Grade

13 Combined Mathematics

Teachers' Guide

(To be Implemented from 2018)

Department of Mathematics

Faculty of Science & Technology

National Institute of Education

Maharagama

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Teachers' Guide

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Department of Mathematics
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Sri Lanka

Combined Mathematics
Grade 13 - Teachers' Guide

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Message from the Director General

With the primary objective of realizing the National Educational Goals recommended by the National Education Commission, the then prevalent content based curriculum was modernized, and the first phase of the new competency based curriculum was introduced to the eight year curriculum cycle of the primary and secondary education in Sri Lanka in the year 2007

The second phase of the curriculum cycle thus initiated was introduced to the education system in the year 2015 as a result of a curriculum rationalization process based on research findings and various proposals made by stake holders.

Within this rationalization process the concepts of vertical and horizontal integration have been employed in order to build up competencies of students, from foundation level to higher levels, and to avoid repetition of subject content in various subjects respectively and furthermore, to develop a curriculum that is implementable and student friendly.

The new Teachers' Guides have been introduced with the aim of providing the teachers with necessary guidance for planning lessons, engaging students effectively in the learning teaching process, and to make Teachers' Guides will help teachers to be more effective within the classroom. Further, the present Teachers' Guides have given the necessary freedom for the teachers to select quality inputs and activities in order to improve student competencies. Since the Teachers' Guides do not place greater emphasis on the subject content prescribed for the relevant grades, it is very much necessary to use these guides along with the text books compiled by the Educational Publications Department if, Guides are to be made more effective.

The primary objective of this rationalized new curriculum, the new Teachers' Guides, and the new prescribed texts is to transform the student population into a human resource replete with the skills and competencies required for the world of work, through embarking upon a pattern of education which is more student centered and activity based.

I wish to make use of this opportunity to thank and express my appreciation to the members of the Council and the Academic Affairs Board of the NIE the resource persons who contributed to the compiling of these Teachers' Guides and other parties for their dedication in this matter.

Dr. (Mrs.) T.A.R.J. Gunasekara
Director General
National Institute of Education

Message from the Director

Education from the past has been constantly changing and forging forward. In recent years, these changes have become quite rapid. The Past two decades have witnessed a high surge in teaching methodologies as well as in the use of technological tools and in the field of knowledge creation.

Accordingly, the National Institute of Education is in the process of taking appropriate and timely steps with regard to the education reforms of 2015.

It is with immense pleasure that this Teachers' Guide where the new curriculum has been planned based on a thorough study of the changes that have taken place in the global context adopted in terms of local needs based on a student-centered learning-teaching approach, is presented to you teachers who serve as the pilots of the schools system.

An instructional manual of this nature is provided to you with the confidence that, you will be able to make a greater contribution using this.

There is no doubt whatsoever that this Teachers' Guide will provide substantial support in the classroom teaching-learning process at the same time. Furthermore the teacher will have a better control of the classroom with a constructive approach in selecting modern resource materials and following the guide lines given in this book.

I trust that through the careful study of this Teachers Guide provided to you, you will act with commitment in the generation of a greatly creative set of students capable of helping Sri Lanka move socially as well as economically forward.

This Teachers' Guide is the outcome of the expertise and unflagging commitment of a team of subject teachers and academics in the field Education.

While expressing my sincere appreciation for this task performed for the development of the education system, my heartfelt thanks go to all of you who contributed your knowledge and skills in making this document such a landmark in the field.

Mr. K. R. Pathmasiri
Director
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Guidlines to use the Teachers' Guide

In the G.C.E (A/L) classes new education reforms introduced from the year 2017 in accordance with the new education reforms implemented in the interim classes in the year 2015. According to the reforms, Teachers' Guide for combine mathematics for grade 12 has been prepared.

The grade 12 Teacher's Guide has been organized under the titles competencies and competency levels, content, learning outcomes and number of periods. The proposed lesson sequence is given for the leaning teaching process. Further it is expected that this teachers' Guide will help to the teachers to prepare their lessons and lessons plans for the purpose of class room learning teaching process. Also it is expected that this Guide will help the teachers to take the responsibility to explains the subject matters more confidently. This teachers' Guide is divided into three parts each for a term.

In preparing lesson sequence, attention given to the sequential order of concepts, students ability of leaning and teachers ability of teaching. Therefore sequential order of subject matters in the syllabus and in the teachers Guide may differ. It is adviced to the teachers to follow the sequence as in the teachers' Guide.

To attain the learning outcomes mentioned in the teachers' Guide, teachers should consider the subject matters with extra attention. Further it is expected to refer extra curricular materials and reference materials to improve their quality of teaching. Teachers should be able to understand the students, those who are entering grade 12 classes to learn combined mathematics as a subject. Since G.C.E (O/L) is designed for the general education, students joining in the grade 12 mathematics stream will face some difficulties to learn mathematics. To over come this short coming an additional topics on basic Algebra and Geometry are added as pre request to learn. For this purpose teachers can use their self prepared materials or "A beginners course in mathematics" book prepared by NIE.

Total number of periods to teach this combined mathematics syllabus is 600. Teachers can be flexible to change the number of periods according to their necessity. Teachers can use school based assessment to assess the students.

The teacher has the freedom to make necessary amendments to the specimen lesson plan given in the new teacher's manual which includes many new features, depending on the classroom and the abilities of the students.

We would be grateful if you would send any amendments you make or any new lessons you prepare to the Director, Department of Mathematics, National Institute of Education. The mathematics department is prepared to incorporate any new suggestions that would advance mathematics education in the upper secondary school system.

S. Rajendram
Project Leader - Grade 12-13 Mathematics
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Common National Goals

The national system of education should assist individuals and groups to achieve major national goals that are relevant to the individual and society.

Over the years major education reports and documents in Sri Lanka have set goals that sought to meet individual and national needs. In the light of the weaknesses manifest in contemporary educational structures and processes, the National Education Commission has identified the following set of goals to be achieved through education within the conceptual framework of sustainable human development.

I Nation building and the establishment of a Sri Lankan identity through the promotion of national cohesion, national integrity, national unity, harmony and peace, and recognizing cultural diversity in Sri Lanka's plural society within a concept of respect for human dignity.

II Recognizing and conserving the best elements of the nation's heritage while responding to the challenges of a changing world.

III Creating and supporting an environment imbued with the norms of social justice and a democratic way of life that promotes respect for human rights, awareness of duties and obligations, and a deep and abiding concern for one another.

IV Promoting the mental and physical well-being of individuals and a sustainable life style based on respect for human values.

V Developing creativity, initiative, critical thinking, responsibility, accountability and other positive elements of a well-integrated and balance personality.

VI Human resource development by educating for productive work that enhances the quality of life of the individual and the nation and contributes to the economic development of Sri Lanka.

VII Preparing individuals to adapt to and manage change, and to develop capacity to cope with complex and unforeseen situations in a rapidly changing world.

VIII Fostering attitudes and skills that will contribute to securing an honourable place in the international community, based on justice, equality and mutual respect.

National Education Commision Report (2003) - December

Basic Competencies

The following Basic Competencies developed through education will contribute to achieving the above National Goals.

(i) Competencies in Communication

Competencies in Communication are based on four subjects: Literacy, Numeracy, Graphics and IT proficiency.

Literacy : Listen attentively, speck clearly, read for meaning, write accurately and lucidly and communicate ideas effectively.

Numeracy : Use numbers for things, space and time, count, calculate and measure systematically.

Graphics : Make sense of line and form, express and record details, instructions and ideas with line form and color.

IT proficiency : Computeracy and the use of information and communication technologies (ICT) in learning, in the work environment and in personal life.

(ii) Competencies relating to Personality Development

- General skills such as creativity, divergent thinking, initiative, decision making, problem solving, critical and analytical thinking, team work, inter-personal relations, discovering and exploring;
- Values such as integrity, tolerance and respect for human dignity;
- Emotional intelligence.

(iii) Competencies relating to the Environment

These competencies relate to the environment : social, biological and physical.

Social Environment :

Awareness of the national heritage, sensitivity and skills linked to being members of a plural society, concern for distributive justice, social relationships, personal conduct, general and legal conventions, rights, responsibilities, duties and obligations.

Biological Environment :

Awareness, sensitivity and skills linked to the living world, people and the ecosystem, the trees, forests, seas, water, air and life-plant, animal and human life.

Physical Environment :

Awareness, sensitivity and skills linked to space, energy, fuels, matter, materials and their links with human living, food, clothing, shelter, health, comfort, respiration, sleep, relaxation, rest, wastes and excretion.

Included here are skills in using tools and technologies for learning, working and living.

(iv) Competencies relating to Preparation for the World of Work.

Employment related skills to maximize their potential and to enhance their capacity

to contribute to economic development,

to discover their vocational interests ad aptitudes,

to choose a job that suits their abilities, and

to engage in a rewarding and sustainable livelihood.

(v) Competencies relating to Religion and Ethics

Assimilating and internalizing values, so that individuals may function in a manner consistent with the ethical, moral and religious modes of conduct in everyday living, selecting that which is most appropriate.

(vi) Competencies in Play and the Use of Leisure

Pleasure, joy, emotions and such human experiences as expressed through aesthetics, literature, play, sports and athletics, leisure pursuits and other creative modes of living.

(vii) Competencies relating to ' learning to learn'

Empowering individuals to learn independently and to be sensitive and successful in responding to and managing change through a transformative process, in a rapidly changing, complex and interdependent world.

Content

FIRST TERM

Combined Mathematics - I

Competency 18 : Interprets the straight line in terms of Cartesian co -ordinates.

Competency level 18.1 : Derives the equation of a straight line.

Number of periods : 04

Learning out comes :

1. Interprets the gradient (slope) of a line, and the x and y Intercepts.
2. Derives various forms of equation of a straight line.

Guidelines to learning - teaching process :

1. • Define the gradient m of a line joining two points (x_1, y_1) and (x_2, y_2) as $\dfrac{y_2 - y_1}{x_2 - x_1}$ provided that $x_1 \neq x_2$.
   • Explain that if θ is the angle between a straight line and the positive direction of the x - axis, then $m = \tan\theta$ provided that $\theta \neq \dfrac{\pi}{2}$.

2. • Equation of the straight line with gradient m and y - intercept c is $y = mx + c$.
   • Equation of the straight line passing through the point (x_1, y_1) with gradient m is $y - y_1 = m(x - x_1)$.
   • Equation of the straight line passing through two points (x_1, y_1) and (x_2, y_2) is $y - y_1 = \left(\dfrac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$.
   • Equation of the straight line with x - intercept a and y - intercept b is $\dfrac{x}{a} + \dfrac{y}{b} = 1$ o r $bx + ay = ab$.
   • The perpendicular form of a straight line is $x\cos\alpha + y\sin\alpha = p$, where p is the length of the perpendicular from the origin and α is the angle which this perpendicular makes with the positive direction of the x - axis.

- General form of a straight line $ax + by + c = 0$.
- In each of the above cases guide students to obtain the equations.
- Guide students to derives equations of various forms using the general form $ax + by + c = 0$.

Competency level 18.2 : Derives the equation of a straight line passing through the point of intersection of two given non - parallel straight lines.

Number of periods : 02

Learning outcomes :

1. Finds the coordinates of the point of intersection of two non-parallel straight lines.
2. Finds the equation of the line passing through the intersection of two given lines.

Guidelines to learning - teaching process :

1. • Guide to Solve the linear simultaneous equation to find the coordinates of point of intersection of the corresponding straight lines.

2. • Show that the equation of a straight line passing through the point of intersection of two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is $l(a_1x + b_1y + c_1) + m\ (a_2x + b_2y + c_2) = 0$, where l, m are parameters.

   • Explain that any such line other than $a_2x + b_2y + c_2 = 0$ can be represented by $a_1x + b_1y + c_1 + \lambda(a_2x + b_2y + c_2) = 0$, where λ is a parameter.

Competency level 18.3 : Describe the relative position of two points with respect to a given straight line.

Number of periods : 02

Learning outcome :

1. Finds the condition for two points to be on the same side or on opposite sides of a given line.

Guidelines to learning - teaching process :

1. Given a straight line is $ax + by + c = 0$ and two points are (x_1, y_1) and (x_2, y_2). Then show that the points lie on the same sides or opposite sides of the given line according as $(ax_1 + by_1 + c)\ (ax_2 + by_2 + c) \gtrless 0$.

Competency level 18.4 : Finds the angle between two straight lines

Number if periods : 02

Learning out comes :
1. Finds the angle between two given straight lines by using their gradients.
2. Finds condition for two lines to be parallels or perpendicular.

Guidelines to learning - teaching process :

1. State that there are two angles between two intersecting lines. Generally one is acute and the other is obtuse.

2. • Derive that the acute angle between the two straight lines $y = m_1x + c_1$ and $y = m_2x + c_2$ is,

   $\tan^{-1}\left|\dfrac{m_1 - m_2}{1 + m_1m_2}\right|$, provided that $m_1m_2 \neq -1$.

   • Two straight lines with slopes m_1 and m_2 are,
     - Parallel to each other if and only if $m_1 = m_2$.
     - Perpendicular to each other if and only if $m_1m_2 = -1$.

   • Discus the following cases.
     - $m_1 = 0$ or $m_2 = 0$
     - m_1 or m_2 is not defined.

Competency level 18.5 : Derives the perpendicular distance from a given point to a given straight line.

Number of periods : 06

Learning outcomes :
1. Derives parametric equation of a straight line.
2. Finds perpendicular distance from a point to a given line using parametric equations of the line.
3. Finds the equations of angle bisectors of two non - parallel straight lines.

Guidelines to learning - teaching process :

1. • Show that the parametric equation of a straight line through the point $P \equiv (x_1, y_1)$ is $x = x_1 + r\cos\theta, y = y_1 + r\sin\theta$, where θ is the angle that the line makes with the positive direction of the x - axis and $Q \equiv (x, y)$, and distance between P and Q is r.

   • For the straight line $ax + by + c = 0$, the parametric equations are $\frac{y - y_1}{a} = -\frac{(x - x_1)}{b} = t$, where t is a parameter and $P \equiv (x_1, y_1)$ is a point on this line. (i.e. $x = x_1 - bt,\ y = y_1 + at$).

2. • Show that the perpendicular distance from a point $P \equiv (h, k)$ to the line $ax + by + c = 0$ is $\frac{|ah + bk + c|}{\sqrt{a^2 + b^2}}$.

   • Deduce that the distance between two parallel lines $ax + by + c = 0$ and $ax + by + d = 0$ is $\frac{|c - d|}{\sqrt{a^2 + b^2}}$.

3. • Show that the equations of the bisectors of the angles between two intersecting straight lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are $\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm\frac{(a_2x + b_2y + c_2)}{\sqrt{a_2^2 + b_2^2}}$.

. • Guide students to use above formulae to solve different types of problems.

Competency 16 : Finds indefinite and definite integrals of functions.

Competency level 16.1 : Deduces indefinite Integrals using anti - derivatives.

Number of periods : 03

Learning outcomes : 1. Finds indefinite Integrals using the results of derivatives.

Guidelines to learning - teaching process :

1. • If $\frac{d}{dx}[F(x)] = f(x)$, then $F(x)$ is an anti - derivatives of $f(x)$

   • Any two anti - derivatives of a function can differ only by a constant.

   • If $\frac{d}{dx}[F(x)] = f(x)$, then we write $\int f(x)dx = F(x) + C$, where C is an arbitrary constant.

   - $\int x^n dx = \frac{x^{n+1}}{n+1} + C,$ where $n \neq -1$
   - $\int \frac{1}{x} dx = \ln|x| + C$, $for \;\; x \neq 0$
   - $\int e^x dx = e^x + C$
   - $\int \sin x \; dx = -\cos x + C$
   - $\int \cos x \; dx = \sin x + C$
   - $\int \sec^2 x \; dx = \tan x + C$
   - $\int \text{cosec } x \; dx = -\cot x + C$
   - $\int \sec x \; \tan x \; dx = \sec x + C$
   - $\int \cot x \; \text{cosec } x \; dx = -\cos ecx + C$
   - $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + C,$ where $(-a < x < a$ and $a \neq 0)$
   - $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C,$ where $(a \neq 0)$

Competency level 16.2 : Uses theorems on integration.

Number of periods : 02

Learning outcomes : 1. Uses theorems on integration to solve problems.

Guide lines to learning - teaching process :

1. If f and g are functions of x, and k is a constant, then
   - $\int k f(x)dx = k\int f(x)dx$
   - $\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$
   - Applications of above theorems.

Competency level 16.3 : Reviews the basic properties of the definite integral using the Fundamental Theorem of Calculus.

Number of periods : 02

Learning outcomes :
1. Uses the Fundamental Theorem of Calculus to solve problems.
2. Uses the properties of definite integral.
3. Solves problems involving definite integral.

Guidelines to learning - teaching process :

1. If f and g are functions of x, and If $\phi(x)$ is an anti - derivative of $f(x)$

   then $\int_a^b f(x)dx = [\phi(x)]_a^b = \phi(b) - \phi(a)$.

2. - $\int_a^b f(x)dx = -\int_b^a f(x)dx$
   - $\int_a^b k\ f(x)dx = k\int_a^b f(x)dx$
   - $\int_a^b \{f(x) \pm g(x)dx\} = \int_a^b f(x)dx \pm \int_a^b g(x)dx$
   - $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$, where $a < c < b$
   - $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

3. Guide students to solves problems involving above results.

Competency level 16.4 : Integrates rational functions using appropriate methods.

Number of periods : 05

Learning outcomes :
1. Uses the formula to find integrals.
2. Uses of partial fraction for integration.

Guidelines to learning - teaching process :

1. - $\int \frac{f^{/}(x)}{f(x)} dx = \ln|f(x)| + C$, where $f^{/}(x)$ is the derivative of $f(x)$.
   - $\int \frac{1}{ax^2 + bx + c}\, dx$, consider the following cases

     (i) $b^2 - 4ac > 0$ (ii) $b^2 - 4ac = 0$ (iii) $b^2 - 4ac < 0$

2. $\int \frac{P(x)}{Q(x)}\, dx$, where $P(x),\ Q(x)$ are a polynominals of degree ≤ 4.
   (maximum 4 unknowns).

Competency level 16.5 : Uses trigonometric identities for integration.

Number of periods : 03

Learning outcomes : 1. Uses trigonometric identities for integration.

Guidelines to learning - teaching process :

1. Using trigonometric identities to obtain the standard integrals of the following.
   - $\int \tan x\, dx$, $\int \cot x\, dx$, $\int \sec x\, dx$, $\int \cos ec\ x\, dx$.
   - $\int \sin^2 x\, dx$, $\int \cos^2 x\, dx$, $\int \tan^2 x\, dx$, $\int \cot^2 x\, dx$.
   - $\int \sin^3 x\, dx$, $\int \cos^3 x\, dx$.
   - $\int \sin mx \cos nx\, dx$, $\int \cos mx \cos nx\, dx$, $\int \sin mx \sin nx\, dx$.

Competency level 16.6 : Uses the method of substitution for integration.

Number of periods : 04

Learning outcomes : 1. Uses suitable substitution to find integrals.

Guidelines to learning - teaching process :

1. Use suitable substitutions.

- $\int \sin^m x \, dx$, where m is an odd positive integer (substitution $t = \cos x$).
- $\int \cos^m x \, dx$, where m is an odd positive integer (substitution $t = \sin x$).
- $\int \sin^m x \cos^n x \, dx$, where m, n are positive intregers.
- $\int \frac{dx}{a\cos x + b\sin x + c}$, (substitution $t = \tan\frac{x}{2}$)
- $\int \frac{dx}{a\cos^2 x + b\sin^2 x + c}$, (substitution $t = \tan x$)
- $\int \sqrt{a^2 - x^2}\,dx$, (substitution $x = a\sin\theta$ or $a\cos\theta$).
- $\int \frac{dx}{\sqrt{a^2 + x^2}}$, (substitution $x = a\tan\theta$).
- $\int \frac{1}{\sqrt{x^2 - a^2}}$, (substitution $x = a\sec\theta$).
- $\int \frac{dx}{(px+q)\sqrt{ax+b}}$, (substitution $t = \sqrt{ax+b}$).
- $\int \frac{dx}{(px+q)\sqrt{ax^2+bx+c}}$, (substitution $px + q = \frac{1}{t}$).
- Guide students to solves problems using other appoopriate substitutions.

Competency level 16.7 : Solves problems using integration by parts.

Number of periods : 03

Learning outcomes : 1. Uses integration by parts to solve problems.

Guidelines to learning - teaching process :

1. - If $u_{(x)}$ and $v_{(x)}$ are differentiable functions, then show that

$$\int u\left(\frac{dv}{dx}\right) dx = uv - \int v\left(\frac{du}{dx}\right) dx$$

- Guide students to solves problems using integration by parts.

Competency level 16.8 : Determines the area of a region bounded by curves using integration.

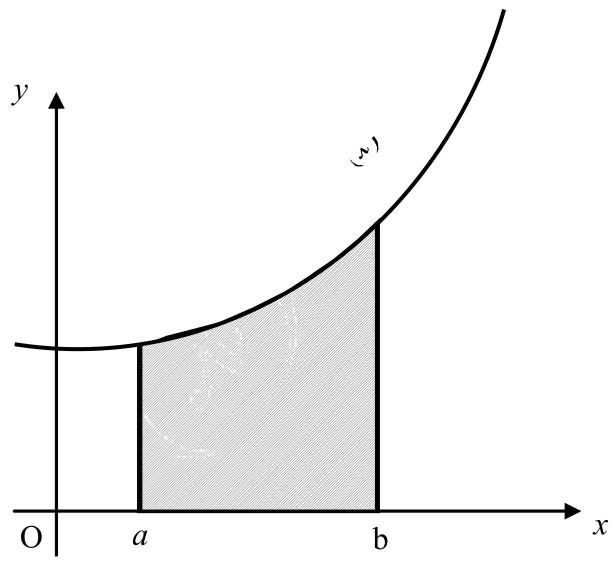
Number of periods : 04

Learning outcomes : 1. Uses definite integrals to find area under a curve and area between two curves.

Guidelines to learning - teaching process :

1. Defines the area under a curve as a definite integral.

- Let $y = f(x)$ be a curve, where $f(x)$ is a non - negative continuous function on $[a, b]$.

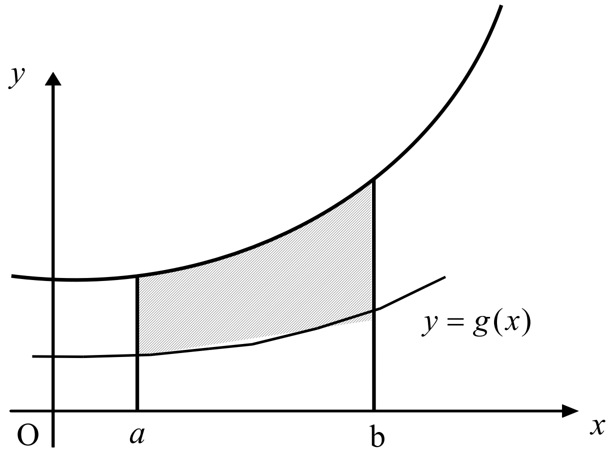


The area of the region bounded by the curve $y = f(x)$, the x - axis, and the lines $x = a$ and $x = b$ is $\int_a^b f(x)dx$.

This is referred to as the area under the curve $y = f(x)$ from $x = a$ to $x = b$.

- Suppose that $y = f(x)$ and $y = g(x)$ are two curves such that

  $f(x) \geq g(x)$ for all x in the interval $[a,b]$.



The area bounded by the two curves and the lines $x = a$ and $x = b$ is $\int_a^b \{f(x) - g(x)\}dx$.

- Only these types of areas are expected.
- Sketching curves are not expected.

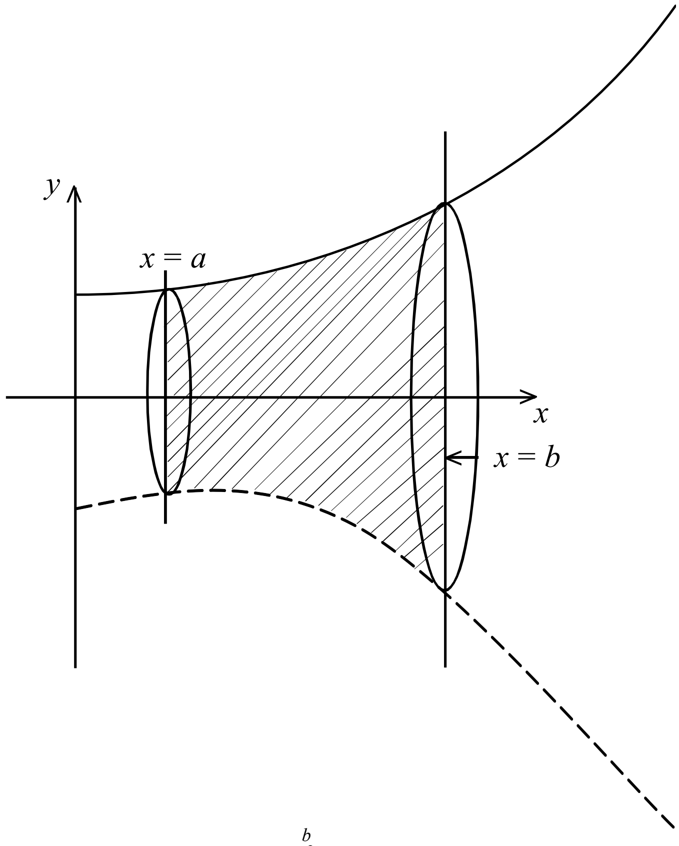
Competency level 16.9 : Determines the volume of revolution

Number of periods : 02

Learning outcomes : 1. Uses integration formula to find the volume of revolution.

Guidelines to learning - teaching process :

1. The volume of revolation obtained by rotaty the region bocended by the were $y = f(x)$ the x - axis at the lines $x = a$ and $x = b$ about the x- oxis through four right anyles is the $\pi\int_a^b \{f(x)\}^2 dx$.



Volume of the shaded area is $\pi\int_a^b \{f(x)\}^2 dx$

- Sketching curves are not expected

Combined Mathematics - II

Competency 2 : Uses systems of coplanar forces.

Competency level 2.10 : Applies the properties of systems of coplanar forces to investigate equilibrium involving smooth joints.

Number of periods : 10

Learning outcomes :

1. States the type simple joints.
2. Describes the movable joints and rigid joints.
3. Marks forces acting on a smooth joints.
4. Solves problems involving jointed rods.

Guidelines to Learning - teaching process :

1. •

- Joints
  - Free joints
    - Pin Joints
      - Smooth Joints
      - Rough Joints
    - Ball Joints
  - Rigid Joints

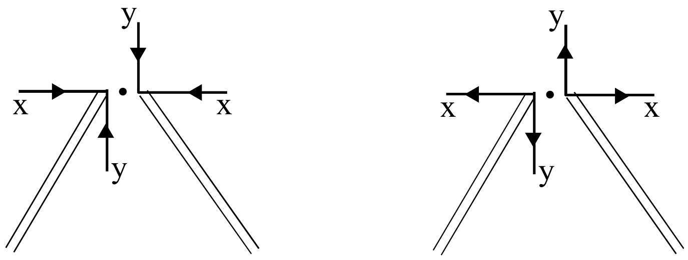
- Discuss each type of joint by considering examples in real life situation.

2. Introduce that

- A joint where the position of rods cannot be changed relative to one another as a rigid joint

- A joint where the position of rods can be changed relative to one another as a movable joint

3. State that the weight of heavy rods, considered are joined by smooth hinges (Nut and Bolt joints). Show that since the joint is smooth the force acting at the joint act in the plane of the two rods and that the reactions between the rods under the action of external forces are equal in magnitude and opposite in direction.



4. • Emphasize that the equilibrium cannot be considered as that of two bodies by disjoining the joint. show that solving problems involving two rods jointed rigidly has to be Considered as single rigid body (Rough hinges are not in the syllabus)
   • Solves problems involving rigid joints.

Competency level 2.11 : Determines the stresses in the rods of a frame work with smooth jointed light rods.

Number of Periods : 10

Learning outcomes :

1. Describes a frame work with light rods.
2. States the condition for the equilibrium at each joint the frame work.
3. Uses Bow's notation.
4. Solves problem involving a frame work with light rod.

Guidelines to learning - teaching process :

1. Introduce a frame work as a structure consisting of three of more straight light rods hinged at their ends such that they are in the same plane.

   Describe using suitable examples.

   - All the rods are smoothly joined at their ends so that no couple or torque is produced.

   - Except for the external forces, all the reactions at the joints act along the rods. These can be either tensions of thrusts.

   - All the rods lie in the same vertical plane. Therefore all the forces in the system including external forces are coplanar.

2. - As the entire frame work is in equilibrium, all the external forces should be in equilibrium.

   - As each joints is in equilibrium under the action of forces acting on it (External forces and the stress meeting in the joint) the principle of polygon of forces can be applied to search joint. Show using suitable examples how the symmetrical property of the system is used in marking the forces and how to use conditions of equilibrium in finding them.

3. **Bows notations**
   - Tell the students that, as this method was found by a mathematician by the name of "Bow" the method is called "Bow's notation".

     - Mark the external forces acting on a frame work, outside this frame work itself.

     - Number all areas bounded by every pair of external forces and rods.

   - Draw closed triangle of forces, or a polygon of forces representing forces at each a joint, ask the students to choose the sence as the like. Note that These figures too should be closed.

- Find unknown stresses by considering the triangles or polygons in the stress diagram, and using trigonometric or algebraic formulae.

- Show that (if necessary) unknown external forces can also be determined, using the stress diagram.

- Draw the attention of students to avoid using arrow heads on the stress diagram, but use these on the frame work diagram.

- Explain how the stress could be classified as tensions and thrust according to the arrow heads inserted on the frame work diagram.

2. guide students to solves problems using bow's notation.

Competency 3 : Applies the Newtonian model to describe the instantaneous motion in a plane.

Competency level 3.9 : Interprets mechanical energy.

Number of Periods : 02

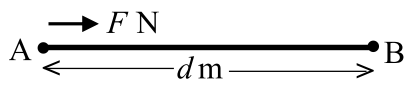
Learning outcomes :

1. Explains the concept of work.
2. Defines work done under a constant force.
3. States dimensions and units of work.
4. Explains energy.
5. States dimensions and units of energy.
6. Explains the mechanical energy.
7. Defines kinetic energy.
8. Defines Potential energy.
9. Explains the gravitational potential energy.
10. Explains the elastic potential energy.
11. Explains conservative forces and dissipative force
12. Writes work - energy equations
13. Explains conservation of mechanical Energy and applies it to solve problems.

Guidelines to learning - teaching process:

1. Explain the idea of work that the point of application moves under the action of a force.

2. Work is defined as the product of the constant force and the distance through which the point of application moves in the direction of the force.



Work done = Fd Nm

3. The unit of force is *Newton* and the unit of distance is *metre*; also that the unit of work done by a force is *Newton metre*.

This unit is called *joule* (J) and its Dimensions are ML^2T^{-2}

4. The energy of a body is its capacity for doing work. The SI unit of energy is the *Joule*

$$1 \text{ kJ} = 1000 \text{ J}$$

5. • States that both work and energy are scalar quantities.

   • Work and energy are interchangeable and so the units and dimensions are same as those of work.

6. Explain that we deal with mechanical energy only, and mechanical energy is of two types;

   Kinetic energy (K.E)

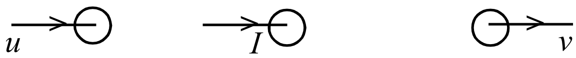
   Potential energy (P.E)

7. • Kinetic energy is the capacity of a body to do work by virtue of its motion. It is measured by the amount of work that the body does in coming to rest.

   • Obtain the formula K.E $= \frac{1}{2}mv^2$, where m is the mass and v is the speed.

   • Explain that work done = change in kinetic energy.

   ΔT - change in kinetic energy



$$\underset{\sim}{I} = m\underset{\sim}{v} - m\underset{\sim}{u}$$

$$\Delta T = \frac{1}{2}mv^2 - mu^2$$

$$= \frac{1}{2}m(\underset{\sim}{v} - \underset{\sim}{u}) \cdot (\underset{\sim}{v} + \underset{\sim}{u})$$

$$\Delta T = \frac{1}{2}I(\underset{\sim}{v} + \underset{\sim}{u})$$

8. The Potential Energy (P.E) of a body is the energy it possesses by virtue of its position. It is measured by the amount of work that the body would do in moving from its actual position to some standard position.

9. Define the gravitational potential energy as when a body of mass m is raised through a vertical distance h, it does an amount of work equal to mgh.

10. • Elastic Potential Energy is *a* property of stretched strings and springs or compressed springs. The amount of Elastic Potential Energy (E.P.E)

- Energy stored in a string of natural length a and modulus of elasticity λ when it is extended by a length x is equivalent to the amount of work necessary to produce the extension.

  Obtain that EPE $= \frac{1}{2}\lambda \cdot \frac{x^2}{a}$

11. Certain forces have the property that the work done by the forces is independent of the path (For an example weight) Such forces are termed as conservative forces.

    e.g:- (i) gravitational force

    (ii) tension in a stretched elastic string

    (iii) tension or thrust in a stretched or compressed spiral spring

12. Writes work - energy equations.

13. • The Principle of conservation of the mechanical energy for a system of bodies in motion under the action of a conservative system of forces.

- Sum of the kinetic energy and the potential energy of the system remains constant.

  K.E. + P.E. = Constant

- Simple applications of the principle of conservation of mechanical energy. (including gravitational and mechanical energy)

  Examples: • Horizontal motion of a particle

  • Vertical motion of a particle

Competency level 3.10 : Solves problems involving power.

Number of periods : 08

Learning outcomes :
1. Defines power.
2. States its units and dimensions.
3. Explains the tractive force.
4. Derives the formula for power
5. Uses tractive force to solve problems when impulse is constant.

Guidelines to learning - teaching process :

1. Define that the power is the rate of doing work.

2. The power is measured in Joule per second (JS^{-1}) and this is called a watt (w).
   Dimensions are ML^2T^{-3}

3. The tractive force is the producing force from the vehicle engine (Driving force).

4. Relationship between power, driving force and velocity.
   If a contant force F N moves a body with a v ms^{-1} in the direction of the force then,
   $P = Fv$ (unit of P is watts)
   Power = rate of work done.

$$= \frac{dw}{dt}$$

$$= \frac{dF \cdot s}{dt}$$

$$= F \cdot \frac{ds}{dt}$$

$$P = F \cdot \underline{v}$$

5. Guide the students to solve problems involving Work, Power and Energy.

Competency level 3.11 : Interprets the effect of an impulsive action.

Number of periods : 05

Learning outcomes :
1. Explains the Impulsive action.
2. States the units and dimensions of impulse.
3. Uses $\underset{\sim}{I} = \Delta(m\underset{\sim}{v})$ to solve problems.
4. Finds the change in kinetic energy due to impulse.

Guide lines to learning - teaching process :

1. Impulsive force

   Define that impluse of a Constant Force as the product of the force and times Δt

   $$\underset{\sim}{I} = \underset{\sim}{F}\Delta t$$

   Hence, obtain $\underset{\sim}{I} = m(\underset{\sim}{v} - \underset{\sim}{u})$ where m is the mass of the particle.

   $$\underset{\sim}{I} = \underset{\sim}{F}\Delta t = \Delta(m\underset{\sim}{v})$$

2. - Units of impulse is Ns Dimension is MLT^{-1}
   - As Units is a vector when applying the formula $\underset{\sim}{I} = \Delta(m\underset{\sim}{v})$ the directions of forces and the velocities must be taken into consideration.

3. Guide the students.

   Uses $\underset{\sim}{I} = \Delta(m\underset{\sim}{v})$

4. - State that the change in K.E. is equal to

     $$\frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

     $$\Delta E = \frac{1}{2}m(v^2 - u^2) = \frac{1}{2}\underline{I}(\underline{v} + \underline{u})$$

   - ` Solve problems involving Impulse in the same line of motion.

Competency level 3.12 : Uses Newton's law of restitution to direct elastic impact.

Number of periods : 10

Learning outcomes :
1. Explains direct impact
2. States Newton's law of restitution
3. Defines coefficient of restitution
4. Explains the direct impact of a sphere on a fixed plane.
5. Calculates change in kinetic energy.
6. Solves problems involving direct impacts.

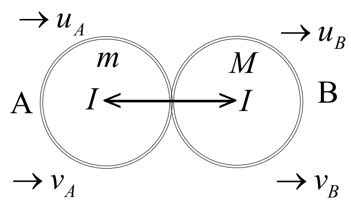
Guidelines to learning - teaching process :

1. Direct Impact

Direct impact occurs when the directions of the velocities of the spheres just before the impact are along the line of centres on impact.

2.
- When two bodies impinge directly, the relative velocity of separation after the impact bears a constant ratio to relative velocity of approach before the impact
- The constant ratio is called coefficient of restitution and denoted by *e*.

3.



$v_B - v_A = e(u_A - u_B)$, provided that $u_A > u_B$

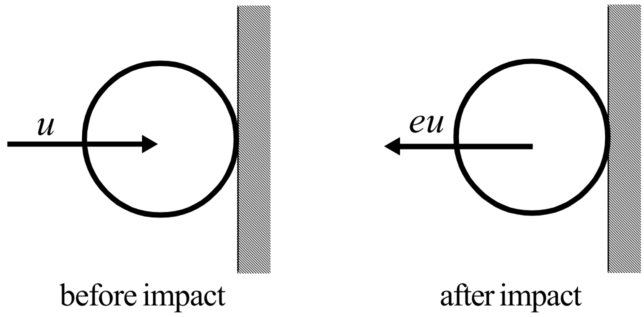
The constant e depends only on the material of which the bodies are made.

$0 \leq e \leq 1$

If $e = 1$ the bodies are said to be perfectly elastic.

If $e = 0$ the bodies are said to be in elastic.

4.



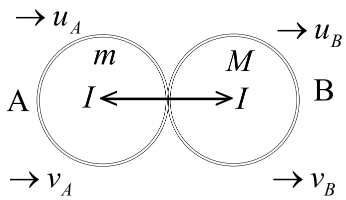
The velocity after impact is equal to e (velocity before impact) and in the opposite direction.

5. During direct impact between two bodies of masses m_1 and m_2 the loss of kinetic energy due to impact is

$\Delta E = \frac{1}{2}\frac{m_1 m_2}{(m_1 + m_2)}(1-e^2)v^2$, where v is the relative velocity at the time of impact.

If $e = 1$, then $\Delta E = 0$

6. Guide the students to solve problems involving direct impacts using $\underline{I} = \Delta\ m\underline{v}$ and Newton's law of restitution.



$v_B - v_A = e(u_A - u_B)$, provided that $u_A > u_B$

Apply $I = \Delta\ mv$

To A $\quad -I = mv_A - Mu_A$ ....(1)

To B $\quad I = mv_B - Mu_B$ ....(2)

(1)+(2) $\quad 0 = mv_A + Mv_B - (Mu_A + mu_B)$

Apply $\underline{I} = \Delta\ m\underline{v}$ to the system

$0 = mv_A + Mv_B - (Mu_A + mu_B)$

Competency level 3.13 : **Solves problems using the conservation of linear joint momentum**

Number of periods: 04

Learning outcomes :

1. Defines linear momentum.
2. Solves problem using the principle of linear momentum.

Guide lines to learning - teaching process :

1. **Principle of conservation of linear momentum**
   If vector sum of external forces is equal to zero or if there are no external forces acting on a system of bodies then the momentum of the system is conserved.

2. Guide the students to solve problems using the principle of conservation of linear momentum.

Competency level 3.14 : **Investigates velocity and acceleration for motion in a circle.**

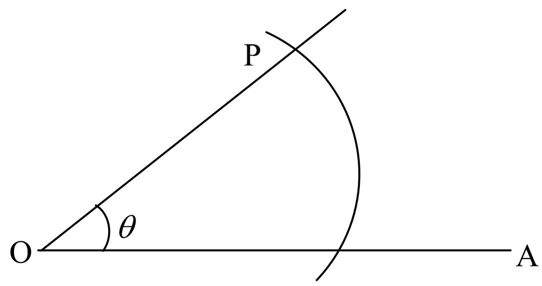
Number of period : 06

Learning outcomes :

1. Defines the angular velocity and acceleration of a particle moving in a circle.
2. Find the velocity and the acceleration of a particle moving in circle.

Guidelines to Learning - teaching outcomes :

1. **Circular Motion**



Let O be a fixed point and OA be a fixed line.

If a particle moves in this plane then the angular velocity of P about O is defined to be the rate at which the angle POA increases and is denoted by

$$\omega = \frac{d\theta}{dt} = \dot{\theta}$$

Its units are given by (rad/s)

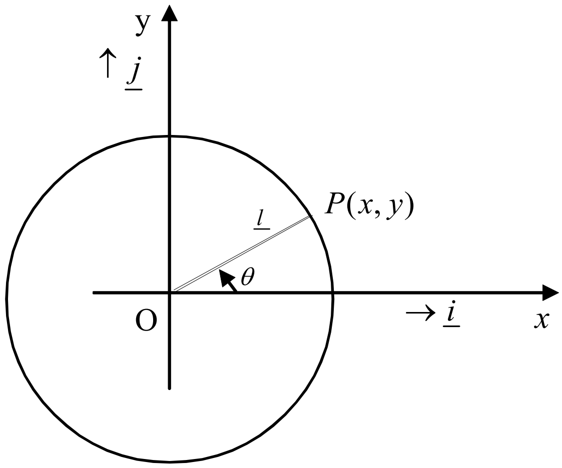
Angular acceleration is defined as the rate of increase of angular velocity.

Angular acceleration is given by

$$\frac{d\left(\frac{d\theta}{dt}\right)}{dt} = \frac{d^2\theta}{dt^2}$$

$$\frac{d\dot{\theta}}{dt} = \ddot{\theta}$$

Its units are given by rad/ s^2



P moves in a circle and $OP \neq a$ (constant)
Position vector of P is $\underline{r}$
$\underline{r} = a\underline{l}$, where $l = \cos\theta\underline{i} + \sin\theta\underline{j}$

2. Velocity $\underline{v} = \frac{dr}{dt} = \frac{da\underline{l}}{dt} = a\frac{dl}{dt}$

$v = a(-\sin\theta\underline{i} + \cos\theta\underline{j})\dot{\theta}$

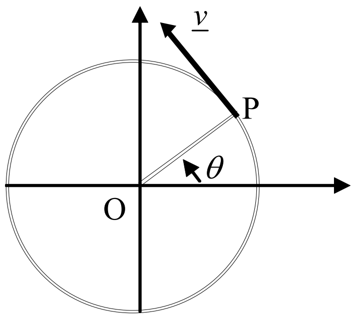
$= a\dot{\theta}\underline{m}$, where $\underline{m} = -\sin\theta\underline{i} + \cos\theta\underline{j}$

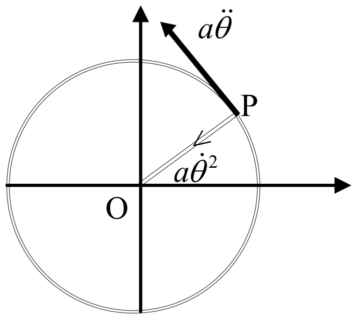
acceleration $f = \frac{dv}{dt} = a\dot{\theta}\left(\frac{d\underline{m}}{dt}\right) + a\ddot{\theta}m$

and $\dfrac{dm}{dt} = (-\cos\theta\,\underline{i} - \sin\theta\,\underline{j})\dot{\theta}$

$= \dot{\theta}\,\underline{l}$

$= -a\dot{\theta}^2\,\underline{l} + a\ddot{\theta}\,\underline{m}$





Velocity

$\underline{v} = a\dot{\theta}$ along the tangent.

Acceleration

1. Component towards the centre is $a\dot{\theta}^2$
2. Along the tangent is $a\ddot{\theta}$

Competency level 3.15 : Investigates motion in a horizontal circle.

Number of periods : 04

Learning outcomes :

1. Finds the magnitude and direction of the force on a particle moving in a horizontal circle with uniform speed.
2. Solves the problems involving motion in horizontal circle
3. Solves problems involving conical pendulum.

Guidelines to learning - teaching process :

1. Explains that since the particle moves with uniform speed, the acceleration is towards the centre and a force must be acting towards the centre and this force is called centrifugal force.
2. Guide students to solve problems involving motion in a horizontal circle.
3. Guide Students to solve the problems involving conical pendulum.

Competency level 3.16 : Investigates the relevant principles for motion on a vertical circle.

Number of periods: 10

Learning outcomes :

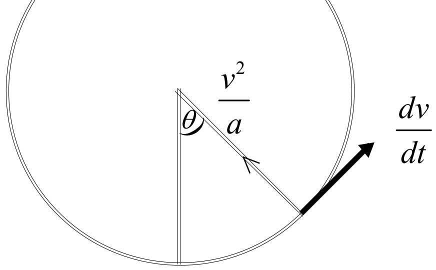
1. Explains motion in a vertical circle.
2. Discusses the motion of a particle on the outer surface of a fixed smooth sphere in a vertical plane.
3. Discusses the motion of a particle on the inner surface of a fixed smooth sphere in a vertical space.
4. Finds the condition for the motion of a particle suspended from an inelastic light string attached to a fixed point, in vertical circle.
5. Explains the motion of a ring threaded on a fixed smooth circular wire in a verticaloe plane.
6. Explains the motion of a particle in a vetical tube.
7. Solves problems including circular motion.

Guidelines to learning - teaching process :

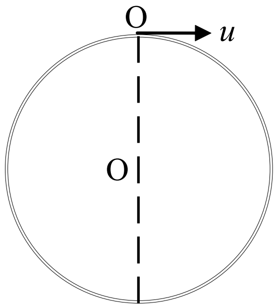
1. When a particle moves in a vertical circle of radius a, with varying velocity v the acceleration towards the centre of the circle is $\frac{v^2}{a}$ and $\frac{dv}{dt}$ in the direction of tangent.

$$\frac{v^2}{a} = a\dot{\theta}^2$$

$$\frac{dv}{dt} = a\ddot{\theta}$$



2.



- Let O be the centre of sphere and a be the radius. A particle is projected with velocity u in horizontal direction from the highest point of a smooth sphere.

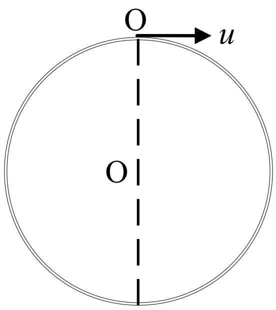
Discuss the motion and show that

- If $u^2 >$ ag, then the particle leaves the sphere at the point of projection highest point)
- If $u^2 < ag$, then particle leaves the sphere, when the radius through the particle makes an angle α with upward

  Vertical, where $\alpha = \cos^{-1}\left(\dfrac{u^2 + 2ag}{3ag}\right)$

  Solve problems leading to vertical circular motion.

3.



- Let O be the centre of sphere and a be the radius. A particle is projected with velocity u in horizontal direction from the highest point of a smooth sphere.

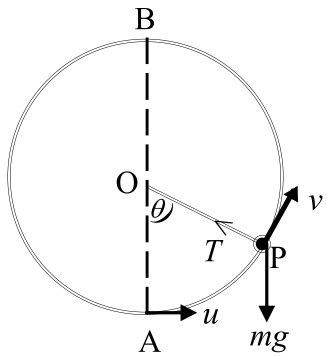
Discuss the motion and show that

- If $u^2 >$ ag, then the particle leaves the sphere at the point of projection (highest point)
- If $u^2 < ag$, then particle leaves the sphere, when the radius through the particle makes an angle α with upward

  Vertical, where $\alpha = \cos^{-1}\left(\dfrac{u^2 + 2ag}{3ag}\right)$

  Solve problems leading to vertical circular motion.

4.



Let u be the velocity of particle m in the horizontal direction at the lowest point. When the string has turned through an angle θ, let v be the velocity and T be the tension. Using conservation of energy and applying

$F = m\underline{a}$ in the radial direction, obtain

$$v^2 = u^2 - 2ag(1-\cos\theta)$$

$$T = \frac{m}{a}\left[u^2 - 2ag + 3ag\cos\theta\right]$$

Discuss the following.

- If $u^2 \leq 2ag$, the string is always taut, particle oscillates below the level of O.

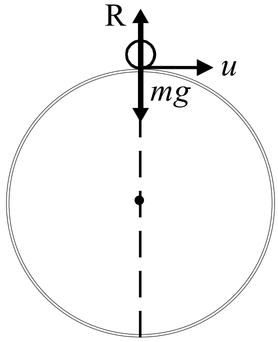
- If $2ag < u^2 < 5ag$, then

  t becomes zero before v becomes zero and hence the string becomes slack.
  when the string is slack $2ag < u^2 < 5ag$,
  once the string is slack, the only force acting on the particle is its own weight and motion continues as that of a projectile.

  If $u^2 > 5ag$ then the particle moves in a complete circle.

Note : That motion of a particle in a vertical circle on the inner surface of a smooth sphere is also same as above.

5.



Motion restricted to circular path. Explain that the only external force acting on the particle is Reaction. As the reaction is perpendicular to the direction of motion, it does no work.

- Law of conservation of energy can be applied.

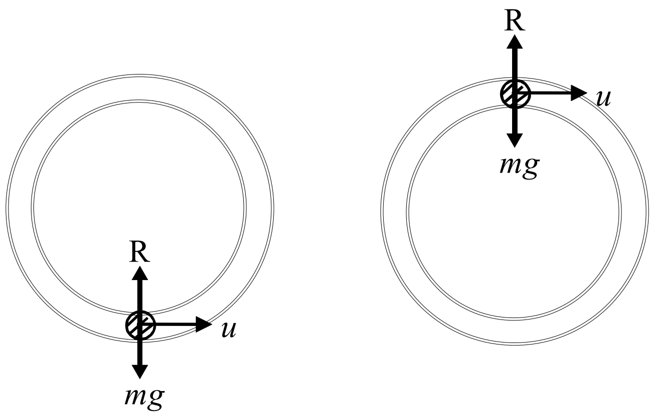
- Applying $\underset{-}{F} = m\,\underset{-}{a}$ in the radial found. Since the ring cannot leave the wire, the only condition necessary for it to describe complete circle is that its velocity is greater than zero at the highest point.

  Let u be the velocity at lowest point.

  - If $u^2 > 4ag$ it describes a complete circle.

  - If $u^2 < 4ag$ come to instantaneous rest before reaching highest point and subsequently oscillates.

6.



Describe the motion of a pareticle inside the tube, which is placed in a vertical plane.

7. Guide Students to solves problems involving circular motion.

SECOND TERM

Combined Mathematics - I

Competency 26 : Interprets the Cartesian equation of a circle.

Competency level 26.1 : Finds the Cartesian equation of a circle.

Number of periods : 03

Learning outcomes :

1. Defines circles as a locus of a variable point in a plane such that the distance from a fixed point is a constant.
2. Finds equation of a circle with origin as the center and with a given radius.
3. Finds equation of a circle with a given centre and a given radius.
4. Interprets general equation of a circle.
5. Finds the equation of the circle having two given points as the end points of a diameter.

Guidelines to learning - teaching process :

1. **Circle :**

   Define a circle as the locus of a point which moves on a plane such that its distance from a fixed point is always a constant. This fixed point is the centre of the circle and the constant distance is the radius of the circle.

2. If centre is the orgin and the radius is r, equation becomes $x^2 + y^2 = r^2$

3. Equaiton of the circle with centre (a, b) and radius $(x-a)^2 + (y-b)^2 = r^2$

4. General equation of a circle is $x^2 + y^2 + 2gx + 2fy + c = 0$.

   Show that the centre is $(-g, -f)$ and radius is $\sqrt{g^2 + f^2 - c}$.

5. Show that the equation of the circle with the points (x_1, y_1) and (x_2, y_2) as the ends of a diameter is $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

Competency 27 : Explores Geometric properties of circles.

Competency level 27.1 : Describes the position of a Straight line relative to a circle.

Number of periods : 02

Learning outcomes :

1. Discuses the position of a straight line with respect to a circle.
2. Obtains the equation of the tangent at a point on a circle.

Guidelines to learning - teaching process :

1. Let $U \equiv lx + my + n = 0$ be the straight line and

   $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ be a circle.

   By considering

   - Discriminant of the quadratic equation

     in x or y, obtained by solving $S = 0$ and $U = 0$,

   - Radius of the circle and the distance between the centre of the circle and the straight line,

   discuss whether,

   - The line intersects the circle,
   - The line touches the circle,
   - The line lies outside the circle,

   .

2. Prove that the equation of the tangent at the $P = (x_0, y_0)$ on the circle.

   $S = x^2 + y^2 + 2gx + 2fy + c = 0$ is

   $xx_0 + yy_0 + g(x + x_0) + f(y + y_0) + c = 0$

Competency level 27.2 : Finds the equations of tangents drawn to a circle from an external point.

Number of periods : 03

Learning outcomes :

1. Obtain the equations of the tangents drawn to a circle from an external point.
2. Obtains the length of tangents drawn from an external point to a circle.
3. Obtains the equation of the chord of contact.

Guidelines to learning - teaching process :

1. Guide students to obtain the equations of tangents drawn to a circle from an external point.
2. Let $S = x^2 + y^2 + 2gx + 2fy + c = 0$ and $\mathrm{P}(x_0, y_0)$ be an external point Show that the length of the tangent is $\sqrt{x_0^2 + y_0^2 + 2gx_0 + 2fy_0 + c}$.
3. Let $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ and $\mathrm{P}(x_0, y_0)$ be an external point. Show that the equation of chord of contact of tangents drawn from P is $xx_0 + yy_0 + g(x + x_0) + f(y + y_0) + c = 0$.

Competency level 27.3 : Derives the general equation of a circle passing through the points of intersection of a given straight line and a given circle.

Number of Periods : 02

Learning outcomes:

1. Interprets the equation $S + \lambda u = 0$, where λ is a parameter.

Guidelines to learning teaching process :

1. Explains that $S + \lambda u = 0$ represents a circle passing through the points of intersection of the circle $S = 0$ and the straight line $U = 0$, where λ is a pameter.

Competency level 27.4 : Describes the position of two circles.

Number of periods: 03

Learning outcomes :

1. Describes the condition for two circles to intersect.
2. Describes the conditions for two circles to touch externally or touch internally.
3. Describes to have one circle lying within the other circle.

Guidelines to learning - teaching process :

Let c_1 and c_2 be centers of two circles with radii r_1 and r_2 .

1. The circles intersect if and only if $C_1C_2 < r_1+r_2$
2.
   - The circles touch externally if an only if $C_1C_2=r_1+r_2$
   - The circles touch internally if and only if $C_1C_2 = |r_1 - r_2|$
3.
   - One circles lies within the other if and only if $C_1C_2 < |r_1 - r_2|$
   - Each circles lies outsides the other if and only if $C_1C_2 > r_1 + r_2$

Competency level 27.5 : Finds the condition for two circle to intersect or thogonally.

Number of periods : 02

Learning outcomes :

1. Finds the condition for two circles to intersect orthogonally

Guidelines to learning - teaching process :

1.
   - Show that two circles $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ and

     $S' \equiv x^2 + y^2 + 2g'x + 2f'y + c' = 0$

     intersect orthogonally if and only if $2gg' + 2ff' = c + c'$
   - Guid stutents to solves problema ivoling orthogonal intersection of two circles.

Competency 24: Uses permutations and combinations as mathematical modeles for sorting and arranging.

Competency level 24.1 : Defines factorials.

Number of periods : 01

Learning outcomes :
1. Defines factorial.
2. States the recursive relation for factorials.

Guidelines to learning - teaching process :

1. Definition of factorial n

   $0! = 1$

   $n! = 1 \cdot 2 \cdot 3 \cdot ... \cdot n$, for $n \in \mathbb{Z}^+$

2. Recursive form $F(0) = 1$

   $F(n) = nF(n-1)$, for $n \in \mathbb{Z}^+$

Competency level 24.2 : Explains fundamental principal of counting.

Number of periods : 02

Learning out comes :
1. Explains the fundamental principle of counting.

Guidelines to learning - teaching process :

1. **Fundamental principles of counting :**

   If one of operation can be perfomed in ***m*** different ways and a second operation can be performed in ***n*** different ways, then there are $m \times n$ different ways of performing the two operations in succession.

Competency level 24.3 : Use of permutations as a technique of solving mathematical problems

Number of periods : 06

Learning out comes :

1. Defines $^{n}p_{\mathrm{r}}$ and obtain the formula for $^{n}p_{\mathrm{r}}$.
2. The number of permutations of n different objects taken r at a time.
3. Finds the number permutations of different objects taken all at a time.
4. Permutation of n objects not all different.
5. Finds number of permutations of n different objects not all different taken r at a time.

Guidelines to learning teaching process :

1. Defines $^{n}p_{\mathrm{r}}$ the number of permutations of r objects taken from n distinct objects $(0 \leq r \leq n)$ taken at a time is $^{n}p_{\mathrm{r}}$.

2. Obtains that the number of permutations of r objects taken fron n different objects is given by $^{n}p_{r} = \dfrac{n!}{(n-r)!}$.

3. Show that the number of permutations of n different objects taken r at time is given by also show that it is given by $^{n}p_{n} = n!$

4. Show that the number of permutations of n objects p of which are one kind and the remaining all are different is $\dfrac{n!}{p!}$.

5. Find the number of permutations of r objects taken from n objects taken from n objects not all distint.

Competency level 24.4 : Uses combinations as a technique of solving mathematical problems.

Number of periods : 05

Learning outcomes :

1. Defines combination .
2. Finds the number of combinations of different objects taken r at a time.
3. Define ${}^{n}C_{r}$ and finds formulae for ${}^{n}C_{r}$
4. Finds the number of combination of n different objects not all different taken r at a time where $r\left(0 \le r \le n\right)$.
5. Explains the difference between permutations and combinations.

Guidelines to learning - teaching process :

1. Define that the number of combinations of n different objects taken r $(0 < r \le n)$ at a time.

2. Introduce ${}^{n}C_{r}$, as the number of combinations of r $(0 < r \le n)$ different objects taken from n different objects.

3. Show that ${}^{n}C_{r} = \dfrac{n!}{(n-r)!r!}$., ${}^{n}P_{r} = r!\,{}^{n}\mathrm{C}_{r}$

4. Guide students to find the number of combination of n different objects not all different taken r at a time, where $0 \le r \le n$.

5. By using examples explain the difference between permutaions and combinations.

Competency 19 : Applies the priniciples of Mathematical Induction as a type of proof for mathematical result for positive integers.

Competency level 19.1 : Uses the principle of Mathematical Induction.

Number of periods : 05

Learning outcomes:
1. States the principle of Mathematical Induction.
2. Proves various results using principles of Mathematical Induction.

Guidelines to learning - teaching process :

1. Principles at Mathematical Induction. Let *P(n)* be a mathematical statement.
   - Prove that the statement is true for $n = 1$.
   - Suppose that the result is true for $n = P$, where P is an any positive integer.
   - Prove that the result is true for $n = P+1$.
   - By combining above result we can induct mathematicaly that the result is true for all $n \in \mathbb{Z}^+$

2. Uses Mathematical Induction to prove divisibility problems, summation of series and etc.

Competency 20 : Finds sums of finite scries.

Competency level 20.1 : Describes finete series and their properties.

Number of periods : 03

Learning outcomes :
1. Describes finite sums.
2. Uses the properties of '$\sum$' notation.

Guidelines to learning - teaching process :

1. If the number terms are write that lype at series can be consider like saves.

2. Show that

- $\sum_{r=1}^{n}(u_r + v_r) = \sum_{r=1}^{n} u_r + \sum_{r=1}^{n} v_r$

- $\sum_{r=1}^{n} ku_r = k\sum_{r=1}^{n} u_r$ where k is a constant

In general, $\sum_{r=1}^{n}(u_r v_r) \neq \sum_{r=1}^{n} u_r \sum_{r=1}^{n} v_r$

Competency level 20.2 : Finds sums of elementary series.

Number of periods : 05

Learning outcomes :
1. Finds general term and the sums of AP and GP.
2. Proves and uses the formulae for me values of the sums $\sum_{r=1}^{n} r,\ \sum_{r=1} r^2$ and $\sum_{r=1}^{n} r^3$ to find the summation of series.

Guidelines to learning - teaching process :

1. A series, which after the first term, the deference between each term and preceding its term is constant then it is called an arithmetic series or arithmetic progression.

Show the general term $Tr = a + (r-1)d$, where a is the first term and d is the common difference and the sum of n terms.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[a + l]$$

Where l is the last term of the series.

- A Series , which after the first term , the ratio between each term and the preceding term is a constant is called geometric series.
  - Show that the general term $T_p = ar^{p-1}$
    where a is the first term and r is the common ratio.
  - Show that the sum of n terms S_n,

$$Sn = \frac{a(1-r^n)}{(1-r)} \quad \text{when } (|r|<1)$$

$$Sn = \frac{a(r^n - 1)}{(r-1)} \quad \text{when } (|r|>1)$$

Determination of $\sum_{r=1}^{n} r, \sum_{r=1}^{n} r^2, \sum_{r=1}^{n} r^3$ and the use of above results and theorem.

Examples: - Find $\sum_{r=1}^{n} r(2r+3)$

Find $\sum_{r=1}^{n} [2r(r+1)(r+2)]$.

Competency 21 : Investigates infinite series.

Competency level 21.1 : Sums series using various methods.

Number of periods : 08

Learning outcomes : 1. Uses various methods to find the sum of a series.

Guidelines to learning - teaching process :

1. Find sums of series using
   - Method of difference.
   - Method of partial fractions.
   - Principles of mathematics induction.

Competency level 21.2 : Uses partial sum to determine convergence and divergence.

Number of periods : 03

Learning outcomes :

1. Interprets sequences.
2. Finds partial sum of an infinite series.
3. Explain the concepts of convergence and divergence using partial sums.
4. Finds the sum of a convergent series.

Guidelines to learning - teaching process :

1. If a_n is the n^{th} term of sequence, the sequence is denoted by $\{a_n\}$

   $\{a_n\}$ is said to be convergent, if $\lim\limits_{n\to\infty} a_n$ exists (finite number).
   Otherwise the sequence is said to be divergent.

2. Connection between a sequence and a series.
   Let $\{a_n\}$ be a sequence.

   Define $S_n = \sum_{r=1}^{n} a_r \quad n = 1, 2, 3...$

   This called the n[th] partial sum.

3. • Let $\sum_{r=1}^{\infty} u_n$ be a series and $s_n = \sum_{r=1}^{n} u_r$

If $\lim_{n \to \infty} S_n = l$ (finite,) then the series $\sum_{r=1}^{\infty} u_n$ is said to be convergent and the sum to infinity is l

i.e. $\sum_{r=1}^{\infty} u_n = l$

If S_n does not tend to a limit , then $\sum_{r=1}^{\infty} u_n$ is said to be divergent.

• Consider a Geometric series with common ratio *r*.

If $|r| < 1$ then the series said to be convergent and its finite sum defined as $\dfrac{a}{1-r}$

4. Guide students to finds the sum of a convergent series

Combined Mathematics - II

Competency 4 : Applies mathematical models to an events on rand om experiment

Competency level 4.1 : Interprets events of a random experiment.

Number of periods : 04

Learning outcomes :

1. Explains random experiment.
2. Defines Sample space and sample point.
3. Defines an event.
4. Explains simple events , compound events, null events and complementary events.
5. Cassifies the union of events and intersection of events.
6. Explains mutually exclusive events and exhaustive events.
7. Explains equally probable events.
8. Explains event space.

Guidelines to learning - teaching process :

1. Discuss what is random experiment.
2. The collection of all possible outcomes for an experiment is called sample space.
3. An event is a subset of a sample space. i.e An event is a collection of one or more of the outcomes of an experiment.
4.
   - An event that includes one and only one of the outcomes for an experiment is called a simple event.
   - A null event is an event which does not consist any outcomes of an random experiment.
   - Let A is an event in the sample space of a random experiment then the event that consist all the outcomes other then the outcomes of A is said to be compliment event.
5. Explains intersection of two events and union of two events.
6. Explains
   - Exhacestive events.
   - Mutualy exclusive events.
7. Explain equal probable events.
8. Set of all events of a random experiment is said to be an event space.

Competency level 4.2: Applies probability models to solve problems on random events.

Number of periods : 06

Learning ourcomes :

1. States classical definition of probability and its limitations.
2. States frequency approximation of probability and its limitations.
3. States the axiomatic definition.
4. Importance of axiomatic definition.
5. Proves the theorems on probability using axiomatic definition.
6. Solves problems using the above theorems.

Guidelines to learning - teaching process :

1. The probability of an event 'A' related to random experiment consisting of N equally likely outcome is defined as $P(A) = \dfrac{n(A)}{N}$;

   where $n\,(A)$ is the number of simple events in the event A.

   Limitations :

   - The above formula cannot be used when the results of the ran dom experiment are not equally probable.
   - When the sample space is infinite the above formulae is not valid.

2. Want be describe the frequany approximation at probability and its limits.

3. Let $\in$ be the event space corresponding to sample space S of a random experiment.

   A function $P : \in \rightarrow [0,1]$

   Satisties the following conditions.

   - $P(A) \geq 0$ for any $A \subseteq S$
   - $P(\mathrm{S}) = 1$

- If A_1, A_2 are two mutually exclusive events

  $P(A_1 \cup A_2) = P(A_1) + P(A_2)$ is said to P be a probability function.

4. Discribe the important of the axiomatic definition.

5. Prove the following using axiomatic definitions.

   - $P(\phi) = 0$
   - $P(A^{/}) = 1 - P(A)$
   - $P(A) = P(A \cap B) + P(A \cap B^{/})$
   - $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
   - If $A \subseteq B$ then $P(A) \leq P(B)$

6. Guide students to solve problems using the above theorems.

Competency 3 : Applies the Newtonian model to describe the instantaneous motion in a plane.

Competency level 3.17 : Analyses simple harmonic motion.

Number of periods : 04

Learning outcomes :

1. Defines simple harmonic motion (SHM)
2. Obtain the differential equation of simple harmonic motion and verifies its general solution.
3. Derives the velocity as a function of displacement.
4. Defines amplitude and period of SHM.
5. Describes displacement as a function of time.
6. Interprets SHM associated with uniform circular motion.
7. Finds times using circular motion associated with SHM.

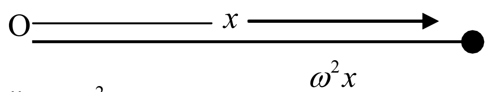
Guidelines to learning - teaching process :

1. **Simple Harmonic Motion:**

   State the Simple Harmonic Motion as a particular type of oscillatory motion.

   Its is defined as a motion of a particle moving in a straight line with a linear acceleration proportional to the linear displacement from a fixed point and is always directed towards that fixed point.

   The fixed point is known as the centre of oscillation.

2. 

   $\ddot{x} = -\omega^2 x$

   The above is the differential equation for linear SHM, where w is a constant.

   Verify that $x = A\cos\omega t + B\sin\omega t$ is the general solution of the above differential equation, where *A, B* are arbitrary constants and *t* is the time.

3. Discuss $x = A\cos\omega t + B\sin\omega t$ implies that

$$\dot{x}^2 = \omega^2\left[(A^2 + B^2) - x^2\right]$$

$$\Rightarrow \dot{x}^2 = \omega^2\left[a^2 - x^2\right], \text{ where } a^2 = A^2 + B^2$$

For the displacement, the following formulae can also be used.

$$x = a\sin(\omega t + \alpha)$$

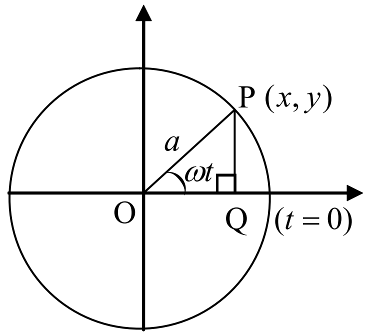
4. State that

(i) The length $a = \sqrt{A^2 + B^2}$ is the amplitude of the SHM

(ii) The time $T = \dfrac{2\pi}{\omega}$ is the period of the SHM.

5. Discuss $x = a\cos\omega t$

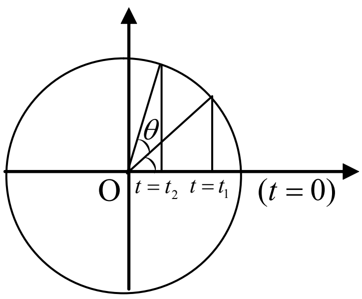
6. Discuss



$$x = a\cos\omega t$$
$$\dot{x} = -a\omega\sin\omega t$$
$$\ddot{x} = -a\omega^2\cos\omega t = -\omega^2 x$$

Let a particle P moves in a circle with uniform angular velocity ω.
Let Q be the foot of the perpendicular from P on a diameter coinciding with the x-axis. When P describes the circular motion with angular velocity ω. Q describes a SHM given be the equation $\ddot{x} = -\omega^2 x$.

7.



Guide students to finds time using circular motion associated with SHM.

Discuss the time duration between two positions of the particle.

$$t_2 - t_1 = \frac{\theta}{\omega}$$

The above time interval can also be derived from the equations of the motion.

Competency level 3.18 : Describes the nature of a simple harmonic motion on a horizontal line.

Number of periods : 06

Learning outcomes :

1. Finds the tension in an elastic string in terms of its extension using Hook's law.
2. Finds tension or thrust in a spring using Hooke's law.
3. Describes the nature of simple Harmonic motion of particle on a horizontal line.

Guidelines to learning - teaching process :

1. State Hookes law and guide students to finds the tension in an elastic string.
2. State Hookes law and guide students to finds tension or thrust in an elastic spring.

State Hookes law for tension or thrust.

$T = \lambda \frac{d}{l}$, where

λ : Modules of elasticity

d : Extension or compression

l : Natural length

Prove by integration that the elastic potential energy is $\frac{\lambda x^2}{2l}$

3. Discuss the SHM of a particle under the action of elastic forces in a horizontal line.

Competency level 3.19 : Describes the nature of a simple harmonic motion on a vertical line

Number of periods : 06

Learning outcomes :

1. Explains the simple harmonic motion on a vertical line.
2. Solves problem with combination of simple harmonic motion and motion under gravity.

Guidelines to learning - teaching process :

1.
   - Simple Harmonic motion of a particle in a vertical line under the action of elastic forces and its own weight.
   - Combination of SHM and a free motion under gravity.
2. Guide students to solve problem with Combination of simple harmonic motion and motion under gravity.

Competency 2 : Uses systems of coplanar forces.

Competency level 2.12 : Applies various techniques to determine the centre of mass of symmetrical uniform bodies.

Number of periods : 04

Learning outcomes :

1. Defines the centre of mass of a system of particles in a plane.
2. Defines the centre of mass of a lamina.
3. Finds the centre of mass of uniform bodies symmetrical about a line.
4. Finds the centre of mass of bodies symmetrical about a plane.
5. Finds centre of mass of Lamminas of different shapes.
6. Finds centre of mass of uniform triangular lamina using thin rectangular stripes.
7. Finds centre of mass of uniform lamina in the shape of parallelogram using thin rectangular stripes.

Guidelines to learning - teaching process :

1. **Centre of Mass (Gravity):**

   Let the mass of the particle at $P_r \equiv (x_r, y_r)$ with respect to rectangular Cartesian coordinate system chosen in the plane of a coplanar system of particles, be m_r where $r = 1, 2, 3, \ldots. n$

   There exists a point $G \equiv (\overline{x}, \overline{y})$ in the plane of the system of particles such that,

   $$\overline{x} = \frac{\sum_{r=1}^{n} m_r x_r}{\sum_{r=1}^{n} m_r} \quad \text{and} \quad \overline{y} = \frac{\sum_{r=1}^{n} m_r y_r}{\sum_{r=1}^{n} m_r}$$

   G is called the centre of mass of the system of particles.

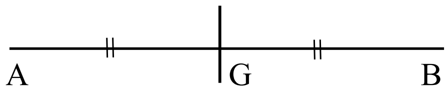
2. • The weight of a body is equal to the weights of its constituent particles and acts vertically downward through a fixed point in the body. The fixed point is called the centre of gravity where the fixed point is independent of the orientation of the body.

- Let a tiny mass at the point $P \equiv (x, y)$ with respect to a Cartesian system of coordinates chosen in the lamina be dm

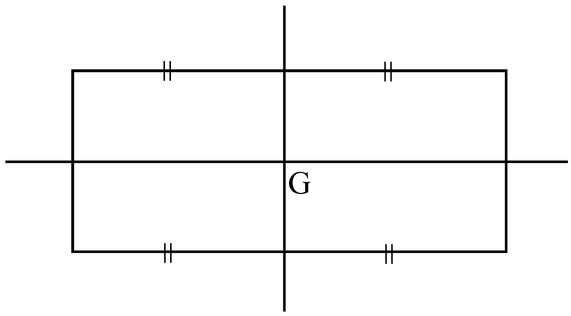
- The point $G \equiv (\overline{x}, \overline{y})$ is such that $\overline{x} = \dfrac{\int x dm}{\int dm}$ and $\overline{y} = \dfrac{\int y dm}{\int dm}$

3. - Bodies in which the masses are distributed with the same constant density are known as uniform bodies.

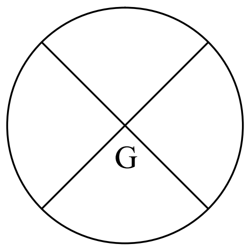
- Centre of gravity of a thin uniform rod,



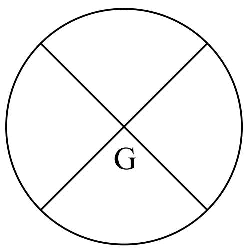
4. - Centre of gravity of a uniform rectangular lamina.



- Centre of gravity of a uniform circular ring



- Centre of gravity of a uniform rectangular disc.



5. • Discuss the centre of gravity of the following uniform bodies.

   - hollow cylinder
   - solid cylinder
   - hollow sphere
   - solid sphere

6. Centre of gravity of a uniform triangular lamina.

   Show that the centre of gravity of a triangle lies at the point of intersection of the medians.

   That is two thirds of the distance from each vertex to the midpoint of the opposite side.

7. Centre of gravity of a uniform parallelogram.

   Show that the centre of gravity of a parallelogram is the point of the intersection of its diagonals.

Competency level 2.13 : Finds the centre of mass of simple geometrical bodies using integration.

Number of periods : 06

Learning outcomes :
1. Finds the centre of mass of uniform bodies symmetric about a line using integration.
2. Finds the centre of mass of uniform bodies symmetrical about a plane using integration.

Guidelines to learning - teaching process :

- When body cannot be divided into a finite number of parts with known centre of gravity it may be divided into infinite number of parts with known centres of gravity.
- Summing the moments of the parts is done by integration.

1. Show by integration that
   - The centre of gravity of a uniform circular arc of radius a subtending an angle

   2α at the centre lies at a distance $\dfrac{a\sin\alpha}{\alpha}$ from the centre along the axis of symmetry.

- The centre of gravity of a uniform circular sector of radius a subtending an angle 2α at the centre lics at a distance $\frac{2a\sin\alpha}{3\alpha}$ from the centre along the axis of symmetry.

2. Show by integration

- Show that the centre of gravity of a solid hemisphere with radius a lies at a distance $\frac{3a}{8}$ from the centre along the axis of symmetry.

- Show that the centre of gravity of a hollow hemisphere with radius a lies at a distance $\frac{a}{2}$ from plane face along the axis of symmetry.

- Show that the centre of gravity of a uniform solid right circular cone of height h lies at a distance $\frac{h}{4}$ from base along the axis of symmetry.

- Show that the centre of gravity of a uniform hollow cone of height h lies at a distance $\frac{h}{3}$ from base along the axis of symmetry.

Competency level 2.14 :	Finds the centre of mass composite bodies and remaining bodies.
Number of periods :	04
Learning outcomes :	1. Finds the centre of mass of composite bodies. 2. Finds the centre of mass of remaining bodies.

Guidelines to learning - teaching process :

1. Find the centre og gravity of comppsite bodies.
2. Discuss the problems involve with remaining bodies and made of two or more bodies with known mass and centre of gravity using priciple of momentum..

Competency level 2.15 : Explains centre of gravity.

Number of periods : 02

Learning outcomes :

1. Explains centre of gravity of a body.
2. States the centre of mass and centre of gravity are same under gravitational field.

Guidelines to learning - teaching process :

1. Introduce the centre of gravity of a rigid body.
2. Discuss the coincidence of the centre of gravity and centre of mass.

Competency level 2.16 : Determines the stability of bodies in equilibrium.

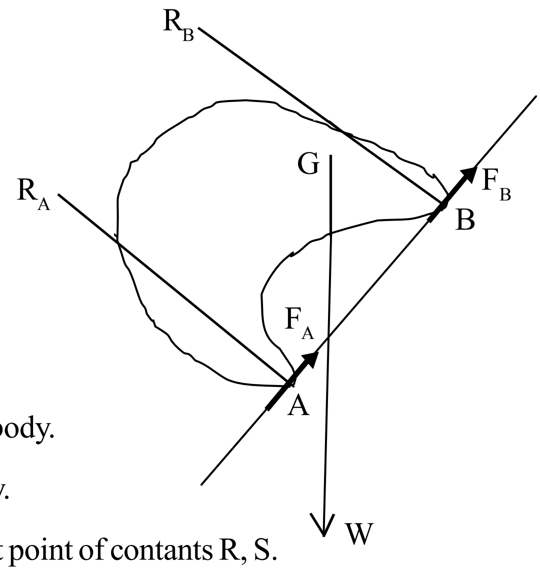
Number of periods : 02

Learning outcomes :

1. Explains the stability of bodies in equilibrium using centre of gravity.

Guidelines to learning - teaching process :

1. Stability of equilibrium of bodies resting on a plane.



Forces acting on the body.

- Weight of the body.
- Normal reaction at point of contants R, S.
- Frictional force at A, B.

To equlibrium

Vertical line passing through the centre of gravity should lies between A and B If this line falls outside of A and B then the equlibrium will collabse by topping.

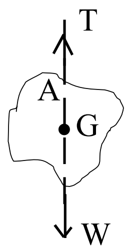
Competency level 2.17 : Determines the angle of inclination of suspended bodies.

Number of periods : 02

Learning outcomes : 1. Solves problem involving suspended bodies.

Guidelines to learning - teaching process :

1. Guide students to solves problem involving suspended bodies.



Hanging bodies

Since the body in equilibrium under the action of two forces they must be equal and opposite to each other.

Example : $T = W$, and AG vertical.

THIRD TERM

Combined Mathematics I

Competency 22 : Explores the binomial expansion for positive integral indices.

Competency Level 22.1 : Describes the basic properties of the binomial expansion.

Number of periods : 06

Learning Outcomes :

1. States binomial theorem for positive integral indices.
2. Writes general term and binomial coefficient.
3. Proves the binomial theoram using Mathematical Induction.

Guidelines to learning - teaching process :

1. States the binomial theorem for a positive integral index,

$$(a+b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1}b + \ldots + {}^nC_r a^{n-r}b^r + \ldots + {}^nC_n b^n,$$

Where ${}^nC_r = \dfrac{n!}{(n-r)!r!}$ for $0 \le r \le n$

2. r^{th} term of the expansion is $T_r = {}^nC_{r-1} a^{n-r+1} b^{r-1}$, for $1 \le r \le n$

3. Guide students to prove the binomial theorem using Mathematical Induction.

Competency Level 22.2 : Applies binomial theorem.

Number of periods : **06**

Learning Outcomes :
1. Writes the relationship among the binomial coefficients.
2. Finds the specific terms of binomial expansions.

Guidelines to learning - teaching process :

1. In the expansion $\left(a+x\right)^n = {}^nC_0 a + {}^nC_1 a^{n-1}x + \cdots + {}^nC_r a^{n-r}x^r + \cdots + C_n x^n$,

   ${}^nC_0,\ {}^nC_1, \cdots,\ {}^nC_n$ are called binomial coefficients.
   ${}^nC_0 a^n,\ {}^nC_1 a^{n-1}, \cdots, {}^nC_n$ are the coefficients of terms in the expansion.

2. • The number of terms in the expansion is $\left(n+1\right)$

   • General term of the expansion is $T_{r+1} = {}^nC_r a^{n-r}x^r$

   • $\left(1+x\right)^n = \sum_{r=0}^{n} {}^nC_r x^r$

   Using the above expansion obtain the properties of binomial coefficients.

   • Guide students to solve problems involving properties of the binomial coefficients.

Competency 23 : Interprets the System of Complex Numbers.

Competency Level 23.1: Uses the Complex Number System.

Number of periods : 02

Learning Outcomes :

1. States the imaginary unit.
2. Defines a complex number.
3. States the real part and imaginary part of a complex number.
4. Uses the equality of two complex numbers.

Guidelines to learning - teaching process :

1. • Introduce the imaginary unit *i* such that $i^2 = -1$
   • The numbers of the form *ai,* where $a \in R$, are called pure imaginary numbers.

2. A complex number is defined as $z = a + ib$, where $a, b \in R$ and $i^2 = -1$.

3. • The number *a* is called the real part of the complex number *z* and it is denoted by Re (*z*).
   • The number and *b* is called the imaginary part of the complex number *z* and denoted by Im(*z*).

4. If $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$ are two complex numbers with $a_1,\ a_2,\ b_1,\ b_2 \in \mathbb{R}$
   then $z_1 = z_2 \Leftrightarrow a_1 = a_2$ and $b_1 = b_2$.

Competency Level 23.2 : Introduces algebraic operation on complex numbers.

Number of periods : 02

Learning Outcomes :

1. Defines algebraic operations on complex numbers.
2. Uses algebraic operations between two complex numbers and verifies that they are also complex numbers.
3. Basic opertions on complex numbers.

Guidelines to learning - teaching process :

1. Let $z_1 = a_1 + ib_2,\ z_2 = a_2 + ib_2$ with $a_1,\ a_2,\ b_1,\ b_2 \in \mathbb{R}$and $\lambda \in R$ then

- $z_2 + z_2 = (a_1 + a_2) + i(b_1 + b_2)$
- $\lambda z = \lambda(a + ib) = \lambda a + i\lambda b$
- $z_1 - z_2 = (a_1 - a_2) + i(b_1 - b_2)$
- $z_1 z_2 = (a_1 a_2 - b_1 b_2) + i(a_1 b_2 + a_1 b_2)$
- $\dfrac{z_1}{z_2} = \dfrac{a_1 + ib_1}{a_2 + ib_2} = \left(\dfrac{a_1 a_2 + b_1 b_2}{a_2^2 + b_2^2}\right) + i\left(\dfrac{a_2 b_1 - a_1 b_2}{a_2^2 + b_2^2}\right)$ for $z_2 \neq 0$

2. Uses algebraic operations between two complex numbers and verifies that they are also complex numbers.

3. Guide students to do basic opertions on complex numbers.

Competency Level 23.3 : Defines and proves basic properties of complex conjugate.

Number of periods : 02

Learning Outcomes:
1. Defines the complex conjugate $\bar{z}$ of a complex number z.
2. States basic properties of complex conjugate.
3. Proves the basic properties of complex conjugate.

Guidelines to learning - teaching process :

1. If $z = a + ib$,

   then the complex conjugate of $z\left(\text{denoted } \bar{z}\right)$ is defined by $\bar{z} = a - ib$.

2. States the following properties of complex conjugates.

- $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$
- $\overline{z_1 - z_2} = \overline{z_1} - \overline{z_2}$
- $\overline{z_1 z_2} = \overline{z_1}.\overline{z_2}$
- $\overline{\left(\dfrac{z_1}{z_2}\right)} = \left(\dfrac{\overline{z_1}}{\overline{z_2}}\right)$

3. • Guide students to prove the above properties.
   • Guide students to applies the basic properties of complex conjugates.

Competency Level 23.4 : Define and proves the properties of modules of a complex number.

Number of periods : 04

Learning Outcomes :
1. Defines the modules $| z |$ of a complex number z,
2. Proves basic properties of modules of a complex number.
3. Applies the basic properties of modules of a complex number.

Guidelines to learning - teaching process :

1. Let $z = x + yi$ be a complex number with $x, y \in \mathbb{R}$.

   $|z| = \sqrt{x^2 + y^2}$

   Then the modulus of a $\mathbb{Z}$ (denoted by $| \mathbb{Z} |$)

2. Proves the following properties.
   - $|z_1 z_2| = |z_1||z_2|$
   - $\left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}$ if $z_2 \neq 0$
   - $z.\overline{z} = |z|^2$
   - $|z_1 + z_2|^2 = |z_1|^2 + 2\,\mathrm{Re}\left(z_1.z_2\right) + |z_2|^2$

3. Guide students to applies the above properties of modules of a complex number.

Competency Level 23.5 : Illustrates algebraic operations geometrically using the Argand diagram.

Number of periods : 04

Learning Outcomes :

1. Represents complex numbers on the Argand diagram.
2. Constructs points representing $z_1 + z_2,\ z_1 - z_2,\ \overline{z}$ and λz_2 where $\lambda \in \mathbb{R}$.
3. Expresses a non - zero complex number in polar form

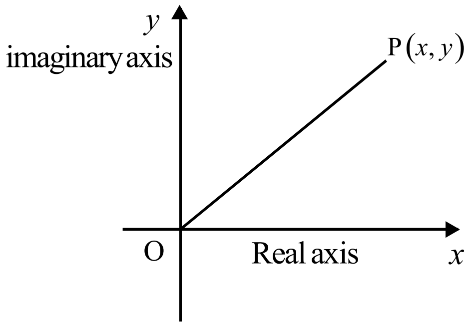
   $z = r(\cos\theta + i\ \sin\theta);\ r > 0$ and $\theta \in \mathbb{R}$
4. Defines the argument of a non - zero complex number.
5. Defines the principal argument of a non - zero complex number.
6. Constructs points representing $r(\cos\alpha + i\sin\alpha)$,

   where $\alpha \in \mathbb{R},\ r \neq 0$
7. Constructs points representing $z_1 z_2$ and $\dfrac{z_1}{z_2}$
8. Constructs points representing $\dfrac{\lambda z_1 + \mu z_2}{\lambda + \mu}$ where

   $\lambda,\ \mu \in \mathbb{R}$ and $\lambda + \mu \neq 0$.
9. Proves the triangle inequality.
10. Deduces the reverse triangle inequality.
11. Uses the above inequalities to solve problems.

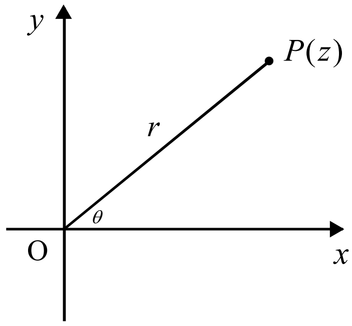
Guidelines to learning - teaching process :

1. Introduces Argand diagram (complex plane)
   Represents a complex number as a point in the Argand diagram.
   Let $z = x + iy$ with $x, y \in \mathbb{R}$ then the point $P(x, y)$ represents z in the Argand diagram.



2. • For a given z construct the points representing
   • λz • $\overline{z}$
   • Given two complex numbers z_1 and z_2 construct the point representing $z_1 + z_2$.

3. Let z be a non- zero complex number.



   Then $z = r(\cos\theta + i\sin\theta)$; $r > 0$ and $\theta \in \mathbb{R}$

4. Let z be a non - zero complex number.
   An angle θ satisfying $z = r(\cos\theta + i\sin\theta)$ is called an argument of z.
   The set of all values of θ for which $z = r(\cos\theta + i\sin\theta)$ is denoted by $\arg z$.

5. Let z be a non-zero complex number.
   The value of θ for which $z = r(\cos\theta + i\sin\theta)$, with $-\pi < \theta \leq \pi$ is denoted by $\operatorname{Arg} z$.
   $\operatorname{Arg} z$ is called the principal value of the argument.

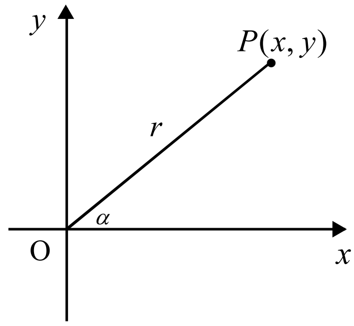
6. Let $z = x + iy$ be a non - zero complex number, then,

$$z = \sqrt{x^2+y^2}\left\{\frac{x}{\sqrt{x^2+y^2}} + i\frac{y}{\sqrt{x^2+y^2}}\right\}$$

$$= r(\cos\alpha + i\sin\alpha)$$

where $r = \sqrt{x^2+y^2}$

$\cos\alpha = \dfrac{x}{r}$ and $\sin\alpha = \dfrac{y}{r}$



7. $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$, $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$ then show that

$$z_1 z_2 = r_1 r_2\left[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\right]$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)\right]$$

Show that the construction of $z_1 z_2$ and $\dfrac{z_1}{z_2}$ in the Argand diagram.

8. Given two complex numbers z_1, z_2, construct the points represent.

$\dfrac{\lambda z_1 + \mu z_2}{\lambda + \mu}$ where $\lambda,\ \mu \in \mathbb{R}$ in the Argand Diagram.

9. Obtain the triangle inequality $|z_1 + z_2| \le |z_1| + |z_2|$ for $z_1, z_2 \in \mathbb{C}$.

10. Deduce that $\big||z_1| - |z_2|\big| \le |z_1 - z_2|$ for $z_1, z_2 \in \mathbb{C}$.

11. Guide students to solve problems, using above inequalities.

Competency Level 23.6 : Uses DeMovier's theorem.

Number of periods : 02

Learning Outcomes :
1. States and prove DeMovier's theorem for a positive integral index.
2. Solves problems involving elementary applications of DeMovier's theorem.

Guidelines to learning - teaching process :

1. Prove DeMovier's theorem which states that if
   $z = r(\cos\theta + i\sin\theta)$ then $z^n = r^n(\cos\theta + i\sin n\theta)$, where $n \in \mathbb{Z}$
   Using the principle of mathetical induetion.

2. Guide students to solves problems involvings elementary applications of DeMovier's theorem.

Competency Level 23.7 : Identifies locus/region of a variable complex number.

Number of periods : 04

Learning Outcomes :
1. Sketches the locus of variable complex numbers in the Argand diagram.
2. Obtains the Cartesian equation of a locus.

Guidelines to learning - teaching process :

1. Let the complex numbers z, z_0, z_1 and z_2 be represented by the points P, P_0, P_1, P_2 respectively.
   Sketches the following locases.
   - the locus of z given by $|z - z_0| = r$ is a circle with centre P_0 and radius r. Obtain the cartesian equation of the locus.
   - the locus of z given by the equation $\text{Arg}(z - z_0) = \alpha$ is the half line PP_0 which make an angle α with positive direction of the x-axis.
   - the locus of z given by the equation $|z - z_1| = |z - z_2|$ is the line which is perpendicular bisector of P_1P_2 and obtain the cartesian equation of the line.
2. Guide to obtain the cartesian equation of the above lici.

Competency 25 : Manipulates Matrices.

Competency Level 25.1 : Describes basic properties of Matrices.

Number of periods : 02

Learning Outcomes :

1. Defines a Matrix.
2. Defines row matrices and columns matrices.
3. Defines the equality of matrices.
4. Defines the multiplication of a matrix by a scalar.
5. Writes the condition for compatibility for addition.
6. Uses the addition of matrices to solve problems.
7. Defines subtraction using addition and scalar multiplication.
8. Writes the condition for compatibility for multiplication.
9. Defines multiplication of matrices.
10. Uses the properties of multiplication to solve problems.

Guidelines to learning - teaching process :

1. **Matrices:**

   Matrix is a rectangular array of numbers. Matrices are denoted by capital letters A, B, C.. etc.

$$\text{A} = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \cdots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$$

If A has m rows and n columns the order of the matrix A is $m \times n$.

Element of a matrix : a_{ij} is the element of i^{th} row and j^{th} column.

A is also written as $\left(a_{ij}\right)_{m\times n}$.

2. • **Row matrix**

   A matrix which has only one row is called a row matrix or a row vector.

   • **Column matrix**

   A matrix which has only one column is called a column matrix or column vector.

   • **Null matrix**

   If every element of a matrix is zero, it is called a null matrix or a zero matrix.

3. If two matrices $\mathrm{A} = \left(a_{ij}\right)_{m\times n}$ and $\mathrm{B} = \left(b_{ij}\right)_{m\times n}$ are of the same order and if

$a_{ij} = b_{ij}$

for $i = 1, 2, 3, ..., m$ and $j = 1, 2, 3, ..., n$ then $\mathrm{A} = \mathrm{B}$

4. If $\mathrm{A} = \left(a_{ij}\right)_{m\times n}$; $\lambda \in \mathbb{R}$, then $\lambda\mathrm{A} = \left(\lambda a_{ij}\right)_{m\times n}$

When $\lambda = -1$, $(-1)\mathrm{A}$ is denoted by $-\mathrm{A}$.

5. State the conditions.
   - Matrices are of the same order,
   - Corresponding element can be added.

     Addition is commutative and associative.

6. Guide students to use addition of matrices to solve problems.

7. Denoted by subtraction using addition and scalor multiplication.

8. Let $\mathrm{A}_{m\times p}$ and $\mathrm{B}_{q\times n}$ be two matrices.
   Multiplication AB is compatible when $p = q$.

9. If $\mathrm{A}=\left(a_{ij}\right)_{m\times p}$ and $\mathrm{B}=\left(b_{ij}\right)_{p\times n}$ then,

$$AB = \left(\sum_{k=1}^{P} a_{ik}b_{kj}\right)_{m\times n}$$ and it is of order $m\times n$

Discuss that even when AB is defined, BA is not necessarily defined.
In general $\mathrm{AB} \neq \mathrm{BA}$

10. Guide student to uses the properties of multiplication to solve problems.

Competency Level 25.2: Explains special cases of square matrices.

Number of periods: 02

Learning Outcomes:
1. Identifies the order of a square matris.
2. Defines special types of matrices.

Guidelines to learning - teaching process:

1. Define a square matrix.
If $m = n$ in a matrix A of order $m \times n$, then A is called a square matrix of order n.

2. $$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix}$$

$(a_{11}, a_{22}, a_{33}, ..., a_{nn})$ is the leading (Principal) diagonal.

- A square matrix A of order n is said to the identity matrix or unit matrix of order n and denoted by I_n if $a_{ij} = \begin{cases} 1 \text{ if } i = j \\ 0 \text{ if } i \neq j \end{cases}$
- A square matrix A is said to be diagonal matrix if $a_{ij} = 0$ for all $i \neq j$.
- A square matrix A is said to be symmetric matrix if $A^T = A$.
- A square matrix A is said to be skew symmetric matrix if $A^T = -A$
- A square matrix A is said to be upper triangular matrix if $a_{ij} = 0$ when $i > j$.
- A square matrix A is said to be lower triangular matrix if $a_{ij} = 0$ when $i < j$.

Competency Level 25.3 : Describes the transpose and the inverse of a matrix.

Number of periods : 04

Learning Outcomes :
1. Finds the transpose of a matrix.
2. Finds the determinant of a 2×2 matrices.
3. Finds the inverse of a 2×2 matrix.

Guidelines to learning - teaching process :

1. Let $\mathrm{A}=\left(a_{ij}\right)_{m\times n}$.

   Then $\mathrm{A}^{\mathrm{T}}=(a_{ji})_{nxm}$

2. Find the value of a 2×2 determinant.

   For a given a matrix $\mathrm{A}=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, determinant of A is denoted by det A or $|\mathrm{A}|$, and it is defined by $\det \mathrm{A}=|\mathrm{A}|=ad - bc$.

3. States that for a given square matrix A, if there exists a matrix B such that $\mathrm{AB}=\mathrm{I}_{\mathrm{n}}=\mathrm{BA}$, then B is said to be the inverse of A and it is denoted by A^{-1}.

   Therefore, $\mathrm{AA}^{-1}=\mathrm{I}_{\mathrm{n}}=\mathrm{AA}^{-1}$.

   Show that
   - $\left(\mathrm{A}^{-1}\right)^{-1}=\mathrm{A}$
   - $\left(\mathrm{A}^{-1}\right)^{\mathrm{T}}=\left(\mathrm{A}^{\mathrm{T}}\right)^{-1}$
   - $(\mathrm{AB})^{-1}=\mathrm{B}^{-1}\mathrm{A}^{-1}$

   Given that $\mathrm{A}=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $|\mathrm{A}|\neq 0$, show that $\mathrm{A}^{-1}=\dfrac{1}{|\mathrm{A}|}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.

Competency Level 25.4 : Uses matrices to solve simultaneous equations.

Number of periods : 06

Learning Outcomes :
1. Examine the solutions of a pair of linear equation.
2. Solves simultaneous equations using matrices.
3. Illuatrates the solution graphically.

Guidelines to learning - teaching process :

1. Given that $a_1x + b_1y = c_1$

   $a_2x + b_2y = c_2$

   write the above equations in the form $AX = C$,

   where $A = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \end{pmatrix}$ and $C = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$.

   If A^{-1} exists, then

   $A^{-1}(AX) = A^{-1}C$

   $\left(A^{-1}A\right)X = A^{-1}C$.

   i.e. $X = A^{-1}C$

   Discuss solutions of simultaneous equations to illustrate the following situations.

   - Unique solution
   - Infinite number of solutions
   - No solution.

2. Guide students to solves simultaneous equations using matrices.

3. Guide students to illustrates the solutions graphically.

Combined Mathematics II

Competency 4 : Applies mathematical models to analysis events of a random experiment.

Competency Level 4.3 : Applies the concept of conditional probability to determine the probability of an event on random experiment under given conditions.

Number of periods : 08

Learning Outcomes :
1. Defines conditional probability.
2. States and proves the theorems on conditional probability.
3. States multiplication rule.

Guidelines to learning - teaching process :

1. Let S be the sample space of a random experiment and A and B be two events where P(A)>0, then the conditional probability of the event B given that the event A has occured, denoted by *P(B|A)*, is defined as $P(B \mid A)=\dfrac{P(B\cap A)}{P(A)}$

2. Prove that,

   (i) If $P(A)>0$, then $P(\phi \mid A)=0$

   (ii) If $A,\ B\in S$ and $P(A)>0$ then $P(B^{/}\mid A)=1-P(B\mid A)$

   (iii) If $A,\ B_1, B_2 \in S$,then

   $$P(B_1\mid A)=P(B_1\cap B_2\mid A)+P(B_1\cap B_2^1\mid A)$$

3. Let A_1, A_2 are any two events in a sample space with $P(A_1)>0$ then, $P\ (A_1\cap A_2)=P(A_1)\cdot P(A_2\mid A_1)$

Competency Level 4.4 : Uses the probability model to determine the independence of two or three events.

Number of periods : 04

Learning Outcomes :
1. Defines independence of two events.
2. Defines pairwise independent.
3. Defines mutually independent.
4. Uses independence of two or three events to solve problems.

Guidelines to learning - teaching process :

1. Let A_1, A_2 be two events in $\mathcal{E}$ and if A_1 and A_2 are independent then $P(A_1 \cap A_2) = P(A_1).P(A_2)$

2. Let A_1, A_2 and A_3 be three events in S and if A_1, A_2 and A_3 are pairwise independent then ,

$$P\ (A_1 \cap A_2) = P(A_1) \cdot\ P(A_2)$$
$$P\ (A_2 \cap A_3) = P(A_2) \cdot\ P(A_3)$$
$$P\ (A_3 \cap A_1) = P(A_3) \cdot\ P(A_1)$$

Pairwise Independent means that each event is independent of every other possible combination.

3. Events A_1, A_2 and A_3 are mutually independent if they are pairwise independent.

$$P\ (A_1 \cap A_2) = P(A_1) \cdot P(A_2)$$
$$P\ (A_2 \cap A_3) = P(A_2) \cdot P(A_3)$$
$$P\ (A_3 \cap A_1) = P(A_3) \cdot P(A_1)$$
and
$$P\ (A_1 \cap A_2 \cap A_3) = P(A_1) \cdot P(A_2) \cdot P(A_3)$$

4. Guide students to solve problems involving independence of events (maximum three)

Competency Level 4.5: Applies Baye's theorem to solve problems.

Number of periods: 06

Learning Outcomes:
1. Defines a partition of a sample space.
2. States and proves theorem of total probability.
3. States Baye's theorem.
4. Solves problems using above theorems.

Guidelines to learning - teaching process :

1. Let $B_1, B_2,..., B_n$ be events in the event space related to sample space S of a random experiment.
   $\{B_1, B_2, B_3,..., B_n\}$ is said to be a partition of S if
   - $\bigcup_{i=1}^{n} B_i = S$
   - $B_i \cap B_j = \phi \quad (i \neq j, 1 \leq i, j \leq n)$

2. Let $\{B_1, B_2,..., B_n\}$ be a partition of the sample space S.
   If $P(B_i) > 0$ and if A is any event in the sample space S then,
   $$P(A) = \sum_{i=1}^{n} P(A \mid B_i) P(B_i)$$

3. Let $\{B_1, B_2,..., B_n\}$ be a partition of the sample space S. If A is any event in S then
   $$P(B_j \mid A) = \frac{P(A \mid B_j) P(B_j)}{\sum_{i=1}^{n} P(A \mid B_i) P(B_i)}$$

4. Guide students to solves problems involving above theorems.

Competency 5 : Applies statistical tools to develop decision making skills.

Competency Level 5.1 : Introduces to nature of statistics.

Number of periods : 01

Learning Outcomes :
1. Explains what is statistics.
2. Explains the nature of statistics.

Guidelines to learning - teaching process :

1. **Statistics :**
   State that statistics is the science of obtaining and analyzing quantitative data with a view to make inferences and decisions.
   A statistic refers to a summary figure computed from a data set.

2. **Statistics can be divided into two areas.**
   - Descriptive statistics
   - Inferential statistics

   Descriptive statistics consists of methods for organizing displaying and describing data by using tables, graphs and summary measures.

   Inferential statistics consists of methods that use sample results to help make decisions or predictions about a population.

Competency Level 5.2 : Describes measures of central tendency.

Number of periods : 03

Learning Outcomes :
1. Describes the mean, median and mode as measures of central tendency.
2. Finds the central tendency measurements.
3. Finds weighted mean.

Guidelines to learning - teaching process :

1. State that mean, median and mode are the measures of central tendency for a set of data.

] **Mean:**

- The mean $\overline{x}$ of a set of data $x_1, x_2, ..., x_n$ is defined by $\overline{x} = \dfrac{\sum_{i=1}^{n} x_i}{n}$.

- Let $x_1, x_2, ..., x_n$ be a set of data with frequencies $f_1, f_2, ..., f_n$ respectively. Mean (Arithmetic mean) is defined as

$$\overline{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}.$$

- Discuss coding method.

- Discuss weighted mean:
- For a grouped data sets with $x_1, x_2, x_3, ..., x_{n,}$ are the mid points of the classes and $f_1, f_2, f_3, ..., f_n$ are the corresponding frequenctes then $\overline{x} = \dfrac{\Sigma f_i x_i}{\Sigma f_i}$.

- Discuss the coding as $y = \dfrac{x_i - a}{b}$ obtain the formula $\overline{x} = a + b\overline{y}$ under this coding

Mode:

State mode as a value of a variable which has the greatest frequency in a set of data.

- Mode may have more than one value.
- For a grouped frequency distribution, mode is given by

Mode = $L_{mo} + c\left(\dfrac{\Delta_1}{\Delta_1 + \Delta_2}\right)$, where

L_{mo} is the lower boundary of the modal class,
c is the size of the class interval,

$\Delta_1 = f_{mo} - f_{mo-1}$,

$\Delta_2 = f_{mo} - f_{mo+1}$ and

f_{mo} is the frequency of the modal class.

- Guide students to obtain the formula and use it.

Median:

Median is the middle value of an ordered set of data.

- Let $x_1, x_2, ..., x_n$ be the ordered set of n data.

Median is the $\left(\frac{n+1}{2}\right)^{th}$ value of the ordered set.

Discuss the cases

- When n is odd,
- When n is even.

- Discuss for ungrouped frequency distribution also.

- For a grouped frequency distribution

Median $= L_m + \dfrac{\left(\frac{N}{2} - f\right)c}{f_c}$, where

L_m is the lower class boundary of the median class.

c is the size of the class interval.

f is the sum of all frequencies below L_m.

f_m is the frequency of the median class.

- N - Total Frequency

3. Discuss the weighted mean:

$x_1, x_2, x_3, ..., x_n$ is the data set and w_i is the weight for the data x_i for $\forall_i$

$$\overline{x} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}.$$

Competency Level 5.3 : Interprets a frequency distribution using measures of relative positions.

Number of periods : 04

Learning Outcomes :
1. Finds the relative position of frequency distribution.
2. Uses Box plot to represent data.

Guidelines to learning - teaching process :

1. • Quartiles for a grouped distribution.

First Quartile (Q_1) :

Q_1 is the $\left(\frac{n+1}{4}\right)^{th}$ value of the data arranged in the ascending order.

Second Quartile (Q_2):

Q_2 is the $\left(\frac{n+1}{2}\right)^{th}$ value of the data arranged in the ascending order.

Third Quartile (Q_3) :

Q_3 is the $\frac{3}{4}(n+1)^{th}$ value of the data arranged in the ascending order.

Note that Q_2 (median) is the 2nd Quartile.

Discuss ungrouped frequency distributions and ungrouped frequency distributions with examples

- Quartiles for grouped frequency distribution.

k^{th} quartile is Q_k is defined as

$$Q_k = L_k + \frac{\left(\frac{KN}{4} - f\right)C}{f_k}, \text{ for } k = 1, 2, 3 \text{ where}$$

- L_k - is the lower class boundary of the guartile class.
- C - is the size of the class interval.
- f - is the sum of all frequencies below L_m and f is the frequency of the k^{th} quartile class.

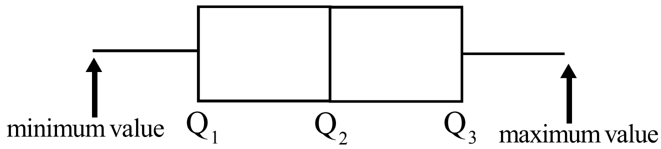
N - Total frequency

Percentile :

p^{th} percentile of the data is given by $\left(\frac{pn}{100}\right)^{th}$ value of the data arranged in the ascending order.

Discuss the integer as well as the non-integer cases.

2. Box plot for represent data.



Competency Level 5.4 : Describes measurs of dispersion.

Number of periods : 08

Learning Outcomes :

1. Uses suitable measures of dispersion to make decisions on frequency distributions.
2. States the measures of dispersion and their importance.
3. Explains pooled mean and pooled variance.
4. Obtains formulas for pooled mean and pooled variance.
5. Describes Z -Score.
6. Applies measures of dispersion to solve problems.

Guidelines to learning - teaching process :

1. Discuss the uses of the measures of central tendency in frequency distributions.

   Explain with suitable examples.

2. Measures of dispersion are indicates the sprad of data and they are used to represent the spread within data.

   Define the following types of measures of dispersion.

   - Range : Range is the difference between the largest value and the smallest value.
   - Inter-quartile Range: Inter-quartile Range = $Q_3 - Q_1$
   - Semi inter-quartile Range = $\frac{Q_3 - Q_1}{2}$

- Mean Deviation :

  For a set of data $x_1, x_2, ..., x_n$, Mean deviation = $\dfrac{\sum_{i=1}^{n} |x_i - \overline{x}|}{n}$.

  For a frequency distribution, Mean deviation is $\dfrac{\sum_{i=1}^{n} f_i |x_i - \overline{x}|}{\sum f_i}$

  where f_i is the freauency of the data x_i.

  for a grouped frequency distribution M.D. = $\dfrac{\sum_{i=1}^{n} f_i |x_i - \overline{x}|}{n}$

Variance :

- For a set of data $x_1, x_2, ..., x_n$,

  Variance = $\dfrac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{n}$, and obtain the following formula for

  variance = $\dfrac{\sum_{i=1}^{n} x_i^2}{n} - \overline{x}^2$.

- For a frequency distribution,

  Variance = $\dfrac{\sum_{i=1}^{n} f_i (x_i - \overline{x})^2}{\sum_{i=1}^{n} f_i}$ and obtain the following formula for a

  grouped frequency distribution variance = $\dfrac{\sum_{i=1}^{n} f_i x_i^2}{\sum_{i=1}^{n} f_i} - \overline{x}^2$.

  for this grouped freqency distribution, x_i is the mid value of the i^{th} class.

- Standard Deviation (S_x) :

  Standard Deviation = $\sqrt{\text{Variance}}$

3. Explains poolmean and pool varience using examples.

4. Pooled mean (Combined mean)

Let $\bar{x}_1$ and $\bar{x}_2$ be the means of two sets of data with sizes n_1 and n_2 respectively.

Show that the pooled mean $\bar{x} = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$.

- Pooled varience

  Let σ_1^2 and σ_2^2 be the variances of two sets of data with sizes n_1 and n_2 respectively. Show that the pooled variance.

$$\sigma^2 = \frac{1}{n_1 + n_2}\left\{n_1\sigma_1^2 + n_2\sigma_2^2\right\} + \frac{n_1 n_2}{\left(n_1 + n_2\right)^2}\left(\bar{x}_1 - \bar{x}_2\right)^2$$

- Guide students to prove and use the above formula.

5. **Z - score**

Let $\bar{x}$ be the mean and S_x be the standard deviation for a set of data $x_1, x_2, ..., x_n$.

For each x_i, corresponding z_i is defined as $z_i = \dfrac{x_i - \bar{x}}{S_x}$

z_i is called z-score of x_i.

For the set of data $z_1, z_2, ..., z_n$ show that the mean is zero and the standard deviation is one.

- Guide students to solve problems involving Z score.

6. Discuss about the linear transformation $y = ax+b$ and prove that,

- $\bar{y} = a\bar{x} + b$
- $\sigma_x = |b|\sigma_y$
- Guide students to solve problems involving linear transformation.

Competency Level 5.5 : Determines the shape of a distribution by using measures of skewness.

Number of periods : 02

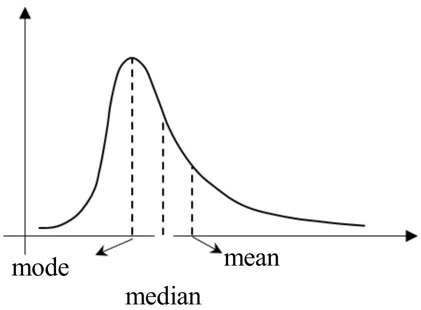
Learning Outcomes :
1. Defines the measure of skewness.
2. Determines the shapes of the distribution using measures of skewness.

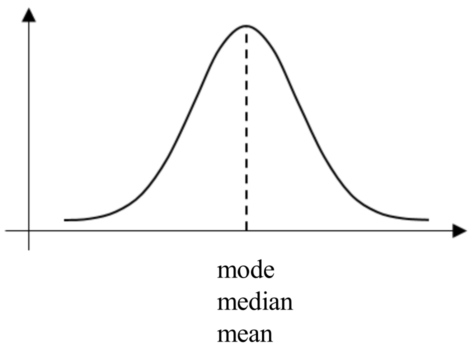
Guidelines to learning - teaching process :

1. Pearson's coefficient of skewness is defined by

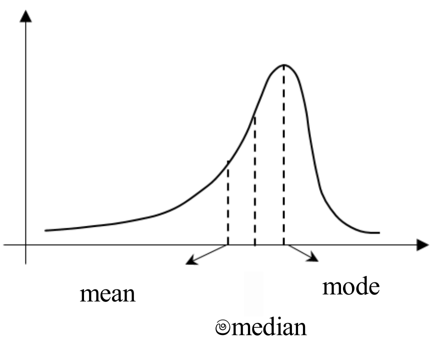
$$K = \frac{\text{Mean} - \text{Mode}}{\text{Standard deviation}} \quad \text{or} \quad K = \frac{3(\text{Mean} - \text{Median})}{\text{Standard deviation}}$$

2. **Positively Skewed** **Symmetric**





Negatively skewed



- Guide students to solve problems involving measures of skewness.

School Based Assessment

Introduction to School Based Assessment

Learning -Teaching and Evaluation are three major components of the process of Education. It is a fact that teachers should know that evaluation is used to assess the progress of learning-teaching process. Moreover, teachers should know that these components influence mutually and develop each other. According to formative assessment (continuous assessment) fundamentals; it should be done while teaching or it is an ongoing process. Formative assessment can be done at the beginning, in the middle, at the end and at any instance of the learning teaching process.

Teachers who expect to assess the progress of learning of the students should use an organized plan. School based assessment (SBA) process is not a mere examination method or a testing method. This programme is known as the method of intervening to develop learning in students and teaching of teachers. Furthermore, this process can be used to maximize the student's capacities by identifying their strengths and weaknesses closely.

When implementing SBA programmes, students are directed to exploratory process through Learning Teaching activities and it is expected that teachers should be with the students facilitating, directing and observing the task they are engaged in.

At this juncture students should be assessed continuously and the teacher should confirm whether the skills of the students get developed up to expected level by assessing continuously. Learning teaching process should not only provide proper experiences to the students but also check whether the students have acquired them properly. For this, to happen proper guiding should be given.

Teachers who are engaged in evaluation (assessment) would be able to supply guidance in two ways. They are commonly known as feed-back and feed- forward. Teacher's role should be providing Feedback to avoid learning difficulties when the students' weaknesses and inabilities are revealed and provide feed-forward when the abilities and the strengths are identified, to develop such strong skills of the students.

Student should be able to identify what objectives have achieved to which level, leads to Success of the Learning Teaching process. Teachers are expected to judge the competency levels students have reached through evaluation and they should communicate information about student progress to parents and other relevant sectors. The best method that can be used to assess is the SBA that provides the opportunity to assess student continuously.

Teachers who have got the above objective in mind will use effective learning, Teaching, evaluation methods to make the Teaching process and learning process effective. Following are the types of evaluation tools student and, teachers can use. These types were introduced to teachers by the Department of Examination and National Institute of Education with the new reforms. Therefore, we expect that the teachers in the system know about them well

Types of assessment tools:

1.	Assignments	2.	Projects
3.	Survey	4.	Exploration
5.	Observation	6.	Exhibitions
7.	Field trips	8.	Short written
9.	Structured essays	10.	Open book test
11.	Creative activities	12.	Listening Tests
13.	Practical work	14.	Speech
15.	Self creation	16	Group work
17.	Concept maps	18.	Double entry journal
19.	Wall papers	20.	Quizzes
21.	Question and answer book	22.	Debates
23.	Panel discussions	24.	Seminars
25.	Impromptus speeches	26.	Role-plays

Teachers are not expected to use above mentioned activities for all the units and for all the subjects. Teachers should be able to pick and choose the suitable type for the relevant units and for the relevant subjects to assess the progress of the students appropriately. The types of assessment tools are mentioned in Teacher's Instructional Manuals.

If the teachers try to avoid administering the relevant assessment tools in their classes there will be lapses in exhibiting the growth of academic abilities, affective factors and psycho- motor skills in the students

Refferences

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Bostock, L. and Chandler, J.(1993). *Applied Mathematics I,* Stanley Thrones (Publishers) Ltd.

Bostock, L. and Chandler, J.(1993). *Applied Mathematics II,* Stanley Thrones (Publishers) Ltd.

Following Resource Books published by Department of Mathematics of National Institute of Education.

Permutation and Combination

Equilibrium of a Particle

Quadratic Function and Quadratic Equations

Polynomial Function and Rational Numbers

Real Numbers and Functions

Inequalities

Statistics

Circle

Probability

Applications of Derivatives

Complex Numbers

Newton's Law

Jointed Rods and Frame Wark

Work, Energy and Power

Centre of Gravity

Circular Motion

Simple Harmonic Motion

Vector Algebra

Straight Line

Derivatives