AL/2022/10/E-I

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கல்வி அமைச்சு
Ministry of Education

G.C.E.(A.L) Support Seminar

සංයුක්ත ගණිතය I
இணைந்த கணிதம் I
Combined Mathematics I

10 E I

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மூன்று மணித்தியாலம்
Three hours

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மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

Index Number							

Instructions:

* *This question paper consists of two parts;*
  Part A (Questions 1 – 10) *and* **Part B** (Questions 11 – 17)
* **Part A:**
  Answer ***all*** *questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
* **Part B:**
  Answer ***five*** *questions only. Write your answers on the sheets provided.*
* *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
* *You are permitted to remove* ***only*** **Part B** *of the question paper from the Examination Hall.*

For Examiners' Use only

(10) Combined Mathematics II		
Part	**Question No.**	**Marks**
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
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B	11	
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	17	
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Total

In Numbers	
In Words	

Code Numbers

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Checked by:	1	
	2	
Supervised by:		

[see page two

Part A

1. Using the Principles of Mathematical Induction prove that $\sum_{r=1}^{n} 6r(r-1) = 2n(n^2-1)$ for all $n \in \mathbb{Z}^+$

2. Sketch the graphs of $y = |2x-2| - x$ and $y = 2|x-2| - 2x$ in the same figure. Hence or otherwise, find all real values of x satisfying the inequality $|2x-4| - |2x-2| > x$

[see page three

3. On a sketch of an Argand diagram, shade the region whose point represent complex number z satisfying the inequality $|z-2-2i| \leq 1$ and $Arg(z-4i) \geq -\frac{\pi}{4}$. Hence find the least of $\text{Im}(Z)$ for points in the region, giving your answer in an exact form.

4. Write the $T_{(r+1)}$ term of the binomial expansion $\left(\sqrt{3}+11^{\frac{1}{5}}\right)^{10}$. Hence, Find the sum of rational terms of the expansion

[see page four

5. Evaluate; $\lim_{x \to \frac{\pi}{6}} \left(\frac{12-12\,Cos\left(2x-\frac{\pi}{3}\right)}{(6x-\pi)^2} \right)$

6. The region enclosed by the curves $y = \sqrt{In|x|}$, $(where\ x > 1, x \in \mathbb{R})$, $y = 0$, $x = 2$ and $x = 4$ is rotated about the x axis through 2π radians. Show that the volume of the solid generated is $\boldsymbol{6\pi In(2) - 2\pi}$ cubic units.

[see page five

7. Show that the coordinates of any point P with parameter θ on the hyperbola $\frac{x^2}{9} - \frac{y^2}{36} = 1$ can be expressed in the form $(3sec\theta, 6tan\theta)$. Show that the equation of the normal to the given hyperbola at the point with parameter $\theta = \frac{\pi}{6}$ is $x + 4y = 10\sqrt{3}$.

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8. Let $l = 0$ be a straight line with gradient $m(\neq 0)$ Show that there are two possible positions for $l = 0$, such that the perpendicular distance from origin O to the line $l = 0$ is 1 unit and find the equation of, each of the line $l = 0$.

A rhombus is formed by above mentioned two lines by opposite sides and the two-coordinator axis as diagonals. Show that the area of the rhombus is $\left|\frac{2(m^2+1)}{m}\right|$

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9. A center of the circle $S \equiv x^2 + y^2 + 2gx + 2fy + p = 0$ lying on the line $y = mx + c$ touches the y axis and the intercept made by circle $S = 0$ on the axis is 8 units. Show that,
$g^2(1 - m^2) + 2gmc = 16 + c^2$

10. By using the identity $sin^2\theta + cos^2\theta = 1$ obtain $Cosec^2\theta = 1 + cot^2\theta$ where $\theta \neq n\pi$, $n \in \mathbb{Z}$. Given that $Cot\theta - Cosec\theta = \frac{5}{4}$ then show that $Cot\theta + Cosec\theta = -\frac{5}{4}$, then show that $Sin\theta = -\frac{40}{41}$

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G.C.E.(A.L) Support Seminar - 2022

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இணைந்த கணிதம் I
Combined Mathematics I

10 E II

Part B

✻ Answer five questions only.

11. (a) Let $f(x) \equiv x^2 + (2a-1)x + (a+1), x \in \mathbb{R}$ where a is real constant. If $(x + 2a - 1)$ is a factor of $f(x)$. Find the value of a.

Write down $f(x)$ for above a value and obtain the roots of $f(x) = 0$.

Hence write down the quadratic expression $F(x) = f(p - 2x)$, where p is a real constant.

Show that the quadratic equation $F(x) = 0$ has real and distinct roots for all $p \in \mathbb{R}$

Find the quadratic equation $G(x) = 0$ whose roots are the reciprocals of roots of $F(x) = 0$ when $p \neq 0$ and $p \neq 3$.

Further, show that both $F(x) = 0$ and $G(x) = 0$ have the same discriminant.

(b) When the polynomial $P(x)$given by $P(x) \equiv x^4 - (1 - \lambda)x^3 + \mu x + 2$ is divided by $x^2 - x - 2$ the remainder is $10(x + 1)$. Find λ and μ.

Show that $(x + 1)$ is a factor of $P(x)$ for above determined λ and μ values.

Express $P(x)$ in the form $P(x) \equiv (x - \alpha)(x^3 - \beta)$ where α and β to be determined.

12. (a) The following table shows some details of a group of persons with their profession.

Profession	Male	Female
Doctor	3	1
Nurse	7	4
Attendant	5	5

A committee of 5 members has to be appointed from this group.
Find the number of different possible committees that can be appointed under each condition.

(i) When there is no any restriction
(ii) All three professions must be participated in the committee and also only for doctors both male and female should be in the committee.
(iii) All the doctors in the group should be participated in the committee.

[see page eight

(b) Let $f(r) = \frac{2}{(2r-1)^2}$, $r\epsilon\mathbb{Z}^+$

Show that $f(r) - f(r+1) = \frac{16r}{(2r-1)^2(2r+1)^2}$

Write down the rth common term U_r in the infinite series

$\frac{1}{1^2.3^2} + \frac{2}{3^2.5^2} + \frac{3}{5^2.7^2} + \frac{4}{7^2.9^2} + \cdots$

Find V_n and W_{2n} which are defined as $V_n = \sum_{r=1}^{n} u_r$ and $W_{2n} = \sum_{r=1}^{2n} u_r$.

Is $W_{2n} - V_n$ convergent? Justify your answer.

13. (a) Let $A = \begin{bmatrix} 2 & 4 \\ 3 & -2 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 2\alpha & \alpha \\ 0 & 0 \\ -1 & -1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 1/2 \\ 2 & 1 \end{bmatrix}$ where α is a real constant.

If $A^TB = 8C$, find $\propto$. Also find B^TA for above $\propto$ value.

Hence show that A^TB+B^TA is a symmetric matrix. Is there exist a 2nd order square matrix P such that $(A^TB)P = I$. Justify your answer. Where I is the 2nd order identity matrix.

(b) Represent the region R on an Argand diagram which satisfies the condition $2 < |Z| \leq 6$ where Z is a complex number. Now let Z_R is the complex number in above region R. Where $Z_R = x + iy$ $(x, y \in \mathbb{R})$

(i) Find Z_0 which is given by $Z_0 = Z_R + \overline{Z_R}$ where $\overline{Z_R}$ is the complex conjugate of Z_R.

(ii) Further show separately the region R' in which, Z_R can exists such that both the complex number Z_R and Z_0 are in the above region R.

(iii) w is the complex number which belongs to the above R' region such that $|w|$ is maximum, $Arg(w)$ is minimum and also in the 1st quadrant. Write down w in $x + iy$ form.

Hence find $w + \overline{w}$ and $w - \overline{w}$ and by using De Moivre's theorem, show that $(|w + \overline{w}| + i|w - \overline{w}|)^{12} = 12^{12}$.

14. (a) Consider the function $y = f(x) \equiv \frac{3x+p}{(x+q)^2}$, $x \in \mathbb{R}$ where p and q are real constants such that $x \neq -q$. $x = 2$ is a vertical asymptote to the curve $y = f(x)$ and the curve has a stationary point at $x = \frac{4}{3}$. Determine p and q. Show that the first derivative of $y = f(x)$ With relative to x can be expressed as $f'(x) = \frac{4-3x}{(x-2)^3}$; $x \neq 2$

Indicating the intercepts on x axis, intercept on y axis, turning points and asymptotes clearly sketch the curve of $y = f(x)$.

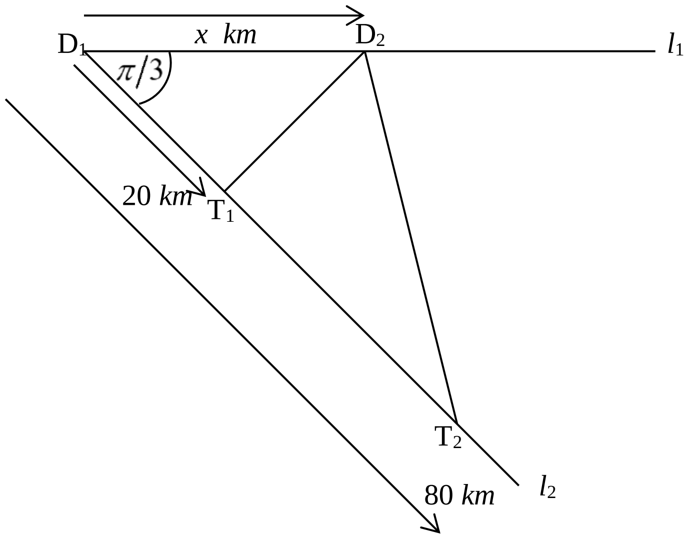
The second derivative of $f(x)$ with relative to x, is given by $f''(x) = \frac{6(x-1)}{(x-2)^4}$, $x \neq 2$

Determine the coordinates of points inflection of the curve $y = f(x)$ and their nature.

[see page nine

(*b*) In the given figure l_1 and l_2 are the two high tension transmission lines starting from the distribution center D_1 which are in an angle $\frac{\pi}{3}$.

Two distribution transformers T_1 and T_2 are located on the line l_2 at distances $20\ km$ and $80\ km$ respectively, from D_1. It is proposed to established another distribution centre D_2 on line l_1 at a distance x km from D_1 and to join it to T_1 and T_2 using straight transmition lines D_2T_1 and D_2T_2.



Obtain $D_2T_1 = \sqrt{x^2 - 20x + 400}km$ and $D_2T_2 = \sqrt{x^2 - 80x + 6400}km.$

State the range of x in above expressions.

What is the distance from D_1 to the point at which the new distribution center D_2 to be constructed so that it makes the total length of D_2T_1 and D_2T_2 is a minimum.

15. (*a*) For $a \in \mathbb{R}$ where $a > 0$, Prove that $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

Let, $I = \int_0^{\frac{\pi}{2}} \frac{d\theta}{Sin\,\theta\,(Sin^2\,\theta - Cos^2\,\theta)}$ and $J = \int_0^{\frac{\pi}{2}} \frac{d\theta}{Cos\,\theta\,(Sin^2\,\theta - Cos^2\,\theta)}$

Show that $I = -J$. Hence evaluate the integral $\int_0^{\frac{\pi}{2}} \frac{d\theta}{Sin\,\theta\,Cos\,\theta\,(Sin\,\theta - Cos\,\theta)}$

(*b*) Determine the real constants A, B and C such that

$x^2 = (Ax + B)(1 + x)^2 + C(1 + x^2)(1 + x) + D(1 + x^2)$

and obtain the result

$$x^2 = \frac{1}{2}x(1 + x)^2 - \frac{1}{2}(1 + x^2)(1 + x) + \frac{1}{2}(1 + x^2)$$

Hence show that,

$$\int \frac{x^2}{(1 + x^2)(1 + x)^2}\,dx = \frac{1}{2}\left[ln\left|\frac{\lambda\sqrt{1 + x^2}}{(1 + x)}\right| - \frac{1}{(1 + x)}\right]$$

for $x \neq -1$, where λ is a real constant.

(*c*) Using a suitable substitution, evaluate the integral $\int_1^{3^{\frac{1}{4}}} \left(\frac{1}{x^3}\right) tan^{-1}\left(\frac{1}{x^2}\right) dx$

[see page ten

16. Show that any point P on the straight line $l = 0$ which passes through $A \equiv (2, 1)$ with the gradient m can be expressed parametrically as $P \equiv (2 + t, 1 + mt)$, where t is a parameter.

The rhombus ABCD is entirely in the first quadrant where ABCD is in the counter clockwise sense. Length of a side of the rhombus is 4 units and $A \equiv (2,1)$. Side AB is parallel to ox axis and $B\hat{A}D = \frac{\pi}{3}$.

(i) Using the above parametric representation itself find the coordinates of the vertices B and D of the rhombus ABCD. Hence obtain the coordinates of vertex C.

(ii) Further by using the same parametric representation, find the gradient of the diagonal AC of rhombus and find the equations of the diagonals AC and BD

(iii) Find the equations of circles $S_1 = 0$ and $S_2 = 0$ where sides AB and BC are as diameters of each circle respectively. Are $S_1\, and\, S_2$ orthogonal. Justify your answer.

(iv) A circle $S_0 = 0$ whose center is on the straight line which passes through the center of rhombus ABCD and parallel to the side AB cuts the circle S_1 orthogonally. Show that S_0 can be expressed as, $S_0 \equiv x^2 + y^2 + 2\lambda x - 2(1 + \sqrt{3})y + (2\sqrt{3} - 11 - 8\lambda) = 0,\ \lambda \in \mathbb{R}$. If the radius of S_0 is $\sqrt{35}$ units then show that there exist such S_0 is circles and find the equations of each circle.

17. (a) Write down $Cos(A + B)$ in terms of $SinA,\ SinB, CosA$ and $CosB$

By selecting A and B properly, obtain the result $Cos[90° + \theta] = -sin\theta.$

Hence show that $Sin110^0 = -Cos200^0$ and $Cos110^0 = -Sin20^0$ and deduce that $tan110^0 + cot20^0 = 0$

(b) Prove that $Cos4\theta - Cos2\theta = 8Cos^4\theta - 10Cos^2\theta + 2$

Hence find the values for$Cos\theta$ such that $Cos4\theta = Cos2\theta$

(c) The medians drawn from the vertices A and B of the triangle ABC, to the opposite sides are AD and BE respectively. The lines AD and BE are perpendicular and meet at G. Also, by usual notation $a = 4\ cm\ and\ b = 3\ cm.$ Using the Cosine rule for appropriate triangles, Show that $A\hat{C}B = Cos^{-1}\left(\frac{5}{6}\right)$

(d) Consider the equation, $tan^{-1}(x + 1) + tan^{-1}(x - 2) = tan^{-1}(2)$

Obtain an equation which satisfies x in above equation.

Hence write down suitable solutions for x in above equation.

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සංයුක්ත ගණිතය II
இணைந்த கணிதம் II
Combined Mathematics II

10 E II

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மூன்று மணித்தியாலம்
Three hours

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மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

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Index Number							

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* In this question paper, g denotes the acceleration due to gravity.

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(10) Combined Mathematics II		
Part	Question No.	Marks
A	1	
	2	
	3	
	4	
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B	11	
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In Numbers	
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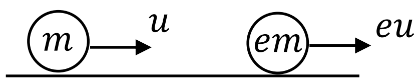
Code Numbers

Marking Examiner		
Checked by:	1	
	2	
Supervised by:		

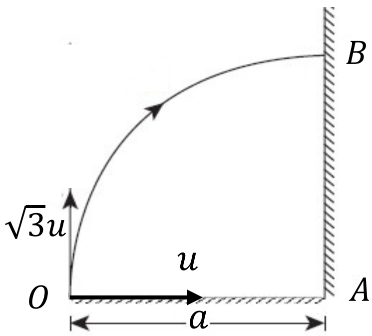
[see page two

Part A

1. The coiefiecint of restitution between two particales A and B is e. The masses of A and B are m and λm respectively. Where $0 < e < 1$. The particals are moving with conatant speeds u and eu in the same horizontal line and in the same direction, as show in the diagram and collied directly.
Show that after the collision the speed of B is independent of e. Find the values of e for which the impulse excerted by B on A due to the collision has magnitude $\frac{6}{25}mu$



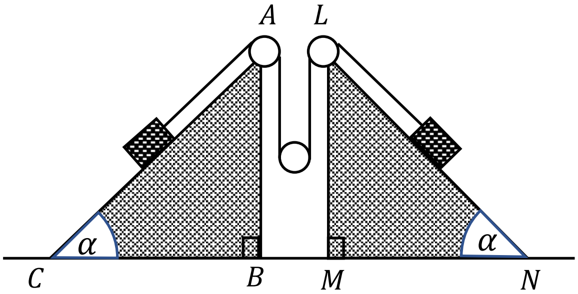
2. A particle is projected from a point O on a horizontal plane with the velocity whose horizontal and vertical components of u and $\sqrt{3}\,u$ respectively. The particle strikes the vertical wall all at the highest point B of its path and bouncing back to the plane OA. The wall is at a distance a from the point O as shown in the figure. Given that the coefficient of of restitution of the particle and the wall is $\frac{1}{2}$, Find



(i) The time taken by the particle to return to the horizontal plane OA

(ii) The distance from the point A to the particle where it reaches the plane OA

3. The right-angle triangles ABC and LMN in the figure, are central vertical cross-sections through the Centre of gravity of two identical fixed smooth wedges, placed in a horizontal plane.
Where $A\hat{C}B = L\hat{N}M = \alpha$ and $A\hat{B}C = L\hat{M}N = \frac{\pi}{2}$. The line AC and LN are the lines of greatest slope of the relevant faces. Two particles P and Q of mass m_1 and m_2 are connected by a light inextensible string passing over to fixed pulleys at A and L, and passed under a smooth movable pulley of mass M. The system is released from rest. Obtain the equations sufficient to determine the tensions on the string and accelerations of the particles.



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4. A car of mass two metric ton is moving upwards a straight road of inclination $Sin^{-1}\frac{1}{10}$ to the horizontal at a steady speed of $32\ km\ h^{-1}$. If the resistance to motor is $400\ N$, find the power develop by the engine in kW.
Now the car moving in a straight horizontal road, given that the engine works in the same rate and resistance to motor is the same, find the acceleration of the car when the speed is $32\ km\ h^{-1}$.
$(g = 10\ m\ s^{-2})$

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[see page four

5. One end of a light inelastic string of length a is attached to a fixed point O and a particle of mass m is attached to the other end. The particle is held at the same level of O at a distance $\frac{a}{2}$ away from O and released. Find the speed of the particle just after the jerked of the string. In the subsequent motion when the particle is vertically below O by using conservation of energy, write an equation to determine the velocity of a particle.

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6. In the usual notation, let $x\underline{\boldsymbol{i}} + y\underline{\boldsymbol{j}}$ and $-2y\underline{\boldsymbol{i}} + 2x\underline{\boldsymbol{j}}$ are the position vectors of two points A and B respectively, with respect to a fixed origin O. Given that point C is the point on AB such that $AC: CB = 1: 2$, find position vector of a point C, if the angle between $\overrightarrow{OC}$ and the $\overrightarrow{Oy}$ is 60^0, then show that $x^2 + y^2 + 4xy = 0$.

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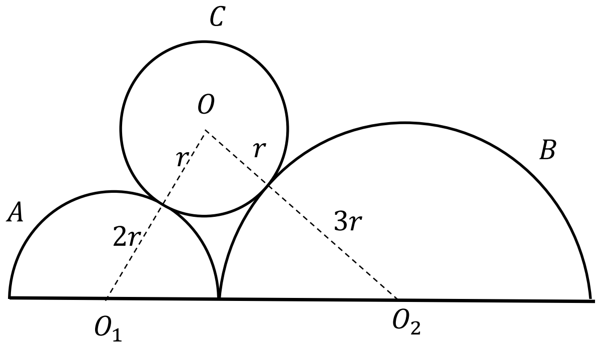
[see page five

7. The figure shows A, B two smooth solid hemispheres of radii $2r$ and $3r$ respectively fixed on a horizontal plane, where O_1, O_2 are centers of A, B respectively. Another uniform Sphere C of radius r and mass m is gently placed on curved surfaces of A, and B. Find the reactions on C which is exerted by A and B.



8. A rod AB with one end A is contact with a rough horizontal plane, and the other end B is contact with a smooth vertical wall. The vertical plane through the rod AB is perpendicular to the wall. The coefficient of friction at point of contact A is μ and the centre of gravity of the rod AB divides $2:1$.Fine $tan\theta$ in the terms of μ when the rod is in limiting equation; when θ is the inclination of the rod to horizontal.

[see page six

9. Given that the two events satisfying the following conditions.

(i) The probability of only A occurring is 0.2

(ii) The probability of only B occurring is 0.1

(iii) The probability of neither A or B occurring is 0.6

Show that $P(A/B) = \frac{1}{2}$

10. The mean of ten numbers is 9.4. The deviations of the ten numbers from the real number k are given bellow.

d_i: -5, -2, -1, -1, -1, 0, 1, 1, 2, 6

Find the median and variance of the numbers.

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Combined Mathematics II

10 E II

Part B

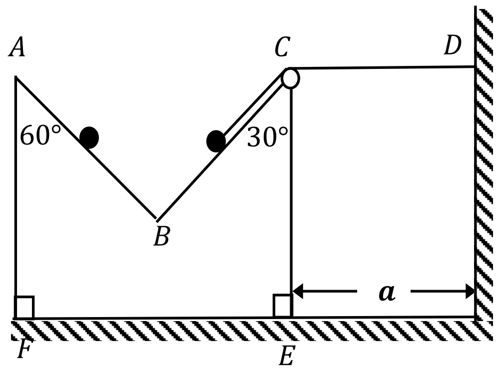
* Answer **five** questions only.

(In this question paper, g denotes the acceleration due to gravity.)

11. (a) A train P travelling in a straight path at a speed of $u\ km\ h^{-1}$ from a station A with uniform acceleration for half an hour and another one hour with a uniform retardation $f\ km\ h^{-2}$ and comes to rest at the station B. At that instant another train Q starting from rest, from the station B with same uniform acceleration of P for half an hour and acquire a maximum velocity. This maximum velocity is maintained and passing the station A. Sketch the velocity-time graph for the motions of P and Q in the same diagram. Hence find the acceleration of the train P and show that the time taken by the train Q, from B to A is $\frac{f}{f-u}$.

(b) A ship S is sailing due North with uniform speed $u\ km\ h^{-1}$ relative to earth. At a certain instant a boat A, at a distance $d\ km$ west of S and another boat B is at a distance $2d\ km$ east of S. The boat A and B travel in straight paths intending to intercept S with uniform speed $\frac{3u}{2}\ km\ h^{-1}$ and $2u\ km\ h^{-1}$ respectively, relative to earth.
By drawing velocity triangles for A and B, find which boat initially intercept the ship S.

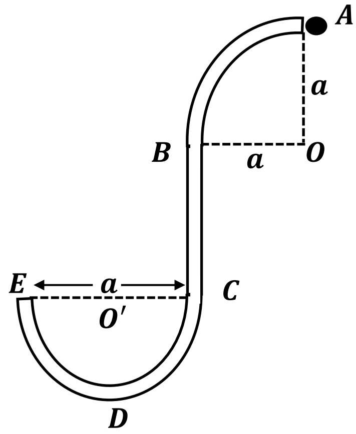
12. (a) The $ABCEF$ is a vertical cross section through the center of gravity of a uniform smooth wedge of mass m. AB and CB are the lines of greatest slope of the respective faces with $F\hat{A}B = \frac{\pi}{3}$, $B\hat{C}E = \frac{\pi}{6}$ and $A\hat{F}E = C\hat{E}F = \frac{\pi}{2}$ such that the face containing FE placed on a smooth horizontal floor and the points A, C and D are in the same horizontal level as shown in the figure. The point D is a fixed point on the vertical wall which is at a distance a from the point C of the wedge.



One end of a light inextensible string passing over a small smooth pulley fixed at C is attached to a particle of mass $2m$ kept on CB and the other end is attached to the fixed-point D on the wall. A particle of mass m is held on AB. The system is released from rest as shown in the figure. Given that the wedge will reach the wall before either particle reaches to the point B. Obtain equations sufficient to determine the time taken by the wedge strikes the wall.

[see page eight

(*b*) A thin tube $ABCDE$ is fixed in a vertical plane as shown in the figure. The portion AB is a thin smooth circular tube with Centre O and radius a and arc AB strendy an angle $\frac{\pi}{2}$ at its Centre O. The portion CD is a thin vertical tube of length a. The portion CDE is a thin semicircular tube with Centre O' and radius $\frac{a}{2}$.



A particle P of mass m is placed inside the tube at A and gently released from rest.

(i) show that the speed V of the particle p when OP makes an angle $0 \leq \theta \leq \frac{\pi}{2}$ with OA is given by $v^2 = 2ga(1 - cos\theta)$ and find the magnitude of the reaction R on the particle from the tube in terms of θ and show that when $\theta = \cos^{-1}\left(\frac{2}{3}\right)$ the direction of the line of action of reaction is changed.

(ii) find the speed of the particle at the point E and further show that magnitude of the normal reaction is 8 mg at that point.

13. On a horizontal smooth table, six points $A,\ B, C, D, E\ and\ F$ lie on straight line such that $AB = BC = CD = DE = l\ and\ EF = 2l$. The points A and F are connected by a light elastic string of length $4l$. A smooth particle P of mass m is fastened to the point D on the string. The particle is pulled along the string on table to the point B and then gently released from rest. When the particle P is at a distance x from the point at time $t\ =\ t$ along AF, write down the equation of motion of particle P for $l \leq x \leq 2$ and show that in the usual notation, that $\ddot{x} + \frac{\lambda}{2ml}(x - 4l) = 0$, where λ is the modulus of elasticity of the string.

(i) By writing $X = x - 4l$ show that $\ddot{X} + \frac{\lambda}{2ml}X = 0$ assuming that the solution of the above equation is of the form $X = \alpha cos(\omega t) + \ \beta sin(\omega t)$

Find the constants $\alpha, \beta\ and\ \omega$. Hence, show that the particle passes the point C after time $\sqrt{\frac{2lm}{\lambda}} \cos^{-1}\left(\frac{2}{3}\right)$ with velocity $\sqrt{\frac{5\lambda l}{2m}}$.

(ii) Show that the equation of motion of the particle P by choosing Y for $2l \leq x \leq 4l$ can be written as $\ddot{Y} + \frac{\lambda}{ml}Y = 0$.

Assuming that the solutions of this equation is of the form,

$Y = \alpha'\cos(\omega'(t - t_0)) + \ \beta'\sin(\omega'(t - t_0))$ find the constants $\alpha',\ \beta'$ and ω'. Where

$$t_0 = \sqrt{\frac{2lm}{\lambda}} \cos^{-1}\left(\frac{2}{3}\right)$$

(iii) Show that the total time for the particle P to reach the point D is,

$$2\sqrt{\frac{l}{m}}\ \left\{\frac{\pi}{2} - \cos^{-1}\left(\frac{2}{7}\right) + \frac{1}{\sqrt{2}}\cos^{-1}\left(\frac{2}{3}\right)\right\}.$$

[see page nine

14. (a) The position vectors of the points P and Q with respect to the point O are $\underline{p}$ *and* $\underline{q}$ respectively. The point L on OP is such that $OL: LP = 3: 4$ and the point N on OQ is such that $ON: NQ = 5: 2$. If M is the point of intersection of the straight lines PN and QL, show that $\overrightarrow{OM} = \underline{q} + \lambda(3\underline{p} - 7\underline{q})$. Here λ is a scaler. Obtain another expression for $\overrightarrow{OM}$ and find the position vector of M in terms of $\underline{p}$ *and* $\underline{q}$.

(b) A system consisting of three forces in the oxy plane act the points indicated below

Point	Position vector	Force
A	$3a\,\boldsymbol{i} + 2a\,\boldsymbol{j}$	$4P\,\boldsymbol{i} + 3P\,\boldsymbol{j}$
B	$-a\,\boldsymbol{i}$	$-P\,\boldsymbol{i} + 4P\,\boldsymbol{j}$
C	$-a\,\boldsymbol{j}$	$5P\,\boldsymbol{i} - P\,\boldsymbol{j}$

Here $\boldsymbol{i}$ *and* $\boldsymbol{j}$ denote unit vectors in the positive direction of coordinate axes ox and oy respectively and P and a are positive quantities measured in Newtons and meters respectively. Show that the system is equivalent to a single resultant force of magnitude $10P\ N$ and find direction and the equation of the line of action.
Also find the moment of the couple and its direction necessary to transfer the line of action to the line with equation $4y = 3x + 6a$

15. (a) Three uniform rods AB, BC and AC are smoothly jointed at their ends to from an equilateral triangle ABC. The rods AB and BC of equal weight W and the rod AC is of weight $2W$. The frame work ABC is freely suspended from vertex A.

Show that the inclination of AC to vertical is θ given by $tan\theta = \frac{\sqrt{3}}{4}$. Write down equations in terms of θ sufficient determine the reaction force at joint B for rod AB

(b) A frame work consists of light rods AB, BC, CD, DA *and* BD smoothly jointed at their ends as shown in the diagram. Here it is given that

$AB = BC, AD = CD, A\hat{D}B = C\hat{D}B = 30°$ *and*

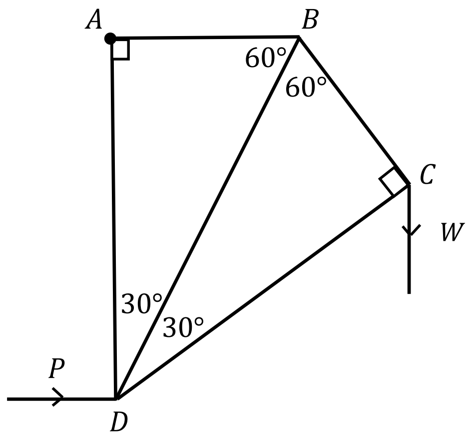
$A\hat{B}D = C\hat{B}D = 60°$

The framework is smoothly hinged at A and carries a weight of w at C. It is held equilibrium in a vertical plane, with AB horizontal and AD vertical by a horizontal force P applied at D. Draw a stress diagram, using Bow's notation, for the joints C, B, D.

Hence find

(i) The stresses in the five rods, stating whether, they are tensions or thrusts

(ii) The value of P and the reaction at A

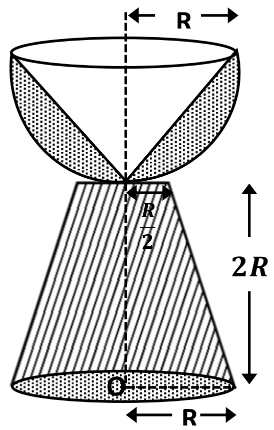


16. Show that the center of mass of

(i) a uniform solid hemisphere of radius a is at a distance $\frac{3}{8}a$ from its Centre

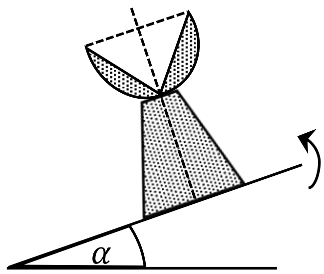
(ii) a uniform solid circular cone of height h is at a distance $\frac{1}{4}h$ from the centre of the base of the cone.

A uniform solid concreate flower Vass is made by rigidly fixing to a uniform solid concreate right circular frustum of radii of upper and lower circular bases $\frac{R}{2}$ and R respectively and height 2R is fastened at its upper circular face to the curved surface of the uniform concreate hemispherical shell, so that the two axes symmetry coincide as shown in the figure.



The hemispherical shell is made by removal of solid right circular cone of base radius R and height R from a uniform solid hemisphere of radius R. Bothe solid blocks are made are made of same material and mass per unit volume is ρ. Show that the distance from O to the Centre of mass of the flower Vass is $\frac{7R}{6}$.

The adjoining figure shows the vertical cross section of the above solid restore on a rough plane through in a line of greatest slope of the plane which is inclined to the horizontal. The inclination of the plane is increased, when the inclination of the plane with horizontal plane is α, show that if $\alpha < \tan^{-1}\left(\frac{6}{7}\right)$ and $\mu \geq \tan\alpha$, the flower Vass is in equilibrium. Where, μ is the coefficient of friction between lower face of the flower vass and the rough plane.



17. (a) Light bulbs manufactured by a company Consists of boxes A, B and C each containing three different standards of bulbs, in respective proportion $1:2:2$. Bulbs are identical in all respects except for their either defective or non-defective. The probability of bulbs of being damaged $0.00, 0.1$ and 0.2 in A, B and C respectively. A box is chosen at random and two bulbs are taken randomly and tested. Find the probability that,

(i) the bulbs are found to be defective

(ii) the box B was chosen, given that the tested bulbs to be non-defective

(b) The following table gives the class mark and corresponding frequency of a grouped frequency distribution of mark obtained by 70 students who passed the examination. In the examination the pass mark is 30

Class Mark	frequency
35	05
45	10
55	15
65	30
75	05
85	05

Using the transformation $y_i = \frac{1}{10}(x_i - 55)$, estimate the mean and the variance of the distribution of marks. The overall mean and the standard deviation of the marks of 100 students including 30 students who didn't pass are 48 and 21.5 respectively. Find mean and standard deviation of the marks of the 30 students who didn't pass.

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