$

Finally let us add 1 to both sides and simplify.

$$\frac{l-a}{d} + 1 = n - 1 + 1$$
$$\frac{l-a}{d} + 1 = n$$
$$n = \frac{l-a}{d} + 1$$

If required, you may simplify the right hand side further, using a common denominator. However it is not essential to do this.

Exercise 17.1

1. Make r the subject of the formula $C = 2\pi r$.

2. Make c the subject of the formula $a = b - 2c$.

3. Make t the subject of the formula $v = u + at$.

4. In the formula $y = mx + c$,
  i. make c the subject.
  ii. make m the subject.

5. Make c the subject of the formula $a = 2\,(b + c)$.

6. Make C the subject of the formula $F = \frac{9}{5}C + 32$.

7. In the formula $l = a + (n - 1)d$,
  i. make a the subject.
  ii. make d the subject.

8. Make y the subject of the formula $\frac{x}{a} + \frac{y}{b} = 1$.

9. Make r_2 the subject of the formula $\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2}$.

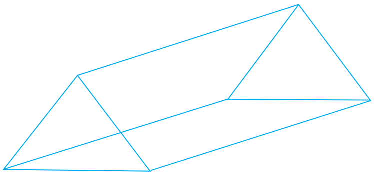
10. Make x the subject of the formula $ax = m\,(x - t)$.

11. Make a the subject of the formula $P = \frac{at}{a - t}$.

17.2 Substitution

Suppose that the values of all the variables in a formula except one are given. By substituting these values in the formula, the value of the unknown can be found.

Let us determine the number of edges in a solid with straight edges, which has 6 vertices and 5 faces.



The triangular prism shown above is an example of such a solid.

We can find the number of edges by substituting the values $V = 6$ and $F = 5$ in the formula $E = V + F - 2$.

Substituting $V = 6$ and $F = 5$ in the formula we obtain,

$$E = 6 + 5 - 2$$
$$= 9$$

Therefore, the solid has 9 edges.

Let us consider more examples.

There are two methods that can be used to find the value of an unknown variable when the values of the remaining variables are given. The first method is to substitute the given values in the formula as it is, and then find the value of the unknown. The second method is to first make the unknown of which the value is to be determined the subject of the formula, and then find its value by substituting the given values.

Let us now consider how the value of an unknown in a formula is found using these two methods.

Example 1

Determine the number of vertices there are in a solid that has 7 faces and 12 edges.

We need to use the formula $E = V + F - 2$ here. The values of F and E are given and we need to find V. We can use either of the above mentioned two methods to find V. That is, we can first substitute the given values in the formula $E = V + F - 2$ and then find the value of V by solving the resulting equation, or we can first make V the subject of the formula and then substitute the given values and simplify.

Let us consider both methods.
Let us take the number of edges as E, the number of vertices as V and the number of faces as F.

Method 1

$$E = V + F - 2$$

Substituting $E = 12$ and $F = 7$ we obtain

$$12 = V + 7 - 2$$
$$12 = V + 5$$
$$12 - 5 = V$$
$$7 = V$$
$$V = 7$$

$\therefore$ The number of vertices is 7.

Method 2

First make V the subject of the formula and then substitute the values.

$$E = V + F - 2$$
$$E + 2 = V + F$$
$$E + 2 - F = V$$
$$V = E + 2 - F$$
$$V = 12 + 2 - 7$$
$$V = 7$$

$\therefore$ The number of vertices is 7.

Note: One reason for changing the subject of a formula is because the value of the unknown can then be found easily by directly substituting the given values.

Example 2

Convert 35ºC into Fahrenheit using the formula $C = \frac{5}{9}\,(F - 32)$.
Consider that the temperature in degrees Celsius is denoted by C and the temperature in degrees Fahrenheit is denoted by F.

$$C = \frac{5}{9}\,(F - 32)$$

Substituting $C = 35$,

$$35 = \frac{5}{9}\,(F - 32)$$

Multiplying both sides by 9

$$35 \times 9 = 5\,(F - 32)$$

Dividing both sides by 5

$$\frac{{}^{7}\cancel{35} \times 9}{\cancel{5}_{1}} = F - 32$$

$$63 = F - 32$$

$$63 + 32 = F$$

$$95 = F$$

i.e., $F = 95$

The given tempreature is 95ºF.

Exercise 17.2

1. Find the value of a when $b = 7$ and c = 6 in the formula $a = (b + c) - 2$.
2. Find the value of C when $F = 104$ in the formula $C = \frac{5}{9}\,(F - 32)$.
3. Find the value of m when $y = 11$, $x = 5$ and $c = -4$ in the formula $y = mx + c$.
4. Find the value of r when $A = 88$ and $\pi = \frac{22}{7}$ in the formula $A = 2\pi r$.
5. Find the value of d when $l = 22$, $a = -5$ and $n = 10$ in the formula $l = a + (n - 1)d$.
6. Find the value of n when $S = -330$, $a = 4$ and $l = -48$ in the formula $S = \frac{n}{2}\,(a + l)$.

Miscellaneous Exercise

1. Consider the formula $P = C\left(1 + \frac{r}{100}\right)$.

(i) Make r the subject of the above formula.
(ii) Find the value of r when $P = 495$ and $C = 450$.

2. Consider the formula $\frac{y - c}{x} = m$.

(i) Make x the subject of the above formula.
(ii) Find the value of x when $y = 20$, $c = -4$ and $m = 3$.

3. Consider the formula $ax = bx - c$.

(i) Make x the subject of the above formula.
(ii) Find the value of x when $a = 3$, $b = 4$ and $c = 6$.

4. Consider the formula $a = \frac{bx + c}{b}$.

(i) Make b the subject of the above formula.
(ii) Find the value of b when $a = 4$, $c = 5$ and $x = 3$.

5. Find the value of f in the formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ when $v = 20$ and $u = 5$.

6. Find the value of b in the formula $\frac{a}{b} = \frac{p}{q}$ when $a = 6$, $p = 3$ and $q = 4$.

7. Consider the formula $S = \frac{n}{2}(a + l)$.
(i) Make l the subject of the above formula.
(ii) Find the value of l when $S = 198$, $n = 12$ and $a = 8$.

8. Consider the formula $y = mx + c$.

(i) Make m the subject of the above formula.
(ii) Find the value of m when $y = 8$, $x = 9$ and $c = 2$.

18 Circumference of a Circle

By studying this lesson you will be able to;

- find the diameter of a circle using different methods,
- find the circumference of a circle and the perimeter of a semicircle using formulae,
- solve problems related to the circumference of a circle.

Do the following exercise to recall what you have learnt about circles.

Review Exercise

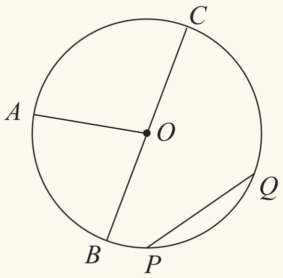
1. a. Fill in the blanks using suitable words.

i. The locus of the points on a plane which are at a constant distance from a fixed point is a. ……………..

ii. The point right at the middle of a circle is known as its ....

b. Copy the two columns A and B given below and using the given figure, join the relevant pairs.

A	B
Point O OA BC OB PQ	Radius Diameter Centre Chord



2.

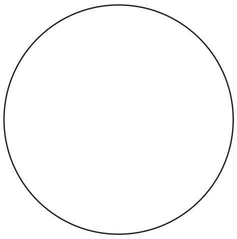
i. What is the length of the diameter of a circle of radius 5 cm?

ii. What is the length of the radius of a circle of diameter 7 cm?

iii. If the radius of a circle is r and diameter is d, write an equation expressing the relationship between d and r.

Measuring the diameter and the circumference of a circle

The total length of the boundary of a circle, or the perimeter of a circle is known as its **circumference**.



A circular ring made from a metal wire of length 25 cm is shown in the above figure. Since the length of the wire is 25 cm, the perimeter or the circumference of the circle is 25 cm.

We cannot directly determine the diameter of a circle.

Do the following activities to identify different methods of finding the diameter of a circle.

Activity 1

(*a*) - Measuring the diameter of a circle using a straight edge with a cm/mm scale.

Step1: Draw any circle using the pair of compasses and mark its centre.

Step2: Draw a diameter and measure its length using a straight edge with a cm/mm scale.

(*b*) - Measuring the diameter by means of an axis of symmetry of a circular lamina.

Step 1: Draw a circle on a piece of paper using an object like a coin or a bangle and cut it out.

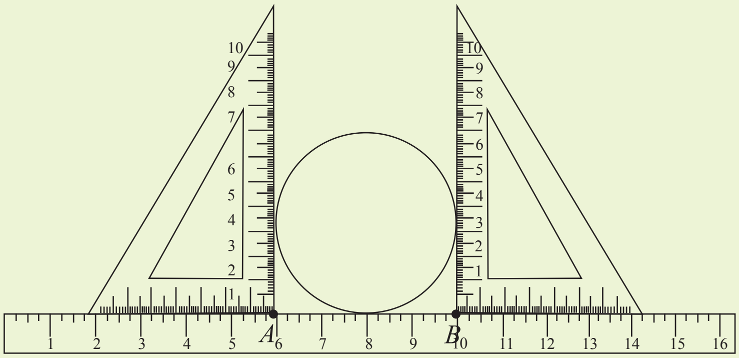
Step 2: Fold the circular lamina into two equal parts (such that the two parts coincide) and mark the axis of symmetry on it.

Step 3: Since the axis of symmetry is a diameter of the circle, measure the length of the axis of symmetry and obtain the length of the diameter.

(*c*)- Measuring the diameter using set squares.

Step 1: Take a ruler, two set squares, a circular coin, a bangle and a cylindrical can.

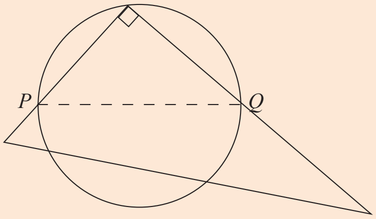
Step 2: Place the bangle and the set squares as shown in the figure, touching the ruler. Find the diameter of the bangle using the two readings denoted by *A* and *B*.



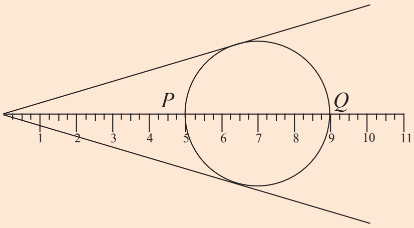
Step 3: Find the diameters of the remaining circular objects too by doing the above activity and note them down in your exercise book.

Different methods of finding the diameter

1. Keep the right angled corner of a piece of paper on a circle as shown in the figure. The distance between the two points(P and Q) where the arms of the 90° angle meet the circle is the length of the diameter of the circle.



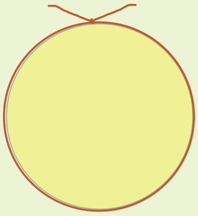
2. Make an instrument as shown in the figure: Draw an angle and its bisector on a Bristol board and calibrate the bisector from the vertex. The length of the diameter of a circle can be measured as shown in the figure.



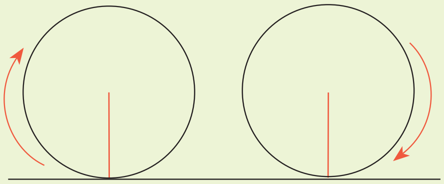
18.1 Measuring the circumference of a circle

Do the following activities in order to find out the methods used to measure the circumference of a circular lamina such as a coin.

Activity 2



1. Mark a point on a piece of thread and with the thread stretched, place it around the circular lamina until it reaches the initial point that was marked. Mark the final point on the thread when it coincides with the initial point, and measure the length between the two points marked on the thread to find the circumference of the coin.



2. Draw a straight line on a piece of paper. Keep it on a flat surface. Mark a point on the circumference of the circular lamina and also on the straight line. Place the circular lamina on the straight line such that the two marks coincide and then role it along the straight line until the point marked on the circular lamina touches the line again. The length of the circumference is obtained by measuring the distance the circular lamina has moved along the straight line.

Developing a formula for the circumference of a circle

Do the following activity to identify the relationship between the circumference and the diameter of a circle.

Activity 3

Complete the table given below by measuring the circumference and the diameter of objects with circular faces, using the methods introduced above.

Object	Diameter d	Circumference c	$\frac{c}{d}$ up to two decimal places
1. Circular lamina made of cardboard			
2. Rs 2 coin			
3. Circular lid of a tin			
4. Compact Disk (CD)			

Compare the values you obtained for $\frac{c}{d}$ in the above activity with the values obtained by your friends and write your conclusion regarding the value of $\frac{c}{d}$.

Through the above activity you would have obtained a value for $\frac{c}{d}$ which is approximately 3.14 for every object you considered. Mathematicians have discovered that $\frac{c}{d}$ is a constant value for all circles. This constant value is denoted by π. It has been shown that this value is 3.14 to the nearest second decimal and is approximately equal to the fraction $\frac{22}{7}$.

Accordingly,

$$\frac{c}{d} = \pi \, .$$

That is,

$$c = \pi d.$$

This is a formula giving the relationship between the circumference and the diameter of a circle. A formula giving the relationship between the radius and the circumference of a circle can be derived as follows.

Since $d = 2r$ we obtain $c = \pi \times 2r$.

i,e.,

$$c = 2\pi r$$

If the circumference of a circle is denoted by c, the diameter by d and the radius by r, then,

$$\boxed{c = \pi d}$$

$$\boxed{c = 2\pi r}$$

Example 1

Find the circumference of a circle of radius 7 cm.

Use $\frac{22}{7}$ for the value of π.

Circumference $c = 2\pi r$

$$= 2 \times \frac{22}{\cancel{7}} \times \overset{1}{\cancel{7}}$$

$$= 44$$

$\therefore$ the circumference is 44 cm.

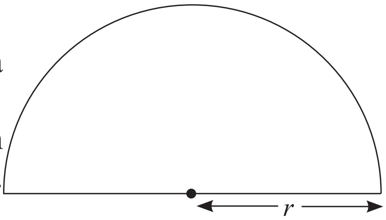
Exercise 18.1

1. Find the circumference of the circle with the measurement given below.
Use $\frac{22}{7}$ for the value of π.

i. radius 7 cm
ii. diameter 21 m
iii. radius 10.5 cm
iv. diameter $17\frac{1}{2}$ m
v. radius $\frac{7}{2}$ m
vi. diameter 28 cm
vii. radius 15.4 cm
viii. diameter $3\frac{1}{9}$ m

18.2 Perimeter of a semicircular lamina

When a circular lamina is separated into two equal parts along a diameter, each part is known as a semicircular lamina (in short, semicircle).
The length of the curved line of a semicircle is known as the arc length. It is exactly half the circumference.



Hence, the arc length of a semicircle of radius $r = \frac{1}{\cancel{2}} \times (\overset{1}{\cancel{2}}\pi r)$

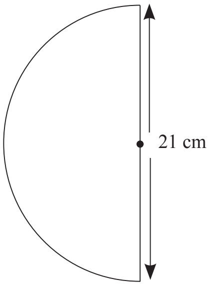
$$= \pi r$$

It is clear from the figure that, to find the perimeter of a semicircle, the diameter should be added to the arc length.

$$\boxed{\therefore \text{ The perimeter of a semicircle} = \pi r + 2r}$$

Example 1

Find the perimeter of the semicircle shown in the figure. Use $\frac{22}{7}$ for the value of π.



Arc length of a semicircle of diameter $d = \frac{1}{2}\pi d$

$\therefore$ Arc length of the semicircle of diameter 21 cm $= \frac{1}{\underset{1}{\cancel{2}}} \times \frac{\overset{11}{\cancel{22}}}{\underset{1}{\cancel{7}}} \times \overset{3}{\cancel{21}}$

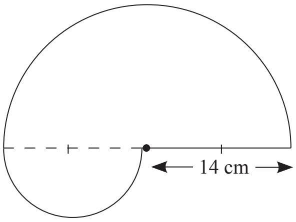
$$= 33$$

$\therefore$ The perimeter of the figure = 33 + 21 = 54 cm

Example 2

A compound figure, consisting of two semicircular laminas of radius 14 cm and diameter 14 cm respectively is shown in the figure. Find its perimeter.

Use $\frac{22}{7}$ for the value of π.



$$\text{Arc length of a semicircle of radius } r = \frac{1}{2} \times 2\pi r$$

$$\therefore \text{Arc length of the semicircle of radius 14 cm} = \frac{1}{\underset{1}{\cancel{2}}} \times \overset{1}{\cancel{2}} \times \frac{22}{\underset{1}{\cancel{7}}} \times \overset{2}{\cancel{14}} \text{ cm} = 44 \text{ cm}$$

$$\text{Arc length of a semicircle of diameter } d = \frac{1}{2}\pi d$$

$$\therefore \text{Arc length of the semicircle of diameter 14 cm} = \frac{1}{\underset{1}{\cancel{2}}} \times \frac{\overset{11}{\cancel{22}}}{\underset{1}{\cancel{7}}} \times \overset{2}{\cancel{14}} \text{ cm} = 22 \text{ cm}$$

$$\therefore \text{Perimeter of the figure} = 44 + 22 + 14 \text{ cm}$$

$$= \underline{\underline{80 \text{ cm}}}$$

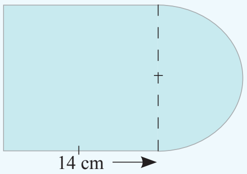
Exercise 18.2

1. Find the perimeter of the semi circular lamina with the measurement given below. Use $\frac{22}{7}$ for the value of π.

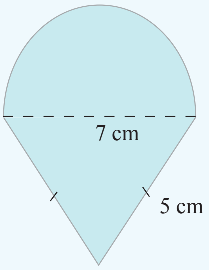
i. r =14cm ii. d =7 cm

2. Find the permeter of the shaded part of each of the figures given below. The curved parts in the figures are semicireles.Use $\frac{22}{7}$ for the value of π.

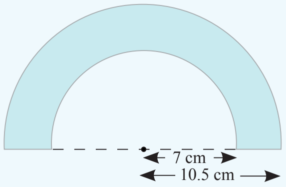
i.



ii.



iii.



18.3 Problems related to the circumference of a circle

Example 1

A wheel of radius 35 cm moves along a straight road.

i. Find in metres, the distance it moves during one full rotation.

ii. What is the distance it moves in metres during 100 rotations?

iii. How many rotation does the wheel undergo when travelling a distance of 1.1 km? (Use $\frac{22}{7}$ for the value of π.)

i. During one full rotation of the wheel, it moves a distance which is equal to its circumference.

$$\text{Circumference} = 2 \times \frac{22}{\overset{}{\underset{1}{\cancel{7}}}} \times \overset{5}{\cancel{35}} \text{ cm} = 220 \text{ cm}$$

$\therefore$ The distance travelled during one full rotation $= \underline{\underline{2.2 \text{ m}}}$

ii. Distance travelled during 100 rotations $= 2.2 \text{ m} \times 100$

$= \underline{\underline{220 \text{ m}}}$

iii. Distance travelled $= 1.1$ km

$= 1100$ m

Distance travelled during one rotation of the wheel $= 2.2$ m

Therefore, number of rotations $= \frac{1100}{2.2}$

$= \underline{\underline{500}}$

Example 2

A circular frame is made by joining the two ends of a wire of length 66 cm. Find its radius.

Let us assume that the radius is r cm.

$$c = 2\pi r$$

$$2 \times \frac{22}{7} \times r = 66$$

$$r = \overset{3}{\cancel{66}} \times \frac{7}{\underset{1}{\cancel{22}}} \times \frac{1}{2}$$

$$= \frac{21}{2}$$

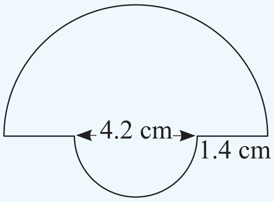
$$= 10.5 \text{ cm}$$

$\therefore$ The radius is 10.5 cm.

Exercise 18.3

Use $\frac{22}{7}$ for the value of π.

1. A lamina composed of two semicircles is shown in the figure. This is pasted on an ornamental box. A gold thread is to be pasted around the lamina.



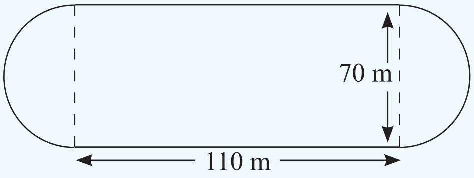
(i) Find the minimum length of thread required to paste around this lamina?

(ii) Find the total length in metres of the thread required to paste around 500 such laminas.

2. The circumference of a circular plot of land is 440 m. Find its radius.

3. The perimeter of a semicircular lamina is 39.6 cm. Find its diameter.

4. A sketch of a playground is shown in the figure. It consists of a rectangular part and two semicircular parts.



(i) Find the perimeter of the playground.

(ii) Show that the distance covered by a runner in completing $2\frac{1}{2}$ rounds of the playground is more than 1 km.

5. A cyclist rides a bicycle along a straight road. The radius of each wheel of the bicycle is 28 cm.

(i) Find the distance the bicycle moves during the period athat the wheels complete one full rotation.

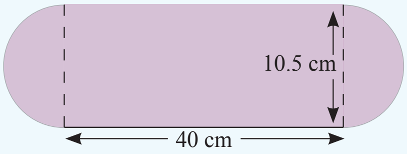
(ii) What is the distance the bicycle moves in meters during the period that the wheels complete 50 rotation?

(iii) The cyclist says that the wheels rotate at least 800 times when it travels a distance of 1500 m. Do you agree with this statement? Give reasons for your answer.

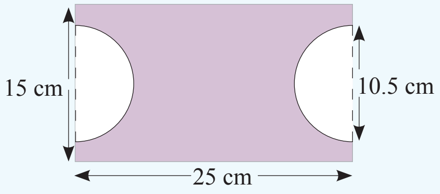
Miscellaneous Exercise

1. Find the perimeter of the shaded part of each figure.

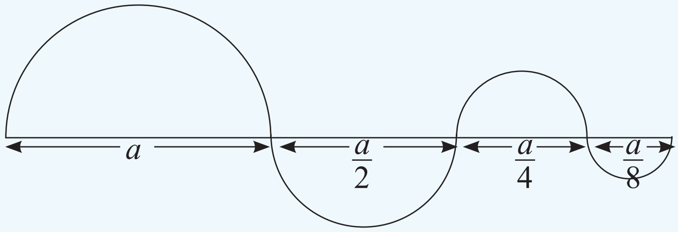
i.



ii.

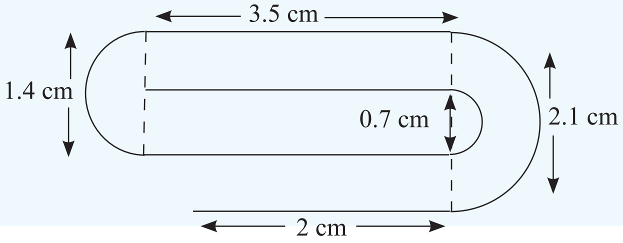


2.



Show that the length of metal wire needed to make the frame shown in the figure which consists of 4 semicircular parts is $\frac{135a}{28}$. (Use $\frac{22}{7}$ for the value of π.)

3. A paper clip with semi circular parts is to be made according to the given measurements. Find the length of the wire needed to make the clip in the figure.



Summary

In acircle of radius r, diameter d and circumference c,

- $c = \pi d$
- $c = 2\pi r$
- The perimeter of a semicircle = $\pi r + 2r$

19

Pythagorean Relation

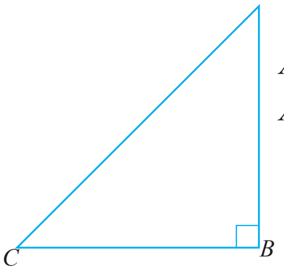
By studying this lesson, you will be able to;

- develop the Pythagorean relation by means of a right angled triangle,
- solve problems related to the Pythagorean relation.

Right angled triangle

If an angle of a triangle is 90^o, it is called a right angled triangle. The side which is opposite(in front of) the right angle and which is the longest side of the triangle is called the hypotenuse. The other two sides are called the sides which include the right angle.

Considering the right angled triangle ABC given below;



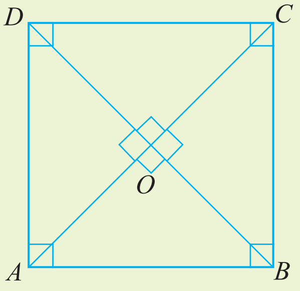
$A\hat{B}C = 90^o$

AC is the hypotenuse,

AB and BC are the sides which include the right angle.

Activity 1

Complete the table given below by identifying all the right angled triangles in the figure.

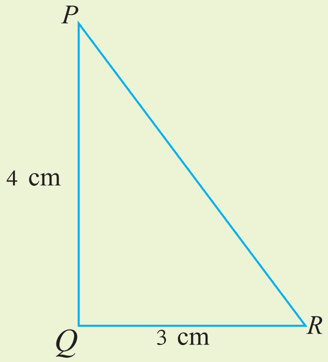


Triangle	Hypotenuse	Sides that include the right angle
AOB	AB	AO, BO
....	....	....
....	....	....
....	....	....
....	....	....
....	....	....

19.2 The Pythagorean relation

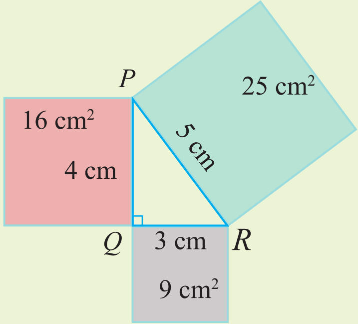
A Greek mathematician named Pythagoras introduced the relationship between the sides of a right angled triangle. Let us understand this relationship through an activity.

Activity 1



Draw a right angled triangle PQR with $QR = 3$ cm and $QP = 4$ cm. You may use a set square to do this. Measure the hypotenuse PR and verify that it is 5 cm in lenghth. Cut three squares of side length 3 cm, 4 cm and 5 cm and paste them on the sides RQ, QP and PR respectively, as shown in the figure given below.

Now let us calculate the area of each square as shown below.



The area of the square pasted on QR = 3 cm × 3 cm = 9 cm^2
The area of the square pasted on QP = 4 cm × 4 cm = 16 cm^2
The area of the square pasted on PR = 5 cm × 5cm = 25 cm^2

Observe the relationship between the above values as given below.

the area of the square on PR	=	the area of the square on QR	+	the area of the square on PQ

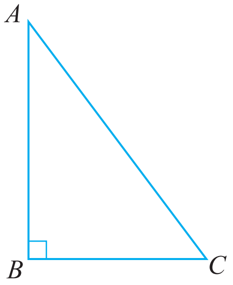
Repeat the above activity by taking the lengths of the sides of the triangle which include the right angle as 6 cm and 8 cm to verify the above relationship for these values too.

The Pythagorean relation for a right angled triangle can be expressed as follows.

> The area of the square drawn on the hypotenuse of a right angled triangle is equal to the sum of the areas of the squares drawn on the remaining two sides.

Though the Pythagorean relation is shown using areas, we can write it simply in terms of the lengths of the sides of the triangle. Let us see how this is done.

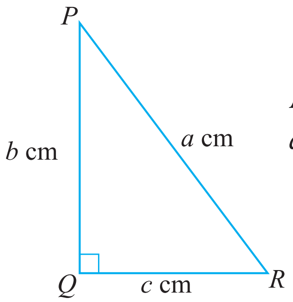
Writing the Pythagorean relation in terms of the lengths of the side



The area of the square drawn on $AB = AB \times AB = AB^2$
The area of the square drawn on $BC = BC \times BC = BC^2$
The area of the square drawn on $AC = AC \times AC = AC^2$
Therefore, according to the Pythagorean relation;

$$AC^2 = AB^2 + BC^2$$

We can express this as follows too.



According to the Pythagorean relation
$a^2 = b^2 + c^2$

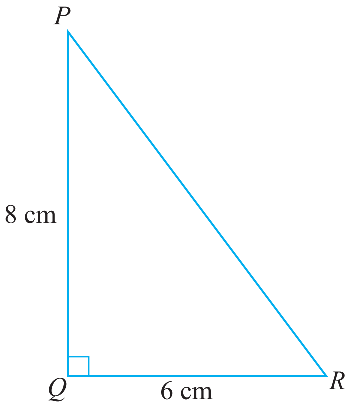
Example 1

In the right angled triangle PQR, $PQ = 8$ cm and $QR = 6$ cm. Find the length of PR.

By applying the Pythagorean relation to the right angled triangle PQR,

$$\begin{aligned} PR^2 &= PQ^2 + QR^2 \\ PR^2 &= 8^2 + 6^2 \\ &= 64 + 36 \\ &= 100 \\ PR &= \sqrt{100} = 10 \end{aligned}$$

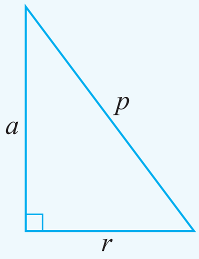
$\therefore$ the length of $PR = 10$ cm.



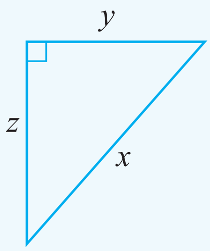
Exercise 19.1

1. Write the Pythagorean relation for each right angled triangle using the given lengths.

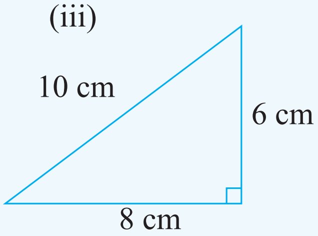
(i)



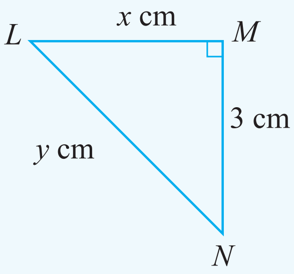
(ii)



(iii)

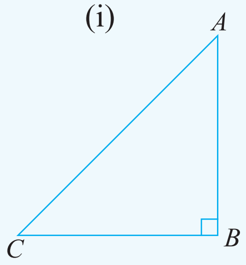


(iv)



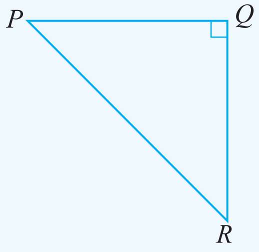
2. Fill in the blanks in the expressions related to the figures shown below.

(i)



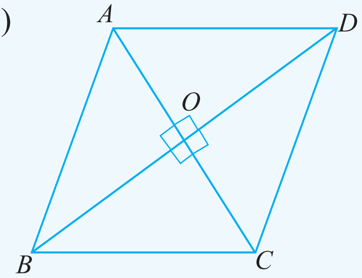
$AC^2 = AB^2 + ...$

(ii)



$PR^2 = ... + ...$

(iii)



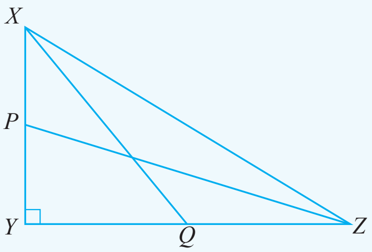
a. $AD^2 = ... + ...$

b. $....... = BO^2 + ...$

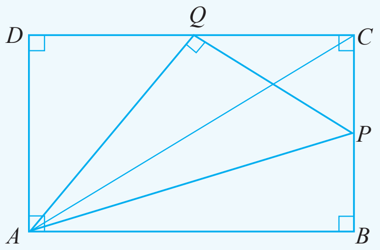
c. $....... = BO^2 + OC^2$

d. $DC^2 = ... + ...$

3. Identify all the right angled triangles in each figure given below and write the Pythagorean relation for each triangle that is identified.

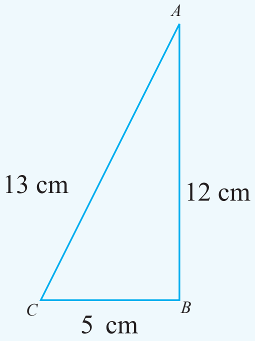


i.



ii.

4. Fill in the blanks in the statements given below which are related to the right angled triangle in the figure.



The longest side of the triangle is ....

The area of the square drawn on the side AB = 12 × 12 = 144 cm^2

The area of the square drawn on the side BC = ... = ... cm^2

The area of the square drawn on the side AC = ... = ... cm^2

The sum of the areas of the squares drawn on the sides BC and BA = ... cm^2 .

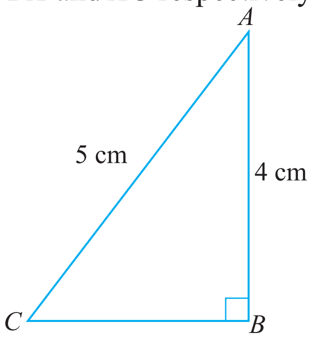
∴ The areas of the square drawn on AC is .... (equal/not equal) to the sum of the areas of the squares drawn on BC and BA.

Now let us consider some problems which can be solved using the Pythagorean relation.

Example 2

A 5m long straight wooden rod is in a vertical plane with one end touching the top of a 4m high vertical wall and the other end in contact with the horizontal ground a certain distance away from the foot of the wall. Find the horizontal distance from the foot of the wall to the point where the rod is in contact with the ground.

We can draw a rough sketch of this as shown below. Here the wall and the wooden rod are represented by BA and AC respectively.



Applying the Pythagorean relation to the right angled triangle ABC

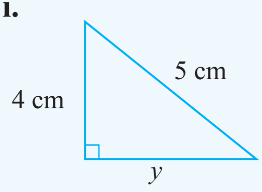
$$AC^2 = AB^2 + BC^2$$
$$5^2 = 4^2 + BC^2$$
$$25 = 16 + BC^2$$
$$\therefore\ BC^2 = 9$$
$$BC = \sqrt{9} = 3$$

∴ The horizontal distance from the foot of the wall to the wooden rod is 3m.

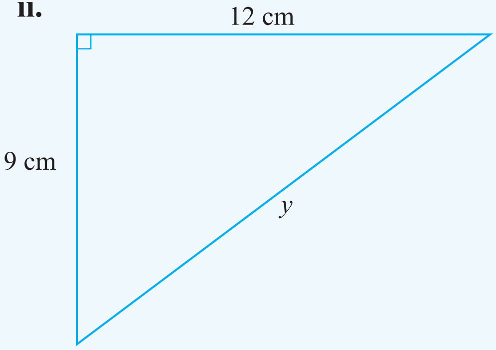
Exercise 19.2

1. Find the length of each side indicated by an algebraic symbol in each figure.

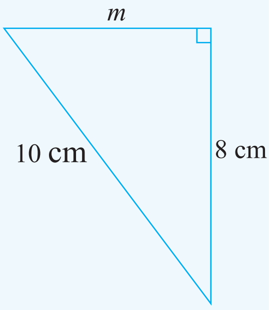
i.



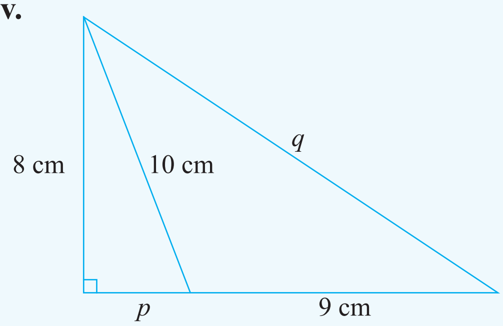
ii.



iii.

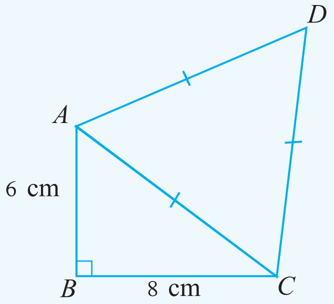


iv.

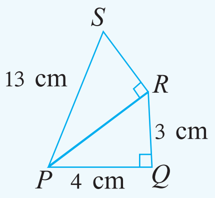


2. Find the perimeter of each figure given below.

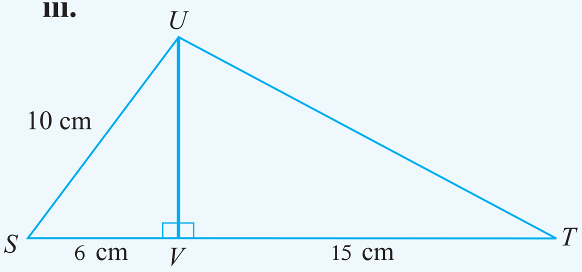
i.



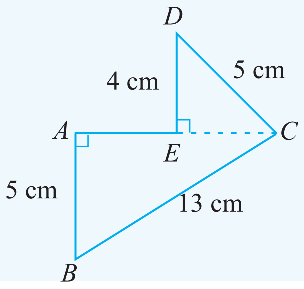
ii.



iii.

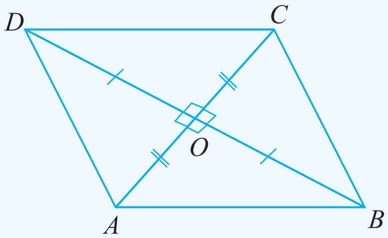


iv.



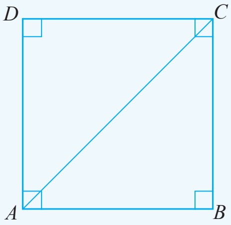
3.

i.



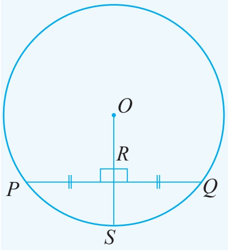
The diagonals BD and AC of the rhombus $ABCD$ bisect each other perpendicularly at O. Moreover, $BD = 16$cm and $AC = 12$cm. Find the perimeter of the rhombus.

ii.



If the length of the diagonal AC of the square $ABCD$ is 10cm, find the area of the square.

iii.



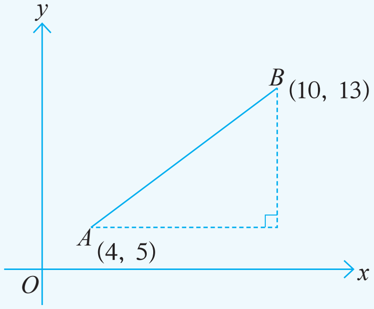
In the circle with centre O shown in the figure, the midpoint of the chord PQ is R. Moreover, OR produced meets the circle at S. If $O\hat{R}P = 90^{\circ}$,$O\hat{R}P$ PQ=12 cm and OR= 8 cm, find

i. the length of RQ,
ii. the radius of the circle,
iii. the length of RS.

4. In the triangle ABC, $A\hat{B}C = 90^{\circ}$, $AB = 8$ cm and $BC = 6$ cm. The mid points of BC and BA are R and P. Find the perimeter of the quadrilateral $APRC$.

Miscellaneous Exercise

1.

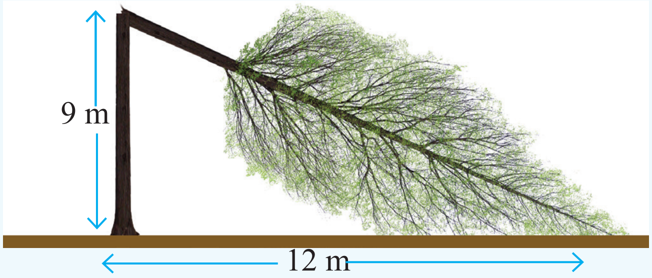


Find the shortest distance between the point $A = (4, 5)$ and $B = (10, 13)$ located on a Cartesian plane.

2. The city Q is located 5 km east of the city P and the city R is located 12 km north of the city Q. Find the distance between the two cities P and R.

3. To keep a 16m tall flag post vertical, one end of a supportive cable is attached to the top of the post while the other end is fixed to a point on the ground (horizontal), 12m from the foot of the flag post. Another cable is fixed from the opposite direction, with one endattached to the flag post, 12m above its foot, and the other to the ground, 9m from its foot. Calculate the total length of the cable that has been used.

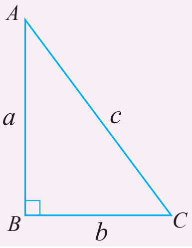
4.



A tree which was struck by a tornado is shown in the figure. Find the height of the tree before it was struck.

Summary

In the ABC right angled triangle



$$AC^2 = AB^2 + BC^2$$

$$c^2 = a^2 + b^2$$

20

Graphs

By studying this lesson you will be able to;

- identify functions,
- draw graphs of functions of the form $y = mx$ and $y = mx + c$ and identify their characteristics,
- identify the gradient and intercept of a straight line graph,
- plot straightline graphs of equations of the form $ax + by = c$
- identify the relationship between the gradients of straight lines which are parallel to each other.

Do the following exercise to recall what has been learnt in previous grades regarding graphs.

Review Exercise

1. i. Draw a coordinate plane with the x and y axes marked from -5 to 5 and mark the points A $(-4,-4)$ and B $(4,-4)$ on it. Mark the points C and D such that $ABCD$ is a square and write the coordinates of C and D.

ii. Write the equation of each side of the plane figure $ABCD$.

2. Draw a coordinate plane with the x and y axes marked from -4 to 4.

i. Draw two straight lines, one parallel to the x axis and the other parallel to the y axis passing through the point $(4, -4)$.

ii. Draw another two straight line, one parallel to the x axis and the other parallel to the y- axis passing through the point $(-3, 2)$.

iii. Write the coordinates of the two points at which the lines in (i) and (ii) above intersect each other.

iv. Write the equations of the axes of symmetry of the plane figure obtained in (iii) above.

20.1 Functions

We have come across relationships between different quantities in various situations. Carefully observe the relationship between the two quantities given below.

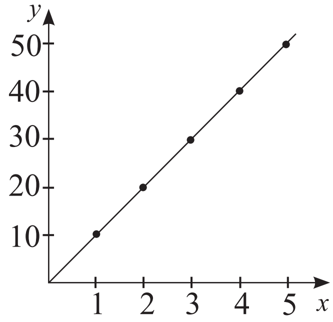
Let us suppose that beads are sold at 10 rupees per gramme. The prices of different amounts of beads is shown below.

Beeds **(g)**		Price (Rs)
1	$\longrightarrow$	$1 \times 10 = 10$
2	$\longrightarrow$	$2 \times 10 = 20$
3	$\longrightarrow$	$3 \times 10 = 30$
4	$\longrightarrow$	$4 \times 10 = 40$

Accordingly, it is clear that the price of x grammes of beeds is Rs $10x$. Moreover, if the price of x grammes of beeds is represented by Rs y, then we can write $y = 10x$.

Let us take the amount of grammes of beeds as x and the corresponding price as Rs y.

By plotting the price (y) against the amount of grammes of beeds (x) for different values of x using the above relationship, we obtain the following graph.



In the above function $y = 10x$, the index of the independent variable x is 1. Therefore it is called a linear function.

When a linear function is given, the values of y corresponding to different values of x can be found as follows.

Example 1

For each of the linear functions given below, calculate the values of y corresponding to the given values of x and write them as ordered pairs.

i. $y = 2x$ (values of x: $-2, -1, 0, 1, 2$)
ii. $y = -\frac{3}{2}x + 2$ (values of x: $-4, -2, 0, 2, 4$)

i. $y = 2x$

x	$2x$	y	Ordered pairs (x, y)
-2	2×-2	-4	$(-2, -4)$
-1	2×-1	-2	$(-1, -2)$
0	2×0	0	$(0, 0)$
1	2×1	2	$(1, 2)$
2	2×2	4	$(2, 4)$

ii. $y = -\frac{3}{2}x + 2$

x	$-\frac{3}{2}x + 2$	y	Ordered pairs (x, y)
-4	$-\frac{3}{2} \times -4 + 2$	8	$(-4, 8)$
-2	$-\frac{3}{2} \times -2 + 2$	5	$(-2, 5)$
0	$-\frac{3}{2} \times 0 + 2$	2	$(0, 2)$
2	$-\frac{3}{2} \times 2 + 2$	-1	$(2, -1)$
4	$-\frac{3}{2} \times 4 + 2$	-4	$(4, -4)$

Exercise 20.1

Find the values of y corresponding to the given values of x and write them as ordered pairs.

i. $y = 3x$ (values of x: $-2, -1, 0, 1, 2$)

ii. $y = 2x + 3$ (values of x: $-3, -2, -1, 0, 1, 2, 3$)

iii. $y = -\frac{1}{3}x - 2$ (values of x: $-6, -3, 0, 3, 6$)

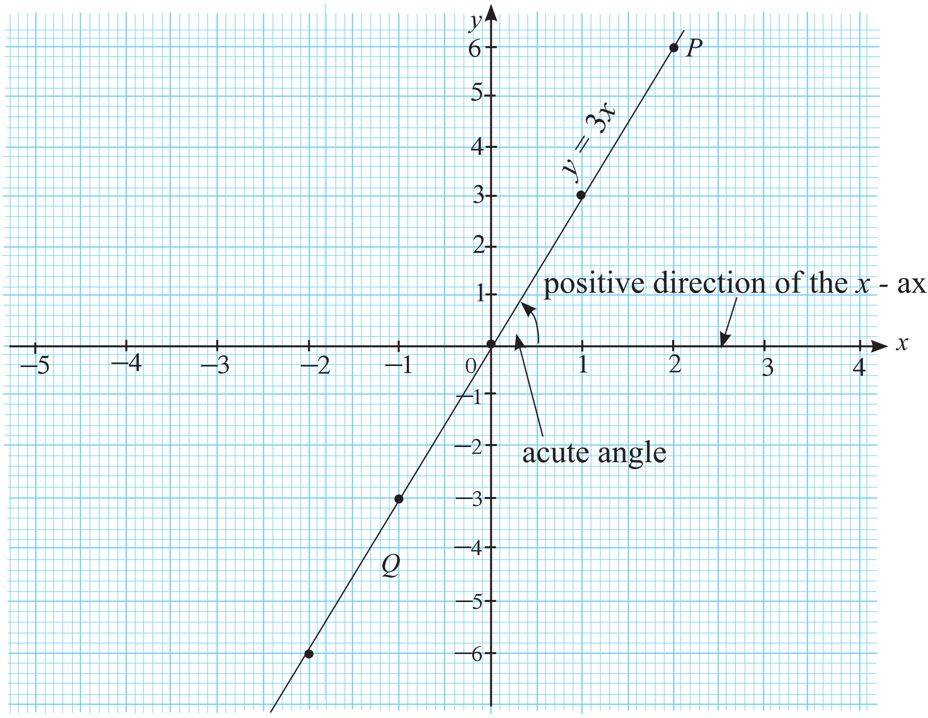
20.2 Functions of the form $y = mx$ and the gradient of the graph of such a function

Linear functions such as $y = 3x$, $y = -2x$ and $y = x$ are examples of functions of the form $y = mx$. Let us obtain ordered pairs of values of x and y by constructiong a table as follows, to draw the graph of $y = 3x$ for the values of x from -2 to $+2$.

$y = 3x$

x	$3x$	y	(x, y)
-2	3×-2	-6	$(-2, -6)$
-1	3×-1	-3	$(-1, -3)$
0	3×0	0	$(0, 0)$
1	3×1	3	$(1, 3)$
2	3×2	6	$(2, 6)$

By plotting the above ordered pairs in a coordinate plane, we obtain the graph of the function $y = 3x$ as shown below.



Let us consider some characteristics of the above drawn graph.

- The graph is a straight line
- It passes through the point (0, 0)
- It makes an acute angle counterclockwise with the positive direction of the x - axis
- When any point on the line other than the origin is considered, the value of $\dfrac{y \text{ coordinate}}{x \text{ coordinate}}$ of that point is a constant (a constant value).

For example,

$$\text{when the point } P \text{ is considered, } \frac{y \text{ coordinate}}{x \text{ coordinate}} = \frac{6}{2} = 3$$

$$\text{when the point } Q \text{ is considered, } \frac{y \text{ coordinate}}{x \text{ coordinate}} = \frac{-3}{-1} = 3$$

Moreover, this constant value is equal to the coefficient m of x of a function of the form $y = mx$. This constant value is called the **gradient** of the graph.

The gradient can be a positive value or a negative value.

Now, let us explain the behavior of functions of the form $y = mx$ through the activity given below.

Activity 1

1. a. Complete the tables given below for the selected positive values of m to obtain coordinates of points to draw the graphs of the given functions of the form $y = mx$, and draw the graphs on one coordinate plane.

(i) $y = x$ (ii) $y = +3x$ (iii) $y = +\frac{1}{3}x$

x	-2	0	2
y	___	___	$+2$

x	-1	0	1
y	-3	___	___

x	-3	0	3
y	___	___	$+1$

b. Complete the tables given below for the selected negative values of m to obtain coordinates of points to draw the graphs of the given functions of the form $y = mx$ and draw the graphs on one coordinate plane.

(i) $y = -x$ (ii) $y = -3x$ (iii) $y = -\frac{1}{3}x$

x	-2	0	2
y	___	___	-2

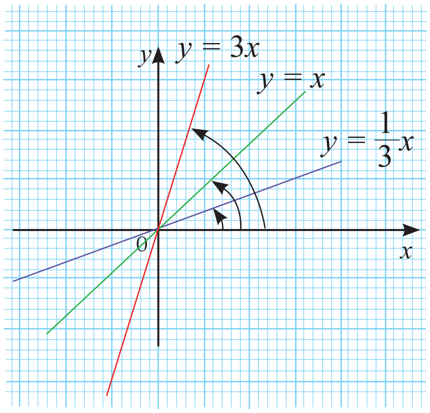
x	-1	0	1
y	___	0	___

x	-3	0	3
y	1	___	___

Observe the relationship between the angle that a graph makes with the positive direction of the x -axis and the gradient (value of m) by considering the graphs obtained in (*a*) and (*b*) above.

By doing the above activity you would have obtained the following graphs.

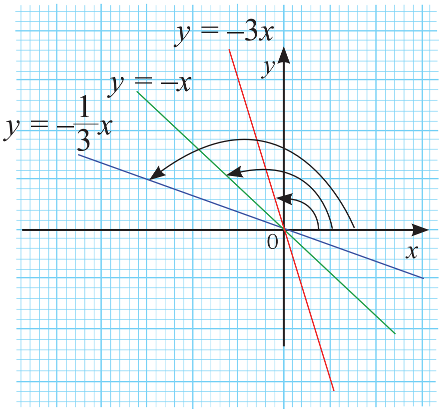
(a) Graphs obtained when the gradient is positive



- ⋆ When the gradient (value of m) is positive, the angle that the graph forms counter clockwise with the positive direction of the x-axis is an acute angle.

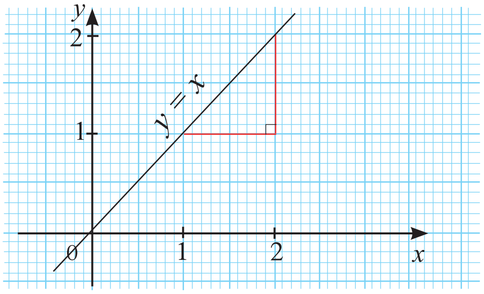
- ⋆ As the value of the gradient increases in the order $\frac{1}{3}$, 1, 3 the magnitude of the angle formed by the graph of the function with the positive direction of the x-axis counterclockwise increases.

(b) Graphs obtained when m is negative

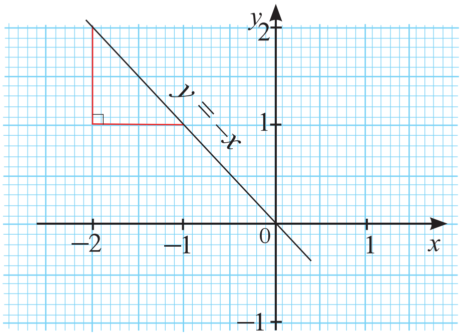


- ⋆ When the value of the gradient (value of m) is negative, the angle formed with the positive direction of the x-axis counterclockwise is an obtuse angle.
- ⋆ As the value of the gradient (value of m) increases in the order $-3, -1, -\frac{1}{3}$ the magnitude of the angle formed by the graph of the function with the positive direction of the x-axis counterclockwise increases.

Note: Gradient of a graph



The gradient of the graph of the function $y = x$ is 1. This means that when the value of x increases by one unit, the value of y also increases by one unit.



The gradient of the graph of the function y = − x is −1. This means that when the value of x increases by one unit, the value of y decreases by one unit.

Example 1

Write the gradient of the graph of each of the functions given below without drawing the graph.

i. $y = 2x$
ii. $y = -5x$
iii. $y = -\frac{1}{2}x$

i. Gradient $(m) = 2$
ii. Gradient $(m) = -5$
iii. Gradient $(m) = -\frac{1}{2}$

Example 2

i. Draw the graphs of the functions $y = 2x$ and $y = -3x$ on the same coordinate plane by selecting suitable values for x.

ii. Using the above drawn graphs, find the value of x when $y = 3$, and the value of y when $x = 2.5$.

i. $y = 2x$

x	-2	-1	0	1	2
$+2x$	2×-2	2×-1	2×0	2×1	2×2
y	-4	-2	0	2	4

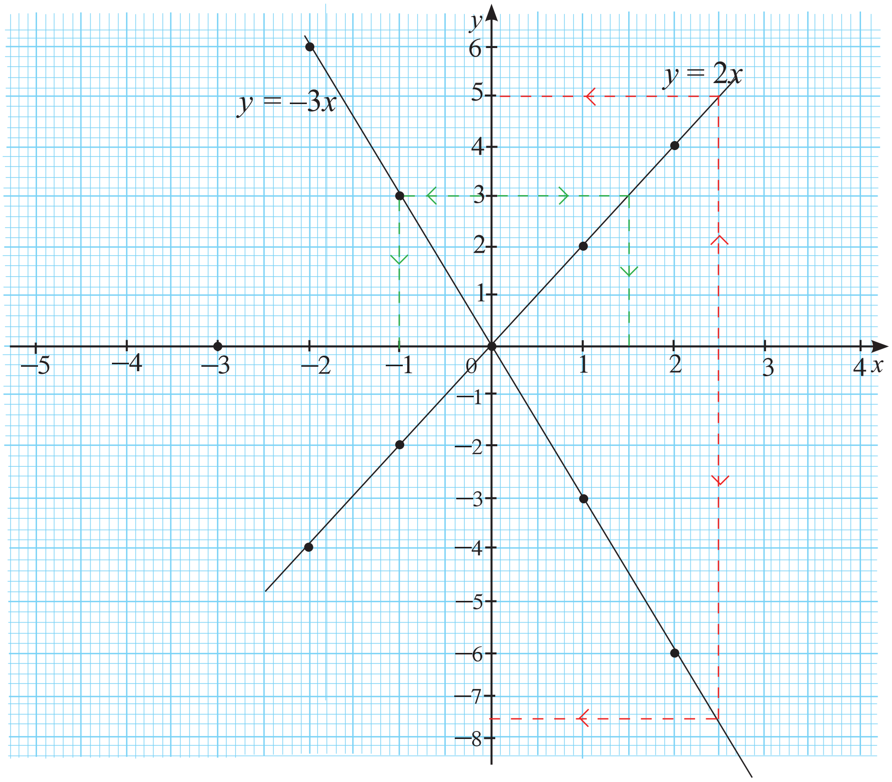
$(-2, -4)$ $(-1, -2)$ $(0, 0)$ $(1, 2)$ $(2, 4)$

$y = -3x$

x	-2	-1	0	1	2
$-3x$	-3×-2	-3×-1	-3×0	-3×1	-3×2
y	6	3	0	-3	-6

$(-2, 6)$ $(-1, 3)$ $(0, 0)$ $(1, -3)$ $(2, -6)$

By plotting the above ordered pairs on the same coordinate plane, graphs of the following form will be obtained.



ii. To find the value of y when $x = 2.5$, the line $x = 2.5$, should be drawn (indicated by the red line) and the y -coordinate of the points of intersection of this line with the two graphs should be obtained.

If we consider the function $y = 2x$, the value of y is 5 when the value of x is 2.5

If we consider the function $y = – 3x$, the value of y is – 7.5 when the value of x is 2.5

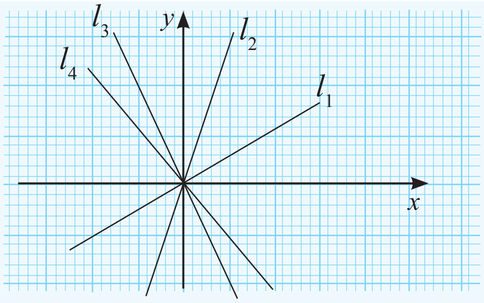
To find the value of x when $y = 3$, the line y = 3 should be drawn (indicated by the green line) and the x - coordinate of the points of intersection of this line with the two graphs should be obtained.

If we consider the function $y = 2x$, the value of x is $1\frac{1}{2}$ when the value of y is 3.

If we consider the function y = – $3x$, the value of x is –1 when the value of y is 3.

Exercise 20.2

1. For each of the functions given by the following equations, select and write the corresponding graph, from the graphs l_1, l_2, l_3, and l_4 .



i. $y = 3x$ **ii.** $y + 2x = 0$

iii. $2y - x = 0$ **iv.** $y + \frac{3}{2}x = 0$

2. The value of a Singapore dollor in Sri Lankan rupees is 100. By taking the amount of Singapore dollors as x and the corresponding value in Sri Lankan rupees as y, the relationship between Singapore dollors and Sri Lankan rupees can be written as a function as $y = 100x$.

- **i.** Prepare a suitable table of values to draw the graph of the above function. (Use the values 1, 2, 3 and 4 for x)
- **ii.** Draw the graph of the above function.
- **iii.** Using the above drawn graph, find the value of 4.5 Singapore dollors in Sri Lankan rupees.
- **iv.** Using the graph, find how much 250 Sri Lankan rupees are in Singapore dollors.

3. For the statements given below, mark a '✓' in front of the correct statements and a '✗' in front of the incorrect statements.

- **i.** For functions of the form $y = mx$, the direction of the graph is decided by the sign of m. (....)
- **ii.** When the graph of a function of the form $y = mx$ is given, the graph of $y = - mx$ cannot be obtained by considering symmetry about the y - axis. (....)
- **iii.** For a straight line passing through the origin, the ratio of the y - coordinate to the x - coordinate of any point on the line other than the origin is equal to its gradient. (....)
- **iv.** Although the point $(- 2, 3)$ lies on the straight line given by $2y + 3x = 0$, it does not lie on the straight line given by $2y - 3x = 0$. (....)
- **v.** The graph of a function of the form $y = mx$ need not pass through the point (0,0). (....)

4. i. Construct a table of values to draw the graphs of the functions given by $y = \frac{1}{3}x$, $3y = 2x$ and $y = -1\frac{1}{3}x$ by taking the values of x to be $-6, -3, 0, 3$ and 6.

ii. Draw the above graphs on the same coordinate plane.

iii. Write the x - coordinate of each of the three points at which the line $y = 1$ intersects the above three graphs.

5. i. Fill in the blanks in the incomplete table given below to draw the graph of the function $y = -\frac{2}{3}x$.

x	-6	-3	0	3	6
y	4	_____	_____	-2	_____

ii. Draw the graph of the above given function using the values in the completed table.

iii. Using the graph, find the value of y when $x = -2$.

iv. Does the point $(-\frac{2}{3}, \frac{2}{3})$ lie on the above graph? Explain your answer with reasons.

v. Using the coordinates of three points on the line, calculate the ratio of the y - coordinate to the x - coordinate. Write the relationship between this value and the gradient.

20.3 Graphs of functions of the form $y = mx + c$ and functions given by $ax + by = c$

Graphs of functions of the form $y = mx + c$

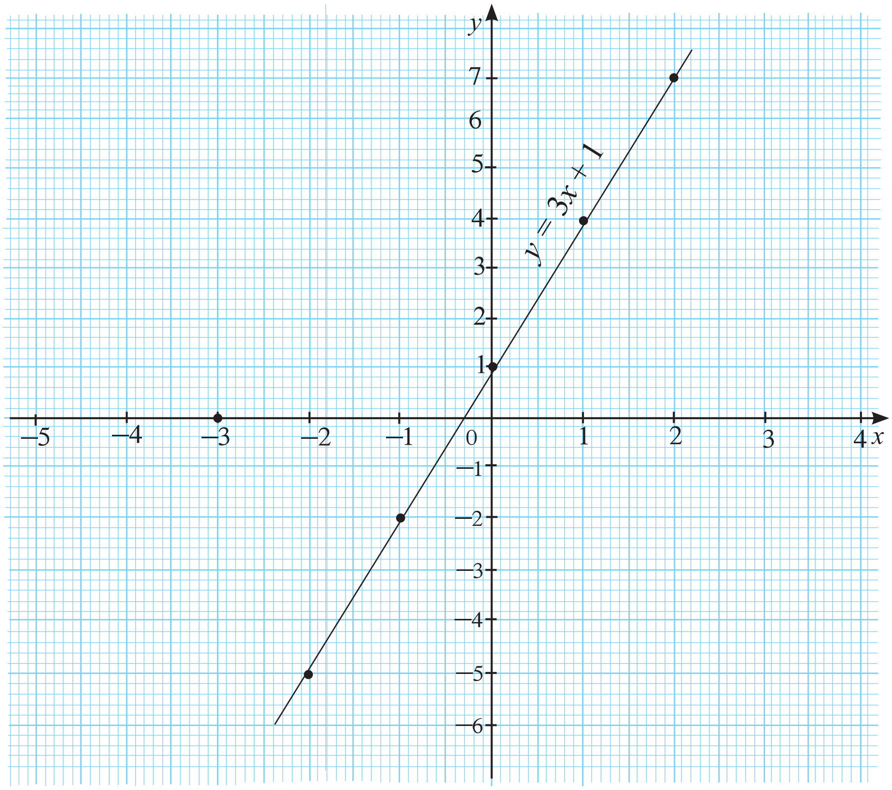
Let us first consider functions of the form $y = mx + c$. Let us draw the graph of the function $y = 3x + 1$.

To do this, let us first develop a lable of values as shown below.

$y = 3x + 1$

x	$3x + 1$	y	(x, y)
-2	$3 \times -2 + 1$	-5	$(-2, -5)$
-1	$3 \times -1 + 1$	-2	$(-1, -2)$
0	$3 \times 0 + 1$	1	$(0, 1)$
1	$3 \times 1 + 1$	4	$(1, 4)$
2	$3 \times 2 + 1$	7	$(2, 7)$

The graph that is obtained when the function is plotted on a coordinate plane using the ordered pairs in the above table of values is shown below.



By observing the above graph, the following characteristics can be identified.

- It is a straight line graph.
- The straight line intersects the y - axis at (0, 1).
- The straight line makes an acute angle counterclockwise with the positive direction of the x- axis. The value of m of this line is + 3. This means that when the variable x increases by 1 unit, the variable y increases by 3 units.
- The value representing c in the equation y = $3x + 1$ is + 1. The y cordinate of the point at which the straight line intersects the y - axis is also 1. These two values are equal.

The distance, from the origin to the point where the straight line intersects the y - axis is known as the intercept. The intercept of this line is + 1.

Accordingly, the gradient of the graph of a function of the form $y = mx + c$ is m and the intercept is c.

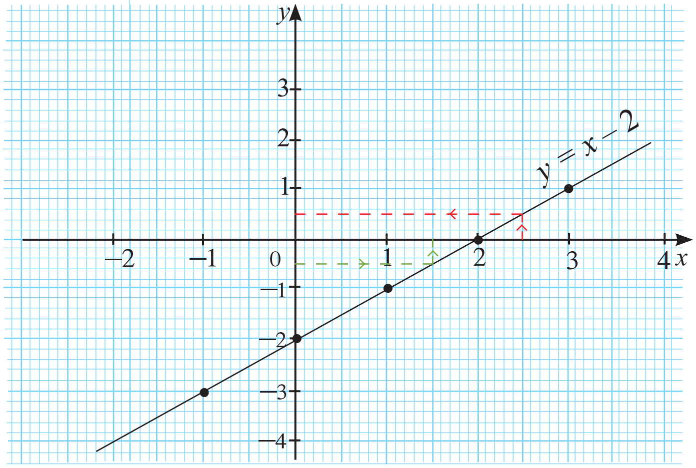
Example 1

Prepare a suitable table of values and draw the graph of the function $y = x - 2$. Using the graph, find the following.

i. The intercept.
ii. The value of y when $x = 2.5$
iii. The value of x when $y = -\frac{1}{2}$

$y = x - 2.$

x	-1	0	1	2	3
$y = x - 2$	-3	-2	-1	0	1



i. Intercept $(c) = -2.$

ii. $y = \frac{1}{2}$ when $x = 2.5.$

iii. $x = 1\frac{1}{2}$ when $y = -\frac{1}{2}.$

Example 2

Without drawing the graph, write the gradient and the intercept of the graph of each of the functions given by the following equations.

i. $y = -2x + 5$
ii. $y + 3x = -2$

i. $y = -2x + 5$ is of the form $y = mx + c$.
Accordingly, the gradient $(m) = -2$
The intercept $(c) = 5$

ii. Let us first write $y + 3x = -2$ in the form $y = mx + c$.
That is, $y = -3x - 2$.
Accordingly, the gradient $(m) = -3$
The intercept $(c) = -2$

Example 3

Prepare suitable tables for the functions $y = 2x$, $y = 2x + 1$ and $y = 2x - 3$, and draw their graphs on the same coordinate plane.

i. Write the gradient and intercept of each graph by observing the equation.
ii. Write a special feature that you can observe in the graphs.

$y = 2x$

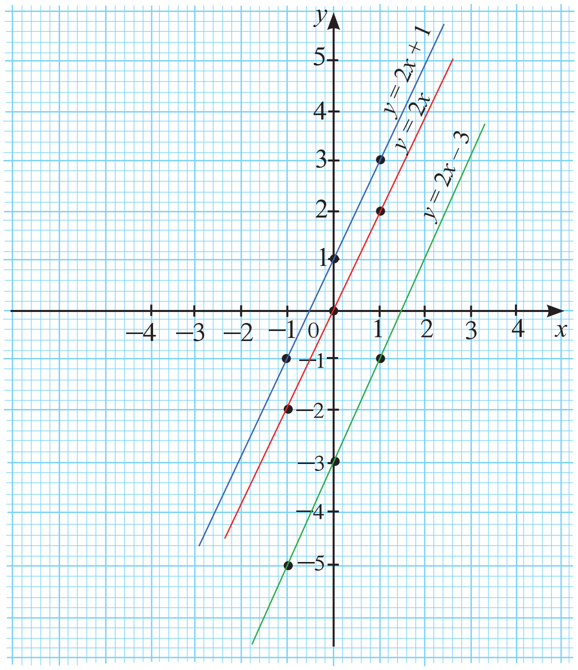
x	–1	0	1
y	–2	0	2

$y = 2x + 1$

x	–1	0	1
y	–1	1	3

$y = 2x - 3$

x	–1	0	1
y	–5	–3	–1



$y = 2x$
gradient = 2; intercept = 0

$y = 2x + 1$
gradient = 2; intercept = + 1

$y = 2x - 3$
gradient = 2; intercept = –3

By observing the equations of the graphs it is clear that the gradients of the three graphs are the same.By observing the graphs it can be seen that the lines are all parallel to each other. Therefore, it is clear that if the gradients of two or more linear functions are equal to each other, then their graphs will be parallel straight lines.

Graphs of functions given by equations of the form *ax* + *by* = *c*

Now let us consider the graphs of functions given by equations of the form $ax + by = c$. It is easy to prepare the table of values by writing this equation in the form $y = mx + c$.

Consider the following example.

Example 1

Prepare a suitable table of values and plot the graph of the function given by the equation $3x + 2y = 6$.

Using the graph that is drawn,

i. write the coordinates of the points at which it intersect the main axes.
ii. writethe gradient and intercept.

First let us change this equation to the form $y = mx + c$

$$3x + 2y = 6$$

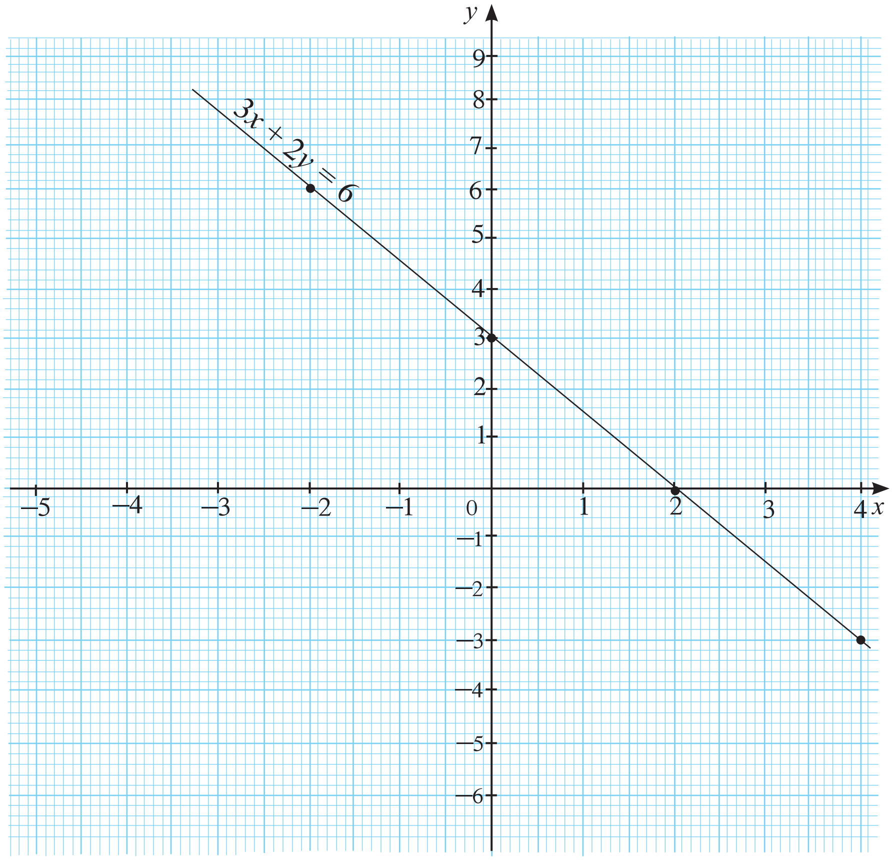
$$2y = -3x + 6$$

$$y = -\frac{3}{2}x + 3.$$

Let us obtain the coordinates of points that lie on the graph of this function from the following table of values and use them to draw the graph.

x	$-\frac{3}{2}x + 3$	y
-2	$-\frac{3}{2} \times -2 + 3$	6
0	$-\frac{3}{2} \times 0 + 3$	3
2	$-\frac{3}{2} \times 2 + 3$	0
4	$-\frac{3}{2} \times 4 + 3$	-3

(– 2, 6) (0,3) (2, 0)(4, – 3)



i. The graph intersects the y - axis at (0, 3) and the x- axis at (2, 0).

ii. Gradient $(m) = -\frac{3}{2}$, intercept $(c) = 3$

Note:

- Observe that the graph of $3x + 2y = 6$ intersects the y axis at the point $(0, 3)$ and that the y coordinate of that point is equal to the coefficient of x in the equation $3x + 2y = 6$.
- Observe that the graph of $3x + 2y = 6$ intersects the x axis at the point $(2, 0)$ and the x coordinate of that point is equal to the coefficient of y in the equation $3x + 2y = 6$.
- Note that the graph of $3x + 2y = 6$ can be plottted by joining the points $(0, 3)$ and $(2, 0)$, without preparing a table of values.

Exercise 20.3

1. For each of the functions given by the equations in parts (a) and (b), write the gradient and intercept without drawing the corresponding graphs and write whether the graph makes an acute or obtuse angle counterclockwise with the positive direction of the x - axis.

(a) **i.** $y = x + 3$ **ii.** $y = -x + 4$ **iii.** $y = \frac{2}{3}x - 2$ **iv.** $y = 4 + \frac{1}{2}x$

(b) **i.** $2y = 3x - 2$ **ii.** $4y + 1 = 4x$ **iii.** $\frac{2}{3}x + 2y = 6$

2. For each of the functions given below, find the coordinates of the points at which the graph of the function intersects the two axes and plot the graph of each function using these two points.

(a) **i.** $y = 2x + 3$ **ii.** $y = \frac{1}{2}x + 2$

(b) **i.** $2x - 3y = 6$ **ii.** $-2x + 4y + 2 = 0$

3. Using the information given below, write the equation of each straight line.

Gradients (m)	Intercept (c)	Equation of the function
i. $+2$	-5	$y = 2x - 5$
ii. -3	$+4$	
iii. $-\frac{1}{2}$	-3	
iv. $\frac{3}{2}$	$+1$	
v. 1	0	

4. An incomplete table of values prepared to draw the graph of the function $y = -3x - 2$ is given below.

x	-2	-1	0	1	2
y	______	______	-2	______	-8

i. Fill in the blanks.
ii. Draw the graph of the above function.
iii. Draw the straight line given by $y = x$ on the same coordinate plane and write the coordinates of the point of intersection of the two lines.

5. By selecting suitable values for x, construct a table of values and draw the graphs of the following functions on the same coordinate plane.

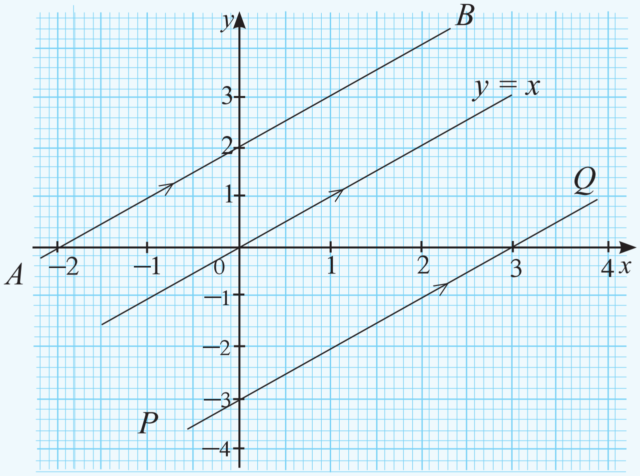
i. $y = x$ **ii.** $y = -2x + 2$ **iii.** $y = \frac{1}{2}x + 1$ **iv.** $y = -\frac{1}{2}x - 3$

6. Draw the graphs of the functions given by the following equations for the values of x from -4 to $+4$.

a. $-3x + 2y = 6$ and $3x + 2y = -6$

b. $y + 2x = 4$ and $-2x + y = -4$

7. Write the equations of the straight lines AB and PQ in the following figure.



Miscellaneous Exercise

1. For the statements given below, mark a "√" in front of the correct statements and a "×" in front of the incorrect statements.

i. For all m, the graph of a function of the form $y = mx + c$ is a straight line which is not parallel to the main axes. (............)

ii. For a function of the form $y = mx + c$, the value of m determines the direction of the straight line graph and the value of c determines the point where the graph intersects the y - axis. (............)

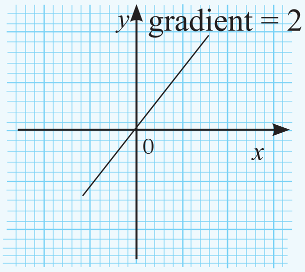
iii. It is not necessary for c to be zero, for the graph of a function of the form $y = mx + c$ to pass through the origin, (............)

iv. The graphs of the functions given by the equations $y_1 = m_1x + c_1$ and $y_2 = m_2x + c_2$ will be parallel when $m_1 = m_2$. (............)

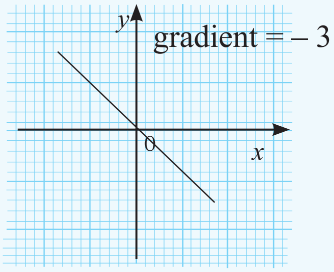
v. A straight line given by $y = mx + c$, intersects the y-axis above the x-axis only when $m > 0$, and $c > 0$. (............)

2. Write the equations of the functions of the graphs sketched below.

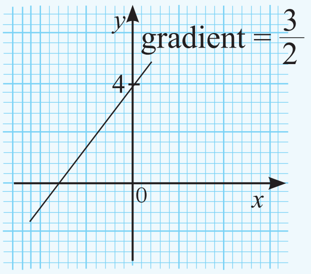
i.



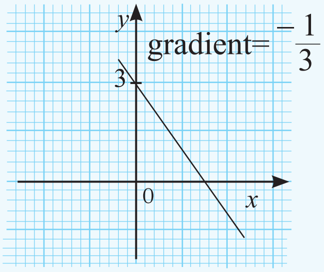
ii.



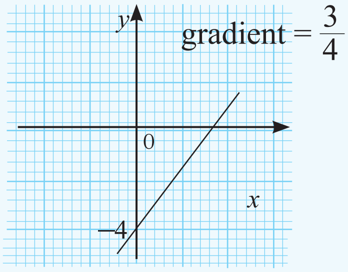
iii.



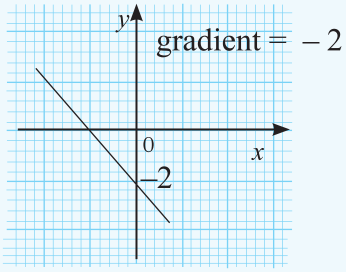
iv.



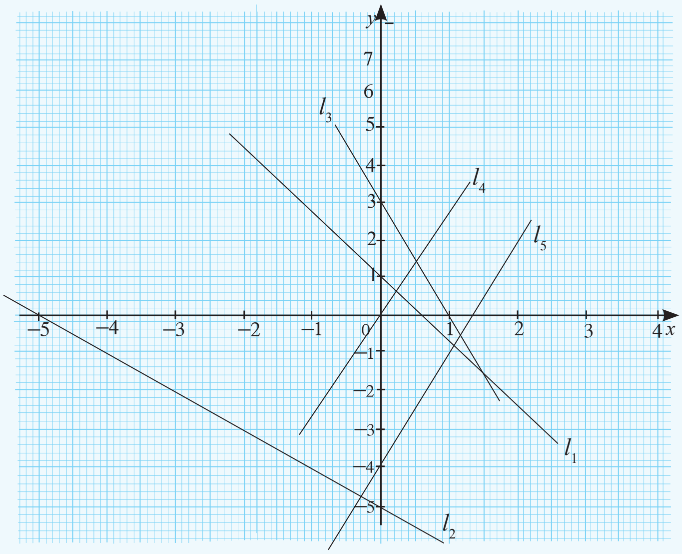
v.



vi.



3. Select and write the graph corresponding to each of the given functions.



Function

i. $y = 3x - 4$

ii. $y = -2x + 1$

iii. $y = -x - 5$

iv. $y = -3x + 3$

v. $y = +3x$

4. The gradient of the straight line given by $4x + py = 10$ is $-\frac{4}{3}$.

i. Find the value of p. **ii.** Write the intercept.

iii. Write the equation of the straight line with gradient –2 which passes through the point at which the above given straight line intersects the y - axis.

Summary

- The gradient of the graph of a function of the form $y = mx + c$ is m and the intercept is c.

- If the gradients of two or more linear functions are equal to each other, then their graphs will be parallel straight lines.

Revision Exercise – Second Term
Part I

1. The price of a dozen books of a certain type is Rs 240. Find the maximum number of books that can be bought for Rs 150.

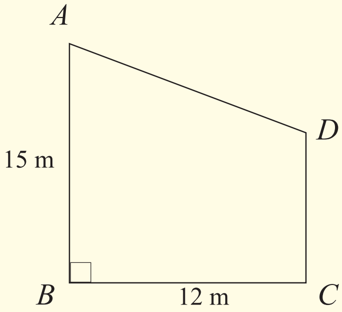
2. The price of a certain item is Rs 85000. If a discount of 20% is given when an outright purchase is made, using calculator find the amount that a customer has to pay when purchasing the item outright.

3. Simplify $\frac{(x^{-3})^0}{(2x^{-1})^2}$.

4. **i.** Round off 12.673 to the nearest second decimal place.
  ii. Round off 4873 to the nearest hundred.

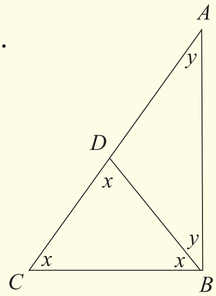
5. **i.** Write 5.62×10^{-3} in general form.
  ii. Write 348 005 in scientific notation.

6. *ABCD* is the side view of a vertical wall of a house. If it is required to fix a bulb at an equal distance from *A* and *D* and 10 m from *B*, indicate its position by a rough sketch.



7. Solve $5\{3(x+1) - 2(x-1)\} = 10$

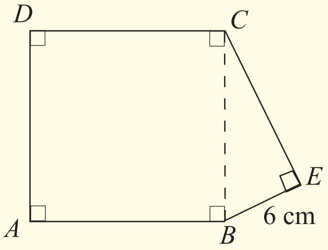
8. Find the values of x and y using the data in the given diagram.



9. Make r the subject of the formula $V = I\,(R + r)$.

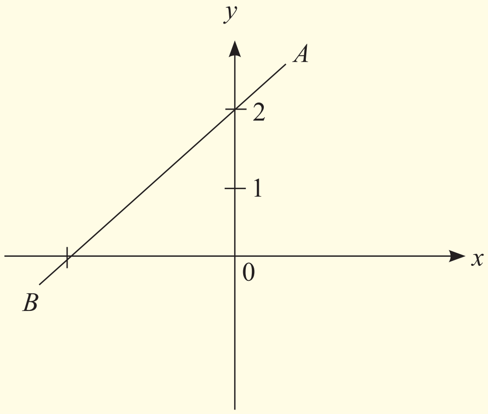
10. A square of side length 11 cm is made from a thin wire. Find the diameter of the largest circular bangle that can be made using this wire. (Use $\frac{22}{7}$ for the value π).

11. The area of the square $ABCD$ is 100 cm^2. If BE = 6 cm, find the perimeter of the figure**.**



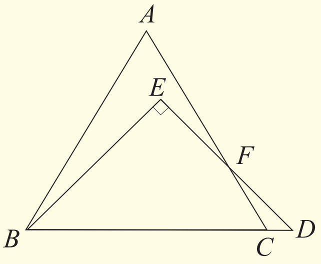
12. The gradient of the straight line AB is 3. Which of the following points are located on AB?

$(1, -5), (-1, -1), (\frac{1}{3}, -3), (-\frac{1}{3}, 1)$



13. A group of Sri Lankans employed in a foreign country sent 25000 US dollars as aid for those affected by floods. How much is this amount in Sri Lankan rupees? (Assume that 1US dollar =150 Sri Lankan rupees)

14. ABC is an equilateral triangle. If $E\hat{F}A = 20^{\circ}$, find the magnitude of $A\hat{B}E$.



Part II

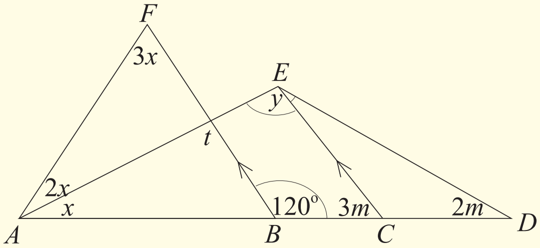
1.

x	-2	-1	0	1	2
y	-5	....	1	....	3

An incomplete table of values prepared to draw the graph of a function of the form $y = 2x - 1$ is given above.

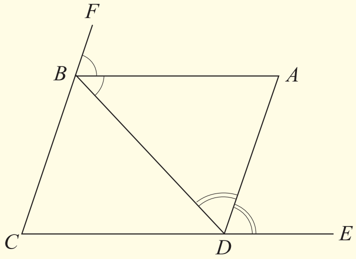
- **i.** Fill the blanks in the table.
- **ii.** Draw the graph of the above function on a suitable coordinate plane.
- **iii.** If the point $(-5, k)$ is located on the above line, find the value of k.
- **iv.** Write the equation of the straight line which is parallel to the line drawn in (ii) above and which passes through the point (0, 2).

2. (a) The straight lines BF and CE are parallel. Using the data given in the figure, find the following.



- **i.** The values of x, t, y and m.
- **ii.** The magnitude of $A\hat{E}D$.

(b) In the following figure, the bisectors of the angles $D\hat{B}F$ and $B\hat{D}E$ intersect at A.



- **i.** Express the magnitude of $A\hat{B}D$ in terms of the magnitudes of $B\hat{D}C$ and $D\hat{C}B$.
- **ii.** Express the magnitude of $A\hat{D}B$ in terms of the magnitudes of $D\hat{C}B$ and $C\hat{B}D$.
- **iii.** Using the results of (I) and (II) above, show that $B\hat{A}D = 90^\circ - \dfrac{B\hat{C}D}{2}$.

3. (a) Simplify the expressions given below and write the answers in terms of positive indices.

(i) $\dfrac{(a^{-3})^2 \times (b^{\frac{1}{2}})^8}{(a^2 \times b^3)^{-2}}$ (ii) $\dfrac{x^3 \times (2y)^2 \times t^3}{(2y^0)^3 \times x^{-2} \times (t^{-\frac{1}{2}})^2}$

(b) Fill in the blanks.

$$\begin{tikzcd}
x^{\dots} \times y^{\dots} & & \dfrac{1}{x^{\dots} \times y^{\dots}} \\
(x^{\dots} \times y^{\dots})^{\dots} & \left(\dfrac{x^2}{y^{-3}}\right)^2 \arrow[ul] \arrow[ur] \arrow[l] \arrow[r] \arrow[dl] \arrow[dr] & \dfrac{y^{\dots}}{x^{\dots}} \\
x^{\dots} \times \dfrac{1}{y^{\dots}} & & x^{\dots} \div y^{\dots}
\end{tikzcd}$$

4. i. Draw a straight line segment *AB* such that *AB* = 8 cm. Mark the point *C* such that $B\hat{A}C = 60^{\circ}$ and *AC* = 5 cm.

ii. Draw the locus of points which are 2 cm from *AB* and located on the same side of *AB* as *C*.

iii. Mark the point *P* which is located on the locus drawn in (ii) above and is equidistant from *AC* and *AB*.

iv. Name the two points on *AB* which are 3 cm from *P* as Q_1 and Q_2. Measure and write the distance between Q_1 *and* Q_2.

5. Write the order in which you need to press the keys of a calculator to simplify the following expressions and obtain their values using a calculator.

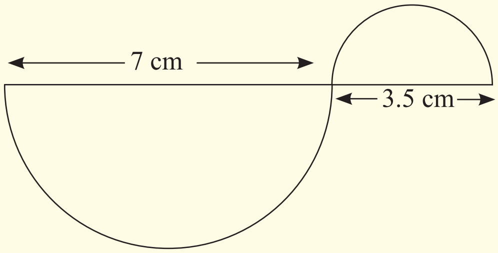
(i) $\dfrac{3.2 \times 5.83}{4.72}$ (ii) $\dfrac{2.5^2 \times 8.3}{4.7}$ (iii) 520 × 20% (iv) $\sqrt{\dfrac{20 \times 9}{5}}$

6. Express the distance to each planet from Earth in scientific notation.

i. The distance from Earth to planet A is 427 000 000 km.

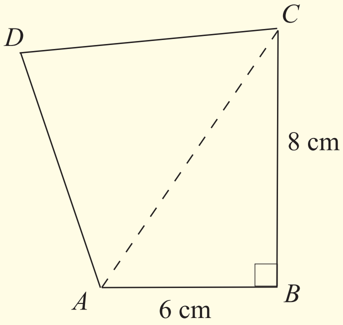
ii. The distance from Earth to planet B is 497 000 000 km.

7. (a) The shape shown below is made using a thin metal wire. (Use $\frac{22}{7}$ for the value π).

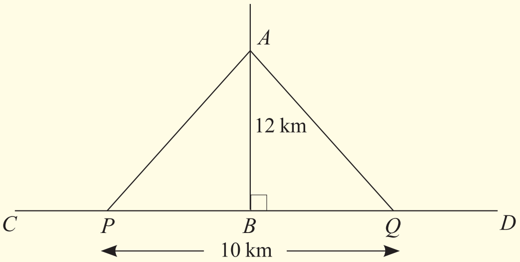


i. Find the total length of metal wire used to make this structure.
ii. If the price of 1m of this metal wire is Rs 120, find the price of the metal wire required for this structure.

(b) If the perimeter of the equilateral triangle *ADC* is 30 cm, find the perimeter of the figure.



8. As shown in the figure, *AB* and *CD* are two straight roads which are perpendicular to each other. *AP* and *AQ* too are two straight roads located as shown in the figure. A factory located at *A* produces two types of items. To store them, the warehouses *P* and *Q* located on *CD* at an equal distance from *B* are used. The distance between *P* and *Q* is 10 km. Giving reasons, explain which roads you would select to transport the items from *A* to *P* and *Q* using the same lorry so that the transport cost is the least.



9. (a) Make a the subject of the $A = \frac{h}{2}(a + b)$ formula. Find the value of a when $A = 70$, $h = 10$, $b = 8$.

(b) Solve.

(i) $2m + 3n = 6$
$2m - 7n = -14$

(c) Solve.

i. $2x + 3\{2(x + 2) + 3(x - 4)\} = 10$

ii. $\frac{2(x + 1)}{3} - 5 = \frac{x - 1}{3}$

iii. $3\left[1 + \frac{(2x - 1)}{3}\right] = 2(3 - x)$

10. **i.** If the price of a dozen eggs is Rs 186, find the price of 25 eggs.

ii. The price of 1 litre of petrol is Rs 117. A certain motorbike requires 3 litres of petrol to travel 180 km. What is the minimum amount he needs to spend on petrol to travel 330 km?

iii. A son employed in a foreign country sends 5000 Sterling Pound to his parents. What is the value of this amount in Sri Lankan rupees?
(Assume that 1 Sterling Pound = 190 Sri Lankan rupees).

Glossary

A

Algebraic form	වීජීය ආකාරය	அட்சரகணித வடிவம்
Axioms	ප්‍රත්‍යක්ෂ	வெளிப்படை உண்மைகள்

B

Bisector	සමච්ඡේදකය	இருகூறாக்கி

C

Capacity	ධාරිතාව	கொள்ளளவு
Circle	වෘත්තය	வட்டம்
Circumference	පරිධිය	பரிதி
Construction	නිර්මාණය	அமைப்பு
Constant distance	නියත දුර	மாறாத் தூரம்
Cube	ඝනකය	சதுரமுகி

D

Diameter	විශ්කම්භය	விட்டம்
Direct Proportion	අනුලෝම සමානුපාතය	நேர்விகிதசமன்
Division	බෙදීම	வகுத்தல்

E

Equal distance	සමාන දුර	சம தூரம்

F

Fixed point	අචල ලක්ෂ්‍යය	நிலையான புள்ளி
Foreign currency	විදේශ මුදල්	வெளிநாட்டு நாணயம்
Formula	සූත්‍රය	சூத்திரம்
Function	ශ්‍රිතය	சார்பு

G

Gradient	අනුක්‍රමණය	படித்திறன்
Graph	ප්‍රස්තාරය	வரைபு

H

Hypotenuse	කර්ණය	செம்பக்கம்

I

Index	දර්ශකය	சுட்டிகள்
Intercept	අන්තඃඛණ්ඩය	வெட்டுத்துண்டு
Interior angles	අභ්‍යන්තර කෝණ	அகக்கோணங்கள்
Intersection	ඡේදනය	இடைவெட்டு தல

K

Key	යතුර	சாவி
Key board	යතුරු පුවරුව	சாவிப்பலகை

L

Locus	පථය	ஒழுக்கு

M

Multiplication	ගුණ කිරීම	பெருக்கல்

P

Parallel	සමාන්තර	சமாந்தரம்
Parallel lines	සමාන්තර රේඛා	சமாந்தரக்கோடுகள்
Perpendicular	ලම්බය	செங்குத்து
Perpendicular bisector	ලම්බ සමච්ඡේදකය	இருசமவெட்டிச் செங்குத்து
Power	බලය	வலு
Proportion	සමානුපාතය	விகிதசமன்
Pythagorean Relation	පයිතගරස් සම්බන්ධය	பைதகரஸ் தொடர்பு

Q

Quantity	රාශිය	கணியம்

R

Radius	අරය	ஆரை
Right angle	සෘජුකෝණය	செங்கோணம்
Right angled triangle	සෘජුකෝණික ත්‍රිකෝණය	செங்கோண முக்கோணி
Rules of indices	දර්ශක නීති	சுட்டி விதிகள்

S

Scientific notation	විද්‍යාත්මක අංකනය	விஞ்ஞான முறைக் குறிப்பீடு
Simple equations	සරල සමීකරණ	எளிய சமன்பாடுகள்
Simultaneous equations	සමගාමී සමීකරණ	ஒருங்கமை சமன்பாடுகள்
Straight line	සරල රේඛාව	நேர்கோடு
Subject	උක්තය	எழுவாய்
Substitution	ආදේශය	பிரதியிடல்

T

Theorem	ප්‍රමේයය	தேற்றம்
Triangle	ත්‍රිකෝණය	முக்கோணம்

U

Unknown	අඥාතය	தெரியாக்கணியம்

V

Verify	සත්‍යාපනය	வாய்ப்புப்பார்த்தல்

Lesson Sequence

Contents	Numbers of Periods
First term	
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2. Binary Numbers	03
3. Fractions	05
4. Percentages	06
5. Algebraic Expressions	05
6. Factors of the Algebraic Expressions	05
7. Axioms	04
8. Angles related to straight lines and parallel lines	07
9. Liquid Measurements	03
Second term	
10. Direct proportion	06
11. Calculator	02
12. Indices	03
13. Round off and Scientific Notation	05
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16. Angles of a triangle	09
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18. Circumference of circle	05
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Third term	
21. Inequalities	03
21. Sets	07
23. Area	05
24. Probability	05
25. Angles of polygons	05
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28. Data representation and prediction	10

MATHEMATICS

Grade 9

Part - III

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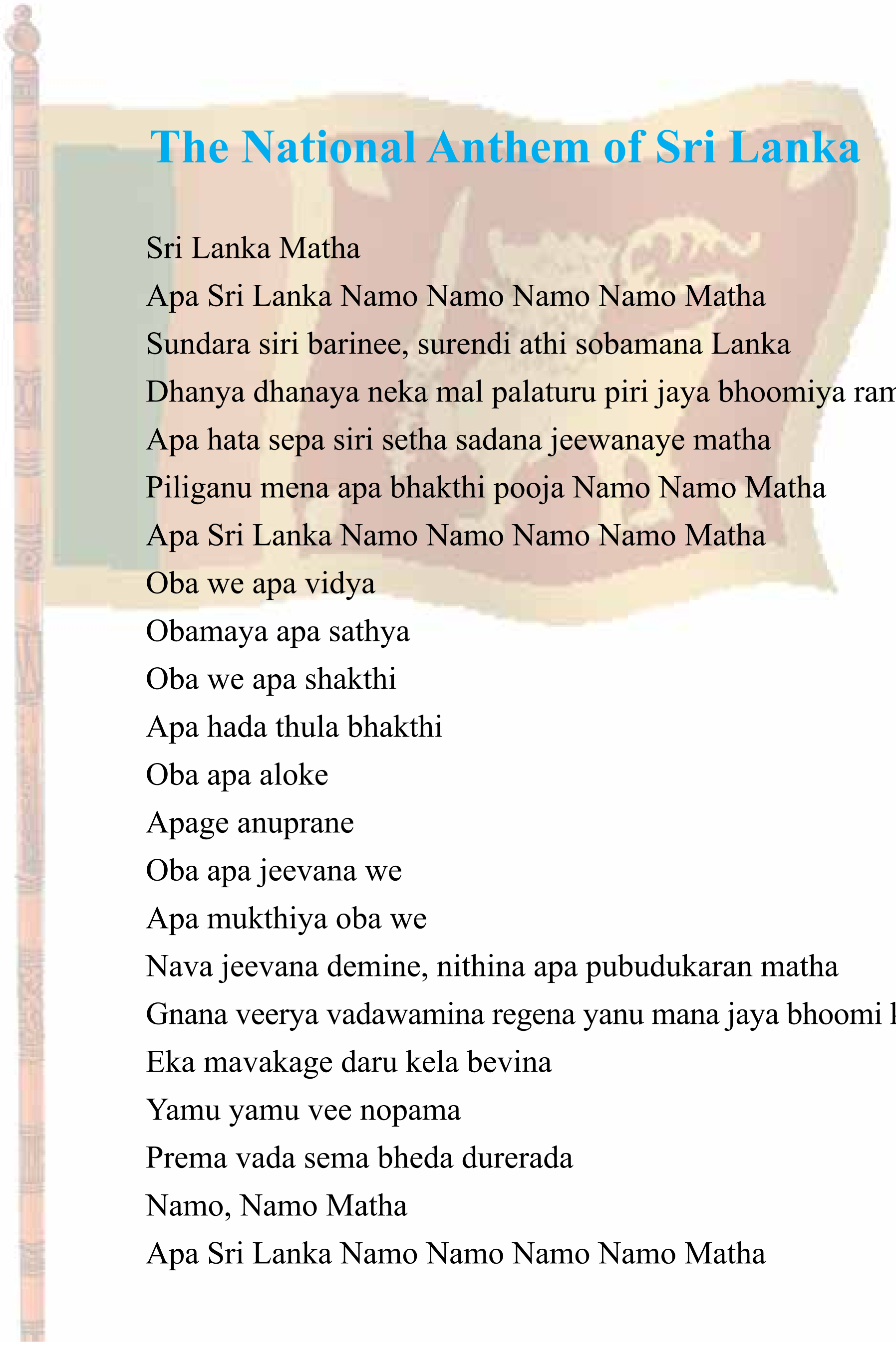

The National Anthem of Sri Lanka

Sri Lanka Matha
Apa Sri Lanka Namo Namo Namo Namo Matha
Sundara siri barinee, surendi athi sobamana Lanka
Dhanya dhanaya neka mal palaturu piri jaya bhoomiya ramya
Apa hata sepa siri setha sadana jeewanaye matha
Piliganu mena apa bhakthi pooja Namo Namo Matha
Apa Sri Lanka Namo Namo Namo Namo Matha
Oba we apa vidya
Obamaya apa sathya
Oba we apa shakthi
Apa hada thula bhakthi
Oba apa aloke
Apage anuprane
Oba apa jeevana we
Apa mukthiya oba we
Nava jeevana demine, nithina apa pubudukaran matha
Gnana veerya vadawamina regena yanu mana jaya bhoomi kara
Eka mavakage daru kela bevina
Yamu yamu vee nopama
Prema vada sema bheda durerada
Namo, Namo Matha
Apa Sri Lanka Namo Namo Namo Namo Matha

අපි වෙමු එක මවකගෙ දරුවෝ
එක නිවසෙහි වෙසෙනා
එක පාටැති එක රුධිරය වේ
අප කය තුළ දුවනා

එබැවිනි අපි වෙමු සොයුරු සොයුරියෝ
එක ලෙස එහි වැඩෙනා
ජීවත් වන අප මෙම නිවසේ
සොඳින සිටිය යුතු වේ

සැමට ම මෙත් කරුණා ගුණෙනී
වෙළී සමගි දමිනී
රන් මිණි මුතු නො ව එය ම ය සැපතා
කිසි කල නොම දිරනා

ආනන්ද සමරකෝන්

ஒரு தாய் மக்கள் நாமாவோம்
ஒன்றே நாம் வாழும் இல்லம்
நன்றே உடலில் ஓடும்
ஒன்றே நம் குருதி நிறம்

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ஒன்றாய் வாழும் வளரும் நாம்
நன்றாய் இவ் இல்லினிலே
நலமே வாழ்தல் வேண்டுமன்றோ

யாவரும் அன்பு கருணையுடன்
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யான்று மழியாச் செல்வமன்றோ.

ஆனந்த சமரக்கோன்
கவிதையின் பெயர்ப்பு.

Being innovative, changing with right knowledge
Be a light to the country as well as to the world.

Message from the Hon. Minister of Education

The past two decades have been significant in the world history due to changes that took place in technology. The present students face a lot of new challenges along with the rapid development of Information Technology, communication and other related fields. The manner of career opportunities are liable to change specifically in the near future. In such an environment, with a new technological and intellectual society, thousands of innovative career opportunities would be created. To win those challenges, it is the responsibility of the Sri Lankan Government and myself, as the Minister of Education, to empower you all.

This book is a product of free education. Your aim must be to use this book properly and acquire the necessary knowledge out of it. The government in turn is able to provide free textbooks to you, as a result of the commitment and labour of your parents and elders.

Since we have understood that the education is crucial in deciding the future of a country, the government has taken steps to change curriculum to suit the rapid changes of the technological world. Hence, you have to dedicate yourselves to become productive citizens. I believe that the knowledge this book provides will suffice your aim.

It is your duty to give a proper value to the money spent by the government on your education. Also you should understand that education determines your future. Make sure that you reach the optimum social stratum through education.

I congratulate you to enjoy the benefits of free education and bloom as an honoured citizen who takes the name of Sri Lanka to the world.

Akila Viraj Kariyawasam
Minister of Education

Foreword

The educational objectives of the contemporary world are becoming more complex along with the economic, social, cultural and technological development. The learning and teaching process too is changing in relation to human experiences, technological differences, research and new indices. Therefore, it is required to produce the textbook by including subject related information according to the objectives in the syllabus in order to maintain the teaching process by organizing learning experiences that suit to the learner needs. The textbook is not merely a learning tool for the learner. It is a blessing that contributes to obtain a higher education along with a development of conduct and attitudes, to develop values and to obtain learning experiences.

The government in its realization of the concept of free education has offered you all the textbooks from grades 1-11. I would like to remind you that you should make the maximum use of these textbooks and protect them well. I sincerely hope that this textbook would assist you to obtain the expertise to become a virtuous citizen with a complete personality who would be a valuable asset to the country.

I would like to bestow my sincere thanks on the members of the editorial and writer boards as well as on the staff of the Educational Publications Department who have strived to offer this textbook to you.

W. M. Jayantha Wickramanayaka,
Commissioner General of Educational Publications,
Educational Publications Department,
Isurupaya,
Battaramulla.
2019.04.10

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Message of the Board of Compilers

This textbook has been compiled in accordance with the new syllabus to be implemented from 2015.

Textbooks are compiled for students. Therefore, we have made an attempt to compile this textbook in a simple and detailed manner making it possible for you to read and understand it on your own.

We have included descriptions, activities and examples to introduce the subject concepts in an attractive manner and to establish them. Moreover, activities are organized from simple to complex to develop an interest to do them.

We have used the terms related to mathematical concepts in accordance with the glossary of technical terms of mathematics compiled by the Department of Official Languages.

Some subject matter learnt during the earlier grades is necessary to learn the subject content in the grade 9 syllabus. Thus, review exercises are included at the beginning of each chapter to revise previous knowledge. You will be prepared by them for the subject content of grade 9.

You will gain maximum benefit from this textbook by reading the chapters and doing the review exercises of each chapter even before your teacher teaches them in the classroom.

We hope that studying mathematics will be an interesting, joyful and productive experience.

Board of Compilers

Contents

21

Inequalities

By studying this lesson you will be able to;

- solve inequalities of the form $x \pm a \gtreqless b$,
- solve inequalities of the form $ax \gtreqless b$,
- find the integral solutions of an inequality,
- represent the solutions of an inequality on a number line.

In Sri Lanka, a person who is 55 years or older is considered to be a senior citizen. Accordingly, if we denote the age of a senior citizen by t, then we can indicate this by the inequality $t \geqslant 55$. This means that the value of t is always greater or equal to 55.

Let us recall what was learnt in grade 8 regarding inequalities.

$x > 3$ is an inequality. This means that the values that x can take are the values that are greater than 3. However, if we write $x \geq 3$, this means that the values that x can take are 3 or a value greater than 3.

Similarly, $x < 3$ means that the values that x can take are the values that are less than 3, and $x \leq 3$ means that the values that x can take are 3 or a value less than 3.

For example, the solution set of $x > 3$ is the set of all numbers which are greater than 3. The set of integral solutions of this inequality is $\{4, 5, 6, 7, \ldots\}$.

The three dots mean that all the integers which belong to the pattern indicated by the first few values belong to the solution set. Therefore, the above inequality has infinitely many integral solutions.

Although in mathematics it is important to represent all the solutions as a set, when indicating the integral solutions of an inequality, it is sufficient to just write the values. For example, the integral solutions of the inequality $x > 3$ can be written as 4, 5, 6, … .

The solution set of an inequality which has an algebraic term, is the set of all values that the algebraic term can take. Let us recall what has been learnt earlier regarding the solution set of an inequality, and how this is represented on a number line, by considering the following examples.

Example 1

Consider the inequality $x > 3$.

i. Write the set of **integral solutions** of the above inequality.
ii. Represent the **integral solutions** of the above inequality on a number line.

i. $\{4, 5, 6, 7, 8, ...\}$

ii.

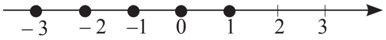
Example 2

Consider the inequality $x \leq 1$.

i. Write the set of integral solutions of the above inequality.
ii. Represent the integral solutions of the above inequality on a number line.

i. $\{..., -3, -2, -1, 0, 1\}$

ii.



Example 3

Represent the solution set of the inequality $x > -3\frac{1}{2}$ on a number line.

Example 4

Represent the solution set of the inequality $x \geq -2$ on a number line.

Example 5

Represent the **solution set** of the inequality $-3 < x \leq 3\frac{1}{2}$ on a number line.

Review Exercise

1. For each of the following inequalities, represent the **set of integral solutions on** a number line.

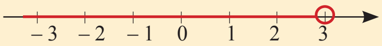
i. $x > 2$ **ii.** $x \geq -1$ **iii.** $x < 4$ **iv.** $x \leq -2.5$ **v.** $x > 1\frac{1}{2}$

2. For each of the following, write an inequality that has the values represented on the number line as its solution set.

i.

ii.

iii.



iv.

3. Represent the solution set of each of the following inequalities on a number line.

i. $-1 < x < 2$ **ii.** $-2 \leq x < 3$ **iii.** $-3 < x \leq 1$

iv. $x < -1$ or $x \geq 2$ **v.** $x \leq -3$ or $x > 0$

21.1 Inequalities of the form $x \pm a \gtreqless b$

The following notice has been placed near a certain bridge.

"This bridge can only bear loads of less than 10 tons"

Suppose a lorry of mass 4 tons carrying a certain load wishes to cross this bridge. If we take the mass of the load carried by the lorry to be x tons, then the lorry can safely cross the bridge only if $x + 4 < 10$. In other words, if the mass of the load carried by the lorry is x tons, then the lorry can safely cross the bridge, only if the inequality $x + 4 < 10$ is satisfied.

We can find the mass of the load that the lorry can safely carry across the bridge by solving the inequality $x + 4 < 10$.

Solving an inequality means, obtaining an inequality equivalent to the given inequality such that only x (or the given variable) is on one side of the inequality.

When solving an inequality, we can adopt the procedure followed in solving equations to a large extent.

For example, we can subtract 4 from both sides of the above inequality $x + 4 < 10$. Accordingly,
$x + 4 - 4 < 10 - 4$.
When we simplify this we obtain
$x < 6$.

Therefore, for the lorry to safely cross the bridge, the load that it carries should be less than 6 tons.

Example 1

Solve the inequality $x + 2 < 7$ and represent the integral solutions on a number line.

$x + 2 < 7$

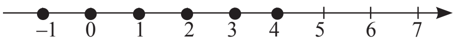
$x + 2 - 2 < 7 - 2$ (subtracting 2 from both sides)

$\underline{\underline{x < 5}}$

The integral solutions of this inequality are the integers that are less than 5. That is, the values 4, 3, 2, 1, 0, –1, –2, … .

These integral solutions can also be expressed as a set as $\{4, 3, 2, 1, 0, -1, -2, \ldots\}$
The solution set can be represented on a number line as follows.

Integral solutions of x



Example 2

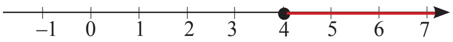
Solve the inequality $x - 3 \geq 1$ and represent the solution set on a number line.

$x - 3 \geq 1$

$x - 3 + 3 \geq 1 + 3$ (adding 3 to both sides)

$\underline{\underline{x \geq 4}}$

Now let us represent the solution set on a number line.



All the solutions (numbers that are greater or equal to 4) are represented here. It is important to remember that not only the integral solutions, but solutions such as 4.5 and 5.02 are also included.

Example 3

The maximum mass that a bag can carry is 6 kg. Nimal puts x packets of rice of mass 1 kilogramme each and 2 packets of sugar of mass 1 kilogramme each into this bag. This information can be represented by the inequality $x + 2 \leq 6$.

i. Solve this inequality.
ii. What is the maximum number of packets of rice that Nimal can carry in this bag?

i. $x + 2 \leq 6$

$x + 2 - 2 \leq 6 - 2$

$\underline{\underline{x \leq 4}}$

ii. Therefore, the maximum number of packets of rice that can be carried in this bag is 4.

Exercise 21.1

1. Solve each of the following inequalities and write the set of integral solutions.

i. $x + 3 > 5$ **ii.** $x - 4 < 1$ **iii.** $x - 7 \geq - 6$ **iv.** $2 + x \leq - 4$
v. $7 + x > 5$

2. Solve each of the following inequalities and represent the solution set on a number line.

i. $x + 1 > 3$ **ii.** $x - 3 \leq 1$ **iii.** $6 + x \geq 2$ **iv.** $x - 7 < - 7$
v. $x + 5 > - 1$

3. Sakindu has 60 rupees. He buys a book for x rupees and a pen for 10 rupees. The total value of the items he bought can be expressed in terms of an inequality as $x + 10 \leq 60$. Solve this inequality and determine the maximum price that the book could be.

4. The maximum number of people that can travel in a certain van is 15. If 3 people get into the van from one location and x number of people from another location, this information can be represented by the inequality $x + 3 \leq 15$.

i. Solve the above inequality.
ii. What is the maximum number of people that can get into the van from the second location?

5. The sum of the ages of Githmi and Nethmi does not exceed 30. Githmi is 14 years old. If Nethmi's age is taken as x years, this information can be represented by the inequality $x + 14 \leq 30$. Solve this inequality and find the maximum age that Nethmi could be.

21.2 Inequalities of the form $ax \gtrless b$

The price of two books of the same type is more than 40 rupees. If we take the price of one of these books as x rupees, we can represent this information by the inequality $2x > 40$ involving x. By solving this inequality, we can find the price that each book could be.

When solving this type of inequalities there are some important facts that we should keep in mind.

Consider the following inequalities.

i. The inequality $3 < 4$ is true.
$2 \times 3 < 2 \times 4$ (multiplying both sides by 2)
The inequality $6 < 8$ is true.

ii. The inequality $8 > 6$ is true.
$\frac{8}{2} > \frac{6}{2}$ (dividing both sides by 2)
The inequality $4 > 3$ is true.

When the two sides of an inequality are either multiplied by the same positive number or divided by the same positive number, the inequality sign does not change.

iii. The inequality $2 < 3$ is true.
$2 \times -2 < 3 \times -2$ (multiplying both sides by -2)
The inequality $-4 < -6$ which is obtained is false. However, $-4 > -6$ is true.

iv. The inequality $9 > 6$ is true.
$\frac{9}{-3} > \frac{6}{-3}$ (dividing both sides by -3)
The inequality $-3 > -2$ which is obtained is false. However, $-2 < -3$ is true.

When an inequality is multiplied or divided by a negative number, the inequality sign changes. That is, the sign $<$ changes to $>$ and the sign $\geqslant$ changes to $\leqslant$, etc .

Through the following examples, let us learn how inequalities are solved taking into consideration the above facts.

Example 1

Solve the inequality $2\,x < 12$ and represent the solutions on a number line.

$$2\,x < 12$$
$$\frac{2x}{2} < \frac{12}{2} \quad \text{(dividing both sides by 2)}$$
$$\underline{\underline{x < 6}}$$

Example 2

Solve the inequality $3x \geq\ 12$.

$$3x \geq\ 12$$
$$\frac{3x}{3} \geq \frac{12}{3}$$
$$\underline{\underline{x \geq 4}}$$

Example 3

Solve the inequality $-\,5x \leq 15$.

$$-\,5x \leq 15$$
$$\frac{-\,5x}{-\,5} \geq \frac{15}{-\,5} \quad \text{(when dividing by a negative number the sign changes)}$$
$$\underline{\underline{x \geq -\,3}}$$

Example 4

Solve the inequality $\frac{x}{3} < 2$.

$$\frac{x}{3} \times 3 < 2 \times 3 \qquad \text{(multiplying both sides by 3)}$$
$$\underline{\underline{x < 6}}$$

Example 5

Solve the inequality $-\frac{2x}{5} > 6$.

$$-\frac{2x}{5} > 6$$

$$-\frac{2x}{5} \times 5 > 6 \times 5 \quad \text{(multiplying both sides by 5)}$$

$$-2x > 30$$

$$\frac{-2x}{-2} < \frac{30}{-2} \quad \text{(the sign changes when dividing by} -2\text{)}$$

$$\underline{\underline{x < -15}}$$

Exercise 21.2

1. Solve each of the following inequalities and write the integral solutions.

i. $2x > 6$ **ii.** $3x \leq 12$ **iii.** $-5x \geq 10$ **iv.** $-7x < -35$

v. $-2x > -5$ **vi.** $\frac{x}{2} \leq 1$ **vii.** $\frac{x}{4} \geq -2$ **viii.** $-\frac{2x}{3} < 4$

2. Solve each of the following inequalities and represent the solutions on a number line.

i. $4x > 8$ **ii.** $7x \leq 21$ **iii.** $-3x \geq 3$ **iv.** $-2x < -6$

v. $\frac{x}{3} \geq 1$ **vi.** $\frac{x}{6} < -\frac{1}{6}$ **vii.** $\frac{2x}{3} \geq 4$ **viii.** $-\frac{3x}{5} < -\frac{1}{6}$

3. The price of 2 mangoes is less than 50 rupees. If the price of one mango is x rupees, this information can be represented by the inequality $2x \leq 50$. Solve this inequality and find the maximum possible price of a mango.

4. The maximum mass that can be carried by an elevator is 560 kilogrammes. Eight men of mass x kilogrammes each are riding this elevator. This information can be represented by the inequality $8x \leq 560$. Find the maximum mass that each man could be.

5.

(a) The amount of money Mahesh has is less than four times the amount that Ashan has. Mahesh as 68 rupees. If the amount that Ashan has is denoted by x rupees, then this information can be represented by the inequality $4x > 68$. Solve this inequality.

(b) If Ashan has only 5 rupee coins, what is the least amount he could be having?

Summary

- When the two sides of an inequality are either multiplied by the same positive number or divided by the same positive number, the inequality sign does not change.
- When an inequality is multiplied or divided by a negative number, the inequality sign changes. That is, the sign $<$ changes to $>$ and the sign $\geqslant$ changes to $\leqslant$, etc.

22

Sets

By studying this lesson you will be able to;

- identify finite and infinite sets,
- write the subsets of a given set,
- identify equivalent sets, equal sets, disjoint sets and universal sets,
- identify the intersection and union of two sets,
- identify the complement of a set,
- represent sets using Venn diagrams.

Introduction to sets

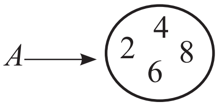
You have learnt earlier that a set is a collection of items that can be clearly identified. The items in a set are called its elements. Curly brackets are used to represent sets in terms of their elements. If a is an element of the set A, we denote this by $a \in A$. Moreover, the number of elements in the set A is denoted by $n(A)$.

Let us recall what was learnt previously about the different ways in which a set can be expressed.

1. Describing the set using a common characteristic by which the elements can be clearly identified
2. Writing the elements within curly brackets
3. Using a Venn diagram

As an example, let us consider how to write the set of all even numbers between 0 and 10 using the above 3 methods in the given order. Let us name this set A.

1. A = {even numbers between 0 and 10}
2. $A = \{2, 4, 6, 8\}$
3. A



The set which has no elements is known as the **null set**. The null set is denoted by { } and also by $\emptyset$. The number of elements in the null set is 0. Therefore, $n(\emptyset) = 0$.

Example 1

If P = {even prime numbers between 5 and 10}, then find $n(P)$.
Since there are no even prime numbers between 5 and 10, $P = \emptyset$ and therefore $n(P) = 0$.

Review Exercise

1. Determine whether each of the following collections is a set.

- **i.** Multiples of four between 0 and 30
- **ii.** The districts of Sri Lanka
- **iii.** Students who are good at mathematics
- **iv.** Triangular numbers
- **v.** The 10 largest integers

2. Write each set given below in terms of its elements and write the number of elements in each set too.

- **i.** A = {multiples of 5 between 0 and 20};
- **ii.** B = {letters of the word "RECONCILLIATION"}
- **iii.** C = {prime numbers between 2 and 13}
- **iv.** D = {numbers between 0 and 20 which are a product of two prime numbers}

3. D = {whole numbers between 5 and 10}.

- **i.** Write the elements of D.
- **ii.** Find $n(D)$.

4. Express the null set in three different ways using the above mentioned first method (That is, by the descriptive method using a special characteristic.)

22.1 Finite sets, infinite sets, equivalent sets and equal sets

Finite sets and infinite sets

Two sets which have been expressed in terms of a common characteristic by which the elements of the set can be clearly identified, are given below.

A = {multiples of 3 between 0 and 20}
B = {multiples of 5}

Let us write the elements of each set within curly brackets.

$A = \{3, 6, 9, 12, 15, 18\}$ $\quad$ $B = \{5, 10, 15, 20, ...\}$

The number of elements in set A is 6. That is, the number of elements in this set is a specific number. Sets with a specific number of elements (that is, sets with a finite number of elements), are known as **finite sets**.

The number of elements in set B however, cannot be stated definitely. That is, the number of elements in this set is infinite. Three dots have been placed at the end of the list of numbers within curly brackets to denote that the set B has an infinite number of elements. Sets that have an infinite number of elements are known as **infinite sets**.

Example 1

For each of the sets given below, write the elements and write whether it is a finite set or an infinite set.

P = {positive multiples of 6 less than 30}
Q = {polygons}

$P = \{6, 12, 18, 24\}$ $\quad$ $n(P) = 4$
Q = {triangle, quadrilateral, pentagon, hexagon, … }

Since the number of elements in the set P is finite, P is a finite set. Since the number of elements in the set Q is infinite, Q is an infinite set.

Equal sets

Consider the two sets given below.

A = {even numbers between 0 and 10}

B = {digits of the number 48268}

These two sets can be written as follows in terms of their elements.

$A = \{2, 4, 6, 8\}$
$B = \{2, 4, 6, 8\}$

Although the two sets A and B have been described differently, when they are written in terms of their elements, we get the same set. Sets which have the same elements are known as **equal sets**. Accordingly, A and B are equal sets. If two sets A and B are equal then we write $A = B$.

Equivalent sets

If the number of elements in the two sets A and B are equal, that is, if $n(A) = n(B)$, then the sets A and B are known as equivalent sets.

If A and B are equivalent sets, we denote this by $A \sim B$.

Example 2

X = {odd numbers between 0 and 10}
Y = {vowels in the English alphabet}

By writing the elements of these sets, show that they are equivalent sets.

$x = \{1, 3, 5, 7, 9\}$ $n(X) = 5$
$y = \{a, e, i, o, u\}$ $n(Y) = 5$

Since $n(X) = n(Y)$, X and Y sets are equivalent sets.

Note :- Although all pairs of equal sets are equivalent, all pairs of equivalent sets need not be equal.

Exercise 22.1

1. From the sets given below, select and write the finite sets and the infinite sets separately.

i. A = {multiples of 5 from 0 to 50}
ii. B = {integers}
iii. C = {numbers that can be written using only the digits 0 and 1}
iv. D = {digits of the number 25265}
v. E = {positive integers which are not prime}

2. Write each of the following sets in terms of their elements and then write all pairs of equal sets and all pairs of equivalent sets.

P = {positive multiples of 3 below 10}
Q = {letters of the word "net"}
R = {odd numbers between 0 and 10}
S = {digits of the number 3693}
T = {vowels in the English alphabet}
v = {letters of the word "ten"}

3. Write 3 examples of finite sets.

4. Write 3 examples of infinite sets.

5. Write three sets which are equivalent to the set $\{2, 3\}$.

22.2 The universal set and subsets

Subsets

When two sets A and B are considered, if all the elements in set B are also in set A, then set B is known as a **subset** of set A.

As an example, let us consider the two sets given below which are expressed in terms of their elements.

$A = \{1, 2, 3, 4, 5, 6\}$
$B = \{2, 4, 6\}$

Since all the elements in set B are in set A, set B is a subset of set A. This is denoted by $B \subset A$ or $A \supset B$, and is read as " B is a subset of A".

Now let us consider another set C. If $C = \{1, 2, 7\}$, then not every element in C belongs to A. Therefore C is not a subset of A. This is denoted by $C \not\subset A$.

Example 1

P = {multiples of 6 between 0 and 20}
Q = {multiples of 3 between 0 and 20}

Write the elements of each of the above sets and select the subset.
$P = \{6, 12, 18\}$

$Q = \{3, 6, 9, 12, 15, 18\}$
As all the elements of P are in Q, P is a subset of Q.

Example 2

Write all the subsets of the set $X = \{1, 2\}$.

It is evident that $\{1\}$ and $\{2\}$ are two subsets. Observe that $\{1, 2\}$ is also a subset. In fact, if two sets A and B are equal, then A is a subset of B and B is a subset of A. Furthermore, the null set is considered to be a subset of every set.

Since the null set and the set itself are subsets of the given set, $\{\ \}$ and $\{1, 2\}$ are subsets of the above set X.

Accordingly, the above set X has 4 subsets which are $\{\ \}, \{1\}, \{2\}, \{1, 2\}$.

Example 3

Write all the subsets of the set $Y = \{3, 5, 7\}$.

$\{\ \}, \{3\}, \{5\}, \{7\}, \{3, 5\}, \{3, 7\}, \{5, 7\}, \{3, 5, 7\}$
There are 8 subsets.

Universal sets

In a study conducted on the students of your school, several subsets may come under consideration.

The following can be given as examples.

{Students in grade 9}
{Female students}
{Students sitting for the G.C.E (O/L) examination this year}

The elements of all the above sets are contained in the set of all students of the school. This set can be considered as the **universal set** relevant to the above study.

Let us consider another example.

When we consider the sets of even numbers, odd numbers, triangular numbers and prime numbers, we see that they are all subsets of the set of integers. Therefore the set of integers can be considered as the universal set.

A **universal set** is a set which contains all the elements under consideration.
Universal sets are denoted by ε.

As another example, suppose that the numbers 1, 2, 3, 4, 5 and 6 are written on the 6 sides of a cubic die. By rolling this die once, the score that can be obtained is one of the numbers 1, 2, 3, 4, 5 and 6.Therefore, we obtain {1, 2, 3, 4, 5, 6} as the set of possible outcomes. This is the universal set of all possible outcomes that can be obtained when a die is rolled once.

This can be expressed as $\varepsilon = \{1, 2, 3, 4, 5, 6\}$. A few subsets of this universal set are given below.

A = {odd numbers} $A = \{1, 3, 5\}$
B = {values greater than 4} $B = \{5, 6\}$
C = {even prime numbers} $C = \{2\}$

Example 4

Write a universal set for A; $A = \{2, 4, 6, 8\}$

ε = {numbers between 1 and 10}

Exercise 22.2

1. Write 8 subsets of the set $A = \{2, 5, 8, 10, 13\}$.

2. Determine whether each of the following statements is true or not.

i. $\{1, 2, 3\} \subset$ {numbers divisible by 5}
ii. $\{4, 9, 16\} \subset$ {square numbers}
iii. {cylinder} $\subset$ {polygons}
iv. {red} $\subset$ {colours of the rainbow}
v. {solution of $2x - 1 = 7$} $\subset$ {even numbers}

3. Write a universal set for the set A; $A = \{a, e, i, o, u\}$

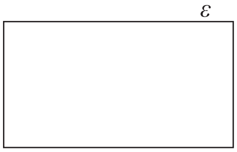
4. For each of the following parts, name a suitable universal set, such that the given sets are subsets.

i. {5, 10, 15, 20, 25} , {10, 100, 100, ...}
ii. {countries with more than 90% literacy},{countries which are not bordered by an ocean}
iii. {January, March, May, August}, {months with 31 days}
{months during which the members of your family celebrate their birthdays}

22.3 Venn diagrams

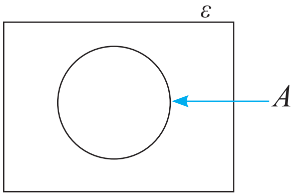
You have learnt how to represent sets in a Venn diagram in earlier grades. In Venn diagrams, sets are represented by closed figures.

The universal set is represented in a Venn diagram by a rectangle as shown below.



The subsets of a universal set are represented using round or oval shaped figures (circles or ellipses)

We represent a subset *A* within the universal set as follows.

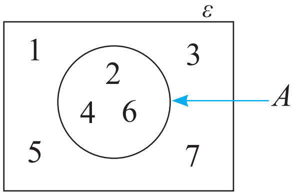


Example 1

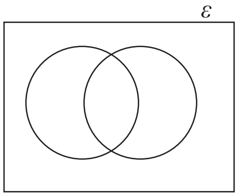
$\varepsilon = \{1, 2, 3, 4, 5, 6, 7\}$

$A = \{2, 4, 6,\}$

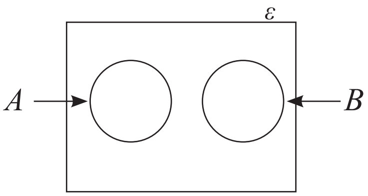
Represent the above sets in a Venn diagram.



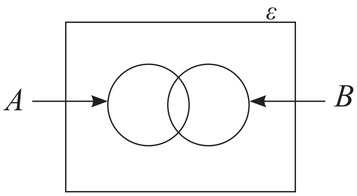
Two subsets of a universal set are generally represented as below.



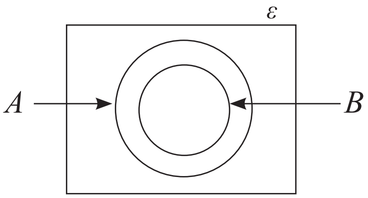
Special instances of two subsets of a universal set are shown below.



When the two sets *A* and *B* have no elements in common



When *A* and *B* have common elements



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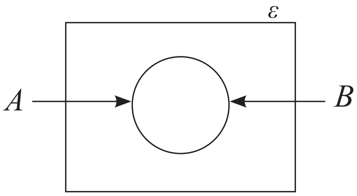
When *B* is a subset of *A*



When *A* and *B* are equal

You will learn more about these four instances and the corresponding regions under the next section on the intersecton of sets, union of sets and disjoint sets.

22.4 Intersection of sets, union of sets and disjoint sets

Intersection of sets

When two or more sets are considered, the set consisting of the elements which are common to all the sets is known as their intersection. When two sets *A* and *B* are considered, their intersection is denoted by $A \cap B$.

As an example, let us consider the pair of sets given below,
$A = \{1, 2, 3, 4, 5, 7\}$
$B = \{2, 5, 6, 7\}$
The set consisting of the elements common to both *A* and *B* is $\{2, 5, 7\}$.

Therefore, the intersection of the sets *A* and *B* is $A \cap B = \{2, 5, 7\}$.

Example 1

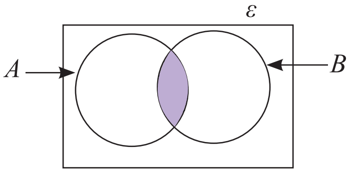
$M = \{$students of Kannangara Vidyalaya who play cricket$\}$
$N = \{$students of Kannagara Vidyalaya who play football$\}$
Write the set $M \cap N$ in descriptive form.

$M \cap N = \{$students of Kannangara Vidyalaya who play both cricket and football$\}$

Now let us consider how the intersection of two sets is represented in a Venn diagram.

Suppose the two sets A and B have common elements.

That is, A and B have a non - empty intersection. There should be a region to represent the set of elements common to both these sets. The Venn diagram depicting this is given below.



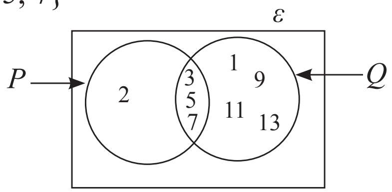
The shaded region in the figure is common to both the sets A and B. Therefore the elements that are common to these two sets can be included here.

Let us consider how to represent two sets with a non-empty intersection in a Venn diagram, through the following example.

Example 2

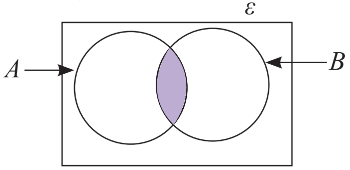
Write the following two sets by listing their elements and then find their intersection. Represent all the elements in a Venn diagram.

P ={prime numbers between 0 and 10}
Q ={odd numbers between 0 and 15}
$P = \{2, 3, 5, 7\}$
$Q = \{1, 3, 5, 7, 9, 11, 13\}$
$\therefore P \cap Q = \{3, 5, 7\}$

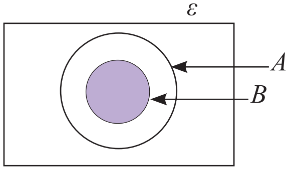


The intersecton of two sets under different conditions are given below.

(i) $A \cap B$

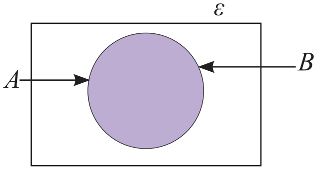


(ii) $A \cap B$ when $B \subset A$



$A \cap B = B$ when $B \subset A$

(iii) $A \cap B$ when $A = B$



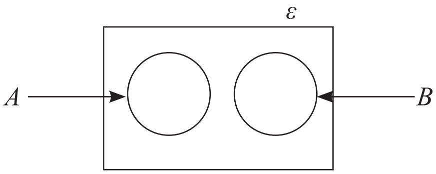
$A \cap B = A = B$ when $A = B$

Let us consider how two sets which do not have common elements represented in a Venn diagram.

Disjoint sets

If two sets have no elements in common, then they are known as disjoint sets. In other words, if two sets A and B are such that $A \cap B = \emptyset$, then A and B are disjoint sets.

Disjoint sets can be represented in a Venn diagram as shown below.

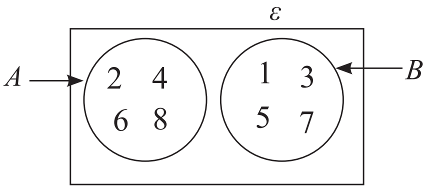


As an example, let us consider the two sets shown below.

$A = \{2, 4, 6, 8\}$

$B = \{1, 3, 5, 7\}$

Since $A \cap B = \emptyset$, A and B are disjoint sets.



$A \cap B = \emptyset$

Union of sets

When two or more sets are considered, the set which consists of all the elements in these sets is known as the **union of these sets**. When two sets A and B are considered, their union is denoted by $A \cup B$.

As an example, let us consider the pair of sets given below.

$A = \{1, 3, 5, 7, 8\}$

$B = \{2, 3, 4, 6, 7, 8\}$

The union of the sets A and B is $\{1, 2, 3, 4, 5, 6, 7, 8\}$.

Accordingly, the union of the sets A and B is $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

Example 1

P = {prime numbers between 0 and 10}
P = {odd numbers between 0 and 10}

Write these sets in terms of their elements and write $P\cup Q$ also in terms of its elements. Furthermore, express the union in terms of a common characteristic of its elements.

$P=\{2, 3, 5, 7\}$
$Q=\{1, 3, 5, 7, 9\}$

$P\cup Q=\{1, 2, 3, 5, 7, 9\}$

The union expressed in terms of a common characteristic;
$P\cup Q$ ={numbers between 0 and 10 which are either prime or odd}

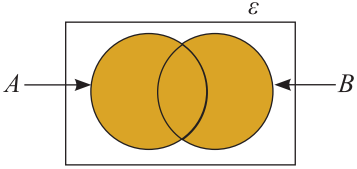
Example 2

X = {students in KannangaraVidyalaya who play cricket}
Y = {students in KannangaraVidyalaya who play football}

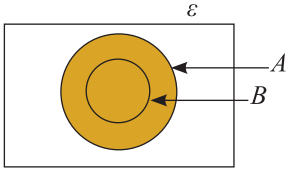
$X\cup Y$={students in Kannangara Vidyalaya who play either cricket or football or both}

Now, let us see how to represent the union of sets in a Venn diagram.

(i) $A\cup B$

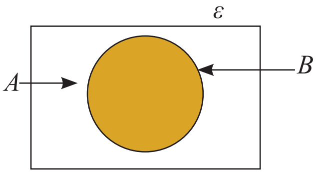


(ii) $A\cup B$ when $B\subset A$



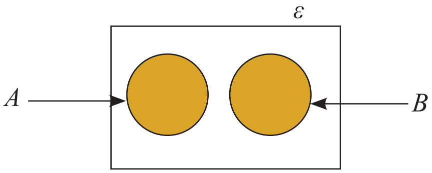
$A\cup B = A$

(iii) $A\cup B$ when $A = B$



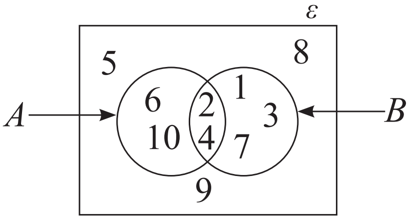
$A\cup B = A = B$ when $A = B$

(iv) $A\cap B = \emptyset$



Example 3

Answer the given questions based on the following Venn diagram.



i. Write set *A* in terms of its elements.
ii. Write set *B* in terms of its elements.
iii. Write the universal set ε in terms of its elements.
iv. Express $A \cap B$ in terms of its elements.
v. Express $A \cup B$ in terms of its elements.

i. $A = \{2, 4, 6, 10\}$
ii. $B = \{2, 1, 4, 7, 3\}$
iii. $\varepsilon = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
iv. $A \cap B = \{2, 4\}$
v. $A \cup B = \{6, 10, 2, 4, 1, 3, 7\}$

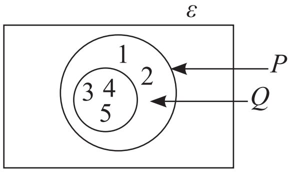
Example 4

$P = \{1, 2, 3, 4, 5\}$
$Q = \{3, 4, 5\}$

i. Represent the above sets in a Venn diagram.
ii. Express $P \cap Q$ and $P \cup Q$ in terms of their elements.

i. $P \cap Q = \{3, 4, 5\}$
$P \cup Q = \{1, 2, 3, 4, 5\}$

ii.



Exercise 22.4

1. The sets P, Q and R are defined as follows.

$P = \{1, 3, 6, 8, 10, 13\}$
$Q = \{1, 6, 7, 8\}$
$R = \{2, 3, 9, 10, 12\}$

Express each of the following sets in terms of its elements.

i. $P \cap Q$ **ii.** $P \cap R$ **iii.** $Q \cap R$ **iv.** $P \cup Q$ **v.** $P \cup R$ **vi.** $Q \cup R$

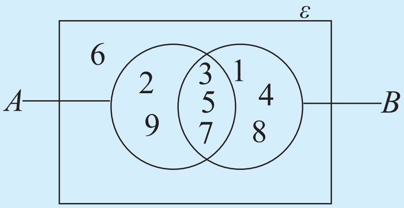
2. The sets A, B and C are defined as follows.

A = {counting numbers from 1 to 12}
B = {prime numbers less than 10}
C = {factors of 12}

i. Write each of the above sets in terms of its elements.
ii. Write each of the following sets in terms of its elements.

i. $A \cap B$ **ii.** $A \cap C$ **iii.** $B \cap C$ **iv.** $A \cup B$ **v.** $A \cup C$ **vi.** $B \cup C$

3. Consider the Venn diagram given below.



Write each of the following sets in terms of its elements.

i. A
ii. B
iii. $A \cup B$
iv. $A \cap B$

22.5 Complement of a set

Let us consider a subset A of a universal set. The set of elements in the universal set which do not belong to the set A is known as the **complement of A**.

Consider the following example.

If we take,
$\varepsilon = \{1, 2, 3, 4, 5, 6, 7\}$ and $A = \{2, 4, 6\}$,
then the set consisting of all the elements in the universal set which are not in set A is $\{1, 3, 5, 7\}$.

This set is the complement of the set A. The complement of the set A is denoted by A'. Accordingly,
we can write, $A' = \{1, 3, 5, 7\}$.

Example 1

By considering the given universal set (ε) and its subset B, write the set B' in terms of its elements.

$\varepsilon = \{5, 10, 15, 20, 25, 30, 35\}$
$B = \{10, 20, 30\}$

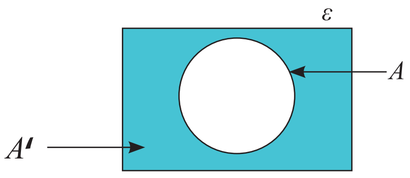
$B' = \{5, 15, 25, 35\}$

Example 2

If $\varepsilon = \{\text{birds}\}$ and
$P = \{\text{birds that make nests}\}$, then write P' in descriptive form.
$P' = \{\text{birds that do not make nests}\}$

Now let us see how to represent the complement of a set in a Venn diagram.

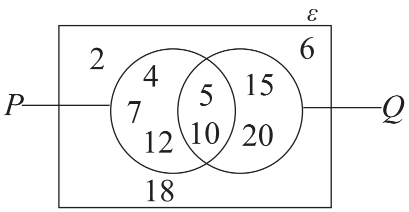
If A is a subset of a universal set, then A' is represented in a Venn diagram as follows.



A' is the set of elements which belong to ε but not to A. Therefore the whole region in the Venn diagram which does not belong to A, belongs to A'.

Example 3

Find the following using the information in the Venn diagram.



i. P' **ii.** Q' **iii.** $P \cap Q$ **iv.** $P \cup Q$

i. $P' = \{2, 6, 15, 18, 20\}$
ii. $Q' = \{2, 4, 6, 7, 12, 18\}$
iii. $P \cap Q = \{5, 10\}$
iv. $P \cup Q = \{4, 5, 7, 12, 15, 20, 10\}$

Exercise 22.5

1. ε = {Sakindu, Ravindu, Sanindu, Pavindu, Nithindu}
A = {Sakindu, Pavindu}
B = {Ravindu,Sanindu, Nithindu}
C = {Sakindu, Sanindu, Pavindu}

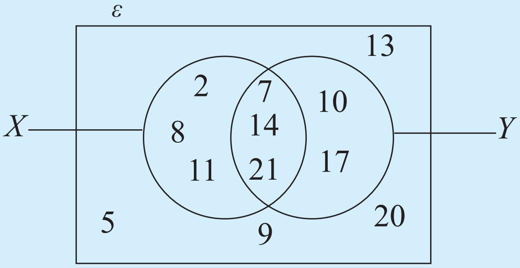
Write each of the following sets in terms of its elements, based on the above given information.

i. A' **ii.** B' **iii.** C'
iv. $A \cap C$ **v.** $A \cap B$ **vi.** $B \cap C$

2. If $\varepsilon = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $P = \{1, 3, 5, 7, 9, 10\}$ and $Q = \{2, 4, 5, 7, 8\}$, represent ε, P and Q in a Venn diagram and using the Venn diagram, write each of the sets given below in terms of its elements.

i. P' **ii.** Q' **iii.** $P \cap Q$ **iv.** $P \cup Q$

3. By considering the Venn diagram given below, write each of the given sets in terms of its elements.



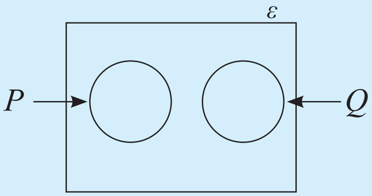
i. X **ii.** Y **iii.** $X \cap Y$

iv. $X \cup Y$ **v.** X' **vi.** Y'

Miscellaneous Exercise

1. Represent the following information in the given Venn diagram.

$\varepsilon = \{1, 2, 3, 4, 5, 6, 7, 8, \}$
$P = \{2, 4, 6\}$
$Q = \{1, 5, 8\}$

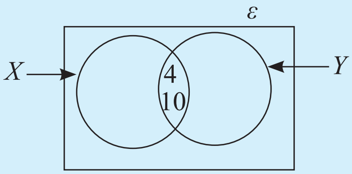


Write each of the sets given below in terms of its elements.

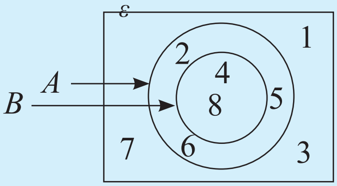
a. $P \cap Q$
b. $P \cup Q$
c. P'
d. Q'

2. Include the elements in the following sets in the given Venn diagram.

$\varepsilon = \{2, 3, 4, 5, 7, 8, 10, 12 \}$
$P = \{2, 4, 10\}$
$Q = \{3, 4, 8, 10\}$



3. Answer the following based on the information in the Venn diagram.



- **i.** Write set A in terms of its elements.
- **ii.** Write set B in terms of its elements.
- **iii.** Write set ε in terms of its elements.
- **iv.** $A \cap B$ set in terms of its elements.
- **v.** $A \cup B$ set in terms of its elements.
- **vi.** A' set in terms of its elements.

Summary

- Sets with a finite number of elements are called finite sets.
- Sets with an infinite number of elements are called infinite sets.
- Sets with the same elements are called equal sets.
- Sets with the same number of elements are called equivalent sets.
- A sets which contains all the elements under consideration is called universal set.

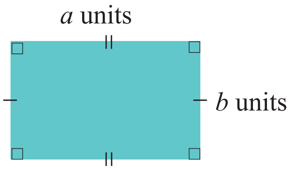
23 Area

By studying this lesson you will be able to;

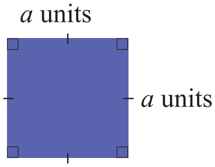
- find the area of a parallelogram,
- find the area of a trapezium,
- find the area of a circle.

Area

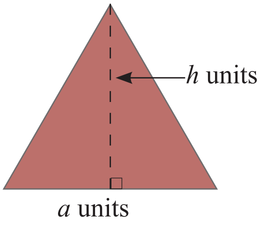
Area can be considered as a quantity that defines the spread of a surface. In grades 7 and 8 you learnt how to find the area of a square lamina, a rectangular lamina and a triangular lamina. Let us recall what was learnt.



If the area of a rectangular lamina of length a units and width b units is A square units, then $A = a \times b$.



If the area of a square lamina of side length a units is A square units, then $A = a^2$.



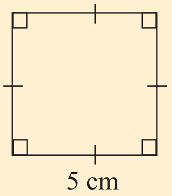
If the area of a triangular lamina of base a units and corresponding perpendicular height h units is A square units, then $A = \frac{1}{2} \times a \times h$.

Do the following review exercise in order to establish these facts further.

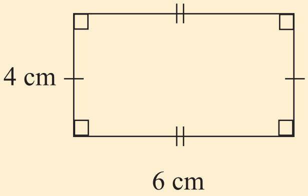
Review Exercise

1. Find the area of each plain figure shown below.

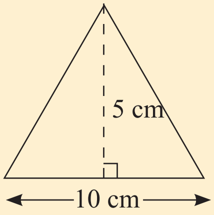
i



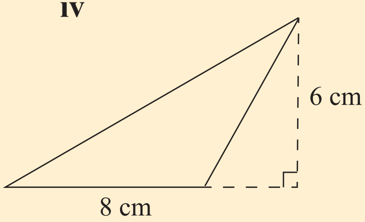
ii



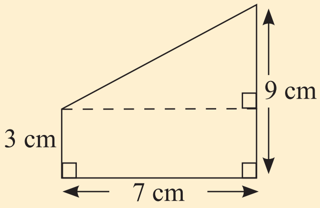
iii



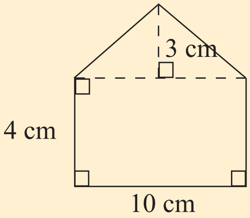
iv



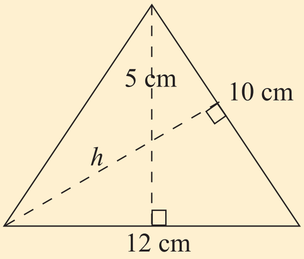
v



vi



2. In the triangle shown in the figure, the perpendicular height corresponding to the base of length 12 cm is 5 cm, and the perpendicular height corresponding to the base of length 10 cm is h cm.



i. Find the area of the triangle.
ii. Find the value of h.

3. (*a*) What is the perimeter of an equilateral triangular lamina of side length 12 cm?

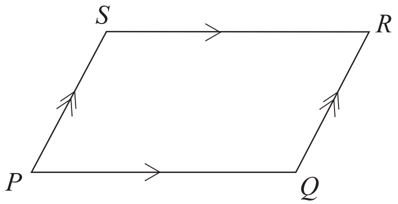
(*b*) Consider a square lamina with the same perimeter as that of the above triangular lamina.

i. Find the side length of the square lamina.
ii. Find the area of the square lamina.

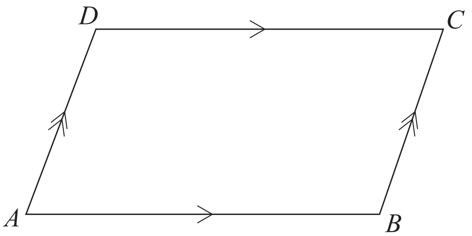
The area of a parallelogram

A quadrilateral with opposite sides parallel to each other is called a parallelogram. We learnt in grade 8 that the opposite sides of a parallelogram are equal. Accordingly, in the parallelogram *PQRS*,

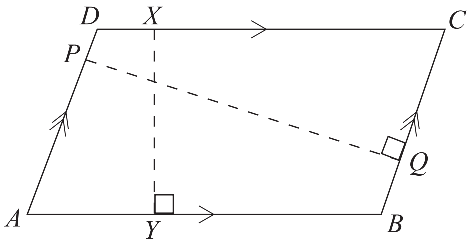
$PQ // SR$ and $PS // QR$.
$PQ = SR$ and $PS = QR$



23.1 The base and height of a parallelogram



Any side of the parallelogram given in the figure can be considered as the base. How the height of the parallelogram corresponding to each base is defined is explained below.

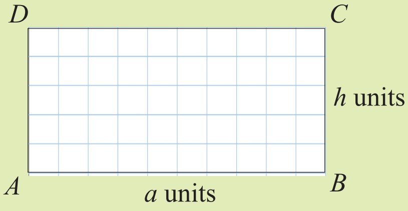


Suppose *AB* is considered as the base of the parallelogram. *AB* and *DC* (the side opposite *AB*) are parallel to each other. According to the figure, the perpendicular distance between these two sides is *XY*. Therefore, *XY* is the perpendicular height corresponding to the base *AB*. When *BC* is considered as the base, the perpendicular distance between the parallel sides *BC* and *AD* according to the figure is *PQ*. Therefore, *PQ* is the perpendicular height corresponding to the base *BC*.

Through the following activity, let us understand how to construct a formula for the area of a parallelogram.

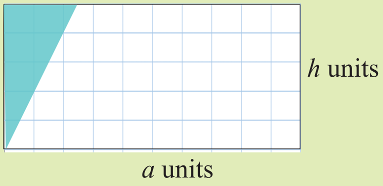
Activity 1

Step 1: In your exercise book, draw a rectangle which is equal in size to that given in the figure. Let us consider its length to be a units and its width to be h units.

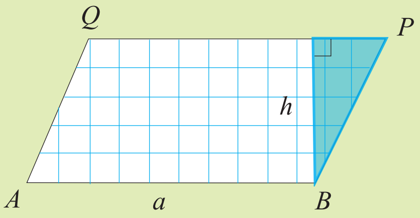


Step 2: Draw another rectangle with the same measurements on another square ruled paper and cut it out.

Step 3: Take that rectangle and cut off the shaded portion as shown below.



Step 4: Create a parallelogram by pasting the triangular portion that was cut off in your exercise book, as shown in the figure. Name it $ABPQ$.



Step 5: Find the area of the rectangle drawn initially, in terms of a and h.

Understand that the area of the parallelogram and the area of the initial rectangle are equal to each other.

Area of parallelogram $ABPQ$ = Area of rectangle $ABCD$

$= a \times h$ square units

Observe that h is the perpendicular height corresponding to the base AB.

According to these facts, a formula for the area of a parallelogram can be given as below.

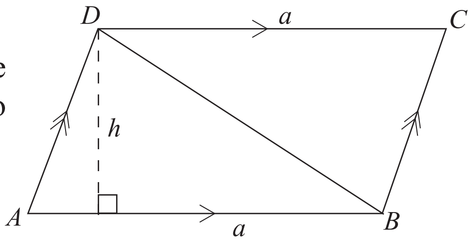
Area of a parallelogram = Length of the base × Perpendicular height corresponding to the base

Let us now consider another method by which the areas of a parallelogram can be found.

The area of a parallelogram can be found by finding the areas of triangles as well.

Consider the parallelogram $ABCD$.

Suppose the length of the base AB is a units and the corresponding height is h units. The parallelogram $ABCD$ is divided into two triangles ABD and BCD by the diagonal DB.



$$\text{The area of triangle } ABD = \frac{1}{2} \times a \times h$$

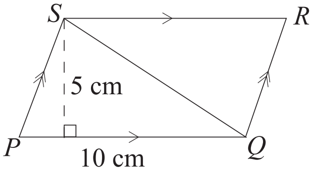
$$\begin{aligned}\text{The area of triangle } BCD &= \frac{1}{2} \times DC \times h \\ &= \frac{1}{2} \times a \times h \text{ (Since } AB = DC\text{)}\end{aligned}$$

$$\begin{aligned}\text{Area of parallelogram } ABCD &= \text{Area of triangle } ABD + \text{Area of triangle } BCD \\ &= \frac{1}{2} \times a \times h + \frac{1}{2} \times a \times h \\ &= \frac{ah}{2} + \frac{ah}{2} = \frac{2ah}{2} \\ &= ah\end{aligned}$$

$\therefore$ The area of the parallelogram *ABCD* is *ah* square units.

Example 1

Find the area of the parallelogram *PQRS*.



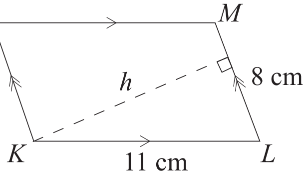
The area of the parallelogram *PQRS* = 10 × 5

= 50

Therefore, the area of the parallelogram is 50 cm^2.

Example 2

If the area of the parallelogram *KLMN* is 48 cm^2, find the value of *h*.



The area of the parallelogram *KLMN* = 48 cm^2

$$\text{Therefore, } 8 \times h = 48$$

$$h = \frac{48}{8}$$

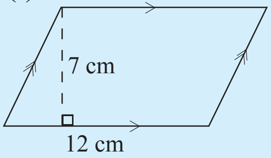
$$h = 6$$

Therefore, *h* = 6 cm.

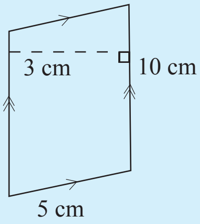
Exercise 23.1

1. Find the area of each parallelogram given below.

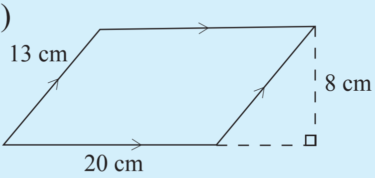
(i)



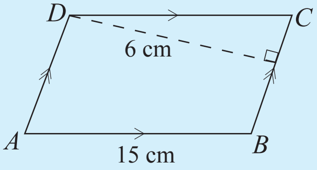
(ii)



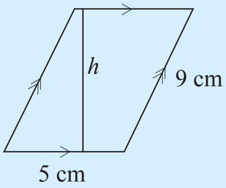
(iii)



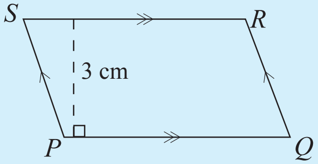
2. If the perimeter of the parallelogram $ABCD$ is 52 cm, find its area.



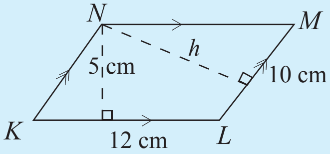
3. If the area of the parallelogram in the figure is 35 cm^2, find the value of h.



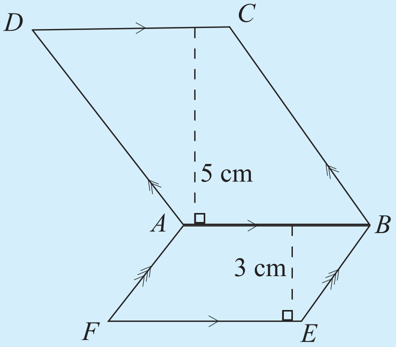
4. If the area of the parallelogram $PQRS$ is 105 cm^2, calculate the length of the side PQ.



5. **i.** Find the area of the parallelogram $KLMN$.
ii. Find the value of h.

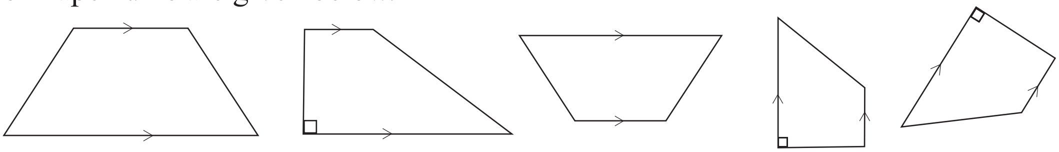


6. If the area of the parallelogram $ABCD$ is 30 cm^2, find the area of the parallelogram $ABEF$.



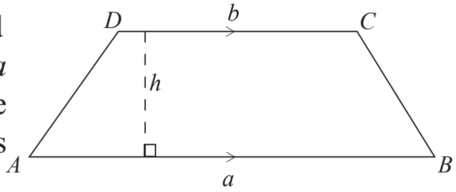
23.2 The area of a trapezium

A quadrilateral with one pair of sides parallel is called a trapezium. Several figures of trapeziums are given below.

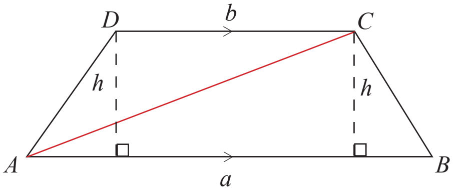


Let us develop a formula for the area of a trapezium.

Let us take the lengths of the parallel sides AB and DC of the trapezium given in the figure as a units and b units respectively and the perpendicular distance between these two sides as h units.



Let us find the area of the trapezium by adding the areas of the two triangles obtained by drawing the diagonal AC of the trapezium.



$$\text{Area of triangle } ABC = \frac{1}{2} \times AB \times h$$

$$\text{Area of triangle } ACD = \frac{1}{2} \times DC \times h$$

$$\text{Area of trapezium } ABCD = \text{Area of triangle } ABC + \text{Area of triangle } ACD$$

$$= \frac{1}{2} \times AB \times h + \frac{1}{2} \times DC \times h$$

$$= \frac{1}{2} \times h \times (AB + DC)$$

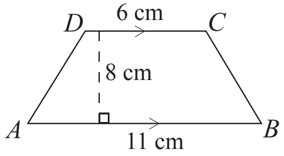
$$= \frac{1}{2} \times (AB + DC) \times h$$

$$= \frac{1}{2} \times (a + b) \times h.$$

The area of a trapezium = $\frac{1}{2} \times$ (The sum of the lengths of the parallel sides) $\times$ (The perpendicular distance between the parallel sides)

Example 1

Find the area of the trapezium *ABCD*.

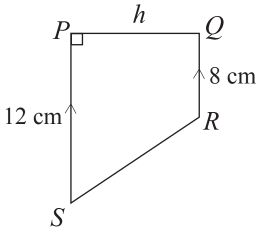


The area of the trapezium *ABCD* $= \frac{1}{2} \times (11 + 6) \times 8$

$$= \frac{1}{\cancel{2}_{1}} \times 17 \times \cancel{8}^{4}$$

$$= \underline{\underline{68 \text{ cm}^2}}$$

Example 2



If the area of the trapezium *PQRS* is 70 cm², find the value of *h*.

The area of the trapezium *PQRS* $= \frac{1}{2} \times (12 + 8) \times h$

$$= \frac{1}{\cancel{2}_{1}} \times \overset{10}{\cancel{20}} \times h$$

Since the area is given as 70 cm²,

$$10\,h = 70$$

$$h = \frac{70}{10}$$

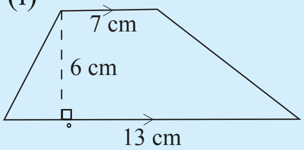
$$h = 7$$

Therefore, $h = 7$ cm.

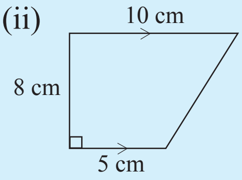
Exercise 23.2

1. Find the area of each trapezium given below.

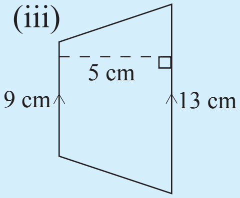
(i)



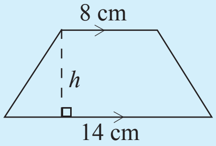
(ii)



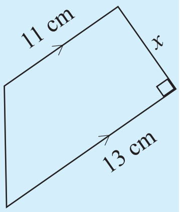
(iii)



2. If the area of the trapezium in the figure is 88 cm^2, find the value of h.

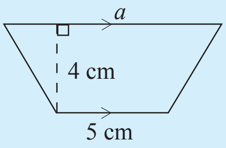


3. If the area of the trapezium in the figure is 60 cm^2, find the value of x.



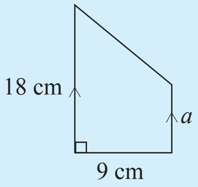
4. Find the length marked as a in each trapezium given below. The area of each trapezium is given below the figure.

i.



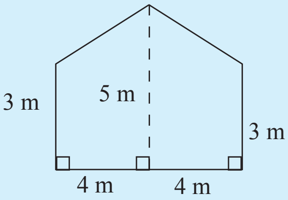
The area is 26 cm^2.

ii.



The area is 135 cm^2.

5.



The side view of a wall is given in the figure. Find the area of the wall according to the given measurements.

6. The area of a trapezium is 30 cm^2. The perpendicular distance between the parallel sides is 3 cm.

i. Give three pairs of integral values that the lengths of the parallel sides can take.

ii. Give three pairs of non – integral values that the lengths of the parallel sides can take.

23.3 The area of a circle

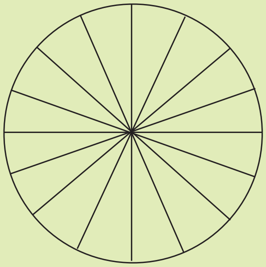
We have learnt how to find the area of a lamina that takes the shape of a rectangle, a square, a triangle, a parallelogram or a trapezium.
Now let us consider how the area of a circular shaped lamina can be found.

To do this, let us first engage in the activity given below.

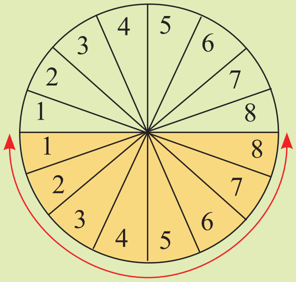
Activity 2

Step 1 : Draw a circle of radius 6 cm on a sheet of paper.

Step 2 : Divide the circle into the maximum possible number of sectors (about 16) by drawing straight lines through the center.



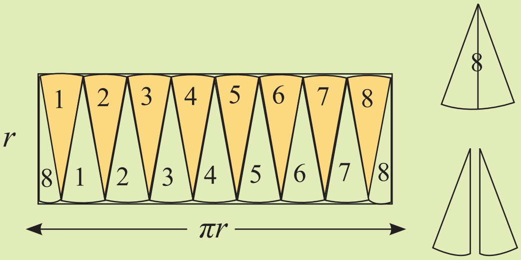
Step 3 : Colour half the circle and number all the sectors consecutively as shown in the figure.



$$\frac{2\pi r}{2} = \pi r$$

Step 4 : Separate out all the sectors by cutting along the drawn lines.

Step 5 : Paste the separated sectors such that a rectangular shape (approximately) is obtained as shown in the figure. (Understand that the accuracy increases as the number of sectors increases.)



Since paper is not wasted, the areas of the circle and the rectangle should be equal. Find the area of the rectangle as shown below by considering the radius of the circle as r.

The length of the rectangle that is obtained $= \text{circumference of the circle} \times \frac{1}{2}$

$$= 2\pi r \times \frac{1}{2}$$
$$= \pi r$$

The width of the rectangle that is obtained $= r$

The area of the rectangle = length × width

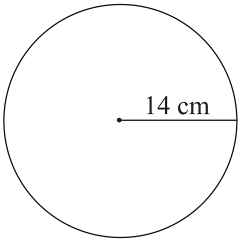
$$= \pi r \times r$$
$$= \pi r^2$$

$\therefore$ Therefore, the area of a circle of radius $r = \pi r^2$

In calculations, we use 3.142 or $\frac{22}{7}$ for the value of π.

Example 1

Find the area of a circular lamina of radius 14 cm.



The area of the circular lamina $= \pi r^2$

$$= \frac{22}{\underset{1}{\cancel{7}}} \times \overset{2}{\cancel{14}} \times 14$$
$$= 616$$

$\therefore$ Therefore, the area of the circular lamina is 616 cm^2.

Example 2

Calculate the radius of a circular lamina of area 154 cm^2.

The area of the circular lamina $= \pi r^2$

$$= \frac{22}{7} \times r^2$$

The area of the circular lamina is given as 154 cm^2.

$$\frac{22}{7} r^2 = 154$$

Therefore, $\frac{22}{\cancel{7}} r^2 \times \cancel{7} = 154 \times 7$

$$\frac{\cancel{22} r^2}{\cancel{22}} = \frac{1078}{22} = 49$$

$$r^2 = 49$$

Therefore, $r = 7$ or $r = -7$.

However, the radius cannot be a negative value.
Therefore, the radius of the circle is 7 cm.

Exercise 23.3

1. The following are the dimensions of some circular laminas. Find the area of each lamina (Use $\frac{22}{7}$ for the value of π).

i. Radius 14 cm **ii.** Radius 21 cm **iii.** Diameter 7 cm **iv.** Diameter 21 cm

2. The following are areas of some circular laminas. Calculate the radius of each lamina.

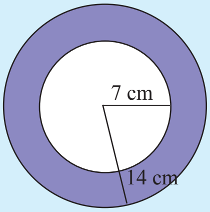
i. 616 cm^2 **ii.** 1386 cm^2 **iii.** $38\frac{1}{2}$ cm

3. Consider the largest circular lamina that can be cut out from a square lamina of area 196 cm^2.

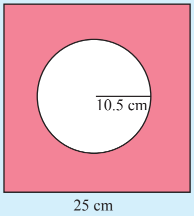
i. What is the radius of this circular lamina?
ii. What is the area of the circular lamina?

4. Find the area of the shaded part in each figure given below.

i.



ii.



5. What is the maximum number of circular laminas of radius 7 cm that can be cut out from a rectangular lamina of length 70 cm and width 14 cm?

Summary

- The area of a parallelogram of base length a and height h is ah.
- The area of a trapezium of which the lengths of the two parallel sides are a and b and the perpendicular distance between the two parallel sides is h is $\frac{1}{2}(a+b)h$.
- The area of a circle of radius r is πr^2.

24

Probability

By studying this lesson you will be able to;

- identify random experiments,
- write the sample space of a random experiment,
- identify the equally likely outcomes of a random experiment,
- find the probability of an event in a sample space when the outcomes are equally likely.

24.1 Random experiment

Let us consider the experiment of an ordinary coin being tossed. When a coin is tossed, we know that the outcome will be either "head turns up" or "tail turns up". That is, we know all the possible outcomes before the experiment is conducted. However, we cannot say with certainly whether head will turn up or tail will turn up. Furthermore, this experiment can be repeated any number of times under the same conditions. Another feature is that we will not be able to identify a pattern in the outcomes when the experiment is repeated. Experiments with the above features are called **random experiments**.

Random experiments have the following common characteristics.

- The experiment can be repeated any number of times under the same conditions.
- All the possible outcomes of the experiment are known before the experiment is carried out.
- The outcome of the experiment cannot be stated with certainty before the experiment is carried out.
- When the experiment is repeated, a pattern cannot be recognized in the outcomes.

Let us consider another example.

Even though all the outcomes of the experiment of rolling an unbiased cubic die with its faces numbered from 1 to 6 and recording the number on the face that turns up are known, it is not possible before carrying out the experiment to state with certainty which outcome will occur. Moreover, this experiment can be repeated any number of times under the same conditions, but a pattern in the outcomes cannot be expected. Therefore, rolling an unbiased cubic die and observing the outcome is a random experiment.

Exercise 24.1

1. For each of the following experiments, in the column to the right, mark "✓" if it is a random experiment and "×" if it is not a random experiment.

Experiment	Random/not random
1. Rolling an unbiased tetrahedral die with its faces numbered from 1 to 4, and recording the number on the face which touches the table.	
2. Drawing a bead from a bag which contains beads of one color and recording its colour.	
3. Throwing a ball at a target and observing whether it hits the target or not.	
4. Planting 5 radish seeds and recording the number of seeds that germinate in 5 days.	
5. Checking whether a door opens when a key picked at random from a bunch of three keys is used.	
6. Tossing a ball in the air and observing whether it falls to the ground.	
7. Drawing out two cards from a box containing three cards, each with one of the numbers 1, 3 and 5 written on it, and observing whether the sum of the two numbers on the two cards that are drawn is an odd number.	

24.2 Sample Space

All the possible outcomes of a random experiment can be written as a set. This set which consists of all the possible outcomes of a random experiment is called its sample space. It is usually denoted by S.

For example,

in the experiment of tossing a coin and observing the side that turns up, the set of all possible outcomes, that is, the sample space is $S = \{\text{Head, Tail}\}$. Here $N(S) = 2$.

Similarly, the sample space of the experiment of observing the number which turns up when an unbiased cubic die with its faces numbered from 1 to 6 is rolled is

$$S = \{1, 2, 3, 4, 5, 6\}$$

Here $N(S) = 6$.

Example 1

Write the sample space for the experiment of rolling an unbiased tetrahedral die with its faces numbered from 1 to 4 and recording the number on the face which touches the table.

$$S = \{1, 2, 3, 4\}$$
$$n(S) = 4$$

Example 2

Write the sample space for the experiment of drawing a bead from a bag which contains two black beads and three white beads marked B_1, B_2, W_1, W_2, W_3 respectively, which are identical in all other aspects. What is the value of $n(S)$?

$$S = \{ B_1, B_2, W_1, W_2, W_3 \}$$
$$n(S) = 5$$

Example 3

There are two cards such that R is written on one side and Y is written on the other side. The cards are tossed simultaniously and the letters turned up is recorded. Write the sample space of this experiment.

Getting R on both cards is denoted by (R, R) and getting R on one card and Y on the other is denoted as (R, Y) etc.. Accordingly,

$S = \{(RR), (RY), (YR), (YY)\}$

Note: An event is a subset of the sample space of a random experiment.

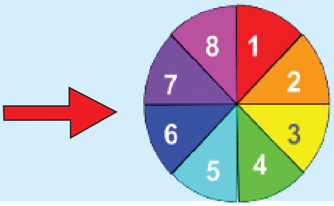
Exercise 24.2

1. Write the sample space of each of the following experiments.

i. Randomly drawing a pen from a bag which contains one pen each of the colors blue, red, black and green and recording the colour. (Assume that the pens are identical in all aspects except the colour)

ii. Recording the number on the card that is drawn at random from a bag containing eleven identical cards numbered 5 to 15.

iii. Recording the number the arrow points to, when the disk shown in the figure is spun and allowed to a stop freely.



iv. A bag contains 4 milk flavoured toffees and 3 orange flavoured toffees of the same size and shape. Randomly drawing a toffee and recording its flavour.

v. Recording the sides that turn up when a coin is tossed twice.

24.3 Equally likely outcomes

When the sample space of a random experiment is considered, if each outcome is equally likely to occur, then that experiment is called an experiment with equally likely outcomes. The outcomes of such an experiment are called equally likely outcomes.

Let us consider a cubic die with its faces numbered 1, 2, 3, 4, 5 and 6. Let us assume that the material it is made of is uniformly distributed throughout the die. Then it is clear that due to symmetry, each face of the die has an equal chance of turning up when the die is rolled. Similarly for a coin. Objects such as these which are symmetrical and are made of a material which is uniformly distributed are called unbiased or fair objects. Experiments such as these, of tossing a fair coin or rolling an unbiased die are considered as important examples when it comes to explaining the theory of probability.

Consider the experiment of rolling a cuboidal die with its faces marked, 1, 2, 3, 4, 5 and 6 and recording the number on the face that turns up. Here, the likelihood of the different sides turning up may not be the same. Therefore such a die is not considered to be a fair die. In such experiments, the outcomes are not equally likely.

Now let us consider another experiment.

It is clear that in the experiment of an unbiased cubic die with four faces painted red and two faces painted blue being rolled and the colour of the face that turns up being recorded, the chance of a red face turning up is greater than a blue face turning up. Therefore, the outcomes of this experiments are not equally likely.

Exercise 24.3

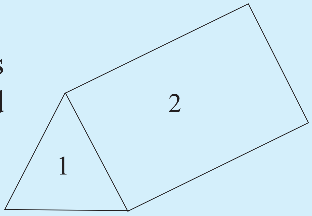
1. For each of the following experiments, determine whether the outcomes are equally likely or not.

i. The four faces of an unbiased tetrahedral die are painted in four different colours, namely, red, blue, yellow and green. Recording the colour of the face which turns up when it is rolled.

ii. Recording the side which turns up when a fair coin is tossed.

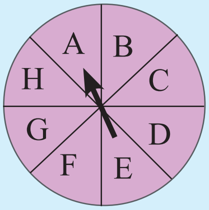
iii. Recording the number on the card which is drawn at random from 10 identical cards which are numbered 1, 1, 1, 1, 2, 2, 2, 3, 3 and 4.

iv. Recording the number on the side which touches the ground when a prism as shown in the figure, with its sides marked with the numbers 1, 2, 3, 4 and 5 is rolled once.



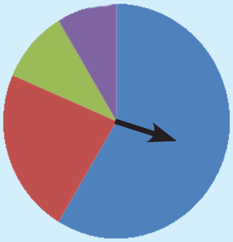
v. Recording the colour of the card drawn randomly from a bag which contains 3 red cards and 4 blue cards which are identical in all other aspects.

vi.



Recording the letter to which the indicator (arrow) which is fixed at the center of a circular disk points, when the disc which has been divided into 8 equal sectors and named A, B, C, D, E, F, G and H as shown in the figure, is spun and allowed to stop freely.

vii.



Recording the color on which the indicator (arrow) which is fixed to the centre of the disc falls when it is spun and allowed to stop freely. Here the disc is divided into unequal sectors and shaded with different colours, and placed on a horizon table top.

24.4 Probability of an event when the outcomes are equally likely

You have learnt that the probability of an outcome of a random experiment with equally likely outcomes is given by the following.

$$\text{Probability of a selected outcome} = \frac{1}{\text{total number of outcomes in the sample space of the random experiment}}$$

Consider the experiment of rolling a fair die. Here the probability of a selected outcome is $\frac{1}{6}$ For example, the probability of getting 3 is $\frac{1}{6}$.

Now consider the event of getting an even number. Its probability can be calculated as follows. Since there are three even numbers and three odd numbers, and the outcomes of this experiment are equally likely, the probability of getting an even number is $\frac{3}{6}$.

The probability of an event in the sample space of a random experiment with equally likely outcomes is given by the following.

$$\text{Probability of the event} = \frac{\text{Number of elements in the event}}{\text{Number of elements in the sample space}}$$

This can be written using symbols as follows.

If the number of elements in the sample space S is $n(S)$, the number of elements in the event A is $n(A)$ and the probability of event A occurring is $p(A)$, then

$$\boxed{p(A) = \frac{n(A)}{n(S)}}$$

Now let us learn more by considering some examples.

Example 1

Consider the experiment of observing the side that turns up when an unbiased coin is tossed once.

i. Write the sample space of this experiment and find $n(S)$.
ii. If the event A is "head" turns up, write the elements in A and find $n(A)$.
iii. Find $p(A)$, the probability that head turns up.

i. S = {head, tail}
$n(S) = 2$

ii. A = {head}
$n(A) = 1$

iii. $p(A) = \frac{n(A)}{n(S)}$

$p(A) = \frac{1}{2}$

Example 2

Consider the experiment of recording the number on the face that touches the table when an unbiased tetrahedral die with its faces numbered 1, 2, 3 and 4 is rolled.

i. Find the probability of getting 2.
ii. Find the probability of getting an even number.
iii. Find the probability of getting a number greater than 1.

Since the sample space is $S = \{1, 2, 3, 4\}$, $n(S) = 4$.

i. Probability of getting 2 = $\frac{1}{4}$

ii. If B is the event of getting an even number,

since $B = \{2, 4\}$, $n(B) = 2$.

$$\therefore p(B) = \frac{n(B)}{n(S)} = \frac{2}{4} = \frac{1}{2}.$$

iii. There are 3 numbers greater than 1. (2, 3, 4)

$\therefore$ the probability of getting a number greater than 1 $= \frac{3}{4}$

Exercise 24.4

1. Consider the experiment of rolling an unbiased cubic die with its faces numbered from 1 to 6 and recording the number on the face that turns up.

- **i.** Write the sample space S of all the possible outcomes of this experiment.
- **ii.** Find the value of $n(S)$.
- **iii.** If A is the event of an even number turning up, write the elements of A and find $n(A)$.
- **iv.** Find $P(A)$, the probability of A occurring.
- **v.** Find the probability of a prime number turning up.

2. Consider the experiment of drawing a card at random from a bag containing 8 identical cards marked with the letters A, B, C, D, E, F, G and H and recording the letter on it.

- **i.** Write the sample space.
- **ii.** Find the probability of drawing the card with the letter B marked on it.
- **iii.** Find the probability of drawing a card with a vowel marked on it.
- **iv.** Find the probability of drawing a card with the letter K marked on it.

3. There are 25 identical cards numbered from 1 to 25 in a box. Consider the experiment of drawing a card at random from the box and recording the number on it.

i. Find the probability of drawing the card with 8 marked on it.

ii. Find the probability of drawing a card with a number which is a multiple of 5 marked on it.

iii. Find the probability of drawing a card with an odd number marked on it.

iv. Find the probability of drawing a card with a square number marked on it.

4.

Consider the experiment of spinning the disc in the figure and recording the colour of the sector in which the arrow head lands when the disc stops spinning.

i. Find the probability of the arrow head landing in the dark blue sector.

ii. Find the probability of the arrow head landing in the red sector.

iii. Find the probability of the arrow head landing in the yellow sector.

5. In a multiple choice question paper, of the 5 answers that are given for a question, only one is correct. A person picks one of the answers randomly for a question to which he does not know the answer. What is the probability of that answer

i. being correct.

ii. being incorrect.

6. In a bag, there are 3 red beads, 2 black beads and 5 white beads which are identical in all other aspects. Consider the experiment of randomly drawing a bead from the bag and recording its colour.

i. Find the probability of drawing a red bead.

ii. Find the probability of drawing a blue bead.

iii. Find the probability of drawing either a red bead or a white bead.

iv. Find the probability of drawing a black bead.

7. Consider the experiment of recording the day of the week on which a student picked at random was born.

i. Find the probability of the student being a person who was born on a Monday.

ii. Find the probability of the student being a person who was born on a Sunday.

iii. Find the probability of the student being a person who was born on either a Wednesday or a Friday.

iv. Find the probability of the student being a person who was born on a day which is neither a Saturday nor a Sunday.

Summary

In a random experiment with equally likely outcomes,

- the probability of a selected outcome $= \dfrac{1}{\text{total number of outcomes in the sample space of the random experiment}}$
- the probability of the event $= \dfrac{\text{number of elements in the event}}{\text{number of elements in the sample space}}$
- $p(A) = \dfrac{n(A)}{n(S)}$

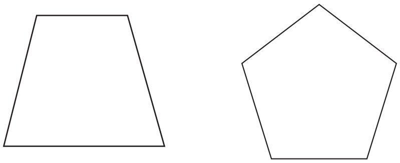
25

Angles of a Polygon

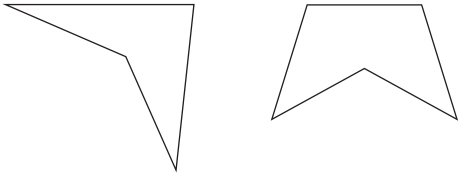
By studying this lesson you will be able to;

- solve geometrical problems related to the interior angles of a polygon,
- solve geometrical problems related to the exterior angles of a polygon,
- solve problems related to regular polygons.

A plane figure bounded by three or more straight line segments is known as a polygon. The two main types of polygons are convex polygons and concave polygons.

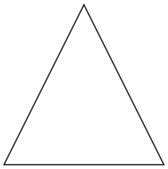


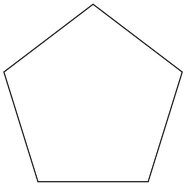
Convex Polygons

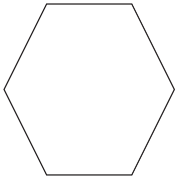


Concave Polygons

Some polygons have special names by which they are identified, which depend on the number of sides they have. Accordingly, polygons which have 3, 4, 5 and 6 sides respectively are known as triangles, quadrilaterals, pentagons and hexagons.







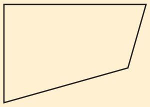
In previous grades you have learnt the following results on the sum of the interior angles of a triangle and of a quadrilateral.

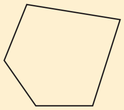
The sum of the interior angles of a triangle is 180°.
The sum of the interior angles of a quadrilateral is 360°.

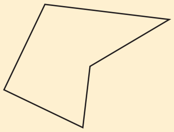
Do the following review exercise to further establish the above facts learnt on polygons in previous grades.

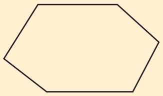
Review Exercise

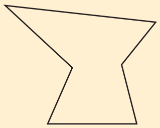
1. From the given figures, select all the convex polygons.


(i)


(ii)


(iii)


(iv)


(v)

2. From the statements given below, select the true statements.

a. A polygon with 7 sides is known as a heptagon.

b. In any polygon, the number of interior angles is equal to the number of sides.

c. A polygon with all sides equal is known as a regular polygon.

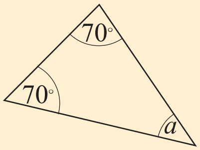
d. At each vertex of a polygon, the sum of the interior angle and the exterior angle is 180°.

e. There are 11 interior angles in a decagon.

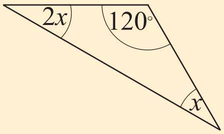
f. The sum of the exterior angles of a quadrilateral is 180°.

3. Find the magnitude of each of the angles denoted by a lower case letter in each of the following figures.

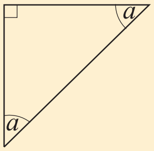
i.



ii.



iii.



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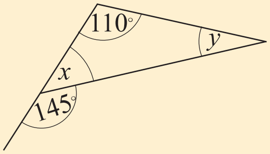

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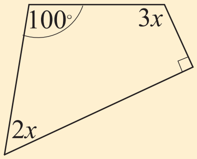

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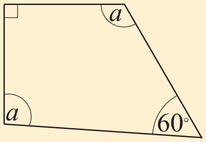
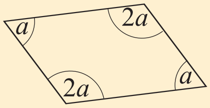
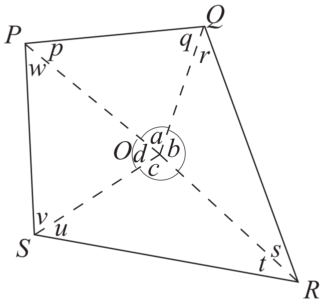


vii.



25.1 The sum of the interior angles of a polygon

Let us first consider how to find the sum of the interior angles of a quadrilateral.



In the given figure, O is any point within the quadrilateral $PQRS$. By joining PO, QO, RO and SO we obtain 4 triangles.

Since the sum of the interior angles of a triangle is 180°,

considering the triangle PQO we obtain, $p + q + a = 180^{\circ}$,

considering the triangle QRO we obtain, $r + s + b = 180^{\circ}$,

considering the triangle RSO we obtain, $t + u + c = 180^{\circ}$,

considering the triangle SPO we obtain, $v + w + d = 180^{\circ}$,

By adding these 4 equations we obtain,

$$(p + q + a) + (r + s + b) + (t + u + c\ ) + (v + w + d) = 180^{\circ} \times 4$$

$$\therefore\ (p + q + r + s + t + u + v + w) + (a + b + c + d) = 720^{\circ}$$

Since a, b, c and d are angles around the point O, $a + b + c + d = 360^{\circ}$.

$$\therefore\ p + q + r + s + t + u + v + w \quad = 720^{\circ} - 360^{\circ}$$

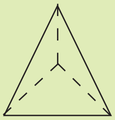
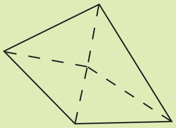
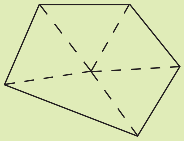
$$= 360^{\circ}$$

$\therefore$ the sum of the interior angles of a quadrilateral is 360°.

Now, to obtain an expression in terms of n for the sum of the interior angles of a polygon which has n sides, let us engage in the following activity.

Activity 1

Copy the following table and complete it.

Polygon	**Figure**	**Number of triangles**	**Sum of the interior angles**
Triangle		3	$180^\circ \times 3 - 360^\circ = 180^\circ$
Quadrilateral		4	$180^\circ \times 4 - 360^\circ = 360^\circ$
Pentagon		5	$180^\circ \times \ldots - 360^\circ = 540^\circ$
Hexagon		....	....
Heptagon		....	....
Octagon		....	....
Polygon with *n* sides		....	....

In the above activity, you must have obtained that when the number of sides is n, the sum of the interior angles of the polygon is $180^\circ \times n - 360^\circ$.

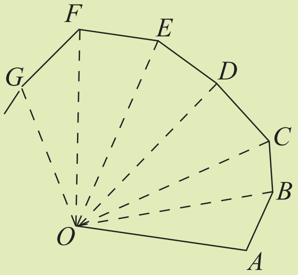
Let us write the expression $180^\circ \times n - 360^\circ$ as follows so that it is easier to remember.

$$\begin{aligned} 180^\circ \times n - 360^\circ &= 90^\circ \times 2n - 90^\circ \times 4 \\ &= 90^\circ\,(2n - 4) \\ &= (2n - 4) \text{ right angles.} \end{aligned}$$

$\therefore$ The sum of the interior angles of a polygon of n sides $= (2n - 4)$ right angles.

Activity 2

2. Let us find a formula for the sum of the interior angles of a polygon in another way.



Polygon	**Number of sides**	**Name of the polygon**	**Number of triangles**	**Sum of the interior angles**
OAB	3	Triangle	1	$180^{\circ} \times 1 = 180^{\circ}$
OABC	4	Quadrilateral	2	$180^{\circ} \times$ .... = ...
OABCD	....	....	....	....
OABCDE	....	....	....	....
OABCDEF	....	....	....	....
OABCDEFG	....	....	....	....

i. Consider the above table and find in terms of n, the number of triangles that are formed by joining one vertex of an n-sided polygon to the other vertices of the polygon.

ii. Show that the sum of the interior angles of a polygon of n sides is $180^{\circ}(n-2)$.

Note: Historically, mathematicians such as the Greek mathematician Euclid, expressed the magnitudes of angles in terms of right angles. For example, the magnitude of the angle on a straight line was said to be 2 right angles and the sum of the interior angles of a quadrilateral was said to be 4 right angles. Accordingly, we can say that the sum of the interior angles of a polygon of n sides is $2n-4$ right angles. However, since we use degrees to measure angles and are familiar with the fact that a right angle is 90°, the sum of the interior angles of a polygon can be remembered as either $90^{\circ}(2n-4)$ or $180^{\circ}(n-2)$ or any other equivalent expression which is easy to recall.

Example 1

Find the sum of the interior angles of a nonagon.

Sum of the interior angles of a polygon of n sides $= 180^{\circ}(n-2)$

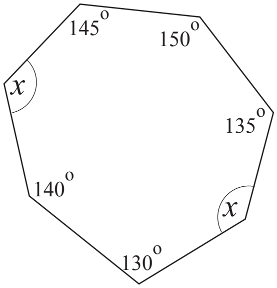
∴ the sum of the interior angles of a polygon of 9 sides $= 180^{\circ}(9-2)$

$= 180^{\circ} \times 7$

$= \underline{\underline{1260^{\circ}}}$

Example 2

Find the value of x based on the information in the figure.



Number of sides in the polygon $= 7$

∴ sum of the interior angles $= 180^{\circ}\ (7-2)$

$= 180^{\circ} \times 5$

$= 900^{\circ}$

$$\therefore\ 145^{\circ} + 150^{\circ} + 135^{\circ} + x^{\circ} + 130^{\circ} + 140^{\circ} + x = 900^{\circ}$$

$$700^{\circ} + 2x = 900^{\circ}$$

$$2x = 900^{\circ} - 700^{\circ}$$

$$2x = 200^{\circ}$$

$$x = \frac{200^{\circ}}{2} = 100^{\circ}$$

Example 3

The sum of the interior angles of a polygon is 1440°. Find the number of sides it has.

If the number of sides is n, the sum of the interior angles $= 180^{\circ}(n-2)$

$$\therefore\ 180^{\circ}(n-2) = 1440^{\circ}$$

$$n-2 = \frac{1440^{\circ}}{180} = 8$$

$$n-2 = 8$$

$$n = 10$$

∴ the number of sides $= 10$.

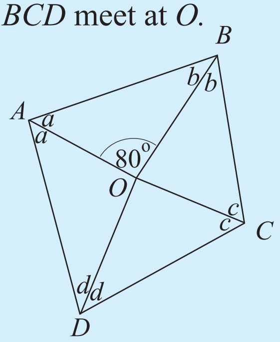
Exercise 25.1

1. Find the sum of the interior angles of each of the polygons given below.

i. Pentagon **ii.** Octagon **iii.** Dodecagon **iv.** Polygon with 15 sides

2. Four of the interior angles of a heptagon are 100°, 112°, 130° and 150°. The remaining angles are equal. Find their magnitude.

3. The bisectors of the interior angles of the quadrilateral $ABCD$ meet at O.



i. Find the value of $a + b + c + d$.

ii. Find the value of $a + b$.

iii. Find the value of $c + d$.

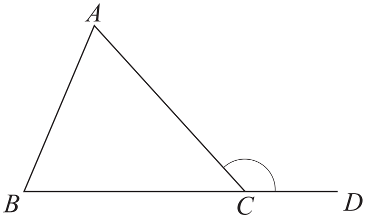
iv. Find the value of $C\hat{O}D$.

4. **i.** If the sum of the interior angles of a polygon is 1620°, find the number of sides it has.

ii. If the sum of the interior angles of a polygon is 3600°, find the number of sides it has.

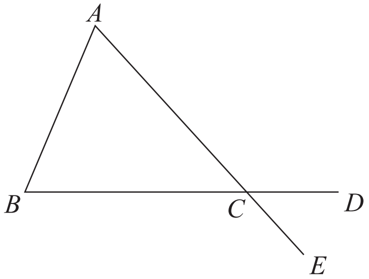
25.2 Sum of the exterior angles of a polygon

First, let us find the sum of the exterior angles of a triangle.



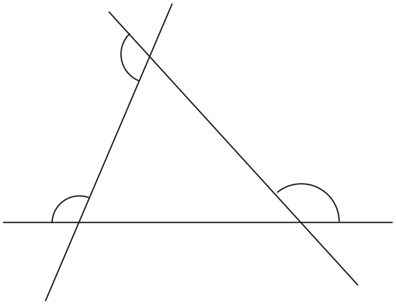
The side BC of the triangle ABC has been produced and point D has been marked on the produced line. The angle $A\hat{C}D$, with the straight line segment CD and the side AC as arms is an exterior angle of this triangle.

As indicated in the figure given below, by producing the side AC too we obtain an exterior angle.

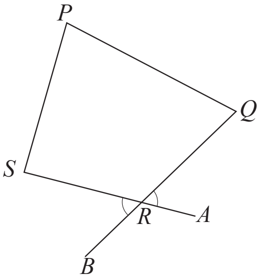


Since vertically opposite angles are equal, this exterior angle is equal in magnitude to the exterior angle $A\hat{C}D$. Either of these two angles can be considered as the exterior angle drawn at the vertex C of the triangle. However, $D\hat{C}E$ is not considered as an exterior angle.

As done above, exterior angles can be drawn at the vertices A and B of the triangle too.



We can define the exterior angles of a quadrilateral similarly.



By producing the side SR of the quadrilateral $PQRS$ up to A we obtain the exterior angle $Q\hat{R}A$ and by producing the side QR up to B we obtain the exterior angle $S\hat{R}B$. Since vertically opposite angles are equal, these two exterior angles are equal.

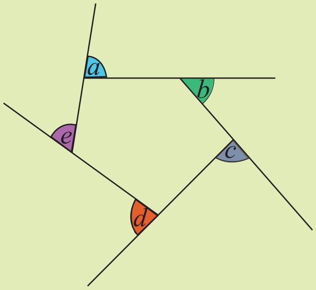
Furthermore, $A\hat{R}B$ is not considered as an exterior angle.

We can define the exterior angles of any polygon similarly.

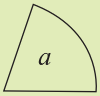
Now, through the following activity, let us determine a value for the sum of the exterior angles of a polygon.

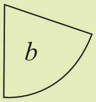
Activity 3

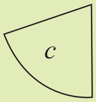
Step 1: Draw a pentagon on a half sheet and name its exterior angles.

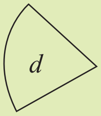


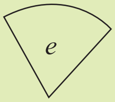
Step 2: Using a blade, cut out the exterior angles as laminas (in the form of sectors of circles) and separate them. (By using the same radius when drawing the sectors, the final outcome will be neat)



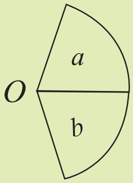








Step 3: On a sheet of paper, paste the laminas which were cut out, such that their vertices meet at one point and such that they don't overlap each other.



Step 4: Carry out the above steps for a hexagon and a heptagon too.

Step 5: Write the common characteristic of the figures obtained by pasting the exterior angles of the polygons and write what can be concluded through this activity too.

You may have observed in the above activity that for each polygon, the exterior angles cover the angle around a point. Accordingly, it can be concluded that the sum of the exterior angles of a polygon is equal to the sum of the angles around a point. As the sum of the angles around a point is 360°, the sum of the exterior angles of the above polygons is also 360°.

Now let us obtain an expression for the sum of the exterior angles of a polygon which has n sides.
We know that the number of exterior angles and the number of interior angles of a polygon with n sides is n.

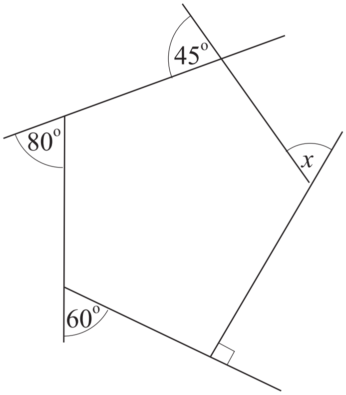
At any vertex of a polygon,
the interior angle + the exterior angle $= 180^{\circ}$.
$\therefore$ sum of n interior angles + sum of n exterior angles $= 180^{\circ} \times n$.
But the sum of n interior angles $= (2n - 4)$ right angles $= 180^{\circ}(n - 2)$. Therefore,
$180^{\circ}(n - 2)$ + sum of n exterior angles $= 180^{\circ}n$

$$\begin{aligned} \therefore \text{ sum of } n \text{ exterior angles} &= 180^{\circ}n - 180^{\circ}(n-2) \\ &= 180^{\circ}n - 180^{\circ}n + 360^{\circ} \\ &= 360^{\circ} \end{aligned}$$

Sum of the exterior angles of a polygon = 360°

Example 1

Find the magnitude of the exterior angle indicated by x, of the given pentagon.



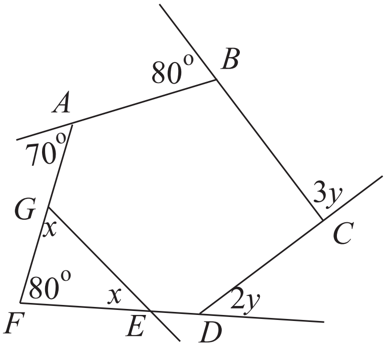
$$\begin{aligned} \text{Sum of the exterior angles} &= 360^{\circ} \\ \therefore\ x + 45^{\circ} + 80^{\circ} + 60^{\circ} + 90^{\circ} &= 360^{\circ} \\ x + 275^{\circ} &= 360^{\circ} \\ x &= 360^{\circ} - 275^{\circ} \\ x &= \underline{\underline{85^{\circ}}} \end{aligned}$$

Example 2

According to the information marked in the figure,

i. find the value of x

ii. find the value of y.



i. Sum of the interior angles of the triangle $EFG = 180^{\circ}$

$$\therefore \quad 80^{\circ} + x + x = 180^{\circ}$$

$$2x = 180^{\circ} - 80^{\circ} = 100^{\circ}$$

$$x = \frac{100^{\circ}}{2}$$

$$x = \underline{\underline{50^{\circ}}}$$

ii. Sum of the exterior angles of the hexagon $ABCDEG = 360^{\circ}$

$$\therefore \quad 70^{\circ} + 80^{\circ} + 3y + 2y + x + x = 360^{\circ}$$

$$70^{\circ} + 80^{\circ} + 5y + 50^{\circ} + 50^{\circ} = 360^{\circ}$$

$$5y = 360^{\circ} - 250^{\circ}$$

$$y = \frac{110^{\circ}}{5}$$

$$y = \underline{\underline{22^{\circ}}}$$

Example 3

The exterior angles of a quadrilateral are in the ratio 2 : 2 : 3 : 3. Find the magnitude of each exterior angle.

Sum of the exterior angles $= 360^{\circ}$

Ratio of the 4 angles $= 2 : 2 : 3 : 3$

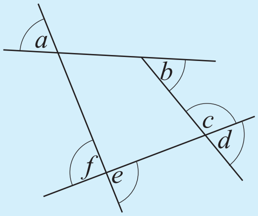
$\therefore$ the smaller angles $= 360^{\circ} \times \dfrac{2}{10} = 72^{\circ}$

The larger angles $= 360^{\circ} \times \dfrac{3}{10} = 108^{\circ}$

$\therefore$ the exterior angles are $72^{\circ}, 72^{\circ}, 108^{\circ}$ and 108°.

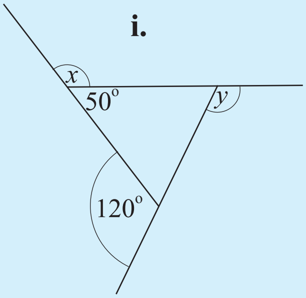
Exercise 25.2

1. From the angles denoted by a, b, c, d, e and f in the figure, select and write the ones which are exterior angles of the quadrilateral.

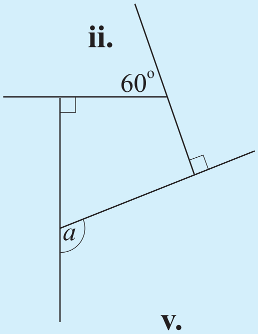


2. For each of the polygons given below, find the magnitude of the angle/angles denoted by the English letter/letters.

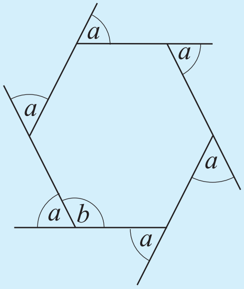
i.



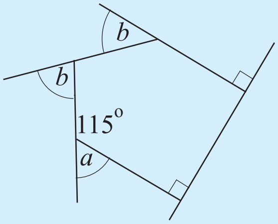
ii.



iii.



iv.



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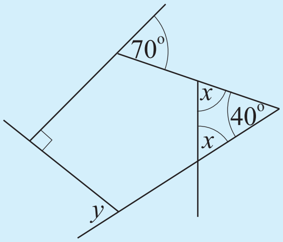


3. The exterior angles of a quadrilateral are x°, $2x^\circ$, $3x^\circ$ and $4x^\circ$.

i. Find the magnitude of each of the exterior angles.

ii. Write the magnitude of each of the interior angles.

4. The exterior angles of a pentagon are in the ratio 1 : 1 : 2 : 3 : 3. Find the magnitude of each of the exterior angles.

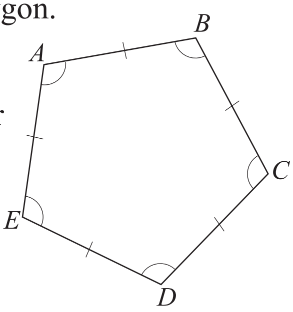
5. The exterior angles of a dodecagon are equal. Find the magnitude of any one of these exterior angles.

6. The magnitude of an exterior angle of a polygon whose exterior angles are equal is 18°. Find the number of sides the polygon has.

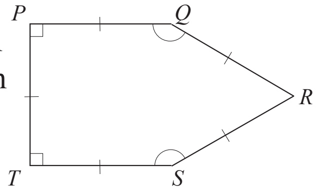
25.3 Regular Polygons

When all the sides of a polygon are equal in length and all the interior angles are equal in magnitude, the polygon is known as a regular polygon.

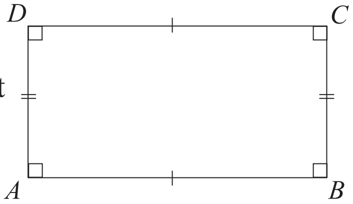
All the sides of the pentagon *ABCDE* in the figure are equal. All the interior angles are also equal. Therefore it is a regular pentagon.



All the sides of the pentagon *PQRST* are equal. However, all the interior angles are not equal. Therefore, the pentagon *PQRST* is not regular.



All the interior angles of the rectangle *ABCD* are equal. However, all the sides are not equal. Therefore, it is not a regular polygon.



Some regular polygons have special names. A regular triangle is called an **equilateral triangle**. A regular quadrilateral is called a **square**.

Example 1

Find the magnitude of an exterior angle of a regular hexagon and thereby find the magnitude of an interior angle.

Sum of the six exterior angles $= 360^{\circ}$

$\therefore$ the magnitude of an exterior angle $= \frac{360^{\circ}}{6} = 60^{\circ}$

exterior angle + interior angle $= 180^{\circ}$

$\therefore$ 60° + interior angle $= 180^{\circ}$

$\therefore$ interior angle $= 180^{\circ} - 60^{\circ}$

$= \underline{\underline{120^{\circ}}}$

Example 2

The magnitude of an interior angle of a regular polygon is 150°. Find,

i. the magnitude of an exterior angle.
ii. the number of sides.

i. exterior angle + interior angle $= 180^{\circ}$

$\therefore$ exterior angle $+ 150^{\circ} = 180^{\circ}$

$\therefore$ exterior angle $= 180^{\circ} - 150^{\circ} = \underline{\underline{30^{\circ}}}$

ii. The number of sides $= \frac{360^{\circ}}{30^{\circ}} = \underline{\underline{12}}$

Exercise 25.3

1. Find the magnitude of an exterior angle of a regular pentagon and thereby find the magnitude of an interior angle.

2. Find the magnitude of an exterior angle of a regular polygon with 15 sides and thereby find the magnitude of an interior angle.

3. **i.** Find the number of sides of a regular polygon with an exterior angle of magnitude 120° and write the special name given to it.

ii. Write with reasons the special name given to the regular polygon which has an exterior angle of magnitude 90°.

iii. Write the name given to the regular polygon which has an exterior angle of magnitude 40°.

4. The magnitude of an interior angle of a regular polygon is four times the magnitude of an exterior angle. Find

i. the magnitude of an exterior angle.
ii. the magnitude of an interior angle.
iii. the number of sides the polygon has.

5. What is the greatest value that an exterior angle of a regular polygon can be? What is the name given to the corresponding regular polygon?

Summary

- Sum of the exterior angles of a polygon = 360^{o}.
- The sum of the interior angles of a polygon of n sides $=$ $(2n - 4)$ right angles.

26

Algebraic Fractions

By studying this lesson you will be able to;

- identify algebraic fractions,
- add and subtract algebraic fractions with integral denominators (equal/ unequal denominators)
- add and subtract algebraic fractions with equal algebraic denominators.

We have already learnt to add and subtract numerical fractions and simplify, expand and factorize algebraic expressions.

Do the following review exercise to recall what has been learnt earlier.

Review Exercise

1. Simplify.

i. $\frac{2}{5} + \frac{1}{5}$ **ii.** $\frac{5}{7} - \frac{2}{7}$ **iii.** $\frac{1}{9} - \frac{7}{9} + \frac{4}{9}$ **iv.** $\frac{12}{13} - \frac{2}{13} - \frac{1}{13}$

2. Fill in each box with the appropriate number.

i. $\frac{1}{2} - \frac{1}{4}$

$= \frac{1 \times \Box}{2 \times 2} - \frac{1}{4}$

$= \frac{\Box \quad - 1}{4}$

$= \underline{\underline{\frac{\Box}{4}}}$

ii. $\frac{3}{4} - \frac{2}{3}$

$= \frac{3 \times \Box}{4 \times 3} - \frac{\Box \times 4}{3 \times 4}$

$= \frac{\Box - \Box}{12}$

$= \underline{\underline{\frac{\Box}{12}}}$

iii. $\frac{4}{5} - \frac{3}{10} - \frac{1}{3}$

$= \frac{4 \times \Box}{5 \times 6} - \frac{3 \times \Box}{10 \times 3} - \frac{1 \times 10}{3 \times \Box}$

$= \frac{\Box - \Box - 10}{30}$

$= \frac{\Box}{30}$

$= \frac{\Box \quad \div 5}{30 \div 5}$

$= \underline{\underline{\frac{\Box}{6}}}$

3. Simplify the algebraic expressions given below.

a. $2x + 3x$ **b.** $3y - y$
c. $5a + 4a + a$ **d.** $5x + 3y + x + 3y$
e. $3y + 2 - y - 2$ **f.** $4n - 1 + 5 - 2n$
g. $-3y + 2 - y - 3 + 2y$ **h.** $5xy - 6xy + 3x + y$

4. Expand and simplify.

a. $2\,(x + y) + 3x$ **b.** $3\,(2x - 4y) + 12y$
c. $-(4 - 3x) - 1$ **d.** $2\,(3x - 2) + 3(x + 2)$
e. $3\,(m + 1) - 2\,(2m - 1)$ **f.** $x\,(\,x - y\,) + 2xy$

5. For each of the statements given below, if it is true, mark a "✓" and if it is false, mark a "✗" in the box to the right of the statement.

a. The value of $\frac{2}{3} + \frac{1}{4}$ is the same as the value of $\frac{2+1}{3+4}$. ☐

b. To obtain the sum or the difference of two fractions, their numerators should be equal; if they are not equal, then they should be made equal. ☐

c. The numerator of the sum of two unit fractions is the sum of the denominators of the original two fractions and the denominator is the product of the denominators of the original two fractions. ☐

d. When adding or subtracting two fractions with unequal denominators, the common denominator that should be used is the L. C. M. of the denominators of the original two fractions. ☐

e. By multiplying the numerator and the denominator of a fraction by the same number, we can convert it into its simplest equivalent form. ☐

f. By dividing the numerator and the denominator of a fraction by the same number, we can convert it into its simplest equivalent form. ☐

g. $-3x - 2x$ can be considered as $(-3x) + (-2x)$. ☐

h. To expand $-3(2x - 5)$, the terms $2x$ and -5 need to be multiplied by 3. ☐

i. When $-x - x$ is simplified we obtain $2x$. ☐

j. When $3x + 4y$ is simplified we obtain $7xy$. ☐

Introduction to algebraic fractions

If the numerator or denominator or both the numerator and the denominator of a fraction contain an algebraic term or expression, then that fraction is known as an algebraic fraction.

Example 1

Write 5 algebraic fractions that have an algebraic term only in the numerator.

$$\frac{x}{2}, \frac{3x}{5}, \frac{7y}{20}, \frac{6mn}{3}, \frac{2t^2}{5}$$

Example 2

Write 5 algebraic fractions that have an algebraic expression only in the numerator.

$$\frac{x+1}{5}, \frac{2x-1}{3}, \frac{x+y}{2}, \frac{m-n}{7}, \frac{3m-2n-1}{10}$$

Example 3

Write 5 algebraic fractions that have an algebraic term only in the denominator.

$$\frac{3}{x}, \frac{2}{3m}, \frac{5}{2y}, \frac{4}{3xy}, \frac{5}{m^2}$$

Example 4

Write 5 algebraic fractions that have an algebraic expression only in the denominator.

$$\frac{3}{2x+1}, \frac{2}{a+b}, \frac{5}{2m-n}, \frac{4}{3x-2y}, \frac{1}{3x+cy+2}$$

Example 5

Write 5 algebraic fractions which have an algebraic term in both the numerator and the denominator.

$$\frac{a}{c}, \frac{2a}{d}, \frac{2m}{3n}, \frac{4x}{5y}, \frac{2xy}{3pq}, \frac{2x^2}{5y^2}$$

Example 6

Write 5 algebraic fractions that have an algebraic expression in the numerator and an algebraic term in the denominator.

$$\frac{x+1}{2x},\ \frac{2a+b}{c},\ \frac{3a+d}{4a},\ \frac{2x-1}{c},\ \frac{4x^2y-a^2}{b}$$

Example 7

Write 5 algebraic fractions that have an algebraic term in the numerator and an algebraic expression in the denominator.

$$\frac{x}{2x+5},\ \frac{a}{5b+d},\ \frac{3c}{a+b},\ \frac{4xy}{5x-3},\ \frac{a^2}{a-b}$$

Example 8

Write 5 algebraic fractions that have algebraic expressions in both the numerator and the denominator.

$$\frac{x+1}{2x-1},\ \frac{x+y}{3x+2y},\ \frac{3x-4}{x+1},\ \frac{4m-3n}{5m+2n},\ \frac{4x-y}{2x+3y-4}$$

26.1 Adding and subtracting algebraic fractions with equal integral denominators

We can add and subtract algebraic fractions in the same way that we added and subtracted fractions with whole numbers in the numerator and denominator.

Example 1

Express $\frac{5x}{9}+\frac{2x}{9}$ as a single fraction.

$\frac{5x}{9}+\frac{2x}{9}=\frac{5x+2x}{9}$ (since the denominators of both fractions are equal)

$$=\underline{\underline{\frac{7x}{9}}}$$

Example 2

Simplify $\frac{5y}{7} - \frac{3y}{7}$.

$\frac{5y}{7} - \frac{3y}{7} = \frac{5y - 3y}{7}$ (since the denominators of both fractions are equal)

$= \underline{\underline{\frac{2y}{7}}}$

Example 3

Simplify $\frac{4x}{15} + \frac{7x}{15} - \frac{2x}{15}$.

$\frac{4x}{15} + \frac{7x}{15} - \frac{2x}{15} = \frac{11x - 2x}{15}$ (since the denominators of both fractions are equal)

$= \frac{9x}{15} = \underline{\underline{\frac{3x}{5}}}$ (by dividing by 3, the highest common factor of 9 and 15)

Example 4

Simplify $\frac{x+1}{5} + \frac{x+2}{5}$.

$\frac{x+1}{5} + \frac{x+2}{5} = \frac{x + 1 + x + 2}{5}$ (since the denominators of both fractions are equal)

$= \frac{x + x + 1 + 2}{5}$

$= \underline{\underline{\frac{2x+3}{5}}}$

Example 5

Simplify $\frac{2b+3}{7} - \frac{b+2}{7}$.

$\frac{2b+3}{7} - \frac{b+2}{7} = \frac{2b + 3 - (b + 2)}{7}$ (the algebraic expression to be subtracted must be written within brackets)

$= \frac{2b + 3 - b - 2}{7}$

$= \frac{2b - b + 3 - 2}{7}$

$= \underline{\underline{\frac{b+1}{7}}}$

Example 6

Simplify $\dfrac{7c+1}{8} - \dfrac{2c+1}{8} - \dfrac{c-2}{8}$.

$$\begin{aligned}\frac{7c+1}{8} - \frac{2c+1}{8} - \frac{c-2}{8} &= \frac{7c+1-(2c+1)-(c-2)}{8}\\ &= \frac{7c+1-2c-1-c+2}{8}\\ &= \frac{4c+2}{8}\\ &= \frac{2\,(2c+1)}{8}\\ &= \underline{\underline{\frac{2c+1}{4}}}\end{aligned}$$

Exercise 26.1

1. Simplify and write the answer in the simplest form.

a. $\dfrac{a}{5} + \dfrac{a}{5}$ **b.** $\dfrac{3d}{15} + \dfrac{2d}{15}$ **c.** $\dfrac{2t}{3} - \dfrac{t}{3}$

d. $\dfrac{7k}{8} - \dfrac{3k}{8}$ **e.** $\dfrac{3k}{7} + \dfrac{2k}{7} + \dfrac{k}{7}$ **f.** $\dfrac{5h}{9} - \dfrac{2h}{9} - \dfrac{h}{9}$

g. $\dfrac{7v}{10} - \dfrac{3v}{10} + \dfrac{v}{10}$ **h.** $\dfrac{x}{8} - \dfrac{3x}{8}$ **i.** $\dfrac{p}{9} - \dfrac{4q}{9} - \dfrac{5p}{9}$

2. Simplify and write the answer in the simplest form.

a. $\dfrac{3y+1}{5} + \dfrac{2y+2}{5}$ **b.** $\dfrac{4m-1}{7} + \dfrac{3m-2}{7}$ **c.** $\dfrac{5n+3}{8} + \dfrac{2n-1}{8}$

d. $\dfrac{5c-2}{10} + \dfrac{3c+4}{10}$ **e.** $\dfrac{6d+1}{10} - \dfrac{2d-3}{10}$

f. $\dfrac{3x+1}{6} - \dfrac{2x-3}{6} + \dfrac{x+4}{6}$

26.2 Adding and subtracting algebraic fractions with unequal integral denominators

Now let us consider how to simplify expressions of algebraic fractions with unequal integral denominators such as $\dfrac{x}{6} + \dfrac{3x}{4}$. These types of fractions can be simplified in the same way that numerical fractions are simplified. A common multiple of the denominators of the fractions can be taken as the common denominator. However simplification is made easier by taking the least common multiple of the denominators.

For example, the denominators of the above two fractions are 6 and 4. Their least common multiple is 12. Therefore, initially, the above fractions need to be converted into fractions with denominator 12. To convert $\frac{x}{6}$ into a fraction with denominator 12, the denominator and numerator of $\frac{x}{6}$ need to be multiplied by 2. (Observe that 2 is obtained from $\frac{12}{6}$). Similarly, to convert $\frac{3x}{4}$ into a fraction with denominator 12, we need to multiply the numerator and denominator of $\frac{3x}{4}$ by 3. (Observe that 3 is obtained from $\frac{12}{4}$). Accordingly, we may write the following to simplify the given expression.

$$\frac{x}{6} + \frac{3x}{4} = \frac{2}{2} \times \frac{x}{6} + \frac{3}{3} \times \frac{3x}{4}$$

When we simplify the numerator and the denominator of these fractions we obtain the following.

$$\frac{2x}{12} + \frac{9x}{12}$$

Now since both fractions have a common denominator, we can write the above as follows.

$$\frac{2x + 9x}{12}$$

By simplifying this we get $\frac{11x}{12}$.

Accordingly, $\frac{x}{6} + \frac{3x}{4} = \frac{11x}{12}$.

Example 1

Simplify $\frac{2y}{5} + \frac{y}{4}$.

$$\frac{2y}{5} + \frac{y}{4} = \frac{4 \times 2y}{4 \times 5} + \frac{5 \times y}{5 \times 4}$$

(Since the L. C. M. of 5 and 4 is 20, equivalent fractions with 20 as the denominator are obtained.)

$$= \frac{8y}{20} + \frac{5y}{20}$$

$$= \frac{8y + 5y}{20} = \underline{\underline{\frac{13y}{20}}}$$

Example 2

Simplify $\frac{2t}{3} - \frac{t}{2}$.

$$\frac{2t}{3} - \frac{t}{2} = \frac{2 \times 2t}{2 \times 3} - \frac{3 \times t}{3 \times 2}$$

(since the L.C.M. of 3 and 2 is 6, equivalent fractions with 6 as the denominator are obtained)

$$= \frac{4t}{6} - \frac{3t}{6}$$

$$= \frac{4t - 3t}{6}$$

$$= \underline{\underline{\frac{t}{6}}}$$

Example 3

Simplify $\frac{3v}{2} - \frac{4v}{5} + \frac{3v}{4}$.

$$\frac{3v}{2} - \frac{4v}{5} + \frac{3v}{4} = \frac{10 \times 3v}{10 \times 2} - \frac{4 \times 4v}{4 \times 5} + \frac{5 \times 3v}{5 \times 4}$$

(since the L.C.M. of 2, 4 and 5 is 20, equivalent fractions with 20 as the denominator are obtained)

$$= \frac{30v}{20} - \frac{16v}{20} + \frac{15v}{20}$$

$$= \underline{\underline{\frac{29v}{20}}}$$

You may have observed in the above examples that when the denominators are not equal, it is easy to simplify by taking the L.C.M. of the unequal denominators as the common denominator.

Now let us consider instances where we have to multiply an algebraic expression by a number. Here it is important to remember to write the algebraic expression within brackets.

Example 4

Simplify $\frac{x + 1}{2} + \frac{2x + 1}{3}$.

$$\frac{x + 1}{2} + \frac{2x + 1}{3} = \frac{3\,(x + 1)}{3 \times 2} + \frac{2\,(2x + 1)}{2 \times 3}$$

(writing the algebraic expression within brackets; L.C.M. of 2 and 3 is 6)

$$= \frac{3x + 3}{6} + \frac{4x + 2}{6} \qquad \text{(expanding)}$$

$$= \frac{7x + 5}{6}$$

Example 5

$$\frac{5y - 1}{6} - \frac{3y - 2}{4}$$

$$\frac{5y - 1}{6} - \frac{3y - 2}{4} = \frac{2\,(5y - 1)}{2 \times 6} - \frac{3\,(3y - 2)}{3 \times 4} \qquad \text{(L.C.M. of 4 and 6 is 12)}$$

$$= \frac{2\,(5y - 1)}{12} - \frac{3\,(3y - 2)}{12}$$

$$= \frac{2\,(5y - 1) - 3\,(3y - 2)}{12}$$

$$= \frac{10y - 2 - 9y + 6}{12}$$

(expanding by multiplying by 2 and by − 3)

$$= \frac{y + 4}{12}$$

Example 6

$$\frac{3m + 2n}{5} - \frac{2m - n}{10} - \frac{3m - 2n}{15}$$

$$= \frac{6\,(3m + 2n)}{6 \times 5} - \frac{3\,(2m - n)}{3 \times 10} - \frac{2\,(3m - 2n)}{2 \times 15} \qquad \text{(L.C.M. of 5, 10 and 15 is 30)}$$

$$= \frac{6\,(3m + 2n)}{30} - \frac{3\,(2m - n)}{30} - \frac{2\,(3m - 2n)}{30}$$

$$= \frac{18m + 12n - 6m + 3n - 6m + 4n}{30}$$

$$= \frac{6m + 19n}{30}$$

Exercise 26.2

1. Simplify and give the answer in the simplest form.

a. $\frac{a}{3} + \frac{a}{6}$ **b.** $\frac{b}{4} + \frac{b}{12}$ **c.** $\frac{5x}{3} - \frac{x}{6}$

d. $\frac{3y}{4} - \frac{5y}{16}$ **e.** $\frac{a}{2} + \frac{a}{3}$ **f.** $\frac{c}{3} - \frac{c}{4}$

g. $\frac{3n}{7} + \frac{n}{5}$ **h.** $\frac{3d}{10} + \frac{2d}{15}$ **i.** $\frac{5m}{6} - \frac{3m}{10}$

2. Simplify and give the answer in the simplest form.

a. $\frac{a}{2} + \frac{a}{3} + \frac{a}{4}$ **b.** $\frac{c}{5} + \frac{3c}{10} + \frac{2c}{15}$

c. $\frac{3x}{5} + \frac{x}{6} - \frac{2x}{15}$ **d.** $\frac{3n}{4} - \frac{3n}{8} - \frac{n}{2}$

3. Simplify and write in the simplest form.

a. $\frac{2a}{5} + \frac{3a-2}{6}$ **b.** $\frac{2b-1}{8} + \frac{3b}{12}$

c. $\frac{3c+2}{6} + \frac{2c-1}{9}$ **d.** $\frac{5t-3}{10} - \frac{3t}{15}$

e. $\frac{2m-n}{12} - \frac{3m+n}{9}$ **f.** $\frac{3y+1}{10} + \frac{2y-1}{5} + \frac{4-y}{20}$

g. $\frac{3x-y}{4} + \frac{2x+y}{6} - \frac{5x-2y}{3}$ **h.** $\frac{3y+2}{3} - \frac{y-1}{4} - \frac{2y-3}{8}$

26.3 Adding and subtracting algebraic fractions with the same algebraic denominator

As an example of this type of algebraic fraction we have $\frac{2}{5x} + \frac{1}{5x}$. Although the denominators of these fractions are algebraic terms, since they are equal, we can simplify this in the same way that we simplify numerical fractions.

Accordingly, we can simplify the above as,

$$\frac{2}{5x} + \frac{1}{5x} = \frac{2+1}{5x}$$

$$= \frac{3}{5x}$$

Example 1

Simplify $\frac{4}{7m} + \frac{2}{7m}$.

$$\frac{4}{7m} + \frac{2}{7m} = \frac{4+2}{7m}$$

$$= \underline{\underline{\frac{6}{7m}}}$$

Example 2

Simplify $\frac{5}{6n} - \frac{1}{6n}$.

$$\frac{5}{6n} - \frac{1}{6n} = \frac{5-1}{6n}$$

$$= \frac{4}{6n}$$

$$= \underline{\underline{\frac{2}{3n}}}$$

(simplifying by dividing by the common factor 2)

Example 3

Simplify $\frac{3a}{4b} + \frac{1}{4b} - \frac{a}{4b}$.

$$\frac{3a}{4b} + \frac{1}{4b} - \frac{a}{4b} = \frac{3a+1-a}{4b}$$ (common denominator is $4b$)

$$= \underline{\underline{\frac{2a+1}{4b}}}$$

Example 4

Simplify $\frac{3}{x+1} + \frac{2}{x+1}$.

Although the denominators are algebraic expressions, since they are equal, this can be simplified in the same manner as above.

$$\frac{3}{x+1} + \frac{2}{x+1} = \frac{3+2}{x+1}$$

$$= \underline{\underline{\frac{5}{x+1}}}$$

Example 5

Simplify $\frac{7}{x-3} - \frac{4}{x-3}$.

$$\frac{7}{x-3} - \frac{4}{x-3} = \frac{7-4}{x-3}$$ (common denominator is $x - 3$)

$$= \underline{\underline{\frac{3}{x-3}}}$$

Exercise 26.3

1. Simplify and give the answer in the simplest form.

a. $\frac{5}{a} + \frac{2}{a}$ **b.** $\frac{8}{x} + \frac{2}{x}$ **c.** $\frac{3}{y} - \frac{1}{y}$

d. $\frac{4}{3y} - \frac{2}{3y}$ **e.** $\frac{3}{5t} + \frac{2}{5t}$ **f.** $\frac{h}{2k} + \frac{5h}{2k}$

g. $\frac{7}{2n} + \frac{3}{2n} - \frac{1}{2n}$ **h.** $\frac{8}{3v} - \frac{4}{3v} - \frac{1}{3v}$ **i.** $\frac{5}{m} + \frac{2}{m} + \frac{1}{m}$

j. $\frac{8}{7xy} - \frac{8}{7xy} + \frac{8}{7xy}$

2. Simplify and give the answer in the simplest form.

a. $\frac{5}{m+3} + \frac{2}{m+3}$ **b.** $\frac{8}{n+5} + \frac{3}{n+5}$ **c.** $\frac{4}{a+b} + \frac{6}{a+b}$

d. $\frac{4x}{x+2y} + \frac{x+y}{x+2y}$ **e.** $\frac{9h}{x+y} - \frac{7h-2}{x+y}$ **f.** $\frac{3x+y}{x-3y} - \frac{2x+4y}{x-3y}$

26.4 Simplifying algebraic fractions with algebraic expressions in the numerator and the denominator

Example 1

Simplify $\frac{5x}{2x+1} + \frac{3x}{2x+1}$.

$$\frac{5x}{2x+1} + \frac{3x}{2x+1} = \frac{5x+3x}{2x+1} \quad \text{(the common denominator is } 2x+1\text{)}$$

$$= \underline{\underline{\frac{8x}{2x+1}}}$$

Example 2

Simplify $\frac{7y}{3y-1} - \frac{2y}{3y-1}$.

$$\frac{7y}{3y-1} - \frac{2y}{3y-1} = \frac{7y-2y}{3y-1} \quad \text{(the common denominator is } 3y-1\text{)}$$

$$= \underline{\underline{\frac{5y}{3y-1}}}$$

Example 3

Simplify $\dfrac{2x-1}{5x+1}+\dfrac{3x+2}{5x+1}$.

$\dfrac{2x-1}{5x+1}+\dfrac{3x+2}{5x+1}=\dfrac{2x-1+3x+2}{5x+1}$ (the common denominator is $5x+1$)

$=\dfrac{5x+1}{5x+1}$

$=\underline{\underline{1}}$

Example 4

Simplify $\dfrac{9m-1}{5m-1}+\dfrac{3m}{5m-1}-\dfrac{2m+1}{5m-1}$.

$\dfrac{9m-1}{5m-1}+\dfrac{3m}{5m-1}-\dfrac{2m+1}{5m-1}=\dfrac{9m-1+3m-(2m+1)}{5m-1}$ (algebraic expressions to be subtracted need to be written within brackets)

$=\dfrac{9m-1+3m-2m-1}{5m-1}$ (multiplying by the – sign and expanding)

$=\dfrac{10m-2}{5m-1}$

$=\dfrac{2\cancel{(5m-1)}}{\cancel{(5m-1)}}$ (separating out the common factor in the numerator and simplifying)

$=\underline{\underline{2}}$

Exercise 26.4

1. Simplify and write the answer in the simplest form.

a. $\dfrac{k}{3k-1}+\dfrac{2}{3k-1}$

b. $\dfrac{2h}{5h-2}-\dfrac{h}{5h-2}$

c. $\dfrac{3t}{3t-1}-\dfrac{1}{3t-1}$

d. $\dfrac{2k+1}{5k+1}-\dfrac{k-2}{5k+1}$

e. $\dfrac{2y}{3y+2} - \dfrac{y}{3y+2} + \dfrac{1}{3y+2}$

f. $\dfrac{2a+1}{5a-2} - \dfrac{3a}{5a-2} - \dfrac{3}{5a-2}$

g. $\dfrac{8m+10}{2m+3} - \dfrac{4m+1}{2m+3} + \dfrac{2m}{2m+3}$

h. $\dfrac{m}{m+n} - \dfrac{m-n}{m+n} - \dfrac{m-n}{m+n}$

27

Scale Diagrams

By studying this lesson you will be able to;

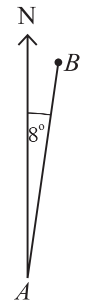
- identify bearings,
- draw a scale diagram of locations in a horizontal plane when bearings and distances are given, and find unknown quantities using the scale diagram.

27.1 Bearing

Bearing is a measurement that is used to indicate a direction in a horizontal plane.

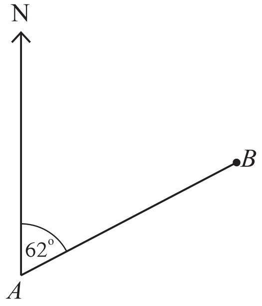
The bearing of the point *B* from the point *A* is the angle that the direction *AB* makes with the direction of North when measured from *A* in a clockwise direction. The following figures illustrate the bearing of *B* from *A* for different locations of *A* and *B*. Observe that the bearing is given in three digits.

i.



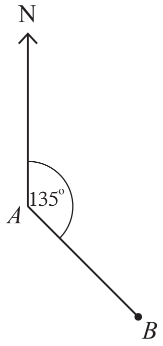
The bearing of *B* from *A* = 008°

ii.



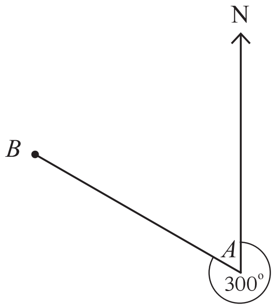
The bearing of *B* from *A* = 062°

iv.



The bearing of *B* from *A* = 135°

v.



The bearing of *B* from *A* = 300°

Since a bearing is always less than 360°, the maximum number of digits it can have is three. Therefore the norm is to always write bearings with three digits. If the angle is one of $1^{\circ}, 2^{\circ}, \ldots, 9^{\circ}$, then the bearing is written as $001^{\circ}, 002^{\circ}, \ldots, 009^{\circ}$ and if the angle is one of $10^{\circ}, 11^{\circ}, \ldots, 99^{\circ}$, then it is written as $010^{\circ}, 011^{\circ}, \ldots, 099^{\circ}$.

Accordingly bearing is,

i. measured starting from the North,
ii. measured in a clockwise direction,
iii. written with three digits.

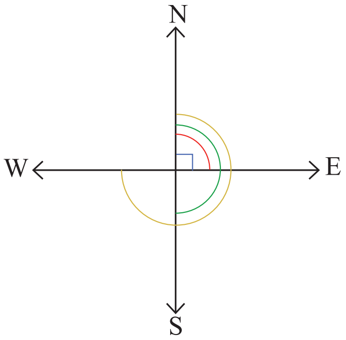
Since the North can be easily identified using a compass, bearings are used widely in sea and air travel.

Let us broaden our knowledge on bearings by studying the examples given below.

Example 1

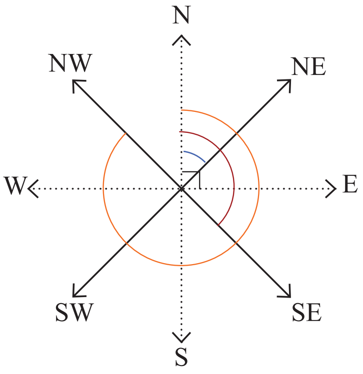
i. Indicate the four main directions (cardinal directions) in terms of bearings.
ii. Indicate the four sub-directions (intermediate directions) in terms of bearings.

i.



Direction	**Bearing**
North	000°
East	090°
South	180°
West	270°

ii.

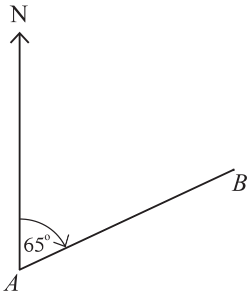


Direction	**Bearing**
Northeast	045°
Southeast	135°
Southwest	225°
Northwest	315°

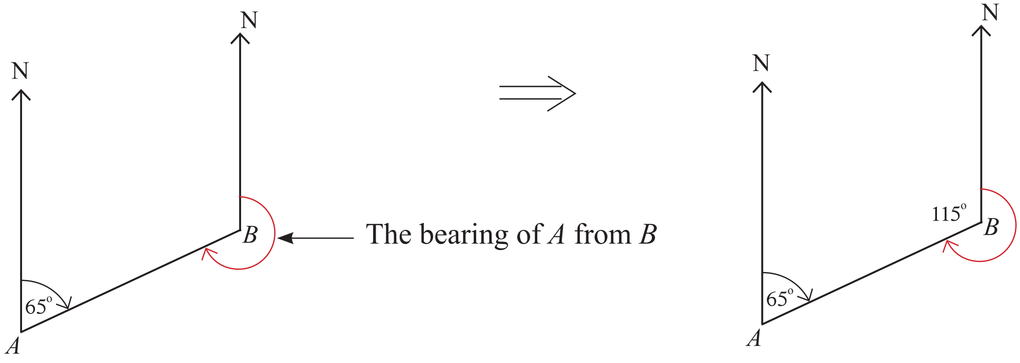
Example 2

The bearing of *B* from *A* is 065°. Illustrate this information in a rough sketch and find the bearing of *A* from *B*.

Since the bearing of *B* from *A* is 065°, the angle drawn from the direction of North at *A* to the directionof *AB* in the clockwise direction is 65°.



Now, to find the bearing of *A* from *B*, a line needs to be drawn in the direction of North from *B*, and the angle that is formed when this line is rotated in a clockwise direction about *B* from the direction of North to the direction of *BA* needs to be found.



The lines drawn at *A* and *B* in the direction of North are parallel. The pair of allied angles formed by the transversal *AB* intersecting these lines are supplementary. Using this fact, the value 115° has been found. The required bearing is indicated in the figure given above.

As the sum of the angles around a point is 360°,

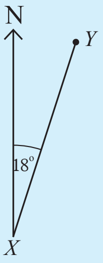
the bearing of *A* from *B* $= 360^{\circ} - 115^{\circ}$

$= \underline{\underline{245^{\circ}}}$

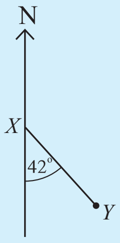
Exercise 27.1

1. In each of the following situations, find the bearing of Y from X.

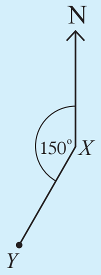
i.



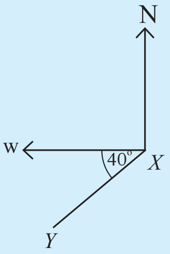
ii.



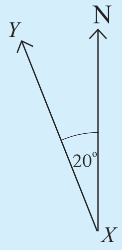
iii.



iv.



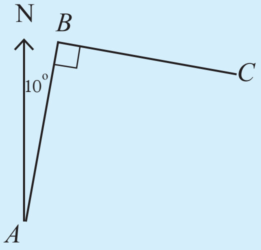
v.



2. Using a protractor to measure the angles, illustrate each of the following bearings by drawing a figure.

- **i.** The bearing of F from E is 005°.
- **ii.** The bearing of Q from P is 075°.
- **iii.** The bearing of N from M is 105°.
- **iv.** The bearing of H from J is 270°.
- **v.** The bearing of D from C is 310°.

3.



Based on the information given in the figure,

- **i.** determine the bearing of B from A,
- **ii.** determine the bearing of A from B,
- **iii.** determine the bearing of B from C.

4. ABC is an equilateral triangle. B is situated to the north of A.

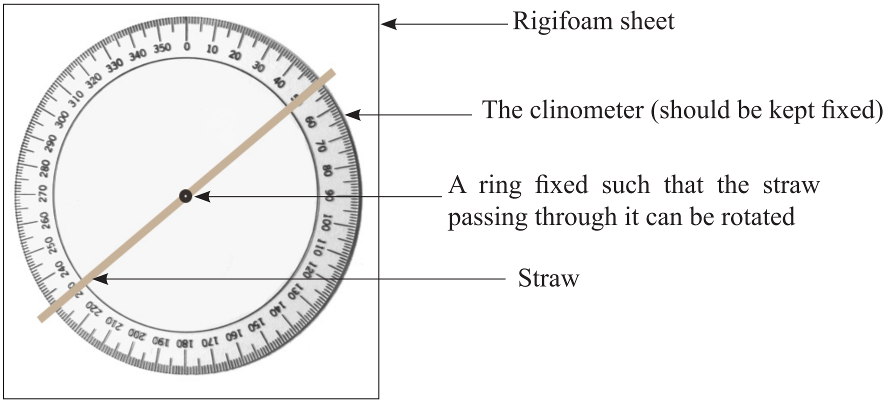
- **i.** Illustrate this information by a rough sketch.
- **ii.** By considering the sketch determine the following.

a. Bearing of B from A
b. Bearing of C from A
c. Bearing of C from B
d. Bearing of B from C
e. Bearing of A from C
f. Bearing of A from B

27.2 Clinometer

Any location in a horizontal plane can be described in terms of bearings and distances. A clinometer can be used to determine bearings.

Clinometer



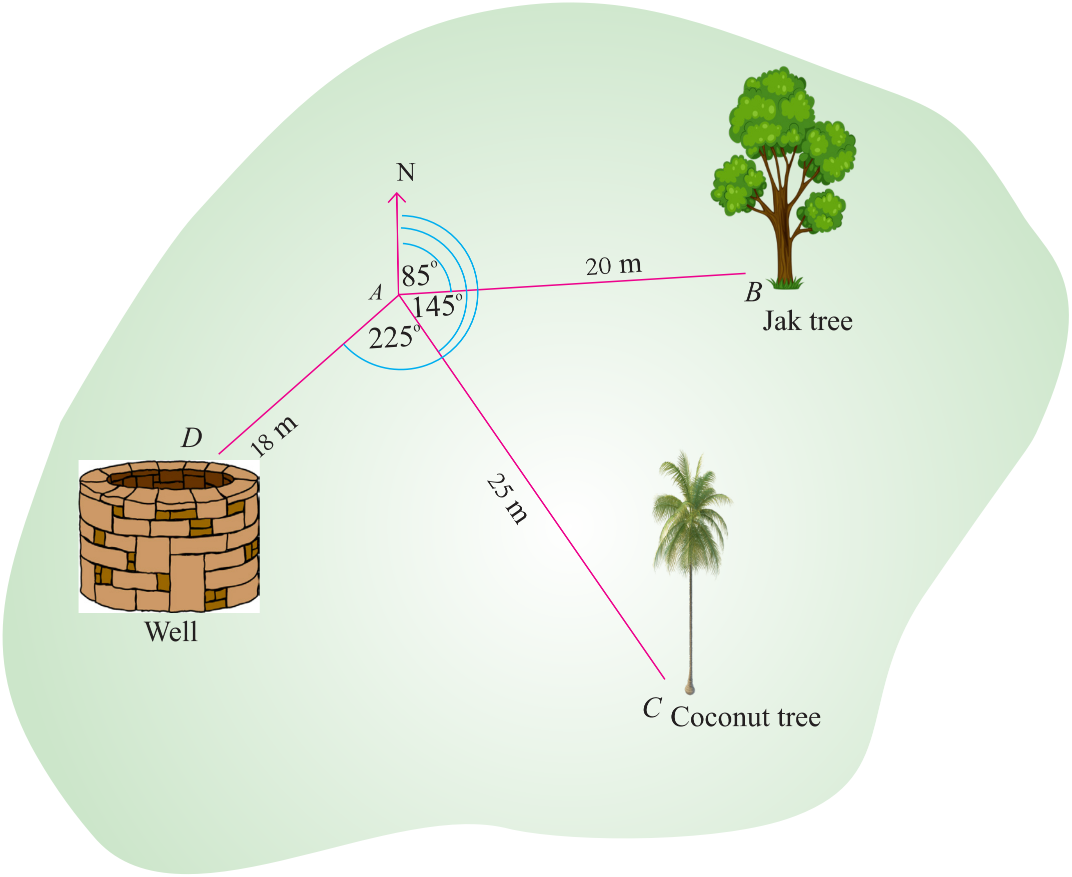
- Place a compass on the horizontal tabletop of a table kept at *A*. Suppose for example that we want to describe the location of *B* with respect to location *A* and mark the direction of North on the tabletop.

- Place the clinometer on the tabletop such that the "0" on the clinometer is towards the North.

- Rotate the straw until the location *B* is observed through the straw and measure the clockwise angle of rotation from the direction of North. By writing it using three digits, the bearing of *B* is obtained.

- By measuring the distance from *A* to *B* using a measuring tape, the position of *B* can be described in terms of the distance and bearing from *A*.

In the following figure the bearings of *B*, *C* and *D* with respect to *A* are given.

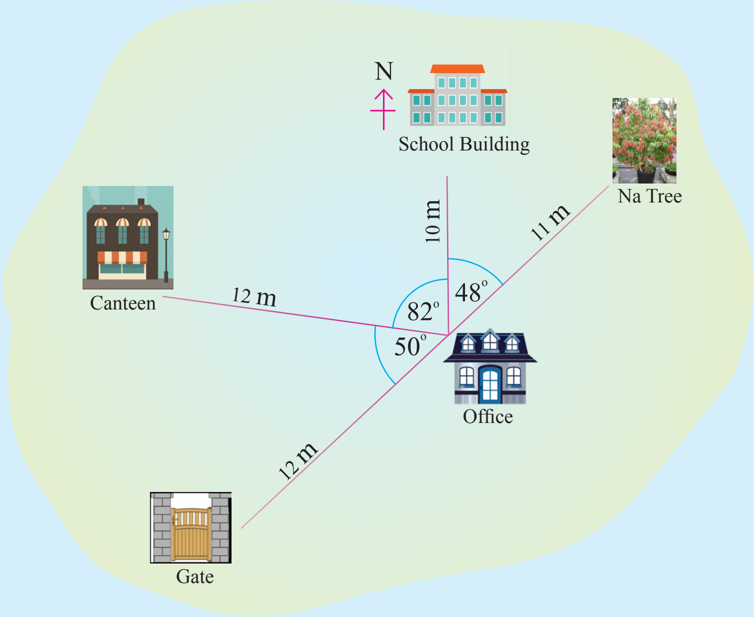


Object which was observed	**Bearing**	**Distance**
Jak tree (*B*)	085°	20 m
Coconut tree (*C*)	145°	25 m
Well (*D*)	225°	18 m

Do the following exercise to broaden your knowledge on this topic.

Exercise 27.2

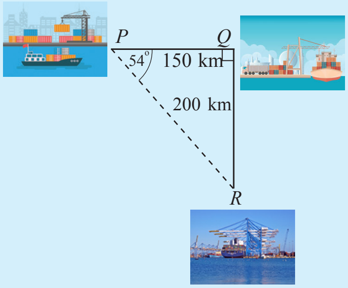
1. A rough school plan is shown below.



Using it describe the following.

i. The location of the Na tree with respect to the School Office
ii. The location of the gate with respect to the School Office
iii. The location of the Canteen with respect to the School Office

2.



P, *Q* and *R* denote three harbours located in the same ocean. *Q* is to the East of *P*. Describe the route in terms of the bearing and distance that a ship needs to take to journey,

i. from harbour *P* to harbour *R* through *Q*.
ii. directly from harbour *P* to harbour *R*.

3. A pilot of a certain air plane which is scheduled to fly from Colombo to a certain airport has been instructed to fly 100 km on a bearing of 020° and then another 100 km on a bearing of 080°.

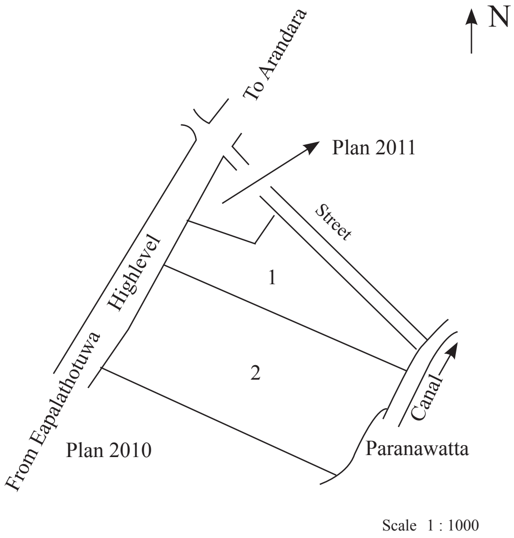
i. Represent this information in a rough sketch

ii. Write a description of the route that the pilot needs to take if he is to fly back to Colombo from that airport along the same path.

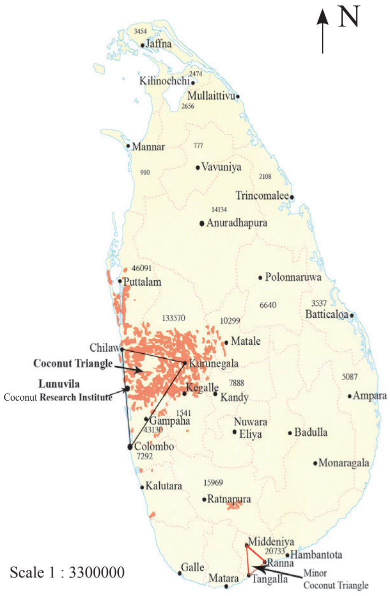
27.3 Scale diagrams in a horizontal plane

Given below are two examples of scale diagrams in a horizontal plane.

Survey Plane No: 103



Coconut Cultivation



In every scale diagram, the scale to which the diagram is drawn is given, and the direction of North is marked. It is very important to understand what is meant by the scale (ratio) given in the scale diagram. For example, a scale of 1 : 500 000 means that a distance of 500 000 cm is represented by 1 cm in the scale diagram. In other words, the distance between two points on the scale diagram is $\frac{1}{500\ 000}$th of the actual distance between the two points. Moreover, since 500 000 cm is equal to 5 km, the actual distance represented by 1 cm in the scale diagram is 5 km.

Now let us learn how to draw scale diagrams by considering some examples.

Example 1

The vertices of a triangular floor area are *A*, *B* and *C*. The positions of the vertices with respect to a point *P* located in this area is given below.

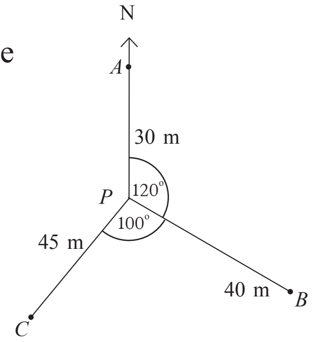
With respect to *P*,

- *A* is located 30 m away on a bearing of 000°
- *B* is located 40 m away on a bearing of 120°
- *C* is located 45 m away on a bearing of 220°

Draw a scale diagram of the floor area using this information and find its perimeter.

Step 1: Mark the direction of North at the top right hand corner of the sheet of paper.

Step 2: Draw a rough sketch as shown, based on the information that is given.



Step 3: To represent the distances 30 m, 40 m and 45 m, select the scale of 1 cm representing 10 m, that is, the scale of 1:1000.(Here, the scale should be selected according to the size of the sheet of paper. Moreover, by selecting a value such as 1000, anyone who is examining the scale diagram can easily get an idea of the actual distances represented in it.)

Step 4: For each distance that is to be represented in the scale diagram, calculate the corresponding length using the selected scale.

$$PA = 3000 \times \frac{1}{1000} \text{ cm} = 3 \text{ cm}$$

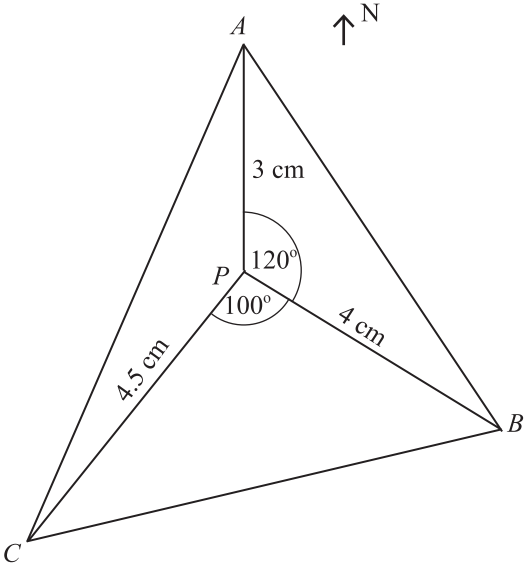
$$PB = 4000 \times \frac{1}{1000} \text{ cm} = 4 \text{ cm},$$

$$PC = 4500 \times \frac{1}{1000} \text{ cm} = 4.5 \text{ cm}$$

Step 5: Using a straight edge with a cm scale and a protractor, draw the scale diagram with a pencil as shown below.

- First draw the line segment *AP* of length 3cm upwards.
- Draw the line segment *PB* of length 4 cm which makes an angle of 120° clockwise with *PA*.
- Draw the line segment *PC* of length 4.5 cm which makes an angle of 100° clockwisewith *PB*.

- Draw the line segments *AB, BC* and *AC.*



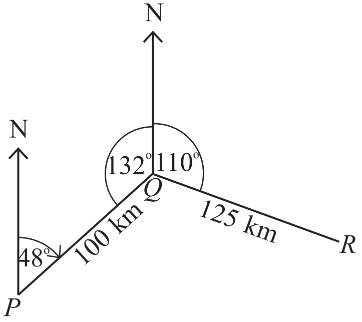
Step 6: Measure the lengths of *AB*, *BC* and *AC*. You will see that *AB* = 6 cm, *AC* = 7.1 cm and *BC* = 6.5 cm. Therefore the perimeter of the scale diagram is 6 + 7.1 + 6.5 = 19.6 cm.

Step 7: Using the scale 1 cm ⟶ 10 m, calculate the actual length.
Perimeter of the floor = 10 × 19.6 = 196 m

Example 2

A ship journeying from harbour *P* approaches harbour *Q* after travelling 100 km on a bearing of 048°. It then travels 125 km on a bearing of 110° and approaches harbour *R*. Draw a scale diagram and describe the position of *R* with respect to *P*.

Step 1: Based on the information given, draw a rough sketch as shown below.



Step 2: Mark a point *P* on a sheet of paper and mark the direction of North upwards.

- Since the bearing of Q from P is 048^{o}, the angle that PQ makes with the direction of North at P is 48^{o} in the clockwise direction.
- Since the bearing of R from Q is 110^{o}, the angle that QR makes with the direction of North at Q is 110^{o} in the clockwise direction.

Since the direction of North at P and the direction of North at Q are parallel, the angle formed between the direction of North at Q and PQ is 132^{o} (allied angles)

Therefore, $P\hat{Q}R = 360^{o} - (132^{o} + 110^{o})$
$= 360^{o} - 242^{o}$
$= 118^{o}$

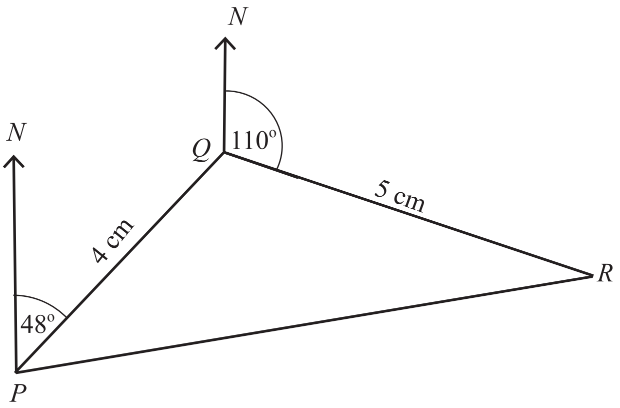
Step 3: Select the scale of 1 cm representing 25 km, that is, 1: 2 500 000 to represent the distances 100 km and 125 km in the scale diagram. (If there is space on the sheet of paper, the scale of 1:1 250 000 can also be used).

Step 4: According to the selected scale, calculate the lengths of by which PQ and QR are to be represented in the scale diagram.

$$PQ = \frac{100}{25} \text{ cm} = 4 \text{ cm}, \quad QR = \frac{125}{25} \text{ cm} = 5 \text{ cm}$$

(When drawing scale diagrams, the magnitudes of the angles do not change.)

Step 5: Draw the scale diagram using a straight edge, a protractor and a pencil, based on the above measurements.



Step 6: When PR is measured, we obtain $PR = 7.7$ cm. When $N\hat{P}R$ is measured, we obtain $N\hat{P}R = 82^{o}$.

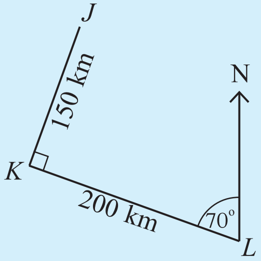
Step 7: Using the scale, calculate the actual length of PR.

Actual length of $PR = 7.7 \times 25$ km
$= 192.5$ km

Step 8: The position of R can be described as follows.
R is situated 192.5 km from P on a bearing of 082°.

Exercise 27.3

1. A rough sketch of the route of a ship travelling from harbour L to harbour K and then from harbour K to harbour J is given.



i. Find the following based on this rough sketch.

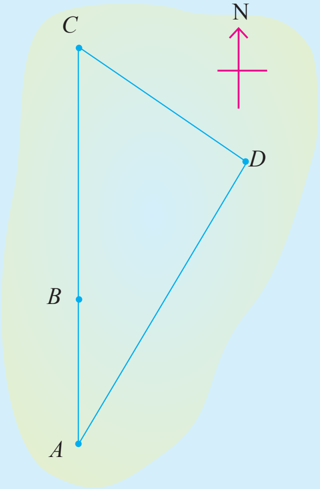
- **a.** Bearing of K from L
- **b.** Bearing of J from K
- **c.** The lengths of LK and KJ in a scale diagram drawn to the scale of 1 cm representing 50 km.

ii. Using the above scale, draw a scale diagram of the route of the ship.

iii. Using the scale diagram,
- **a.** find the distance from harbour L to harbour J
- **b.** find the bearing of harbour L from harbour J.

vi. Using the Pythagorean relation, calculate the distance from harbour L to harbour.
J and check whether the answer you obtained in (iii) (a) above is correct.

2.



A portion of a map drawn to the scale of 1 : 50 000 is shown here. While the cities A, B and C are situated on the same straight line, C lies directly north of A.

i. Measure and write the lengths of the line segments AB, BC, CD and AD and the magnitudes of the angles $A\hat{C}D$, $A\hat{D}C$ and $C\hat{A}D$.

ii. Calculate the actual distances of AB, BC, CD and AD.

iii. Describe the locations of B, C and D with respect to A in terms of bearings and distances from A.

3. The School Office is located at a distance of 10 m and on a bearing of 025° from the school flag post. The Main Hall is located at a distance of 12 m and on a bearing of 310° from the school flag post.

i. Draw a rough sketch based on the above information.

ii. Draw a scale diagram based on the sketch, using the scale of 1 cm representing 2 m.

iii. Using the scale diagram, find the shortest distance between the Office and the Main Hall.

iv. Describe the location of the Main Hall with respect to the Office.

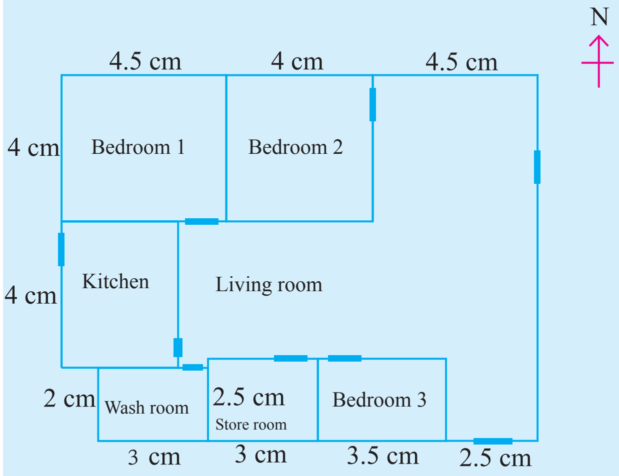
4. A pilot flies a plane 80 km on a bearing of 150° and then 150 km on a bearing of 200° and arrives at airport B from airport A.

i. Draw a rough sketch based on the above information.

ii. Draw a scale diagram using a suitable scale and find,

- **a.** the bearing of B from A
- **b.** the distance from A to B
- **c.** the bearing of A from B.

5. The floor plan of a house to be constructed, which is drawn to scale is shown below. Answer the questions given below using the scale diagram.



i. If the actual length of bedroom 2 is 4 m, express the scale to which this plan is drawn, as a ratio.

ii. Find the actual breadth of the house.

iii. Find the actual area of the washroom in square meters.

6. A person standing on a straight road that runs from East to West across a carnival ground, observes a flag post on a bearing of 115°. When he travels 220 m to the East along the road, he see the flag post on a bearing of 210°.

i. Describe the final location of the person with respect to the flag post.

ii. By drawing a scale diagram, find the shortest distance from the flag post to the road.

7. A portion of a road map of Sri Lanka drawn to the scale of 1 : 1 000 000 is shown here. The main "A" road is highlighted in red.

- **i.** Find the actual length in kilometres that is represented by 1 cm in the map.
- **ii.** With the aid of a string, find the length of the portion of the "A" road which falls between *X* and *Y* in the scale diagram and find the actual distance from *X* to *Y* along this road in kilometres.

28

Data Representation and Interpretation

By studying this lesson you will be able to;

- construct an ungrouped frequency distribution from given raw data,
- find the mode, median and mean of data in the form of an ungrouped frequency distribution,
- construct a grouped frequency distribution from given raw data,
- identify the modal class and median class from a grouped frequency distribution.

In Grade 8 you learnt how to find the mode, median and mean of given raw data. Do the following review exercise to recall what was learnt.

Review Exercise

1. The ages of the members of a school cricket team (rounded off to the nearest year) are given below.

15, 14, 15, 14, 14, 19, 17, 18, 17, 16, 18

For the above set of data, find the following.

i. The range
ii. The mode
iii. The median
iv. The mean

2. Data collected by a certain weather station on the highest temperature (in degrees Celsius) recorded during each day of the first two weeks of a certain month is given below.

26, 28, 28, 29, 27, 28, 29, 30, 31, 28, 30, 31, 32, 27

For the above set of data, find the following.

i. The range
ii. The mode
iii. The median
iv. The mean

28.1 Ungrouped frequency distribution

To extract the information we require from a given set of raw data, we need to first organize the data in a suitable way. For example, to find a representative value such as the median of a set of raw data, the data needs to be arranged in ascending or descending order.

When there are only a few values, they can easily be arranged in ascending or descending order. However when the number of data is large, arranging them in order and extracting information is not that easy. In such instances it is more appropriate to use tables.

Let us consider such an instance.

The marks obtained in a test by the students of a certain class are given below.

42, 70, 68, 68, 56, 62, 74, 74. 74, 56, 62, 85, 91, 91, 74, 74, 56, 68, 68, 68, 74

This information can be tabulated as follows.

Note: This table can be easily and accurately constructed by using tally marks.

Marks	Tally Marks	Number of Students (Frequency)
42	/	1
56	///	3
62	//	2
68	~~////~~	5
70	/	1
74	~~////~~ /	6
85	/	1
91	//	2

The frequencies are shown in the third column of this table.
Let us first consider what is meant by frequency.

In the above data set, the value 42 occurs once, the value 56 occurs three time, etc. The **frequency** of a value is the number of times that value occurs in the data set.

Accordingly, if we consider the above data set,
the frequency of 42 is 1,
the frequency of 56 is 3,
the frequency of 62 is 2, etc.

An **ungrouped frequency distribution** is a table containing the values of a data set and their respective frequencies.
The following is an ungrouped frequency distribution prepared using the above data set.

Marks	**Number of Students (Frequency)**
42	1
56	3
62	2
68	5
70	1
74	6
85	1
91	2

The mode of the data in an ungrouped frequency distribution

You have learnt in Grade 8 that the **mode** of a data set is the value that is repeated the most in that data set. The largest value in the frequency column of the above table is 6. The value corresponding to the frequency 6 is 74. Therefore the mode of the above data set is 74.

The median of the data in an ungrouped frequency distribution

You have learnt that the **median** of a data set is the value that occurs in the middle when the data is arranged in ascending or descending order.

There are 21 data in the above example. Therefore, when the data is arranged in ascending or descending order, the value that occurs in the middle is the 11th value. Now we need to find out what the 11th value is.

Let us consider how this is done.
Observe from the above table that,

the 1st value is 42,
the 2nd value is 56,
the 3rd value is also 56,
.
.
.
the 6th value is 62.

Accordingly, the 11th value can be found by considering the sums of the values in the frequency column as shown below.

Let us write the sums of the values in the frequency column by the side of the frequency table.

Marks	Frequency
42	1
56	3
62	2
68	5
70	1
74	6
85	1
91	2
	21

Sum of the frequencies

1
3 + 1 = 4
2 + 3 + 1 = 6
5 + 2 + 3 + 1 = 11

It can easily be seen by considering the sums of the frequencies in the frequency column that the value in the 11th position is 68.

When there is a large number of data, arranging it in ascending or descending order and identifying the middle value may not be very easy. The following method can be used to identify the middle position (the position of the median).

Note: When the total number of data is an odd number, the middle position is obtained from $\frac{\text{number of data} + 1}{2}$.

The number of data in the above data set = 21
When the data is arranged in ascending order,

the position where the median is located $= \dfrac{21 + 1}{2}$

$= 11$

The value in the 11th position is 68. Therefore, the median of the data set is 68; that is, the median of the marks is 68.

The mean of the data in an ungrouped frequency distribution

You have learnt in Grade 8 that to find the mean of a data set, the sum of all the data values needs to be divided by the number of data values.

Let us see how the mean of the data in an ungrouped frequency distribution is found by considering the above example.

As indicated previously, the value 42 occurs once, the value 56 occurs 3 times, etc. To find the mean, the sum of all the values has to be found.

Let us use a table of the following form to find this sum.

Marks	**Frequency** f	fx
42	1	$42 \times 1 = 42$
56	3	$56\times 3 = 168$
62	2	$62 \times 2 = 124$
68	5	$68 \times 5 = 340$
70	1	$70 \times 1 = 70$
74	6	$74 \times 6 = 444$
85	1	$85 \times 1 = 85$
91	2	$91 \times 2 = 182$
	21	1455

The sum of the data values = 1455

The mean of the data set $= \dfrac{1455}{21}$

$= 69.29$

≈ 69 (rounding off to the nearest whole number)

$\therefore$ the mean of the marks that the students obtained is 69 to the nearest whole number.

Example 1

The masses of 36 grade 3 students of a primary school are given below.
(Mass in kilogrammes)

27	25	20	23	21	26	20	23	21	22	24	25
26	24	23	23	26	24	26	20	24	22	24	25
26	22	23	26	22	24	23	25	24	21	27	27

i. Find the range of the above set of data.
ii. Construct an ungrouped frequency distribution using the above information.
iii. For the above data set, find the following using the frequency distribution.
  (a) Mode
  (b) Median
  (c) Mean

i. The largest value of the data set = 27
The smallest value of the data set = 20
$\therefore$ the range of the data set = 27 − 20
$= \underline{\underline{7}}$

ii.

Mass x (Kg)	**Frequency** f	**Sum of the frequencies**
20	3	3
21	3	6
22	4	10
23	6	16
24	7	23
25	4	27
26	6	33
27	3	36

iii. **a.** The mode of the data set = 24 kg

There are 36 values in this data set. Since 36 is an even number, when this data set is arranged in ascending or descending order, we obtain two middle values. In such a case, the median of the data set is the average of the middle two values.

Let us first find the positions of the middle two values.

Note: When the total number of data is even, the positions of the middle two values are obtained from $\frac{\text{number of data}}{2}$ and $\frac{\text{number of data}}{2} + 1$.

---

b. The positions of the middle two values $= \frac{36}{2}$ and $\frac{36}{2} + 1$

$= 18$ and 19

Therefore the middle two values are in the 18th and 19th positions.

The value in the 18th position = 24
The value in the 19th position = 24

$$\therefore \text{ the median of the data set } = \frac{24 + 24}{2}$$

$$= \frac{48}{2}$$

$$= \underline{\underline{24 \text{ kg}}}$$

c.

Mass x (Kg)	**Frequency** f	$f \times x$
20	3	60
21	3	63
22	4	88
23	6	138
24	7	168
25	4	100
26	6	156
27	3	81
Sum of the data values	36	854

Sum of the data values = 854

Number of data values = 36

$$\therefore \text{mean of the data set} = \frac{854}{36} \text{ kg}$$

$$= \underline{\underline{23.72 \text{ kg}}} \text{ (to the nearest second decimal place)}$$

Exercise 28.1

1. The data collected at a certain weather station on the highest temperature (in degrees Celsius) recorded on each day of the month of December in the year 2016 is given below.

28 26 28 28 29 30 28 26 27 27
28 26 25 24 24 25 25 26 27 28
28 27 26 28 27 28 29 30 28 27 27

i. What is the range of this data set?
ii. Construct an ungrouped frequency distribution to find the mode, median and mean of the data set.
iii. Find the mode of the data set using the above constructed frequency distribution.
iv. Find the median of the above set of temperatures.
v. Find the mean of the above set of temperatures.

2. In a certain market, bags containing lime of mass 100 g each are available for sale. The number of limes in each bag is given below.

5 3 4 6 2 3 4 5 3 4 6 5 3 4
4 2 4 3 5 3 3 4 2 5 3 2 4 3

i. What is the range of this data set?
ii. Construct an ungrouped frequency distribution using this data.
iii. Find the mode of the data set.
iv. Find the median of the data set.
v. Find the mean number of limes in a bag (to the nearest whole number).

3. Information on the number of units of electricity consumed daily during a certain period by a certain business establishment is given in the following ungrouped frequency distribution.

Number of units of electricity consumed in a day	8	9	10	11	12	13	14
Number of days	3	5	8	6	4	3	1

i. What is the range of the above data set?
ii. Find the mode of the above data set.
iii. Find the median of the above data set.
iv. Find the mean number of units of electricity consumed per day during the period in which the data was collected.

4. An ungrouped frequency distribution prepared with the information collected on the number of patients who received treatment in the Out Patient Department of a certain hospital each day during a certain period is given below.

Number of patients who received treatment during a day	29	30	31	32	33	34	35
Number of days	2	4	6	8	12	6	2

i. Find the range of this data set.
ii. Find the following for this data set.

a. Mode
b. Median
c. Mean

28.2 Grouped frequency distributions

In this section we will identify what a grouped frequency distribution is, the need for grouped frequency distributions and how they are constructed.

To do this, let us consider the following example.

The marks obtained by a group of students in a certain test is given below.

21	26	28	32	34
36	36	38	39	39
39	40	41	41	41
41	42	45	48	48
52	53	56	66	68
70	75	80	81	83

The highest mark obtained is 83 and the lowest mark obtained is 21.
Therefore the range = 83 – 21 = 62.

Since the range is large and there are many distinct data values, if we try to prepare an ungrouped frequency distribution, we will end up with a fairly long table. In such instances we consider the range of the data set and prepare a table of intervals such that each data value belongs to exactly one of the intervals. These intervals are called **class intervals**. A frequency distribution prepared using class intervals is called a **grouped frequency distribution**.

The following is an example of a grouped frequency distribution.

Class Interval	Frequency
10 - 19	3
20 - 29	6
30 - 39	5
40 - 49	2

This distribution has four class intervals.

Any data value which is equal to one of 10, 11, 12, 13, 14, 15, 16, 17, 18 and 19 belongs to the class interval 10 – 19.

Since there are 10 values in the class interval 10 – 19, the **class size (or class width)** is considered to be 10. The class sizes of the other class intervals are defined similarly.

The frequency corresponding to the class interval 10 – 19 is 3. This means that the data set has only 3 values belonging to this class interval.

Now let us consider how a grouped frequency distribution is prepared.

When preparing a grouped frequency distribution, we need to first decide on either the size of the class intervals or the number of class intervals we want to have.

When we have decided on the size of the class intervals, the number of class intervals can be obtained as follows.

- Find the range of the data set.
- Divide the range by the size of a class interval.
- The number of class intervals is the nearest whole number greater or equal to the above obtained value.

Consider the following example which was discussed earlier.

The marks obtained by a group of 30 students in a certain test are given below.

21	26	28	32	34	36	36	38	39	39
39	40	41	41	41	41	42	45	48	48
52	53	56	66	68	70	75	80	81	83

Suppose we want to separate this data set into class intervals of size 10.

Let us first find the number of class intervals.

The largest value of this data set $= 83$
The smallest value of this data set $= 21$
The range $= 83 - 21$
$= 62$

Since we want the size of the class intervals to be 10,

$$\text{the number of class intervals } = \frac{62}{10}$$

$= 6.2$
≈ 7 (when rounded off to the nearest whole number greater than the obtained value)

Accordingly, if we take the class size to be 10, we obtain a grouped frequency distribution with 7 class intervals.

Since the smallest value in the data set is 21, let us prepare the frequency distribution starting with the value 20. The first class interval will then consist of the ten integers 20, 21, 22, 23, 24, 25, 26, 27, 28 and 29. The next class interval will consist of the next 10 integers and so on.

Accordingly, we obtain the following class intervals.

20 - 29

30 - 39

40 - 49

50 - 59

60 - 69

70 - 79

80 - 89

Note: Although we commenced the first class interval from 20, we could have started with the value 21 too (or some other suitable value). If we started with 21, the class intervals would have been 21 – 30, 31 – 40, 41 – 50, etc.

Now, let us find the number of values that fall into each class interval by using tally marks.

Class Interval (Marks)	Tally Marks	Frequency
20 - 29	///	3
30 - 39	𝍸 ///	8
40 - 49	𝍸 ////	9
50 - 59	///	3
60 - 69	//	2
70 - 79	//	2
80 - 89	///	3

Note: It is not necessary to include the tally marks column in a frequency distribution.

When we have decided on the number of class intervals, we can find the size of the class intervals (class size) as follows.

- Find the range of the data set by subtracting the smallest value of the data set from the largest value.
- Divide the range by the number of class intervals. (In general, the number of class intervals is taken to be less than 10.)
- Round off the value that is obtained to the nearest whole number greater or equal to it and take this value to be the size of the class intervals.

Let us consider how to construct a grouped frequency distribution with 5 class intervals using the above data set. Let us first find the size of the class intervals.

$$\begin{aligned}\text{The largest value of this data set } &= 83\\ \text{The smallest value of this data set } &= 21\\ \text{The range } &= 83 - 21\\ &= 62\end{aligned}$$

Since we require 5 class intervals,

$$\text{the size of each class interval} = \frac{62}{5}$$

$$= 12.4$$

≈ 13 (nearest whole number greater than the obtained value)

Accordingly, we prepare a grouped frequency distribution with 5 class intervals of size 13.

Class Interval	**Frequency**
20 - 32	4
33 - 45	14
46 - 58	5
59 - 71	3
72 - 84	4

As shown above, we can construct grouped frequency distributions according to our requirements, based on the given data set.

Consider the first grouped frequency distribution we constructed. We took 20 - 29 as the first class interval, 30 – 39 as the second class interval, etc. We were able to do this because there were no values between 29 and 30 or between 39 and 40, etc. Observe that this feature is seen in the second grouped frequency distribution we constructed too.

However, if we have a data set consisting of values which are lengths or times or masses, it is necessary to start the second class interval with the value that the first class interval ends, to start the third class interval with the value that the second class interval ends, and so on.

Let us now consider such an example.

The masses of 20 students in a class are given below to the nearest kilogramme.

31	31	31	32	32
32	32	33	33	34
34	34	35	36	36
38	39	39	40	41

Let us construct a grouped frequency distribution with 4 class intervals of size 3 each.

Let us take the first class interval as 30 – 33, the next class interval as 33 – 36, etc.

30 - 33

33 - 36

36 - 39

39 - 42

Here, the second class interval commences with the same value that the first class interval ends. The reason is because the data set consists of masses and masses need not be integral values. For example, we may have students whose masses are 33.2 kg, 33.5 kg, 33.8 kg, etc., which are between 33 kg and 34 kg, or 36.5 kg, 36.9 kg, etc., which are between 36 kg and 37 kg etc. Therefore, in such situations, each class interval needs to commence with the same value that the previous class interval ends (except for the first class interval).

Here, the first class interval ends with 33 and the second class interval commences with the same value 33. A question arises as to which class interval the value 33 belongs. The value 33 can be taken to belong to either one of these two intervals. However, it is important to state the convention that is being used.
In this lesson we will consider the class intervals to be as follows.

Here,

the values greater than 30 but less than or equal to 33 belong to the class interval 30 -33,
the values greater than 33 but less than or equal to 36 belong to the class interval 33 – 36,
the values greater than 36 but less than or equal to 39 belong to the class interval 36 – 39, and
the values greater than 39 but less than or equal to 42 belong to the class interval 39 – 42.

The grouped frequency distribution prepared according to this convention is given below.

Class Interval	Frequency
30 - 33	9
33 - 36	6
36 - 39	3
39 - 42	2

Note: When constructing a grouped frequency distribution, it should be remembered that the class intervals need to be selected by taking the nature of the data into consideration.

Exercise 28.2

1. The data collected by an electricity metre reader on the electricity consumption of each of the households in a certain housing scheme during the month of January 2017 is given below.

63	68	75	54	56	58	85
90	73	63	76	62	69	78
50	74	64	58	88	85	72
71	53	82	68	73	67	75
74	67	69	62	66	74	70
84	72	69	59	67	78	72

Construct a grouped frequency distribution using the above data.

2. The marks obtained in a mathematics test by a group of Grade 9 students of a certain school are given below.

34	27	45	12	63	35	54	29
42	68	73	54	26	11	63	54
33	69	62	38	53	48	63	61
60	44	67	61	79	65	47	

i. Find,
(a) the highest mark obtained by a student
(b) the lowest mark obtained by a student.

ii. Find the range of the data set.
iii. For the above data set, construct a grouped frequency distribution with 7 class intervals.

3. The heights (in centimetres) of the Grade 4 students of a certain primary school are given below. Construct a suitable grouped frequency distribution.

124 124 138 125 122 129 122 128 131 127 125 120 125
120 121 125 120 132 127 124 126 130 125 131 122 130
129 128 125 122 133 138 125 123 126 125 135 126 132

28.3 Finding the modal class and median class from a grouped frequency distribution

We learnt how to construct a grouped frequency distribution in the previous section. Now let us consider how the modal class and the median class can be found from a grouped frequency distribution.

When we are given a grouped frequency distribution, we will not be able to identify the mode and the median as the raw data is not available to us. In such situations we consider the modal class and the median class.

The **modal class** is the class interval with the highest frequency. The **median class** is the class interval to which the median belongs.

Example 1

A grouped frequency distribution prepared with the marks obtained by a group of students in a certain test is given below.

From this distribution, find

i. the modal class
ii. the median class.

Marks	Frequency	Sum of the frequencies
10 - 20	3	3
21 - 30	4	7
31 - 40	6	13
41 - 50	7	20
51 - 60	11	31
61 - 70	4	35

i. Since the highest frequency is 11, the modal class is 51 – 60.

ii. The position of the median of the data set $= \dfrac{35 + 1}{2}$

$$= 18$$

The median class is the class interval to which the 18th value belongs. Therefore, the median class is 41 - 50.

Example 2

A grouped frequency distribution prepared using the ages of the employees of a certain establishment is given below.

Find,

i. the modal class

ii. the median class.

Age	Frequency	Sum of the frequencies
20 - 27	3	3
27 - 34	5	8
34 - 41	11	19
41 - 58	6	25
48 - 55	3	28

i. The highest frequency $= 11$

$\therefore$ the modal class $= 34 - 41$

ii. The positions of the middle two values $= \dfrac{28}{2}$ and $\dfrac{28}{2} + 1$

$$= 14 \text{ and } 15$$

The class interval that contains the 14th value $= 34 - 41$
The class interval that contains the 15th value $= 34 - 41$
$\therefore$ the median class 34 - 41.

Exercise 28.3

1. The number of sweep tickets sold each day during the month of March of year 2016 by a certain sweep ticket seller is given below.

380 390 379 402 370 385 397 386 377 405
400 381 390 375 392 384 391 385 387 395
390 393 373 386 378 395 379 396 395 391
373

i. What is the maximum number of sweep tickets that were sold on a day during this period?
ii. What is the minimum number of sweep tickets that were sold on a day during this period?
iii. Find the range of this data set.
iv. Construct a grouped frequency distribution of class size 6.
v. Using the table,

a. find the modal class
b. find the median class.

2. The number of books loaned by a school library during 30 days of the first term of the year 2016 is given below.

27 20 33 37 40 25 15 29 33 32
29 32 25 36 16 35 37 28 34 27
41 36 40 28 27 23 32 33 24 38

i. What is the range of this data set?
ii. Using this data set, construct a grouped frequency distribution consisting of the class intervals 15 – 19, 20 – 24, etc., of class size 5.
iii. Using the table, find the number of days in which 30 or more books have been loaned.
iv. How many days are there in which more than 25 but less than 30 books were loaned?
v. What is the modal class?
vi. To which interval does the median of the number of books loaned each day during this period belong?

Miscellaneous Exercise

1. The ungrouped frequency distribution given below has been prepared with the information collected on the number of coconuts that were plucked from each coconut tree in an estate, during a certain season.

Number of coconuts	Frequency
8	3
10	5
12	8
13	7
14	5
15	2

i. Find the mode of the data set.
ii. Find the median of the data set.
iii. Find the mean number of coconuts plucked from a tree in this estate.

2. The circumferences (in centimetres) of a pile of rubber tree trunks that were purchased to cut planks are given below.

95 112 118 86 103 102 94 98 80 97
87 105 85 103 95 106 98 94 110 102
103 105 90 110 96 100 89 104 98 114
106 98 98 112 86 105 97 107 96 92
115

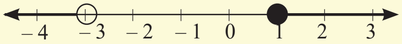
i. Prepare a grouped frequency distribution consisting of 8 class intervals.
ii. Find the modal class from this distribution.
iii. Find the median class.

Revision Exercise – Third term.

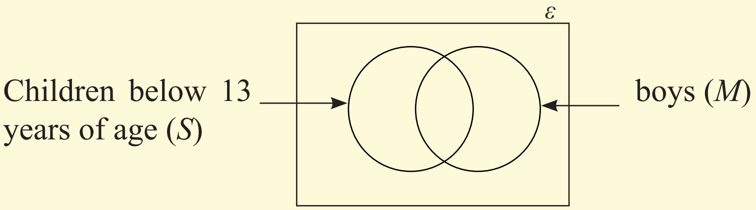
Part – I

1. Represent all the solutions of $x - 3 < -1$ on a number line.

2. What is the inequality represented on the number line.

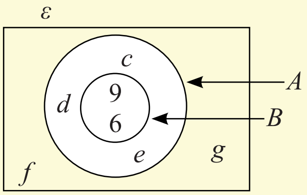


3.



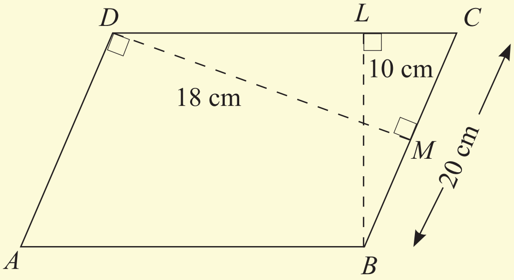
In the Venn diagram drawn to illustrate the information on the grade 9 students of a certain school, shade the region which represents the girls who are below 13 years of age and express it in terms of S and M.

4.



Write the elements in B' based on the information in the Venn diagram.

5.

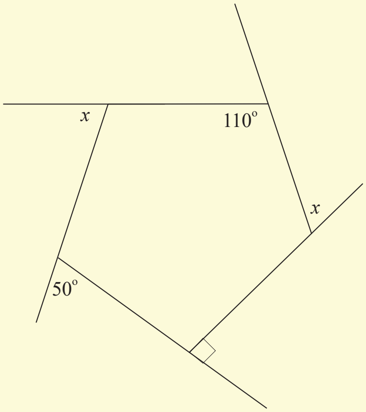


In the parellelogram $ABCD$, $BC = 20$ cm, $BL = 10$ cm and $DM = 18$ cm. Calculate the perimeter of $ABCD$.

6. A number is picked randomly from a set of 20 identical cards numbered 1 to 20. What is a probability of drawing a triangular number?

7. Let A denote the set of the letters of the word "*numbers*". If a letter is drawn randomly from this set, what is the probability of it being "*m*"?

8. Find the value of x based on the information in the figure.

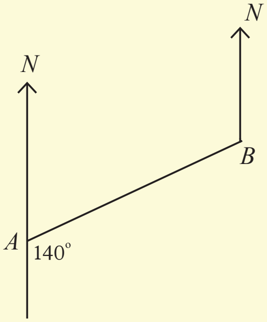


9. In a certain regular polygon, an interior angle is 150° more than an exterior angle. Find the number of sides it has.

10. Simplify $\frac{x+1}{2} - \frac{3x-2}{6}$.

11. Simplify $\frac{a+1}{a-3} - \frac{4-2a}{a-3}$.

12.



According to the information in the given figure, find

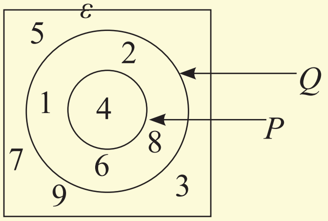
(i) the bearing of B from A
(ii) the bearing of A from B.

13. A scale diagram is drawn to the scale of 1: 50 000. If the direct distance between the two cities A and B is 8 km, what is the length of the line segment in the scale diagram that represents this distance?

14. If the mean of the collection of data 12, 8, x, 5, 10 is 10, find the median.

Revision Exercise – Third Term
Part - II

1. (*A*) Fill in the blanks in the following, by using either the symbol $\subset$ or $\in$ based on the information in the given Venn diagram.



i. 4 ——— Q **ii.** 7 ——— Q
iii. P ——— ε **iv.** P ——— Q
v. $P \cap Q$ ——— P

(*B*) **i.** Write $n(P')$.
ii. How may subsets does Q' have? Write four of these subsets.

(*C*) ε = {counting numbers from 1 to 20}
A = {multiples of 3 from 1 to 20}
B = {multiples of 2 from 1 to 20}

i. List out the elements in the above three sets.
ii. Represent the above sets in a suitable manner in a Venn diagram.
iii. Write the elements of the sets given below using the set (ii) above.

a. A **b.** B **c.** $A \cap B$ **d.** $A \cup B$ **e.** A' **f.** B'

2. The number of milk packets that were sold in a canteen of a certain school during 50 days is given below.

31	34	38	40	44	43	45	47	45	50
53	52	58	55	54	53	61	63	65	66
66	68	64	63	66	67	62	63	66	70
71	73	74	75	76	72	73	72	74	81
82	82	82	83	83	84	8	85	92	96

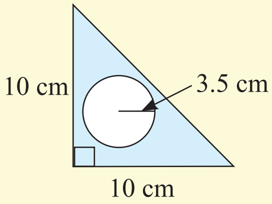
i. Write the range of the data.

ii. By taking the class intervals as 30-40, 40-50, 50-60 etc., construct a grouped frequency distribution using all the above data.

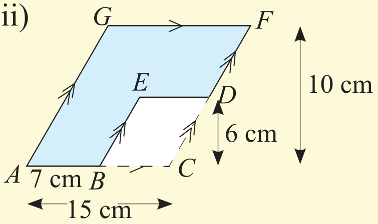
iii. Find the modal class and the median class using the above frequency distribution.

3. (*a*) calculate the areas of the figures given below.

(i)



(ii)



4. (1) An interior angle of a certain regular polygon is 100° more than an exterior angle.

i. Calculate the magnitude of an exterior angle

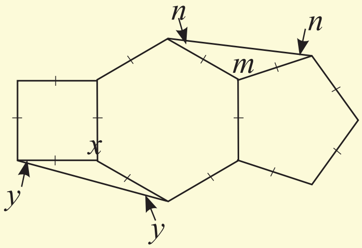
ii. Calculate the number of sides the polygon has.

(2) The ratio of an interior angleto an exterior angle of a regular polygon is 3:1. Find the sum of the interior angles

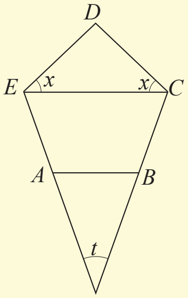
(3) The sum of the exterior angles of a regular polygon is five times the sum of the interior angles. Find the number of sides the polygon has.

(4) Four interior angles of a certain polygon are 160°, 140°, 130° and 110° respectively. The exterior angles corresponding to the remaining interior angles are 30° each. Calculate the number of sides the polygon has.

(5) A certain creation is made up of square, a regular hexagon and a regular pentagon connected together as shown below. Calculate the values of the angles x, y, m, and n.(Thanuja, x is not placed properly.)



(6) *ABCDE* is a regular pentagon.



i. Find the value of a vertex angle of the regular pentagon.

ii. Find the value of x.

iii. Show that the straight lines EC is parallel to AB.

iv. Find the value of t.

5. ***A.*** **i.** Solve the inequality $x - 1 \leq -3$; and represent the set of solutions on a number line.

ii. solve the inequality $\frac{2x}{3} > -2$; and represent all the silutions on a number line.

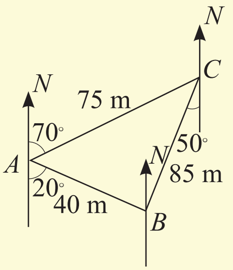
B. A container of capacity 10 l has 3 l of water in it. If another x litres of water is poured into it, the volume of water in the container satisfiesthe inequality $3 + x \leq 10$. Solve this inequality and find the maximum volume of water that can be poured into the container.

C. Simplify.

i. $\frac{m+1}{3} - \frac{1+2m}{2} + \frac{3m+2}{4}$

ii. $\frac{a+5}{a+3} - \frac{2-a}{3+a} + \frac{a}{a+3}$

6. ***A.*** The location of three points A, B and C on a horizontal ground is shown in the sketch given below.



i. Find the bearing of B from A.
ii. Find the bearing of A from B.
iii. Find the bearing of A from C.

B. A water tank in a school located to the left of a straight road running from North to South is observed on a bearing of 230° from the point A which is located on the straight road. The same tank is observed from the point B which is 140 m to the South of point A, on a bearing of 300°

i. Draw a sketch depicting the above information.
ii. Draw a scale diagram by using a suitable scale and find the distance from the water tankto the points A and B.
iii. Find the minimum distance from the road to the water tank.

7. The information on the number of customers that came to a certain bank on several days is given below.

The number of customers	65	66	67	68	69	70	71	72
Number of days	2	5	8	10	12	8	6	4

i. Find the range of the above data.
ii. Find the mode and the median of the data.
iii. Prepare a suitable table to find the mean of the above distribution.

Glossary

A

Algebraic fractions	වීජීය භාග	அட்சரகணிதப் பின்னம்

B

Bearing	දිගංශය	திசைகோள்

C

Circle	වෘත්තය	வட்டம்
Class Intervals	පන්ති ප්‍රාන්තර	வகுப்பாயிடைகள்
Clockwise	දක්ෂිණාවර්ත	வலஞ்சுழி
Common Denominator	පොදු හරය	பொதுப் பகுதி
Compass	මාලිමාව	திசையறிகருவி
Compliment of sets	කුලක අනුපූරකය	நிரப்பித் தொடை

D

Data	දත්ත	தரவு
Denominator	හරය	பகுதி
Disjoint sets	වියුක්ත කුලක	மூட்டற்ற தொடைகள்
Distance	දුර	தூரம்

E

Equal	සමාන වේ	சமன்
Equal sets	සමකුලක	சம தொடைகள்
Equivalent fractions	තුල්‍ය භාග	சமவலுப் பின்னம்
Equivalent sets	තුල්‍ය කුලක	சமவலுத் தொடைகள்
Equally likely outcomes	සමසේ භව්‍ය ප්‍රතිඵල	சமமாய் நிகழத்தக்க
Event	සිද්ධිය	நிகழ்ச்சி
Exterior angle	බාහිර කෝණය	புறக்கோணம்

F

Finite sets	පරිමිත කුලක	முடிவுள்ள தொடைகள்
Frequency distribution	සංඛ්‍යාත ව්‍යාප්තිය	மீடிறன் பரம்பல;

G

Greater than	විශාල වේ	பெரிது
Grouping	සමූහනය	கூட்டமாக்கல்

H

Hexagon	ෂඩස්‍රය	அறுகோணி
Horizontal Plane	තිරස්තලය	கிடைத்தளம்

I

Inequality	අසමානතා	சமனிலி
Infinite sets	අපරිමිත කුලක	முடிவில் தொடைகள்
Interior angle	අභ්‍යන්තර කෝණය	அகக்கோணம்
Intersection of sets	කුලක ඡේදනය	தொடை இடைவெட்டு

L

Less than	කුඩා වේ	சிறிது
Least commom multiple	කුඩා ම පොදු ගුණාකාරය	பொது மடங்குகளுள் சிறிது
Location	පිහිටීම	அமைவு

N

Numerator	ලවය	தொகுதி

O

Out come	ප්‍රතිඵලය	பேறு

P

Parallelogram	සමාන්තරාස්‍රය	இணைகரம்
Pentagon	පංචාස්‍රය	ஐங்கோணி

Q

Quadrilateral	චතුරස්‍රය	நாற்பக்கல்

R

Random experiments	අහඹු පරීක්ෂණ	எழுமாற்றுப் பரிசோதனை
Rectilinear closed plane	සරල රේඛීය සංවෘත	நேர்கோட்டுத்
Regular polygons	සවිධි බහු අස්‍ර	ஒழுங்கான பல்கோணி

S

Sample space	නියැදි අවකාශය	மாதிரி வெளி
Sub sets	උපකුලක	உபதொடைகள்

T

Trapezium	ත්‍රපීසියම	சரிவகம்
Triangle	ත්‍රිකෝණය	முக்கோணி

U

Union of sets	කුලක මේලය	தொடை ஒன்றிப்பு

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