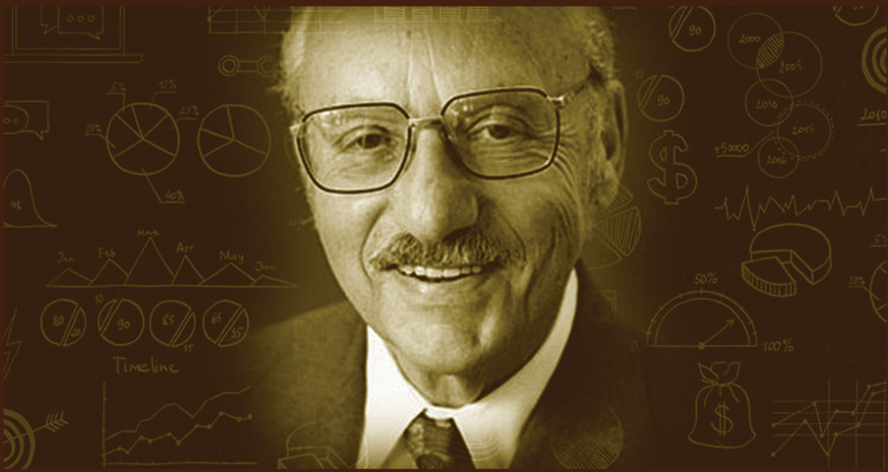
Grade 13 Mathematics

Teachers' Guide

(To be Implemented from 2018)



Department of Mathematics

Faculty of Science & Technology

National Institute of Education

Sri Lanka

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Mathematics

Teachers' Guide

Grade 13

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Department of Mathematics
Faculty of Science and Technology
National Institute of Education
Maharagama

Mathematics
Grade 13 - Teachers' Guide

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Message from the Director General

With the primary objective of realizing the National Educational Goals recommended by the National Education Commission, the then prevalent content based curriculum was modernized, and the first phase of the new competency based curriculum was introduced to the eight year curriculum cycle of the primary and secondary education in Sri Lanka in the year 2007

The second phase of the curriculum cycle thus initiated was introduced to the education system in the year 2015 as a result of a curriculum rationalization process based on research findings and various proposals made by stake holders.

Within this rationalization process the concepts of vertical and horizontal integration have been employed in order to build up competencies of students, from foundation level to higher levels, and to avoid repetition of subject content in various subjects respectively and furthermore, to develop a curriculum that is implementable and student friendly.

The new Teachers' Guides have been introduced with the aim of providing the teachers with necessary guidance for planning lessons, engaging students effectively in the learning teaching process, and to make Teachers' Guides will help teachers to be more effective within the classroom. Further, the present Teachers' Guides have given the necessary freedom for the teachers to select quality inputs and activities in order to improve student competencies. Since the Teachers' Guides do not place greater emphasis on the subject content prescribed for the relevant grades, it is very much necessary to use these guides along with the text books compiled by the Educational Publications Department if, Guides are to be made more effective.

The primary objective of this rationalized new curriculum, the new Teachers' Guides, and the new prescribed texts is to transform the student population into a human resource replete with the skills and competencies required for the world of work, through embarking upon a pattern of education which is more student centered and activity based.

I wish to make use of this opportunity to thank and express my appreciation to the members of the Council and the Academic Affairs Board of the NIE the resource persons who contributed to the compiling of these Teachers' Guides and other parties for their dedication in this matter.

Dr. (Mrs.) T.A.R.J.Gunasekara
Director General
National Institute of Education

Message from the Director

Education from the past has been constantly changing and forging forward. In recent years, these changes have become quite rapid. The Past two decades have witnessed a high surge in teaching methodologies as well as in the use of technological tools and in the field of knowledge creation.

Accordingly, the National Institute of Education is in the process of taking appropriate and timely steps with regard to the education reforms of 2015.

It is with immense pleasure that this Teachers' Guide where the new curriculum has been planned based on a thorough study of the changes that have taken place in the global context adopted in terms of local needs based on a student-centered learning-teaching approach, is presented to you teachers who serve as the pilots of the schools system.

An instructional manual of this nature is provided to you with the confidence that, you will be able to make a greater contribution using this.

There is no doubt whatsoever that this Teachers' Guide will provide substantial support in the classroom teaching-learning process at the same time. Furthermore the teacher will have a better control of the classroom with a constructive approach in selecting modern resource materials and following the guide lines given in this book.

I trust that through the careful study of this Teachers Guide provided to you, you will act with commitment in the generation of a greatly creative set of students capable of helping Sri Lanka move socially as well as economically forward.

This Teachers' Guide is the outcome of the expertise and unflagging commitment of a team of subject teachers and academics in the field Education.

While expressing my sincere appreciation for this task performed for the development of the education system, my heartfelt thanks go to all of you who contributed your knowledge and skills in making this document such a landmark in the field.

Mr. K. R. Pathmasiri
Director
Department of Mathematics.

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Guidlines to use the Teachers' Guide

In the G.C.E (A/L) classes new education reforms introduced from the year 2017 in accordance with the new education reforms implemented in the interim classes in the year 2015. According to the reforms, Teachers' Guide for Mathematics for grade 13 has been prepared.

The grade 13 Teacher's Guide has been organized under the titles competencies and competency levels, content, learning outcomes and number of periods. The proposed lesson sequence is given for the leaning teaching process. Further it is expected that this teachers' Guide will help to the teachers to prepare their lessons and lessons plans for the purpose of class room learning teaching process. Also it is expected that this Guide will help the teachers to take the responsibility to explains the subject matters more confidently. This teachers' Guide is divided into three parts each for a term.

In preparing lesson sequence, attention given to the sequential order of concepts, students ability of leaning and teachers ability of teaching. Therefore sequential order of subject matters in the syllabus and in the teachers Guide may differ. It is adviced to the teachers to follow the sequence as in the teachers' Guide.

To attain the learning outcomes mentioned in the teachers' Guide, teachers should consider the subject matters with extra attention. Further it is expected to refer extra curricular materials and reference materials to improve their quality of teaching. For this purpose teachers can use their self prepared materials .

Total number of periods to teach this mathematics syllabus is 600. Teachers can be flexible to change the number of periods according to their necessity. Teachers can use school based assessment to assess the students.

The teacher has the freedom to make necessary amendments to the specimen lesson plan given in the new teacher's manual which includes many new features, depending on the classroom and the abilities of the students.

We would be grateful if you would send any amendments you make or any new lessons you prepare to the Director, Department of Mathematics, National Institute of Education. The mathematics department is prepared to incorporate any new suggestions that would advance mathematics education in the upper secondary school system.

S. Rajendram
Project Leader
Grade 12-13 Mathematics
National Institute of Education

Common National Goals

The national system of education should assist individuals and groups to achieve major national goals that are relevant to the individual and society.

Over the years major education reports and documents in Sri Lanka have set goals that sought to meet individual and national needs. In the light of the weaknesses manifest in contemporary educational structures and processes, the National Education Commission has identified the following set of goals to be achieved through education within the conceptual framework of sustainable human development.

I Nation building and the establishment of a Sri Lankan identity through the promotion of national cohesion, national integrity, national unity, harmony and peace, and recognizing cultural diversity in Sri Lanka's plural society within a concept of respect for human dignity.

II Recognizing and conserving the best elements of the nation's heritage while responding to the challenges of a changing world.

III Creating and supporting an environment imbued with the norms of social justice and a democratic way of life that promotes respect for human rights, awareness of duties and obligations, and a deep and abiding concern for one another.

IV Promoting the mental and physical well-being of individuals and a sustainable life style based on respect for human values.

V Developing creativity, initiative, critical thinking, responsibility, accountability and other positive elements of a well-integrated and balance personality.

VI Human resource development by educating for productive work that enhances the quality of life of the individual and the nation and contributes to the economic development of Sri Lanka.

VII Preparing individuals to adapt to and manage change, and to develop capacity to cope with complex and unforeseen situations in a rapidly changing world.

VIII Fostering attitudes and skills that will contribute to securing an honourable place in the international community, based on justice, equality and mutual respect.

National Education Commision Report (2003) - December

Basic Competencies

The following Basic Competencies developed through education will contribute to achieving the above National Goals.

(i) Competencies in Communication

Competencies in Communication are based on four subjects: Literacy, Numeracy, Graphics and IT proficiency.

Literacy : Listen attentively, speck clearly, read for meaning, write accurately and lucidly and communicate ideas effectively.

Numeracy : Use numbers for things, space and time, count, calculate and measure systematically.

Graphics : Make sense of line and form, express and record details, instructions and ideas with line form and color.

IT proficiency : Computeracy and the use of information and communication technologies (ICT) in learning, in the work environment and in personal life.

(ii) Competencies relating to Personality Development

- General skills such as creativity, divergent thinking, initiative, decision making, problem solving, critical and analytical thinking, team work, inter-personal relations, discovering and exploring;
- Values such as integrity, tolerance and respect for human dignity;
- Emotional intelligence.

(iii) Competencies relating to the Environment

These competencies relate to the environment : social, biological and physical.

Social Environment :

Awareness of the national heritage, sensitivity and skills linked to being members of a plural society, concern for distributive justice, social relationships, personal conduct, general and legal conventions, rights, responsibilities, duties and obligations.

Biological Environment :

Awareness, sensitivity and skills linked to the living world, people and the ecosystem, the trees, forests, seas, water, air and life-plant, animal and human life.

Physical Environment :

Awareness, sensitivity and skills linked to space, energy, fuels, matter, materials and their links with human living, food, clothing, shelter, health, comfort, respiration, sleep, relaxation, rest, wastes and excretion.

Included here are skills in using tools and technologies for learning, working and living.

(iv) Competencies relating to Preparation for the World of Work.

Employment related skills to maximize their potential and to enhance their capacity

to contribute to economic development,

to discover their vocational interests ad aptitudes,

to choose a job that suits their abilities, and

to engage in a rewarding and sustainable livelihood.

(v) Competencies relating to Religion and Ethics

Assimilating and internalizing values, so that individuals may function in a manner consistent with the ethical, moral and religious modes of conduct in everyday living, selecting that which is most appropriate.

(vi) Competencies in Play and the Use of Leisure

Pleasure, joy, emotions and such human experiences as expressed through aesthetics, literature, play, sports and athletics, leisure pursuits and other creative modes of living.

(vii) Competencies relating to ' learning to learn'

Empowering individuals to learn independently and to be sensitive and successful in responding to and managing change through a transformative process, in a rapidly changing, complex and interdependent world.

Content

First Term

Mathematics - I

Competency 13 : **Uses the derivative of a function to solve problems.**

Competency level 13.1 : Interprets the derivative of a function.

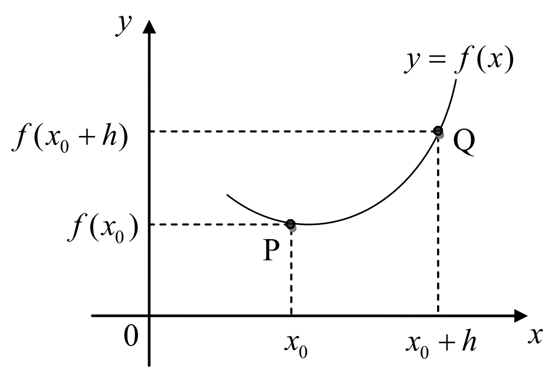
Number of periods : 04

Learning outcomes :

1. Defines the derivative at a point
2. Obtains the slope and tangent line of a curve at a point.
3. Describe the rate of change as a derivatives.
4. Applies rate of change.

Guidelines to learning teaching process :

1. Let y be a function of x, given by $y = f(x)$



Let P be a fixed point on the curve $y = f(x)$ with x co - ordinate is x_0

So $\mathrm{P} = \left(x_0, f(x_0)\right)$.

Let Q be a nearby point to P on the curve $y = f(x)$. Suppose that the x coordinate of Q is $x + h$. Then $\mathrm{Q} \equiv \left(x_0 + h, f(x_0 + h)\right)$

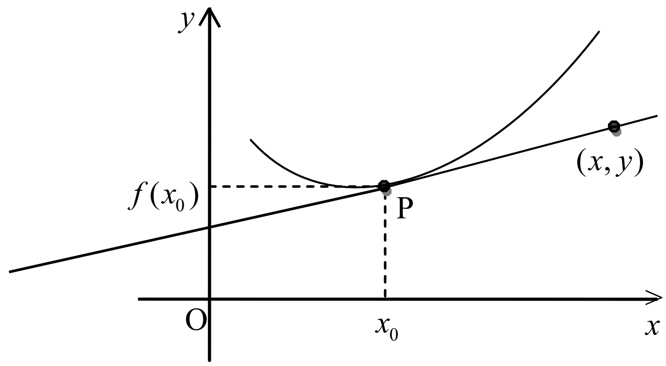
Let m_{PQ} denotes the slope of the secant line PQ.

Then $m_{PQ} = \dfrac{f(x_0 + h) - f(x_0)}{x_0 + h - x_0}$ for $h \neq 0$

$$= \frac{f(x_0 + h) - f(x_0)}{h} \quad \text{for } h \neq 0$$

when $\lim_{h \to 0} M_{PQ}$ exists as a real numbers, then it is defined to be the slope m of the tangent line to the graph of $y = f(x)$ at P.

So, $m_{PQ} = \lim_{h \to 0} \dfrac{f(x_0 + h) - f(x_0)}{h}$



The equation of the tangent line at P to the curve $y = f(x)$ is $y - f(x_0) = m(x - x_0)$.

2. The limit $\lim_{h \to 0} \dfrac{f(x_0 + h) - f(x_0)}{h}$ which was used to define the slope of the tangent line is given a name and a notation since it occurs in many other situations. It is called the derivative of $f(x)$, at $x = x_0$, and is denoted by $f'(x_0)$ provided the limit exists (as a real numbers). If $f'(x_0)$ exists, then $f(x)$ is differentiable at $x = x_0$

Using suitable examples, explain that the derivative of $f(x)$ does not exist at $x = x_0$ in the instances given below.

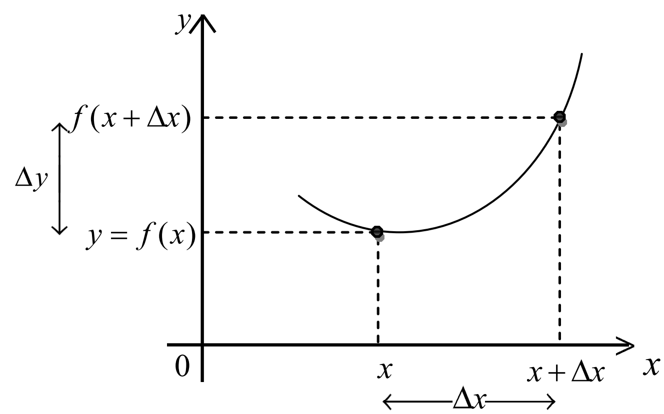
- When $\lim_{h \to o} \dfrac{f(x_0 + h) - f(x_0)}{h}$ does not exist.
- When $\lim_{h \to 0} \dfrac{f(x_0 + h) - f(x_0)}{h}$ is not finite

The function f' whose domain consists of all values of x at which the derivative exists is called the derivative function of $f(x)$.

So $(f')(x) = f'(x)$ and

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}.$$

3. Let y be a function of x given by $y = f(x)$. Take any x and consider an increment Δx of x. Then Δx the change in x in the closed interval with x and $x + \Delta x$ as its end points.



The corresponding change in y values denoted by Δy is equal to $f(x+\Delta x) - f(x)$

Therefore the average rate of change of y with respect to x in the closed interval with x and $x+\Delta x$ as its end points is $\dfrac{\Delta y}{\Delta x}$

Then $\dfrac{\Delta y}{\Delta x} = \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$

Emphasize that Δx is a symbol and not a product of Δ and x. The (Instantaneous) rate of change of y with respect to x is defined to be

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Provided the limit exist as real number.

Note that $\lim\limits_{\Delta x \to 0} \dfrac{\Delta y}{\Delta x} = f'(x)$.

$\lim\limits_{\Delta x \to 0} \dfrac{\Delta y}{\Delta x}$ is also denoted by $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

Hence $f'(x)$ is the same as $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

4. Guide students to solves problems involving applications of derivatives.

Competency level 13.2 **:** Find the derivatives of polynomials, exponential and logarithmic functions

Number of periods **:** 06

Learning outcomes **:** 1. State the following formulae.

- $\dfrac{\mathrm{d}x^n}{\mathrm{d}x} = nx^{n-1}$

- $\dfrac{\mathrm{d}e^x}{\mathrm{d}x} = e^x$

- $\dfrac{\mathrm{d}\ln(x)}{\mathrm{d}x} = \dfrac{1}{x}$; For $x > 0$

Guidelines to learning teaching process :

1. • Obtains the derivatives of simple functions such as x^2, x^4.

Obtains the derivatives of x^n using

$$\lim_{x \to a}\left(\frac{x^n - a^n}{x - a}\right) = na^{n-1}$$ where n is a rational number.

• States that the sum of the infinite series.

$1 + \dfrac{x}{1!} + \dfrac{x2}{2!} + \ldots + \dfrac{x^n}{n!} + \ldots$ is donated by e^x

- e^x is called the natural exponential function.

- State that
  - $\lim_{x \to \infty} e^{-x} = 0$
  - $\lim_{x \to \infty} e^{x} = \infty$
  - $\dfrac{de^x}{dx} = e^x$

Guide students to solves problems involving natural exponantial functions.

- Explain that $\ln x$ defined for $x > 0$ by

  $y = \ln x$ if and only if $x = e^{y}$

  $\ln x$ is called the natural logarithimic function of x

  - $\ln x$ is dfined only for $x > 0$
  - $\ln\,(e^{x}) = x$ for $x \in \mathbb{R}$
  - $e^{\ln x} = x$ for $x > 0$
  - Obtain that

    $$\frac{\mathrm{d}\ln x}{\mathrm{d}\,x} = \frac{1}{x},\ \text{for}\ x > 0$$

    deduce that $\dfrac{\mathrm{d}(a^{x})}{\mathrm{d}x} = \ln x.\,a^{x}\ ; x > 0$

  Guide students to solves problems involving $\ln x$.

Competency level 13.3 : Uses the formulae for a derivative of the sum, product and quotient of two functions and their applications.

Number of periods : 05

Learning outcomes :

1. Derive formulae for sum, product and quotient of two functions and applies to differentiate functions.
2. Uses above rules to solves problems.

Guidelines to learning teaching process :

1. States that,
   - when K is a constant
     - If $f(x)=K$, then $f'(x)=0$
     - If $f(x)=K\,g(x)$ then $f'(x)=K\,g'(x)$
   - Sum rule or difference rule.

     If $f(x)=g(x)\pm h(x)$ then $f'(x)=g'(x)\pm h'(x)$
   - Product rule

     $$\frac{d}{dx}[g(x)\cdot h(x)]= f(x)\frac{d}{dx}[g(x)]+g(x)\frac{d}{dx}[f(x)]$$
     $$= f(x).g'(x)+g(x).f'(x)$$
   - Quotient rule

     $$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)\frac{d}{dx}[f(x)]-f(x)\frac{d}{dx}[g(x)]}{\{g(x)\}^2}\text{; whenever } g(x)\neq 0$$
     $$=\frac{g(x).f'(x)-f(x).g'(x)}{\{g(x)\}^2}$$

2. Guids students to differentiate functions using sum rule, product rule and quotient rule wherever it is applicable.

Competency level 13.4 : Uses the chain rule to find the derivatives.

Number of periods : 06

Learning outcomes : 1. Applies chain rule to find the derivatives of composite functions.

Guidelines to learning teaching process :

1. If y is a differenctiable function of u and u is a differentiable function of x then y is a differentiable function of x, and

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

This is said to be the chain rule.

- A function defined by $y = f(x)$, which statisfies the equation $g(x, y) = 0$ is called an implicit function.

  Explain this using examples.

- To obtain the derivatives of an implicit function $y = f(x)$ defined by an equation $g(x, y) = 0$, it is not necessary. (sometimes not possible) to solve $g(x, y) = 0$, explicity for y in terms of x and then get the derivative. we instead differentiate $g(x, y) = 0$ using the chain rule.

  Explain using examples.

- A curve is defined by the parametic equations $x = f(t)$ and $y = g(t)$, where t is a parameter,

  Guide students to obtain $\frac{dy}{dx}$.

- Solves problems involving above applications.

Competency level 13.5: Determines the behaviour of a function using derivatives.

Number of periods : 04

Learning outcomes :
1. Describes increasing and decreasing function by using differentiation.
2. Finds stationary Points.
3. Finds local maximum and local minimum.

Guidelines to learning teaching process :

1.
- Explain that a function $f(x)$ is increasing on an interval I, if $f(x_1) \le f(x_2)$ whenever $x_1, x_2 \in I$ with $x_1, < x_2$
- Explain that it $f'(x) > 0$ for $x \in I$, then $f(x)$ is strictly increasing on I.
- Explain that a function $f(x)$ is decreasing on an interval I, if $f(x_1) \ge f(x_2)$ whenever $x_1, x_2 \in I$ with $x_1, < x_2$
- Explain that it $f'(x) < 0$ for $x \in I$, then $f(x)$ is strictly decreasing on I.

2. A point at which the derivative of a function is zero is defined as a stationary point. So, $f(x)$ has a stationary point when $x = c$ provided that $f'(c) = 0$ explains with suitable examples.

3.
- State that if there exists $\delta > 0$ such that $f(x) \le f(c)$ for all $x \in (c - \delta, c + \delta)$, then $f(x)$ is said to have a maximum at $x = c$
- State that if there exists $\delta > 0$ such that $f(x) \ge f(c)$ for all $x \in (c - \delta, c + \delta)$, then $f(x)$ is said to have a minimum at $x = c$
- Describes the first derivative test for local maximum and local minimum.
- States that there exists stationary point which are neigther local maximum or local minimum.
- Discuss with examples in which $f'(c) = 0$ but the point which neither local maximum or local minimum
- Introduce the point of inflection.

Competency level 13.6 : Sketches simple curves using derivatives.

Number of periods : 07

Learning outcomes :

1. Sketches simple curves using derivatives.
2. States vertical and horizontal asymptotes.

Guidelines to learning teaching process :

1. Guide students to sketch graphs of functions using derivatives.
2. Guide students to sketche graphs which have asymtotes.
   (Only vertical and horizontal asymtotes are expected.)

Competency level 13.7 : Uses derivatives to solve problems involving related rates.

Number of periods : 08

Learning out comes :

1. Solve problems involving related rates.

Guidelines to learning teaching process :

1. Guide students to solve problems involving relaled rates.

Mathematics - II

Competency 4 : **Analyses random phenomena mathemtically.**

Competency level 4.3 : Describes the possibility of an event in terms of conditional Probablity.

Number of periods : 08

Learning outcomes :

1. Defines conditional probability.
2. States and proves the theorems on conditional probability.
3. Solves problems involving conditional probability.
4. Uses chain rule for more than two events.

Guidelines to learning teaching process :

1. Let **S** be the sample space of a random experiment and **A** and **B** be two events in the Sample space S.
   Then the conditional probability of event **A,** given that event **B** has occured, is denoted by $P(A \mid B)$ and it is defined as $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$, provided that $P(B) > 0$
2. Prove the following results

- If $P(A) > 0$ then $P(\phi \mid A) = 0$
- $P(\phi \mid A) = \dfrac{P(\phi \cap A)}{P(A)} = \dfrac{P(\phi)}{P(A)} = \dfrac{0}{P(A)} = 0$
- If $A,\ B \in S$ and $P(B) > 0$ then $P(A'|B) = 1 - P(A|B)$
- $P(A' \cap B) = P(B) - P(A \cap B) \quad [\because P(B) = P(A \cap B) + P(A' \cap B)]$

Hence $P(A'|B) = \dfrac{P(B) - P(A \cap B)}{P(B)} = 1 - P(A \mid B)$

therefore $P(A'|B) = 1 - P(A|B)$

3.
   - Guide students to solves problems involving conditional probability.
   - Use tree diagram to explan conditional probability.
   - Guide students to solve problems involving conditional probability using tree diagram.

4. Use chain rule for more than two events.

Competency level 4.4 : Interprets the independence of two random events.

Number of periods : 04

Learning outcomes :
1. Defines independent of two events.
2. Defines pairwise independent events.
3. Defines mutually independent events.
4. Uses independent events to solves problems.

Guidelines to learning teaching process :

1. Let A_1, A_2 be two events in a sample space.

   A_1 and A_2 are said to be independent if and only if $P(A_1 \mid A_2) = P(A_1)$

   Then we can show that if A_1 and A_2 are independent.

   $$P(A_1 \cap A_2) = P(A_1).P(A_2)$$

   - If A_1 and A_2 are independent events then
     - A_1 and A_2'
     - A_1' and A_2
     - A_1' and A_2' are also independent events.

2. Let A_1, A_2, A_3 are three events in the same sample space and if

   $$P(A_1 \cap A_2) = P(A_1).P(A_2)$$

   $$P(A_2 \cap A_3) = P(A_2).P(A_3)$$

   $$P(A_3 \cap A_1) = P(A_3).P(A_1)$$

   then A_1, A_2 and A_3 are said to be pairwise independent events.

3. • Let $A_1,\ A_2,\ A_3$ be three events in a sample space, S of a random experiment and if

$$P(A_1 \cap A_2) = P(A_1).P(A_2)$$

$$P(A_2 \cap A_3) = P(A_2).P(A_3)$$

$P(A_3 \cap A_1) = P(A_3).P(A_1)$ and $P(A_1 \cap A_2 \cap A_3) = P(A_1).P(A_2).P(A_3)$

then $A_1,\ A_2$ and A_3 are said to be mutualy independent events.

• **Note that** Pairwise independence does not imply mutual independence.

4. • Guide students to discuss the following results related to conditional probabilities.

- Multiplication rule for two events.

  Let A_1, A_2 be any two events in an random experiment and $P(A_1) > 0$ then,

  $$P(A_1 \cap A_2) = P(A_1).P(A_2|A_1)$$

  Multiplication rule for three events.

  Let $A_1, A_2,\ A_3$ be any three events in an random experiment and $P(A_1) > 0$. $P(A_1 \cap A_2) > 0$

  $$P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2|A_1).P(A_3 \mid A_1 \cap A_2)$$

- Explains the multiplication rule for two events and multiplication rule for three events using tree diagram.
- Guide students to solves problems involving multiplication rule.

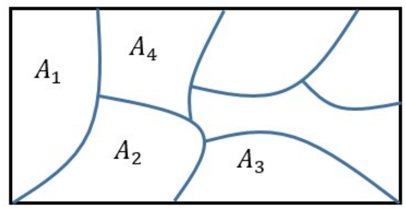
Competency level 4.5 : Use Bayes' theorem as a deduction of the total probability theorem.

Number of periods : 08

Learning outcomes :

1. Defines partition of a sample space
2. States total probability theorem
3. Proves total probability theorem
4. States and proves Baye's theorem
5. Solves problems involing Baye's theorem

Guidelines to learning teaching process :



1. The events A_1, A_2, A_3, ..., A_n are defined on a particular sample space S are said to be a partion of the sample space S, if then they satisfy

   - $A_i \cap A_j = \phi$ for all $i \neq j$ (mutually exclusive)
   - $\overset{n}{\underset{i=1}{U}} A_i = S$ (Collectively exhastive)
   - $A_j \neq \phi$ for all i

2. Let $\{A_1, A_2, ... A_n\}$ be partition of sample space S, of a random experiment. B is an event in the sample space S then

   $$P(B) = \sum_{i=1}^{n} P(B|A_i).P(A_i)$$

   This is said to be total probability theorem.

3. Proof $\quad B = (B \cap A_1) \cup (B \cap A_2) \cup ....\cup (B \cap A_n)$

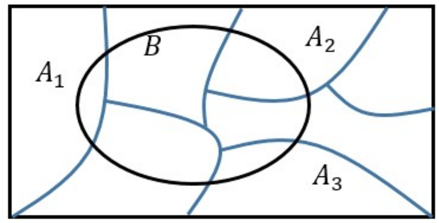
   $$= \overset{n}{\underset{i=1}{U}} B \cap A_i$$

   $$\therefore\ P(B) = P\left\{\overset{n}{\underset{i=1}{U}} (B \cap A_i)\right\}$$

   $$= \sum_{i=1}^{n} P(B \cap A_i) \text{ (since } (A_i \cap B) \text{ are mutually exclusive)}$$

   $$= \sum_{i=1}^{n} P(B|A_i).P(A_i)$$

4. Baye's Theorem



Let $A_1, A_2, \ldots\ldots A_n$ be are partitions of the sample space S of a random experiment and **B** is an any event in S and $P(B) > 0$ then.

$$P(A_i|B) = \frac{P(B|A_i)\,P(A_i)}{\sum_{j=1}^{n} P(B|A_j).P(A_j)}$$

Proof :

$$P(A_i|B) = \frac{P(A_i \cap B)}{P(B)}$$

$$P(A_i|B) = \frac{P(B|A_i)\,P(A_i)}{\sum_{j=1}^{n} P(B|A_i).P(A_i)}$$ — Using theorem on total probability

5. Guide students to solve the problems involving total probability and Baye's theorem. (Mximum 3 partions problems)

Competency level 4.6 : Interprets random variables.

Number of periods : 02

Learning outcomes :

1. Defines random variable.
2. Describes possible values of a random variable.
3. Defines discrete random variables.
4. Defines continuous random variables.

Guidelines to learning teaching process :

1. Let S be the sample space of a random experiment. A random variable is a function from sample space S to set of real numbers in the real line.

   Random variables are usually denoted by X, Y, Z, ... etc.

   X is a function from S to $\mathbb{R}$

   $X : S \to \mathbb{R}$

   $X(s) = x$; where $s \in S$ and $x \in \mathbb{R}$

2. Let us consider a rondom experiment of tossing 3 coins simultaneously. The corresponding sample space S is given below.

   S = { (H, H, H), (H, H, T), (H, T, H), (H, T, T), (T, H, H), (T, H, T),
   (T, T, H), (T, T, T)}

   Let us define the random variable X as the number of heads in a particular trial as,

   X (H, H, H) = 3
   X (H, H, T) = 2
   X (H, T, H) = 2
   X (H, T, T) = 1
   X (T, H, H) = 2
   X (T, H, T) = 1
   X (T, T, H) = 1

This kind of example may be related to a game where the winning price is depand on the no of heads. In such case, we don't worry about the exact outcome. We need to know only the number of heads obtained.

3. Let **X** be a random variable If the set of values of **X** (range of X) is finite or countably infinite, then the random variable is said to be discrete.

4 Let **X** be a random variable If the set of values of X is an any value in an interval, then X is said to be continuous.

Competency level 4.7 : Analyses the properties of a probability distribution of a random variable.

Number of periods : 12

Learning outcomes :

1. Describes probability distribution of a discete random variable. (Probability mass function)
2. Describes probability distribution of a continuous random variable. (Probability density function)

Guidelines to learning teaching process :

1. • Let S be the sample space of a random experiment and **X** be a discrete random variable defined on S.

Let the values of X be $\{x_1, x_2, x_3, \ldots, x_n\}$.

A function **P** is defined on $\{x_1, x_2, \ldots, x_n\}$ as follows :

i.e $$\mathrm{P}(x) = \begin{cases} \mathrm{P}(\mathrm{X} = x); \;\; x = x_i, \;\; i = 1, 2, \ldots\ldots., n \\ 0 \text{ ; otherwise} \end{cases}$$

$\mathrm{P}(X = x)$ means the probability of $X = x$

- $\mathrm{P}(x)$ is said to be probability mass function of **X** (p.m.f)
- The set of ordered pairs $\{(x_1, \mathrm{p}(x_1)) : i = 1, \; 2, \ldots, n\}$

is the probability distribution

It can be shown in a table as follows :

X	x_1	x_2		x_n
$P(X = x)$	$p(x_1)$	$p(x_2)$		$p(x_n)$

Properties of a probability mass function.

- $P(X = x_i) \geq 0 \quad \text{for } (i = 0, 1, 2, \ldots n)$
- $\sum_{i=1}^{n} P(X = x_i) = 1$

Eg:- Suppose we have data on mobile phone sales on 30 days number of mobile phones sold per day is given below.

Number of phones	0	1	2	3	4
Number of days	3	5	12	6	4

We can construct a frequency distribution for the no. of mobile phones sold per day X.

X	Number of days	Relative frequency
0	3	$\frac{3}{30}$
1	5	$\frac{5}{30}$
2	12	$\frac{12}{30}$
3	6	$\frac{6}{30}$
4	4	$\frac{4}{30}$
Total	30	1

Using relative frequency approach to probability. Probality mass function of the random variable X, can be found

Therefore, the probability mass function of X can be given as follows.

X	0	1	2	3	4
P(X)	$\frac{3}{30}$	$\frac{5}{30}$	$\frac{12}{30}$	$\frac{6}{30}$	$\frac{4}{30}$

2. The probability density function corresponds to a "smoothed out" relative frequency histogram for which the area under the curve represent the probability. Hence the total area under probability density function must be one.

Probability density function of a random variable X is denoted by $f(x)$

Properties of $f(x)$:

- $f(x) \geq 0$ for all x and
- $\int_{-\infty}^{+\infty} f(x)\,dx = 1$
- $P(a < X < b) = \int_{a}^{b} f(x)\,dx$

Consider the following probability density function

$$f(x) = \begin{cases} \frac{6}{29}(x^2 - 5x + 6) & ; \quad 2 \leq x \leq 3 \\ 0 & ; \quad otherwise \end{cases}$$

To show that $f(x)$ is a probability density function, we have to show that

- $f(x) \geq 0$ for all x (by sketching graph of it)
- $\int_{-\infty}^{+\infty} f(x)dx = 1$

The probability of X when its valus lies between in between a and b is

$$\int_{b}^{a} f(x)dx$$

Competency level 4.8 : Interprets the mathematical expectation of a random variable.

Number of periods : 12

Learning outcomes :

1. Defines mathematical expectation of a discreate random variable.
2. Defines mathematical expectation of a continuous random variable.
3. Defines variance of a discreate random variable.
4. Defines variance of a continuous random variable.

Guidelines to learning teaching process :

1. Let $P(x)$ be the Probability mass function corresponding to a discrete random variable **X**.

$$P(x) = \begin{cases} P(X = x); & (i = 1, 2, ..., n) \\ 0 \ ; & \text{otherwise} \end{cases}$$

Then the mean of **X** or expected value of **X** is denoted by

$E(X)$ *or* μ and $E(X) = \sum_{i=1}^{n} x_i \, P(x_i)$

Note that E(X) is a constant (X_i 's and $P(X_i 's)$ are constant)

2. Let $f(x)$ be a probability density function for a continuous random variable **X**.

Mean of **X** or expected value of **X**, denoted by $E(X)$ or μ

$$\mu = E(X) = \int_{-\infty}^{+\infty} x\, f(x)\, dx$$

3. Let $P(x)$ be the Probability mass function corresponding to a discrete random variable **X**.

$$P(x) = \begin{cases} P(X = x); & (i = 1, 2, ..., n) \\ 0 \ ; & \text{otherwise} \end{cases}$$

Variance of X, denoted $\mathrm{Var}(X)$ or σ^2

$$\sigma^2 = \mathrm{var}(X) = E\left[X - E(X)\right]^2$$

also it can be shown that, $\mathrm{E}\left[\mathrm{X}-\mathrm{E}(\mathrm{X})\right]^2=\mathrm{E}\left(\mathrm{X}^2\right)-\left[\mathrm{E}(\mathrm{X})\right]^2$

where $\mathrm{E}\left(\mathrm{X}^2\right)=\sum_{i=1}^{n} x_i^{\,2}\,\mathrm{P}(x_i)$

Standard derivation (σ) is defined as the positive square root of variance.

4. Let $f(x)$ be a Probability density function for a continuous random variable **X**. Mean of **X** or expected value of **X**, denoted by E(X) or μ variance of **X**, denoted Var**(**X**)** and $\mathrm{Var}(\mathrm{X})=\mathrm{E}\left[\mathrm{X}-\mu\right]^2$

show that $\mathrm{E}\left[X-\mu\right]^2=\mathrm{E}\left(\mathrm{X}^2\right)-\left[\mathrm{E}(\mathrm{X})\right]^2$

$$\mathrm{E}\left(\mathrm{X}^2\right)=\int_{-\infty}^{+\infty} x^2 f(x)\,dx$$

Standard derivation (σ) is defined as the positive square root of variance.

- Guide students to obtain the following results fora discrtere random variable

- If *a, b* are constant
  $\mathrm{E}(a\mathrm{X}+b)=a\,\mathrm{E}(\mathrm{X})+b$ and $\mathrm{Var}(a\mathrm{X}+b)=a^2\mathrm{Var}(\mathrm{X})$

Competency level 4.9 : Determines the cumulative distribution function of a random Variable.

Number of periods : 20

Learning outcomes :
1. Defines Cumulative distribution function of discrete randam variable.
2. Defines cumulative distribution function of a continuous random variable.
3. Finds cumulative distribution function (c. d. f.) for a given probability mass function (p.m.f)
4. Finds c. d. f. for a given p. d. f.
5. Plot c. d. f. for discrete randam variable
6. Plot c. d. f. for continuous randam variable.

Guidelines to learning teaching process :

1. In a Probability distribution the probabilities up to a certain values of **X** are summed to give a cumulative probability. The cumulative probability function is written as $F(x)$

   For a discrete random variable **X** with Probability mass function $\mathrm{P}(x)$

   the Cumulative distribution functions give by $F(x) = \sum_{\mathrm{X} \le x} \mathrm{P}(\mathrm{X} = x)$

2. For a continuous random variable **X** with probability density function $f(x)$,

   the cumulative distribution function is given by $F(x) = \int_{-\infty}^{\infty} f(x)dx$

3. By giving suitable examples guide students to find cumulative distribution function for a given probabilty mass distribution.
4. By giving suitable examples guide students to find cumulative distribution function for a given probabilty density distribution
5. By giving suitable examples guide students to plot the graph of cumulative distribution function for a continuous random varible.

Second Term

Mathematics - I

Competency 14 : Finds indefinite and definite integrals of functions

Competency level 14.1 : Indentifies integration as the reverce process of differentitation (anti - derivatives of functions)

Number of periods : 02

Learning out comes :
1. finds intergrals using results of derivative
2. Uses the therems on integration

Guidelines to learning teaching process :

1.
- For a given function $f(x)$, if there exists a function

  $F(x)$ such that $\dfrac{dF(x)}{dx} = f(x)$, then $F(x)$ is said to be the anti derivative of $f(x)$ and the process is called anti - differentiation.

- If $F(x)$ an anti-derivative of $f(x)$, then if $\dfrac{d}{dx}\{F(x)+c\} = f(x)$,

  Hence, $\int f(x)dx = F(x)+C$,
  where C is an arbitary constant.

- Any two anti-derivatives of a function can differ only by a constant.

2.
- Explain the following theorems
  - $\int \{f(x)+g(x)\}dx = \int f(x)dx + \int g(x)dx$
  - $\int K f(x)dx = K\int f(x)dx$,

  where $f(x)$ and $g(x)$ are function of x and K is a constant

- Guide students to integrals of functions using above theorems.

Competency level 14.2 : Identifies integrals of elementary functions, and results of integratian

Number of periods : 10

Learning out comes :

1. Solves integral problems using standard results
2. Uses the formulae to find integrals
3. Uses Partial fractions to find integrals

Guidelines to learning teaching process :

1. State the following integrals of elementary functions.

- $\int x^n dx = \dfrac{x^{n+1}}{n+1} + c \quad (n \neq -1)$

- $\int \dfrac{1}{x} dx = \ln |x| + c \qquad (x \neq 0)$

- $\int e^x dx = e^x + c$

2. Direct students to apply the following standard results.

- $\int [f(x)]^n f'(x) dx = \dfrac{[f(x)]^{n+1}}{n+1} + c$

- $\int \dfrac{f'(x)}{f(x)} dx = \ln |f(x)| + c$, where $f(x)$ is the anti derivative of $f'(x)$.

3. Guide stuents to use partial fraction to find integrals.

Competency level 14.3 : Detemines definite integrals using the fundamental theorem of calculus.

Number of periods : 06

Learning out comes :
1. States the fundamental theorem of calculus
2. Finds the values of definite integrals
3. Uses the properties of definite integrals.

Guidelines to learning teaching process :

1. Define

   $\int_a^b f(x)dx = \left[F(x)\right]_a^b = F(b) - F(a)$, where $F(x)$ is the anti derivative of $f(x)$

2. Guide to calculate the values of definite intgrals.

3. Discuss the following theorems on define integrals.

   - $\int_a^b \{f(x) + g(x)\}dx = \int_a^b f(x)\,\mathrm{dx} + \int_a^b g(x)dx$
   - $\int_a^b k\ f(x).dx = k\int_a^b f(x)dx$, where k is a constant.
   - $\int_a^b f(x)dx = -\int_b^a f(x)dx$
   - $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$, where $a < c < b$

   Solves problems involving the applications of above results.

Competency level 14.4 : Uses different methods for integration

Number of periods : 04

Learning out comes : 1. Solves problems using partial fractions

Guidelines to learning teaching process :

1. - Intergrales rational functions using partial fractions

     $\int \frac{p(x)}{q(x)} dx$ where $p(x)$ and $q(x)$ are polinomials and $q(x)$ is a factorizable polinomials with degree ≤ 4. (Partial fractions with maximum 4 unknowns)

   - Guide students to use partial fractions to find intgrals.

Competency level 14.5 : Integration using the method of integration by parts

Number of periods : 04

Learning out comes : Uses Intrergration by parts to integrate suitable problems.

Guidelines to learning teaching process :

1. - Student directed to apply the method of integration by parts.

     $\int u\left(\frac{dv}{dx}\right)dx = u.v - \int v\left(\frac{du}{dx}\right)dx$, where u and v are functions of x.

   - Guide students to find intgrals using integration by parts.

Competency level 14.6 : Determines the area of a region bounded by curves using integration

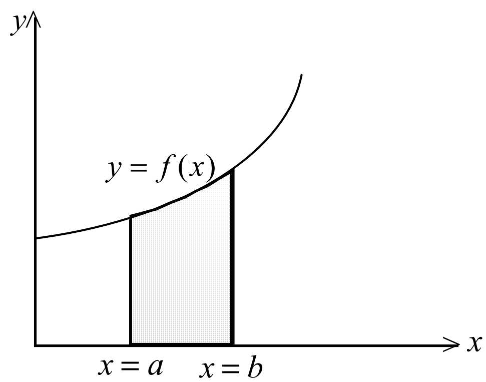
Number of periods : 08

Learning out comes :
1. Uses definite integral to find area under a curve.
2. Uses definite integral to find area between two curves.

Guidelines to learning teaching process :

1. Define the area under a curve as a definite integral.

   Let $f(x)$ be a continuous function, with $f(x) \geq 0$, for $x \in [a,b]$

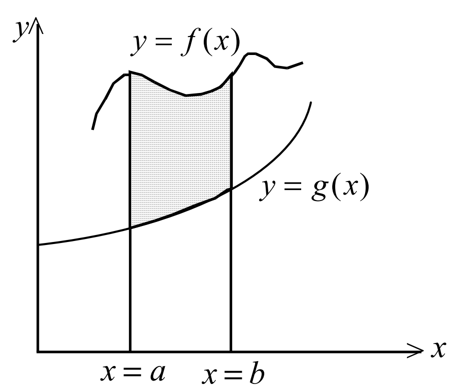


   The area bounded by the curve $y = f(x)$ and x axis and the lines $x = a$ and $x = b$ is given by $\int_a^b f(x)dx$

   This is referrd to as the area under the curve $y = f(x)$ from $x = a$ and $x = b$

2. • Direct students are to find the area between two curves.
   Let $f(x)$ and $g(x)$ be continuous functions. such that $f(x) \geq g(x)$ in the interval $[a,b]$. The area bounded by the two curves $y = f(x), y = g(x)$ and the lines $x = a, x = b$ is given by

   $\int_a^b (f(x) - g(x))dx$.



   • Only these types of graphs are expected.

Competency level 14.7 : Uses Method of approximation to solve Problems.

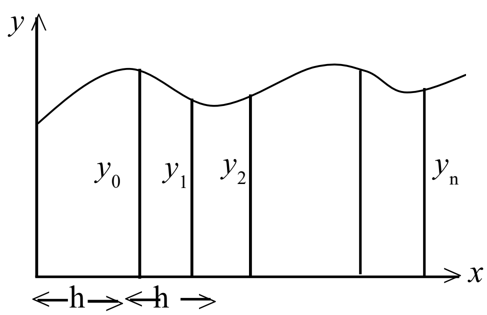
Number of periods : 08

Learning out comes :

1. Solves integral problems by using trapezium rule
2. Solves integral problems by Simpson's rule

Guidelines to learning teaching process :

1. • The trapezoidal rule :



Suppose the area represented by $\int_a^b f(x)dx$ is divided into n equal strips each of width h

Then, trapezoidal rules is

$$\int_a^b f(x)dx = \frac{1}{2}h(y_0 + y_1) + \frac{h}{2}(y_1 + y_2) + \ldots + \frac{h}{2}(y_{n-1} + y_n)$$

$$= \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + \ldots y_{n-1})]$$

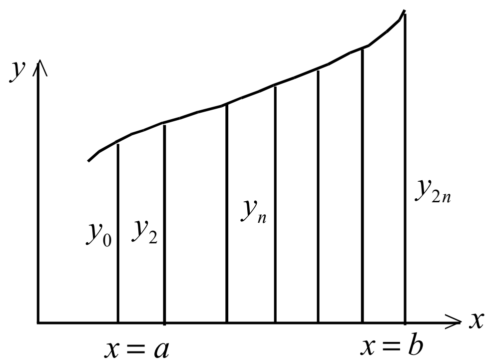
where $h = \dfrac{b-a}{n}$

• Guide students to evaluate area by using trapezoidal rule.

2. • Simpon's Rule

Suppose that the area represented by

$\int_a^b f(x)dx$ is divided into $2n$ equal with strips, each of width h.



- Then area given by the Simpson's rule is

$$\int_a^b f(x)dx \approx \frac{h}{3}[(y_0 + y_{2n}) + 4(y_1 + y_3 + .. + y_{2n-1}) + 2(y_2 + y_4 + .. + y_{2n-2})]$$

where $h = \dfrac{b-a}{2n}$

- Note that Simpson's rule requires even number of strips (or odd number of ordinates)

Mathematics - II

Competency 4 : Analyses random phenomena mathematically.

Competency level 4.1 : Determines the events of a random experiment.

Number of periods : 14

Learning out comes :
1. Describes Bernoullies distribution
2. Discribes uniform distribution
3. Discribes binomial distribution
4. Discribes Poison distribution
5. Solves problems involving above distributions

Guidelines to learning teaching process :

1. **Bernoulli trial**

If a random experiment can have only two outcomes then such an experiment is known as a Bernoulli trial. The two possible outcomes are usually known as success and a failure.

e.g. 1: In tossing a coin getting a head can be considered as a success

e.g. 2: In tossing a die getting an even number can be considered as a successes

Note that we define success depending on our requirement and Bernoulli trials are considered to be the building blocks of many discrete distributions such as Binomial, Geometric.

Bernoulli distribution

If we define a random variable X based on a Bernoulli trial where X =1 if the outcome is a success and X=0 where the outcome is a failure. If the probability of getting a successes is p then the probability of getting a failure is 1-p. Therefore the probability distribution of random variable X can be given as

$P(X = x) = p^x (1- p)^{1-x} ; x = 0,1$

Illustrate with example.

Suppose that a bag contains 6 white balls and 3 red balls of same size. A ball is taken randomly from the bag . Let X be the random variable which represent the number of red balls.

Now $p(x) = \left(\frac{2}{3}\right)^x \left(1 - \frac{2}{3}\right)^{1-x}$ *if* $x = 0, 1$

$= 0$, Otherwise

x	0	1
$p(x)$	$\frac{1}{3}$	$\frac{2}{3}$

2. Let the random variable X be defind over the set of n distinct values $x_1, x_{2,3}, \ldots x_n$ which are all equally likely

Then X follows a discrete uniform distrubution .

The probability mass function is given by

$$p(x) = \frac{1}{n} \text{ for } x = x_1, x_{2,3}, \ldots x_n$$

$= 0$, otherwise.

Illustrate with example.

Consider throwing, an unbiassed die once.

Let random variable X be the number appearing on the uppermost face.

Now $p(x) = \frac{1}{6}$ *for* $x = 1, 2, 3, 4, 5, 6$

$= 0$, otherwise

x	1	2	3	4	5	6
$p(x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

3. Binomial distribution

If we repeat n Bernoulli trials independently we can define a random variable X as the number of successes in n trials. We assume that probability of success is constant for all trials and result of one trial in independent of the others. Then the probability distribution of random variable X can be given as,

$P(X = x) = {}^nC_x \, p^x (1-p)^{n-x}$; $x = 0, 1, 2, \ldots.. n$

This is known as a Binomial probability distribution with parameters n and p.

Note that the probability of X changes with n and p.

e.g.1: If a fair coin is tossed 15 times, let us find the probability of getting exactly 5 heads.

Here we define getting a head as a success since we are interested in finding the number of heads. Then the probability of getting a head p=0.5, and the number of trials n=15. Therefore the required probability,

$$P(X=5) = {}^{15}C_5\,(0.5)^5(1-0.5)^{15-5} = 3003 \times 0.00003052 = 0.09165$$

e.g.2: Any item produced by a particular machine may be defective with 1% probability. If a random sample of 10 items produced by this machine is selected for examination, the probability that more than one items will be defective can be found as follows.

Let us define getting a defective item as a success since we are interested in the number of defectives. Then p = 0.01, n = 10. Required probability,

P(X >1) = 1 - [P (X = 0) + P (X = 1)] = 1 - [0.9044 + 0.0914] = 0.0042

4. Poisson distribution

If a random variable X has the following probability distribution, X is said to have a Poisson probability distribution.

$P(X=x) = \dfrac{e^{-\lambda}\lambda^{x}}{x!};\ x = 0, 1, 2, \ldots\ldots$ where λ is the mean of X and $m>0$, λ

e is approximately equal to 2.718.

This probability distribution can be applied to many counting processes such as no. of telephone calls received to an office during an hour, number of misprints on a page.

e.g: If the number of defects on a finished item follows a Poisson distribution with mean m = 2, probability of having less than three items can be found as follows.

$$P(X<3) = P(X=0) + P(X=1) + P(X=2) = \frac{e^{-2}2^0}{0!} + \frac{e^{-2}2^1}{1!} + \frac{e^{-2}2^2}{2!}$$

$$= 0.1353 + 0.2706 + 0.2706 = 0.6765$$

5. Guide students to solves problems involving above distributions

Competency 5 : Determines the optimum solution of a linear programming problem

Competency Level 5.1 : Constructs a linear programming model

Number of periods : 10

Learning out comes :

1. Constructs Linear Programming Model
2. States decision variable
3. Constructs objective functions
4. Defines constraints
5. States conditions

Guidelines to learning teaching process :

1. Explain what a Linear Programming Model

   Linear Programming (LP) is a problem solving approach developed to help managers make decisions. LP is a mathematical method that will be used to optimize (maximize or minimize) functions under constraints. Consider a situation where a furniture manufacturer produces desks, chairs and tables by using the same materials and machines. This company cannot produce furniture as much as they like because of limited resources. Such a situation LP can be used to determine how many units from each furniture must be produced in under to maximize profit.

   LP can be used to maximize (profit, revenue etc.) or minimize (cost, time etc.) a function subject to constraints.

2. Discuss the components of an LP model.

   Use a sample question to explain the components.

   A company manufactures tablesandchairs. Each table yields a profit of Rs. 400 and each chair yields a profit of Rs. 500. A table requires 4 hours of processing at lathe machine and 2 hours at cutting machine. A chair requires 6 hours at lathe machine and 1 hour at cutting machine. Lathe machine is available for a maximum of 120 hours per month and cutting machine is available for a maximum of 72 hours per month.

   Now the problem is to find out how many chairs and tables are to be produced in a month to maximize profit.

   In order to solve this problem it is necessary to convert this description into a mathematical model.

Step 1 : Define ***decision variables***.

x - Number of tables to be produced per month

y - Number of chairs to be produced per month

Step 2 : Define ***objective function***

Then the profit function can be written as

$Z = 400\,x + 500\,y$

This is called the ***objective function***. Since this is a case of profit, objective is to maximize profit. Therefore it can be written as

Maximize $Z = 400\,x + 500\,y$

Step 3 : Identify ***constraints***

Total machine time requirements at the lathe machine and cutting machine are $4x + 6y$ and $2x + 1y$ respectively. Since maximum availability of machines are 120 and 72 hours per month, the mathematical expression will be as follows.

$4x + 6y \leq 120$

$2x + y \leq 72$

Step 4 : Identify ***non-negativity constraints***

There is another constraint for any LP model called *non-negativity constraint.* Since number of chairs and tables cannot be negative, $x \geq 0$ and $y \geq 0$. Then the complete model will be as follows.

Maximize $Z = 400\,x + 500\,y$

Optimize Objective function

Subject to Constraints

$$\left.\begin{array}{l} 4x + 6y \leq 120 \\ 2x + y \leq 72 \end{array}\right\} \text{Constraints}$$

Non-negativity Constraints

$x \geq 0$ and $y \geq 0$

Constraints may be $\geq, \leq$ ” or $=$ type.

Competency 5 : Determines the optimum solution of a linear programming problem.

Competency level 5.2 : Determines the solution of a linear programming problem graphically

Number of periods : 15

Learning out comes :

1. Identifies the feasible region
2. Finds the solutions of maximizing model and minimising model
3. Obtains unfeasible solutions single solutions and multiple solutions in problems
4. Solves problems involving linear programming

Guidelines to learning teaching process :

1. Explain the Graphical Method

   Explain that LP models with 2 decision variables can be solved by using Graphical Method. Explain how the graphical method works.

2. Explain how to find the feasible solution.

   Explain by using a suitable example how the constraints are shown in a Cartesian plane. Then explain how to identify the feasible region, i.e. the area that will satisfy all the constraints (inequalities). It is to be mentioned that all drawings are made only in 1st Quadrant because of non-negativity constraint. i.e. $x \geq 0$ and $y \geq 0$.

   If a common area cannot be identified then it is called an infeasible solution. For example consider the following constraints.

   $x \geq 3$

   $x \geq 5$

   In this case it is clear that a common area that will satisfy both constraints cannot be identified. In such a situation an answer for the given problem cannot be found unless otherwise the constraints are modified.

3. Explain how to find the answer for the LP problem

   If the solution is feasible, maximum (minimum) value of the objective function is given by the coordinates of one of the corner points of the feasible region. It is necessary to explain how to find the corner points (points of intersection), how to substitute x and y values of corner points to the objective function and to identify the corner point with the highest value (for a maximization problem) or lowest value (for a minimization problem).

   Explain iso-profit line/ iso-cost line method as well.

   The answer is obtained in the format of values of x and y. Then it is necessary to explain how to interpret the answer in the context of original problem.

Note : Explain that models with any number of variables can be solved by using a method called Simplex Method. Solution procedure has become simple with the development of computers. Solver tool in MS_excel can be used to solve any LP problem. No need to discuss the solution procedure. Objective is to let the students know that there are other methods to solve problems with more than two variables.

Competency 8 : Manipulates Daterminants as a Mathematical model of solving Problems.

Competency Level 8.1 :

Number of periods : 04

Learning out comes :

1. Defines determinant
2. Finds value of a determinant.
3. States properties of determinant.
4. Solves problems involving determinant and its properties.

Guidelines to learning teaching process :

1. - State the forms of 2x2 and 3x3 determinants.

   Expansion of a 2x2 determinant

   If $\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$

   then $\Delta = a_1 b_2 - a_2 b_1$,

   where a_1, a_2, b_1, b_2 are real numbers.

   - Expansion of a 3 x 3 determinant

   Let $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_2 & c_3 \end{vmatrix}$

   then $\Delta = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$

   $= a_1 (b_2 c_3 - b_3 c_2) - b_1 (a_2 c_3 - a_3 c_2) + c_1 (a_2 b_3 - a_3 b_2)$

   where $a_1, a_2, a_3, b_1, b_2, b_{3,} c_1, c_2, c_3$ are real numbers.

Note: We can expand determinant along a row or along a coloumn. We get the same result.

2. By using examples guide stdents to find determinants

3. Discuss the following properties for 2 x 2 and 3 x 3 determinants.

- If Δ_2 is obtained from Δ_1 by interchanging two rows (columns) of Δ_1, then $\Delta_2 = -\Delta_1$
- If two rows (columns) of a determinant are equal, then the determinant is zero.
- The value of the determinant is unaltered if a multiple of any row (column) is added to any other row (column).
- If one row (column)of a determinant (Δ) is multiplied by a scalar λ, the resulting determinant is equal to $\lambda\ \Delta$.
- If all the elements in a row (or column) are zero the value of determinant is zero.
- Let $\Delta = \begin{vmatrix} x_1 & y_1 & a_1 + b_1 \\ x_2 & y_2 & a_2 + b_2 \\ x_3 & y_3 & a_3 + b_3 \end{vmatrix}$

$$\Delta_1 = \begin{vmatrix} x_1 & y_1 & a_1 \\ x_2 & y_2 & a_2 \\ x_3 & y_3 & a_3 \end{vmatrix}, \Delta_2 = \begin{vmatrix} x_1 & y_1 & b_1 \\ x_2 & y_2 & b_2 \\ x_3 & y_3 & b_3 \end{vmatrix}$$

Then $\Delta = \Delta_1 + \Delta_2$

4. Solves problems involving determinant

Competency Level 8.2 : Solves equation in two or three variable.

Number of periods : 06

Learning out comes : :
1. Discuss the solutions of two simaltaneous equations
2. Solves simultaneous equations using matrices
3. Solves problems involving matrix multiplications.

Guidelines to learning teaching process : :

1. • Let $a_1x + b_1y = c_1$ ——— (i)

   $a_2x + b_2y = c_2$. ——— (ii)

   by writing the simultaneous equations in thr fprm $AX = C$

   $$A = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}, \ X = \begin{pmatrix} x \\ y \end{pmatrix} \text{ and } C = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} .$$

   if A^{-1} exist

   $A^{-1}(AX) = A^{-1}C$

   $(A^{-1}A)X = A^{-1}C$

   $X = A^{-1}C$

   explains the following

   - a unique solution
   - infinite solutions
   - no solutions

2. Guide the students to solves simultaneous equations using matrices
3. Guide the students to solves problems involving matrix multiplications.

Competency 9 : Manipulates Matrices using Matrix Alegbra

Competency Level 9.1 : Describes Matrix Algebra

Number of periods : 08

Learning out comes :

1. Defines matrices.
2. Identify row, column and order of a matrix.
3. Identify row matrix , column matrix.
4. Describes compartible for addition of two matrices.
5. States closure property.
6. Uses commutative law and associative laws for addition.
7. Multiplies matrices by a scalar.
8. Uses distributive law for addition over scalar multiplication.
9. Solves problems involving matrices, addition.

Guidelines to learning teaching process :

1. Matrix is a rectangular array of numbers. Matrices are denoted by capital letters in the alphabet A, B, C, ...

$$\text{Let } A = \begin{pmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ a_{m1} & a_{m2} & . & . & . & a_{mn} \end{pmatrix}$$

2. The Matrix A has m rows and n colomns The size (order) of the matrix A is $m \times n$.

   A can be written as $\left(a_{ij}\right)_{m\times n}$

   Element of a matrix:

   a_{ij} is the element of a matrix A in the i th row and j th colomn.

3. Row matrix:
   A matrix which has only one row is called a row matrix or row vector.

   Coloumn matrix:
   A matrix which has only one coloumn is a column matrix or column vector.

   Null matrix:
   A matrix where every element is Zero, is called null matrix.

4. • Let A and B two matrices of same order.

$A=(a_{ij})_{m\times n}, B=(b_{ij})_{m\times n}$

if $a_{ij}=b_{ij}$ for all i, j

then $A=B$

• State the condition for two maxtrices to be added.

Maxtrices are in the same order.

Then corresponding elements are added.

Let $A=(a_{ij})_{m\times n}, B=(b_{ij})_{m\times n}$

Then $A+B=(a_{ij})_{m\times n}+(b_{ij})_{m\times n}$

$=(a_{ij}+b_{ij})_{m\times n}$

5. States and explains the closure property for addition

6. Note that

- Addition is closed.
- Addition is commulative

$A+B=B+A$

Addition is associative.

$(A+B)+C=A+(B+C)$

7. Explains scalar multiplication of a matrice

Let $A=(a_{ij})_{m\times n}$ and $\lambda\in\mathbb{R}$

$\lambda A=(\lambda a_{ij})_{m\times n}$, for all $i.j$

When $\lambda=-1$

$(-1)A=-A$ is called the negative of the matrix A.

Let *A, B* be two matrices of same order.

Then, $A-B=A+(-1)B$.

8. Guide students to solves problems involving matix addtion and propertice of addtion

Competency 9.2 : Investigate preperties of square matrices.

Number of periods : 12

Learning out comes :

1. Verifies square matrices by using the definition
2. Define the compatibility for multiplication of two matrices
3. Verifies the property $AB \neq BA.$ for two matrices
4. Defines the unit and diagonal matrices.

Guidelines to learning teaching process :

1. In a matrix A of order $m \times n$ when $m = n$ A is defined as a square matrix of order n Let A be a square matrix of order n

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \\ a_{n1} & a_{n2} & \cdots & & a_{nn} \end{bmatrix}$$

$(a_{11}, a_{22}, a_{33}, ..a_{nn})$ is the leading (principal) diagonal.

2. Defines the compartibility for multiplication

Let $A = (a_{ij})_{m \times p}$ and $B = (b_{ij})_{q \times n}$

When $p = q,$ the product AB is defined.

If A $= (a_{ij})_{m \times p}$ and B $= (b_{ij})_{p \times n}$

then AB $= (\sum_{k=1}^{p} (a_{ik} b_{kj})_{m \times n}$

is order of $m \times n$

3 Discuss that

- Even if AB is defined, BA may not necessarily be defined.
- In general $AB \neq BA.$

4. • A squre matrix A of order n is said to be identity matrix if

$a_{ij} = 1$ when $i = j$

0 when $i \neq j$

and denoted by I_n

• A square matrix A is said to be a diagonal if matrix $a_{ij} = 0$ for all $i \neq j$

• A square matrix is said to be a zero matrix if $a_{ij} = 0$ for all i, j

For square matrices A,B and C, of the same order

A(BC) = (AB) C (Associatove) under multiplication.

A(B+C) = AB + AC (Disributive)

(B+C)A = BA + CA (Disributive)

A + 0 = A = 0 + A (where o is the zero square matrix of order n)

A x I = A = I x A, whwre I is the identity matrix of oeder n.

• Let A be a matrix of order $m\,x\,n$

$$A = (a_{ij})_{mxn}$$

Transpose of A, denoted A^T, is defined by

$$A^T = (b_{ij})_{nxm}$$

Where $b_{ij} = a_{ji}$ for all i , j.

Properties of matrix transpose

$(A + B)^T = A^T + B^T$

$(kA)^T = k A^T$, $k \in \mathbb{R}$

$(A^T)^T = A$

$(AB)^T = B^T A^T$

Third Term

Mathematics - I

Competency 9 : Explares the binomial expansion for positive integral indices.

Competency level 9.1 : Describes the basic preperties of the binomial expansion.

Number of periods : 08

Learning out comes :
1. Defines ${}^{n}c_{r}$ and obtain the for formula for it
2. expands $(a+b)^{n}$ using binomlal theorem
3. writes the general term in the expension of $(a+b)^{n}$

Guidelines to learning teaching process :

1. $${}^{n}c_{r} = \frac{n!}{(n-r)!\,r!}$$

2. statement of the binomial theorem for positive integral index.

   $$(a+b)^{n} = {}^{n}C_{o}a^{n} + {}^{n}C_{1}a^{n-1}b + {}^{n}C_{2}a^{n-2}b^{2} + \dots\dots\dots\dots\dots\dots + {}^{n}C_{n}B^{n}$$

   $$= \sum_{r=0}^{n} {}^{n}C_{r}a^{n-r}b^{r} \quad \text{Where } {}^{n}c_{r} = \frac{n!}{(n-r)!\,r!}\ (0 \le r \le n)$$

   In the expansion
   - ${}^{n}C_{1}, {}^{n}C_{2}, {}^{n}C_{3}, \dots {}^{n}C_{n}$ are called binomial coefficients.
   - ${}^{n}C_{0}a^{n} + {}^{n}C_{1}a^{n-1} + {}^{n}C_{2}a^{n-2} + \dots\dots + {}^{n}C_{n}$ are called terms of the expansion.
   - The number of terms in the expansion is $n+1$
   - Guide the students to expand the binomials.

3. General term T_{r+1} is given by in the binomial expansion of $(a+b)^{n}$.

   $T_{r+1} = {}^{n}C_{r}a^{n-r}b^{r}$, $T_{r} = {}^{n}C_{r-1}a^{a-r+1}b^{r-1}$

   Note that the power of b are in ascending order.
   - Guide the students to solves problems involving binomial expansion.

Competency level 9.2 : Applies binomial therems

Number of periods : 08

Learning out comes :

1. Expands $(1+x)^n$ using binomial therorem
2. Writes the gerneral term in the expansion of $(1+x)^n$
3. Solves the problems involving binomial expansion

Guidelines to learning teaching process :

1. Guide the students to expands $(1+x)^n$ as

$$(1+x)^n = {}^nC_o + {}^nC_1x + {}^nC_2x^2 + {}^nC_3x^3 + ... + {}^nC_nx^n$$

$$= \sum_{r=0}^{n} {}^nC_rx^r \qquad \text{Where } {}^nC_r = \frac{n!}{r!(n-r)!} \qquad (0 \le r \le n)$$

${}^nC_0, {}^nC_1, {}^nC_2, {}^nC_3, ... {}^nC_n$ are binomial Coefficiants.

2. Guide the students to get the General term T_{r+1}, is givenby

$T_{r+1} = {}^nC_rx^r$ and $T_r = {}^nC_{r-1}\,x^{r-1}$

3. Guide the students to solves problems involving binomial expansion.

Competency 10 : Finds the sum of the finite series.

Competency level 10.1 : Describes finite series and their preperties

Number of periods : 08

Learning out comes : 1. Finds the sum of arthmatic and geomatric series

2. Finds the general term of arithmatic and geometric series

Guidelines to learning teaching process :

1. Definition of an arithmetic series
A series, which after the first term, the difference between a term and the preeding term is constant, then this series is called an Arithmetic series or Arithmetic Progression.

   - Show that the general term T_r

     $T_r = a + (r-1)d$, Where a is the first term and d is the common differnce.
   - The sum of first n terms is

     $$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[a + l]$$

     where l is n th term of the series.

2. Definition of a geometric series
A series which after the firsl term, the ratio between a term and the preceeding term is constant, is called geometric term

   - Show that the general term

     $T_p = ar^{p-1}$ where a is the first term and r is the common ratio

   - Show that the sum of n terms S_n,

     Application of the above formulae

     $$S_n = \frac{a(1-r^n)}{1-r} \quad where\ |r| < 1$$
     $$= \frac{a(r^n - 1)}{(r-1)} \quad when\ |r| > 1$$

   - $\underset{n\to\infty}{Lt}\ S_n = \frac{a}{1-r} \quad when\ |r<1|$
   - Guide students to solves problems involving arithmetic,geometric series.

Competency level 10.2 : Solves problems involving arithmetic and geometric series

Number of periods : 08

Learning out comes : 1. write the series using $\sum$ notation and find the sum

2. Application of arithmatic and geometric series by using sigma notation.

Guidelines to learning teaching process :

1. State the general term of a series is Ur and denote the sum of n terms as,

$$\sum_{r=1}^{n} U_r$$

State that

(i) $\sum_{r=1}^{n}(u_r + V_r) = \sum_{r=1}^{n}(u_r + \sum_{r=1}^{n} V_r$

(ii) $\sum_{r=1}^{n} kU_r = k\sum_{r=1}^{n} U_r$, where k is a constant.

state that $\sum_{r=1} U_r V_r \neq \left(\sum_{r=1}^{n} U_r\right)\left(\sum_{r=1}^{n} V_r\right)$

2. Guide students to solve problems involving arithmetric and Geometric series.

Competency level 10.3: Finds sum s of elementary series

Number of periods : 10

Learning out comes :

1. proves and uses the formulae by principle of mathematical Induction for values $\sum_{r=1}^{n} r, \sum_{r=1}^{n} r^2, \sum_{r=1}^{n} r^3,$
2. Applies the above formulae to find the sumation of series
3 uses the method of diferece to find sum of a series
4. Determine convergent of a series

Guidelines to learning teaching process :

1. Determination of $\sum_{r=1}^{n} r, \sum_{r=1}^{n} r^2, \sum_{r=1}^{n} r^3$

Guide students to prove the following results using Principle of Mathematical Induction.

$$\sum_{r-1}^{n} r = \frac{n}{2}(n+1)$$

$$\sum_{r-1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1)$$

$$\sum_{r-1}^{n} r^3 = \left[\frac{n}{2}(n+1)\right]^2$$

2. Guide students to find the sum of series using the above results and their combinations.

3 .Explains method of difference to find the sum of a series

4. Guide students to determine convergent of a series

Mathematics - II

Competency 4 : Analyses random phenomena mathematically

Competency level 4.11 : Calculates probabililty using theoritical models and interprets the density function of special Continuous distribution

Number of periods : 15

Learning out comes :

1. Describes uniform distrubutions
2. Describes normal distributions.
3. Describes standard normal distributions.
4. Solves Problems involving above distributions.

Guidelines to learning teaching process :

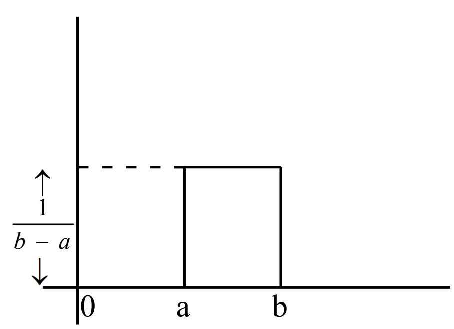
1. Let X be a continuous random variable and values of X lies between a and b.

   If the probability density function distributed uniformly in the intervel $a \leq x \leq b$ this distribution called Uniform distribution and it is denoted by

   $X \sim U_{(a,b)}$ where a, b parameters of this distributon.

   Probability density function is $f(x) = \dfrac{1}{(b-a)}$.

   This can be shown as



Note that total area of the rectangle

$$= \frac{1}{(b-a)} \times (b-a)$$
$$= 1$$

2. Let X be a continuous random variable.

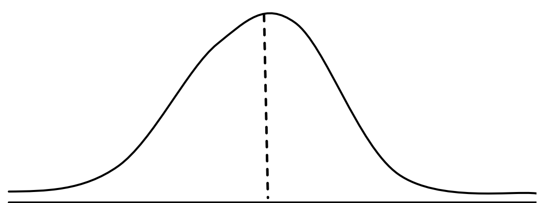
If X is normal distribution with mean μ and standard deviation σ^2

Then X has a probability density function, given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}, -\infty < x < \infty$$

$X \sim N(\mu, \sigma^2)$, μ - mean σ^2 - Variance

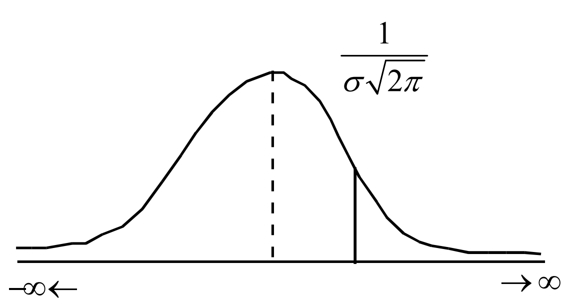
If I can written as this normal distribution can be described by a curve. This curve is called Nomal distribution curve.



Nomal distribution curve have the following features.

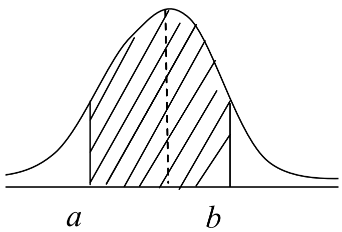
- It is bell shaped.
- It is symmetrical about mean (μ).
- It extends from $-\infty$ to $+\infty$
- The maximum value of $f(x)$ is $\frac{1}{\sigma\sqrt{2\pi}}$
- The total area under the curve is 1 unit

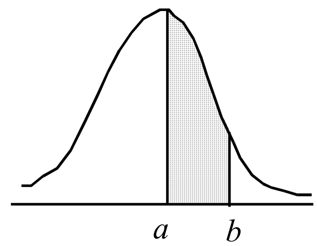
  If $X \sim N\left(\mu, \sigma^2\right)$

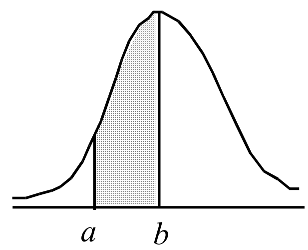


- approximately 95% of the distribution has within two standard deviation of the mean.
- approximately 99.75% of the distribution lies within three standard deviation of the mean.

- Probability that X lies between a and b is written $P(a < x < b) =$ Area under the normal curve between a and b $(a < b)$. It can be one of the following.







3. Let X be a normal distribution with mean μ and standard deviation σ
   $X \sim N(\mu, \sigma^2)$.

   X is standardised so that the mean is 0 and the standard deviation is 1.

   Define $Z = \dfrac{x - \mu}{\sigma}$

   $$\bar{Z} = \frac{\bar{x} - \mu}{\sigma}$$

   $$\bar{Z} = 0$$

   $$\sigma_Z^2 = \frac{\sigma_X^2}{|\sigma^2|}$$

   $$\sigma_Z^2 = 1$$

   $$\therefore\ Z \sim N(0,1)$$

   Probability density function of Z is $\phi(Z) = \dfrac{1}{\sqrt{2\pi}} e^{\frac{1}{2}Z^2}$

   This distribution is said to be standard nomal distribution.

Eg. : Let $X \sim N(40, 9)$

then define $Z = \dfrac{X - 40}{\sqrt{9}}$

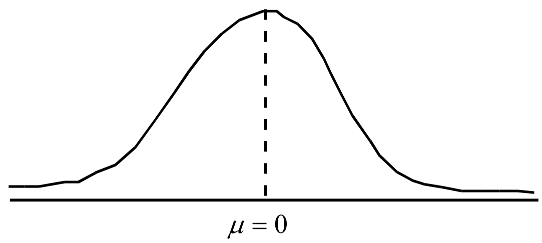
$$\bar{Z} = \frac{\bar{X} - 40}{3}$$

$$\bar{Z} = 0$$

$$\sigma_Z^2 = \frac{\sigma_Z^2}{3^2}$$

$$\sigma_Z^2 = 1$$

$$\therefore\ \overline{Z} \sim N(0,1)$$



- Guide students to find area between two values.
- Guide students to solves problems involving distributions.

4. Guide students to solves problems involving above distributions

Competency 7 : Analyses Projects by using networks

Competency level 7.1 : Describes networks

Number of periods : 10

Learning out comes : 1. Defines network and applies it in problems

Guidelines to learning teaching process :

1. Explain what a network is, and discuss the components of a network.

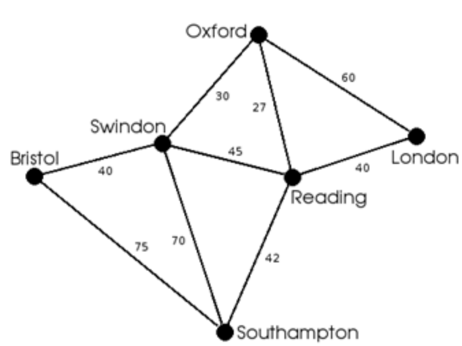
Network :

Many business problems (including some special LP models) can be solved by using networks. A ***network*** is a graphical representation of the problem and it consists of ***nodes*** and ***arcs***.Nodes usually indicate locations or junction points and are represented by numbered or labelled circles in the diagram. Nodes are used to represent cities, junctions, routers, start or end of an activity of a project etc. Arcs (branches) are used to connect nodes to each other and are represented by a line. Arcs may represent highways,pipe lines, rivers or activities of a project (in project management) etc.

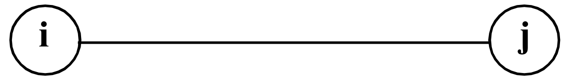
Simple examples of networks:

Nodes	**Arcs**	**Flow**
Cities	highways	vehicles
call switching centers	telephone lines	telephone calls
pipe junctions	pipes	water

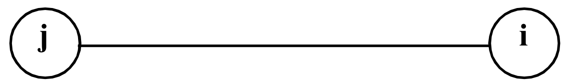
An example of a network :



An undirected arc represents flow or movement between two nodes in both directions.



A directed arc assumes the flow or movement only in one direction. For example a river or one way road.



2. Explain the meaning of network techniques and discuss the application of network techniques.

Network Technique:

Network techniques can be used to solve many different problems. Some of the problems are briefly discussed below.

- **Transportation problems**

  Transportation models are used to minimize the cost of distributing a product from a number of sources (such as factories or warehouses) to a number of destinations (such as retail outlet).

- **Transshipment problems**

  This is a special type of transportation problem with the possibility of sending goods through an intermediary source (for example transshipment through a port)

- **Minimal Spanning Tree Problems**

  A special network technique called minimal spanning tree modelcan be used to find the optimal way to distribute water to all the required areas (such as fields or towns) at a minimum cost. This method also can be used to develop telecommunication networks to all the points of a network.

- **Shortest route problems**

  Another popular network method is shortest route problem. This method can be used to find the route with the shortest distance among many routes from one point to another point. This method also can be used to determine the best time to replace the equipment when both maintenance cost and replacement cost are considered.

- **Maximal Flow problems**

  This is a special network that can be used to determine the maximum capacity that can be passed through a network at a given time. For example this technique can be used to determine maximum number of jeeps that should be allowed to enter into a sanctuary like Yala.

- **Project Management**

  There are specific network techniques is a technique for planning, scheduling (programming) and controlling the progress of projects. This is very useful for projects which are complex in nature or where activities are subject to considerable degree of uncertainty in performance time.This technique provides an effective management, determines the project duration more accurately, identifies the activities which are critical at different stages of project completion to enable to pay more attention on these activities, analyses the scheduling at regular interval for taking corrective action well in advance, facilitates in optimistic resources utilization, helps management for taking timely and better decisions for effective monitoring and control during execution of the project.

  Note : Minimal Spanning Tree problem, Maximum Flow problem and Project Management are discussed in detail under 7.2

School Based Assessment

Introduction - School Based Assessment

Learning -Teaching and Evaluation are three major components of the process of Education. It is a fact that teachers should know that evaluation is used to assess the progress of learning-teaching process. Moreover, teachers should know that these components influence mutually and develop each other. According to formative assessment (continuous assessment) fundamentals; it should be done while teaching or it is an ongoing process. Formative assessment can be done at the beginning, in the middle, at the end and at any instance of the learning teaching process.

Teachers who expect to assess the progress of learning of the students should use an organized plan. School based assessment (SBA) process is not a mere examination method or a testing method. This programme is known as the method of intervening to develop learning in students and teaching of teachers. Furthermore, this process can be used to maximize the student's capacities by identifying their strengths and weaknesses closely.

When implementing SBA programmes, students are directed to exploratory process through Learning Teaching activities and it is expected that teachers should be with the students facilitating, directing and observing the task they are engaged in.

At this juncture students should be assessed continuously and the teacher should confirm whether the skills of the students get developed up to expected level by assessing continuously. Learning teaching process should not only provide proper experiences to the students but also check whether the students have acquired them properly. For this, to happen proper guiding should be given.

Teachers who are engaged in evaluation (assessment) would be able to supply guidance in two ways. They are commonly known as feed-back and feed- forward. Teacher's role should be providing Feedback to avoid learning difficulties when the students' weaknesses and inabilities are revealed and provide feed-forward when the abilities and the strengths are identified, to develop such strong skills of the students.

Student should be able to identify what objectives have achieved to which level, leads to Success of the Learning Teaching process. Teachers are expected to judge the competency levels students have reached through evaluation and they should communicate information about student progress to parents and other relevant sectors. The best method that can be used to assess is the SBA that provides the opportunity to assess student continuously.

Teachers who have got the above objective in mind will use effective learning, Teaching, evaluation methods to make the Teaching process and learning process effective. Following are the types of evaluation tools student and, teachers can use. These types were introduced to teachers by the Department of Examination and National Institute of Education with the new reforms. Therefore, we expect that the teachers in the system know about them well

Types of assessment tools:

1.	Assignments	2.	Projects
3.	Survey	4.	Exploration
5.	Observation	6.	Exhibitions
7.	Field trips	8.	Short written
9.	Structured essays	10.	Open book test
11.	Creative activities	12.	Listening Tests
13.	Practical work	14.	Speech
15.	Self creation	16	Group work
17.	Concept maps	18.	Double entry journal
19.	Wall papers	20.	Quizzes
21.	Question and answer book	22.	Debates
23.	Panel discussions	24.	Seminars
25.	Impromptus speeches	26.	Role-plays

Teachers are not expected to use above mentioned activities for all the units and for all the subjects. Teachers should be able to pick and choose the suitable type for the relevant units and for the relevant subjects to assess the progress of the students appropriately. The types of assessment tools are mentioned in Teacher's Instructional Manuals.

If the teachers try to avoid administering the relevant assessment tools in their classes there will be lapses in exhibiting the growth of academic abilities, affective factors and psycho- motor skills in the students

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Statistics

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