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Department of Education – Western Province

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2022 (2023)

General Certificate of Education (Adv. Level) Examination, 2022 (2023)

සංයුක්ත ගණිතය I
Combined Mathematics I

10 | E | I

පැය තුනයි මිනිත්තු දහයයි.
Three hours and ten minutes.

2022.11.24 / 07:50-11:00

Index Number: ____

Name : .... Class : ....

Instruction :

- *This question Paper consists of two parts.*
- ***Part A** (Questions 1 - 10) and **Part B** (Questions 11 - 17)*
- ***Part A***
  Answer all questions. Write your answer to each question in the space provided. You may use additional sheets if more space is needed.
- ***Part B***
  Answer five questions only. Write your answers on the sheets provided.
- *At the end of the time allotted, tie the answer scripts of the two parts together so that **Part A** is on top of **Part B** and hand them over to the supervisor.*
- *You are permitted to remove only **Part B** of the question paper from the Examination Hall.*

For Examiner's Use only.

(10) Combined Mathematics II

Part	Question No.	Marks
A	01	
	02	
	03	
	04	
	05	
	06	
	07	
	08	
	09	
	10	
B	11	
	12	
	13	
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	17	
	Total	
	Percentage	

Part I	
Part II	
Total	
Final Marks	

Final Marks

In Numbers	
In Words	

Code Numbers

Marking Examiner	
Checked by: 1	
2	
Supervised by:	

Part - A

Answer **all** questions

01. Using the **Principle of Mathematical Induction**, prove that $\sum_{r=1}^{n} 2^{r-1} = 2^n - 1$ for all $n \in \mathbb{Z}^+$.

02. In a suitable cartesian plane, sketch the graph of $y = |3x-2| - |x-2|$. **Hence or otherwise,** find all real values of x satisfying the inequality $|3x+4| \le |x| + 4$.

03. Sketch, in an Argand diagram, the locus of the points that represent the complex number z satisfying $|1+(z+2)i|=1$. Hence or otherwise, show that $n-1\le|z|\le n+1$. Here n is an irrational real number to be determined. Also, show that the minimum value of $\text{Arg}(z)$ is $\pi-2\sin^{-1}\left(\frac{1}{n}\right)$.

04. Let $a\in\mathbb{R}^{+}$ It is given that the coefficient of x^{4041} in the binomial expansion of $\left(ax^2+\frac{1}{x}\right)^{2022}$ is 2022. Find the value of a. **Hence or otherwise,** find the value of b such that the sum of the coefficients of the trinomial expansion $\left(ax^2+bx+a\right)^{2022}; b\in\mathbb{Z}$ is zero.

05. Show that $\lim_{x\to 0}\left(\frac{\sqrt[3]{3x+1}-(x-1)^4}{\sin x}\right)=5$.

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06. Let S be the region enclosed by the curves $y=\frac{1}{\sqrt[4]{(3-2x)(1+2x)}}$, $y=0, 2x-3=0$ and $x=1$.

The region S is rotated about the $x-axis$ through 2π radians. Show that the volume of the solid thus generated is $\frac{\pi^2}{6}$ cubic units.

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07. Consider the curve C , parametrically given by $x=\sec\theta-\tan\theta$ and $y=\sec\theta+\tan\theta$ for $0<\theta<\frac{\pi}{2}$. Show that the gradient of the normal to the curve at the point corresponding to $\theta=\frac{\pi}{4}$ is $3-2\sqrt{2}$. Also, show that the equation of the above normal is $k^2x-y=k^3-k-2$ where $k=\sqrt{2}-1$.

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08. Let $A\equiv(3,2)$ and $B\equiv(0,-1)$. P is a point lies on the line $y=x$ .The angle subtended by the points A and B ,at point P is $\tan^{-1}\left(\frac{3}{4}\right)$. Show that, there are two possible positions for P and find them.

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09. Let where $S \equiv x^2 + y^2 + 2gx + 2fy - 1$ where $g, f \in \mathbb{Z}$. It is given that the chord of contact of tangents relative to $A(1,2)$ to the circle $S \equiv 0$ is $2x + 3y + 2 = 0$ find g and f .

Also show that the equation of the circle passes through the above two contacting points and A is given by $x^2 + y^2 - y - 3 = 0$.

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10. Sketch the graph of $y = \sin x$ and $y = \frac{x}{10}$ on the same diagram. **Hence or otherwise,** find the number of solutions of the equation $\sin x = \frac{x}{10}$ in the domain $[-3\pi, 3\pi]$.

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Department of Education – Western Province

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2022 (2023)
General Certificate of Education (Adv. Level) Examination, 2022 (2023)

සංයුක්ත ගණිතය I
Combined Mathematics I

10 E I

Part -B

Answer **five** questions only

11.a. Let $F(x) \equiv (a-b+c)x^2+(2c-b)x+c$ such that $a \neq 0, b \neq a+c$ and $a,b,c \in \mathbb{R}$. Show that the equation $F(x)=0$ for $x \neq -1$ can be expressed in the from $ay^2-by+c=0$.

Here y is a rational function of x to be determined.

Let $\frac{\alpha}{1-\alpha}$ and $\frac{\beta}{1-\beta}$ be the roots of $F(x)=0$.

If α and β be the roots of the quadratic equation $G(x)=0$, find $G(x)$.

Without solving the equation $G(x)=0$, find the constant k such that $\alpha+\beta=\frac{b}{ak}$ and $\alpha\beta=\frac{c}{ak}$.

When $a=1$, show that the quadratic equation whose roots are α^3 and β^3 is $x^2+b(3c-b^2)x+c^3=0$.

Also, when $a=1$, **deduce** that the quadratic equation whose roots are $\frac{\alpha^3}{1-\alpha^3}$ and $\frac{\beta^3}{1-\beta^3}$.

b. Let $H(x)$ is a polynomial of degree 3 . When $H(x)$ is divided by (x^2-3) and $(7x+2)$ separately, remainder is 5. $(x+1)$ is a factor of the polynomial $H(x)+15$. Find $H(x)$. Also, solve the inequality $H(x) \geq -15$.

12.a. Let $U_r=\frac{2(r+8)}{r(r+2)(r+4)}$ for $r \in \mathbb{Z}^+$. Find the values of the constants A, B and C such that $U_r=\frac{A}{r}+\frac{B}{r+2}+\frac{C}{r+4}$. **Hence,** show that $\sum_{r=1}^{n} U_r=\frac{29}{12}-\frac{2}{n+1}-\frac{2}{n+2}+\frac{1}{n+3}+\frac{1}{n+4}$ for $r \in \mathbb{Z}^+$

Deduce, that the infinite series $\sum_{r=1}^{\infty} U_r$ is convergent and find its sum. Now, Let $t_n=\sum_{r=n}^{2n} U_r$ for $r \in \mathbb{Z}^+$, deduce that $\lim_{n\to\infty} t_n=0$.

b. Find the number of different arrangements that can be made by using all the letters of the word ACCOMPANY. Also find the number of different arrangements which can be made by using 5 letters from the 9 letters of the word ACCOMPANY. How many of these arrangements, do start and end with the letter A.

13.a. Let $A=\begin{pmatrix} a & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}, B=\begin{pmatrix} 2 & 1 & 3 \\ 1 & -a & 4 \end{pmatrix}$ and $C=\begin{pmatrix} b & -2 \\ -1 & b+1 \end{pmatrix}$. Find the constants a and b such that $AB^{T}=C$, where B^{T} is the transpose of B.

Show that C^{-1} does **not** exist. Let D be a square matrix of order 2 such that $D=C+I$ where is I the identity matrix of order 2. Find the inverse matrix of D. Hence, solve the simultaneous equation $x-y=2$ and $3y-x=0$.

b. Let $z=x+iy$ and $w=z+i\sqrt{3}z$ where $x, y \in \mathbb{R}$. . State $|w|$ interms of z.

Show that $|w|^2+|\overline{w}|^2=8|z|^2$ for all $z \in \mathbb{C}$,

Using the algebraic method, show that

i. $2|\mathrm{Re}(z)||\mathrm{Im}(z)| \le |z|$

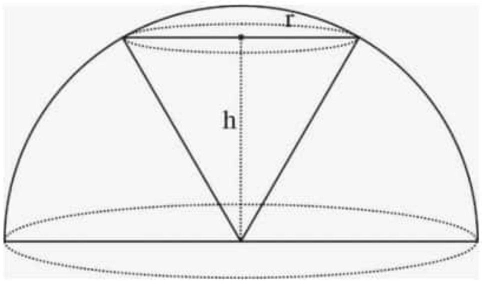
ii. $|z| \le |\mathrm{Re}(z)|+|\mathrm{Im}(z)| \le \sqrt{2}|z|$

Hence, deduce that $2 \le \dfrac{|\mathrm{Re}(z)|+|\mathrm{Im}(z)|}{|z|} \le 2\sqrt{2}$ Let n be an even number and $k \in \mathbb{Z}$ for $\theta \ne k\pi$. **Using De Moivre's theorem** show that $(\cot\theta+i)^n-(\cot\theta-i)^n=2i\,\mathrm{cosec}^n\theta.\sin n\theta.$

14.a. Let $f(x)=\dfrac{ax^2+1}{3x^3}$ where a is a real constant and $x \ne 0$. Find the value of a such the derivative of $f(x)$ is given by $f'(x)=\dfrac{x^2-1}{x^4}$ for $x \ne 0$. **Hence,** find the intervals on which $f(x)$ is increasing and decreasing. Also, find the coordinate of the turning points of $f(x)$. Let $f''(x)=\dfrac{-2x^2+4}{x^5}$ for $x \ne 0$. Find the **abscissa** (x coordinate) of the points of inflexion of the graph of $y=f(x)$. Hence, find the intervals on which $f(x)$ is concave up and concave down.

Sketch the graph of $y=f(x)$ indicating the asymptotes, the turning points and the points of inflexion.

b. The adjoining figure shows a **paperweight** is made by scooping out a solid cone with radius and height from a solid glass hemisphere with unit radius. The apex of the cone and the center of the hemisphere are coincident and the circular surfaces of the hemisphere and the cone are parallel.



Show that the volume of the paperweight V is given by $V=\frac{\pi}{3}\left(2-\sin^2\theta.\cos\theta\right)$. Here θ be the semi-vertical angle of the cone.

When $\theta=\tan^{-1}\left(\sqrt{2}\right)$, show that the weight of the paperweight is minimum. If the weight per unit volume of the glass material is ρ , show that the **minimum** weight of the paperweight is $\frac{\left(3\sqrt{3}-1\right)2\pi\rho}{9\sqrt{3}}$

15.a. Find the integers A, B and C such that $8x^5-11x^4+8x^3+3x^2+2\equiv A\left(x^2+1\right)\left(x-1\right)^2+Bx\left(4x^2+1\right)\left(x-1\right)^2+C\left(4x^2+1\right)\left(x^2+1\right)$ for all $x\in\mathbb{R}$. **Hence,** write down $\frac{8x^5-11x^4+8x^3+3x^2+2}{\left(4x^2+1\right)\left(x^2+1\right)\left(x-1\right)^2}$ in partial fractions and find

$$\int\frac{8x^5-11x^4+8x^3+3x^2+2}{\left(4x^2+1\right)\left(x^2+1\right)\left(x-1\right)^2}.dx$$

b. Using **integration by parts** or otherwise, show that $\int\sec^3x\cdot dx=\frac{1}{2}\sec x\tan x+\frac{1}{2}\ln\left|\sec x+\tan x\right|+c$ where C is a real constant. **Hence,** find $\int\sqrt{9x^2-6x+5}.dx$.

c. Let $I=\int_0^{\frac{\pi}{2}}\frac{dx}{1+\cos\alpha.\sin x}$ and $J=\int_{\frac{\pi}{2}}^{\pi}\frac{dx}{1+\cos\alpha.\sin x}$ Show that $I=J$. Using the substitution $\tan\left(\frac{x}{2}\right)=t$ or otherwise, obtain $I=\int_0^1\frac{2dt}{t^2+\left(2\cos\alpha\right)t+1}$ then evaluate I .

Establish the formula $\int_0^a f(x).dx=\int_0^a f(a-x).dx$ such that $a>0$.

Hence, deduce $\int_0^{\pi}\frac{x.dx}{1+\cos\alpha.\sin x}=\pi\left(\frac{\alpha}{\sin\alpha}\right)$.

16. In triangle OPQ, O be the origin and P and Q be the points lie on the first and second quadrants respectively such that $OP = 3OQ$.

The equations of the sides OP and OQ are $3x-4y=0$ and $4x+3y=0$ respectively.

If the line PQ passes through the point $(0,5)$, find the equation of the line PQ .

Also show that $P \equiv (12,9)$ and $Q \equiv (-3,4)$.

By finding the perpendicular distance from O to the line PQ, show that the area of the triangle OPQ is $\frac{75}{2}$ square units.

Find the equation of the circle S_1, whose end points of diameters of the circle are P and Q. Show that this circle passes through O.

Let $S_2 \equiv x^2+y^2-10x-10y+15=0$

Find the equation of the common chord of the circle $S_1 \equiv 0$ and $S_2 \equiv 0$.

The tangents drawn to the circle $S_2 \equiv 0$ at P and Q meet at R . Find the coordinates of the point R .

17.a. Write down $\sin(A+B)$ in terms of $\sin A, \sin B, \cos A$ and $\cos B$. **Hence,** obtain a similar expression for $\sin(A-B)$. Deduce that $\sin(A-B)\cdot\sin(A+B) \equiv \sin^2 A - \sin^2 B$.

Show that $\sin 20^0 \cdot \sin 40^0 \cdot \sin 60^0 \cdot \sin 80^0 = \frac{3}{16}$

b. Let $T(x) \equiv 1-\sqrt{3}+2\sin x\left(\cos x+\sqrt{3}\sin x\right)$ for $x \in \mathbb{R}$. Find the real constants for A, B and α such that $T(x) \equiv A+B\sin(2x-\alpha)$ where $0<\alpha<\frac{\pi}{2}$. Find the range of the function T then sketch the graph of $y=T(x)$ for the domain $\left[-\frac{\pi}{2},\frac{\pi}{2}\right)$.

Solve the equation $T(x)=2$.

c. In the usual notation, state the sine rule for a triangle ABC. Hence, **deduce** the cosine rule.

Let $\frac{a+b}{13}=\frac{b+c}{14}=\frac{c+a}{15}$ Find the value of k such that $\frac{\sin A}{k}=\frac{\sin B}{k-1}=\frac{\sin C}{k+1}$

Also, show that $\frac{\cos A}{17}=\frac{\cos B}{11}=\frac{\cos C}{8}$.

- **The Paper discussion of this paper will be done in the online revision program "Nestamalt Buddhi Prabodha" If anyone wishes to participate for this discussion, register by scanning the following QR code.**

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Department of Education – Western Province

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2022 (2023)

General Certificate of Education (Adv. Level) Examination, 2022 (2023)

සංයුක්ත ගණිතය II

Combined Mathematics II

10	E	II

පැය තුනයි මිනිත්තු දහයයි.

Three hours and ten minutes.

2022.11.25 / 07:50-11:00

Index Number | | | | | | | | |

Name : .... Class : ....

Instruction :

- *This question Paper consists of two parts.*
- ***Part A** (Questions 1 - 10) and **Part B** (Questions 11 - 17)*
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  Answer five questions only. Write your answers on the sheets provided.
- *At the end of the time allotted, tie the answer scripts of the two parts together so that **Part A** is on top of **Part B** and hand them over to the supervisor.*
- *You are permitted to remove only **Part B** of the question paper from the Examination Hall.*

For Examiner's Use only.

(10) Combined Mathematics II

Part	Question No.	Marks
A	01	
	02	
	03	
	04	
	05	
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	08	
	09	
	10	
B	11	
	12	
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	Total	
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Part I	
Part II	
Total	
Final Marks	

Final Marks

In Numbers	
In Words	

Code Numbers

Marking Examiner	
Checked by: 1	
2	
Supervised by:	

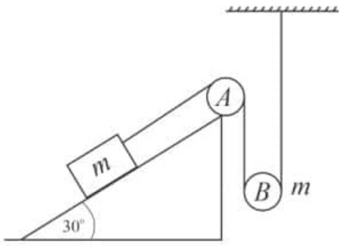
Part - A

Answer **all** questions

01. Two particles of masses M and m are moving in opposite directions on a **smooth horizontal** table, so as to collide directly with speeds u and v respectively. Just after the collision, if the velocity of mass M is **half** of its initial velocity, show that $e=\dfrac{Mu-m(u+2v)}{2m(v+u)}$, where e is the coefficient of restitution.

02. A particle is projected from a point O on the horizontal ground. The particle just clears a point where the horizontal and vertical displacements are a and b. If R is the horizontal range of the projectile on the ground, show that the angle of projection is $\tan^{-1}\left(\dfrac{bR}{a(R-a)}\right)$.

03. In the diagram, a light inextensible string attached to a particle of mass m, placed on a fixed smooth plane inclined at 30^0 to the horizontal. The string passes over a fixed small pulley at A and under a smooth moving pulley B of mass m. The other end is fixed to a ceiling as shown in the diagram.



Show that the system is in equilibrium. Now, a vertical downward impulse mu is given to B. Just after the impulse, find the velocity of the pulley B.

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04. A truck of mass $1000\,\text{kg}$ is towing a damaged car of mass $500\,\text{kg}$, along a straight road, by means of a light inextensible cable which is parallel to the direction of motion of the truck and the car.

The resistance to the motion of the two vehicles are proportional to their masses. At a certain instant, the power generated by the engine of the truck is P kW and the speed of the truck and the car is $v\ \text{ms}^{-1}$. Show that the tension of the cable at that instant is $\dfrac{1000P}{3v}$ newtons.

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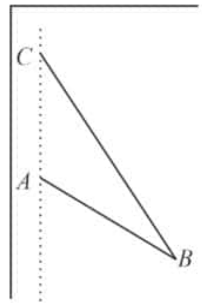
05. A uniform rod AB of length a and weight W is kept in equilibrium in a vertical plane with the end A against a smooth vertical wall and one end of a light inextensible string is attached to the B and the other end is attached to a fixed point C vertically above A such that $AC = a$ (see the figure). If the magnitude of the reaction on the rod by the wall is $\frac{W}{\sqrt{3}}$, show that the rod makes an angle with the **vertical** is $\frac{\pi}{3}$.



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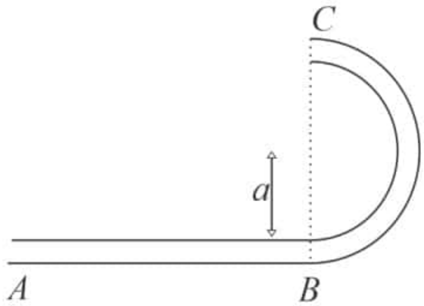
06. A narrow smooth tube ABC is bent into the form indicated in the figure below. The portion AB of the tube is straight and BC is of semicircular shape with radius a and AB is perpendicular to BC . A particle is projected from the point A with velocity u towards AB and coalesce with a similar particle at B and **coalesces** with it after the collision. If the composite particle leaves the tube at C , show that $u = 2\sqrt{5ag}$.



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07. In the usual nation, Let $\mathbf{p} = 6\mathbf{i} + 8\mathbf{j}, \mathbf{q} = 4\mathbf{i} + 3\mathbf{j}$ and $\mathbf{r} = (\alpha - 1)\mathbf{i} + (2 + \alpha)\mathbf{j}$ where $\alpha \in \mathbb{R}$. Find,

i. $|\mathbf{p}|$ and $|\mathbf{q}|$ ii. $\mathbf{p.r}$ and $\mathbf{q.r}$ interms of α .

If **q** and **r** are **perpendicular** to each other, find the value of α , then show that $\mathbf{r} = \frac{-9}{7}\mathbf{i} + \frac{12}{7}\mathbf{j}$.

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08. A uniform rod is kept under the **limiting** equilibrium inside a rough hollow hemisphere in a position by subtending a **right angle** at the center of the hemisphere by the rod. The coefficient of friction between the rod and the hemisphere is $\frac{1}{3}$. . Show that the inclination of the rod to the horizontal is $\tan^{-1}\left(\frac{3}{4}\right)$.

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09. Let A and B be two events of a sample space Ω. In the usual notation, it is given that $P(A)=\frac{p}{2}, P(B)=\frac{p}{6}$ and $P(A\cup B)-P(A\cap B)=\frac{p}{3}$ where $p>0$. Find $P(A\cap B)$ interms of p . Are those events A and B **independent**?

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10. The **median** of the following 40 observations is 30.

05-15	15-25	25-35	35-45	45-55
4	x	14	y	5

Find the value of x and y , then find the **mean** of the above distribution.

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Department of Education – Western Province

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2022 (2023)
General Certificate of Education (Adv. Level) Examination, 2022 (2023)

සංයුක්ත ගණිතය II
Combined Mathematics II

10 E II

Part -B

Answer **five** questions only

11.a. A car starting from rest moves along a straight road with an acceleration and then it moves with a deceleration and comes to rest. The maximum acceleration, maximum deceleration and maximum velocity the car can be obtained are a, f and v.

Sketch the velocity time graph for the motion of the car. By considering the parts of the motion separately,

i. If the car could be able to get the maximum velocity, show that it can travel **atleast** $v^2\dfrac{(a+f)}{2af}$ distance.

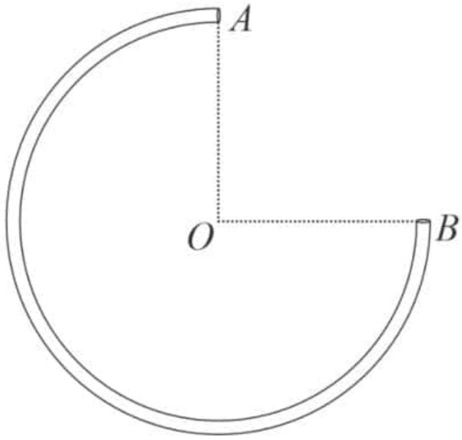
ii. If the maximum velocity acquired by the car is $\dfrac{2v}{3}$, , show that it can traval at least $\dfrac{2v^2(a+f)}{9af}$ distance during the time $\dfrac{2v(a+f)}{3af}$.

iii. If the car travels $\dfrac{2v^2(a+f)}{3af}$, show that the time taken by it is $\dfrac{7v(a+f)}{6af}$.

b. A ship is sailing due North with uniform speed v in **still** water. The caption of the ship observes a helicopter is moving $2\sqrt{d}$ distance from the ship towards North - West and returning to the ship. The velocity of the helicopter relative to the earth is $u(>v)$. The time taken by the helicopter to change the direction is negligible. Find the total time taken by the helicopter (for both journeys) to reach the ship again is $\dfrac{2d\sqrt{2u^2-v^2}}{u^2-v^2}$.

12.a. A thin smooth tube in the shape of a circular arc of radius a that subtends an angle $\frac{3\pi}{2}$ at its centre O is fixed in a vertical plane with AO vertical as shown in the figure.

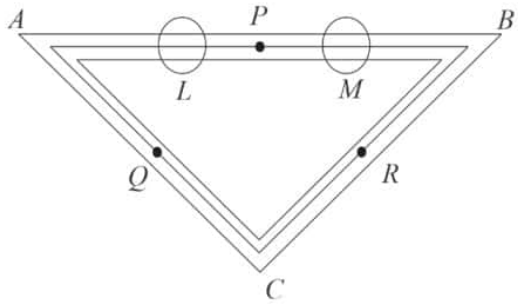
A smooth particle P of mass m is placed inside the tube at A and gently released from rest. Show that the speed of the particle P when OP makes an angle $\theta\left(0\le\theta\le\frac{3\pi}{2}\right)$ with OA is given by $v^2=2ga(1-\cos\theta)$. **Hence,** deduce the velocity of the particle P when it leaves the tube at B. At the moment it leaves the tube at B , another smooth similar particle Q of mass m with velocity $\sqrt{2ga}$ towards $\overrightarrow{BO}$ strikes P and coalesces with P . Find the horizontal and vertical velocity components of the composite particle. Also show that the locus of the point of the composite particle passes through O.



b. The diagram shows a light smooth thin tube ABC in the shape of a triangle such that $\hat{A}=\hat{B}=\alpha$ The tube stands such that horizontal and C is below AB by using two smooth rings L and M as shown in the diagram. Three particles P,Q and R of masses $m, 2m$ and m are in the three light inextensible strings PQ, QR and RP such that P,Q and R are inside $AB.AC$ and BC respectively. The system is released from rest with string taut.

Show that the acceleration of the tube is $\frac{g}{3}\frac{(1-3\cos\alpha)\sin\alpha}{(1+\cos\alpha)(5-3\cos\alpha)}$ towards $\overrightarrow{AB}$.

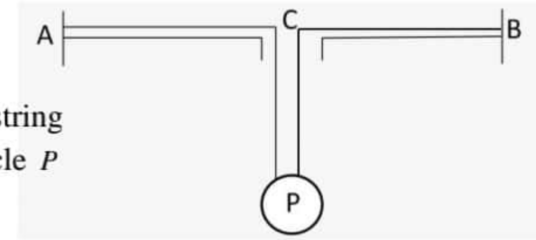
Also show that the magnitude of the acceleration of the particles relative to tube $\frac{4g}{3}\frac{\sin\alpha}{(1+\cos\alpha)(5-3\cos\alpha)}$.



When $\alpha=\frac{\pi}{4}$, show that the resultant of the reactions at L and M is $4\left(\frac{20+3\sqrt{2}}{21+3\sqrt{2}}\right)mg$.

13. A Practice of mass $2m$ is connected with two ends of two equal light elastic strings each of natural length l and modulus of elasticity $3mg$. As shown in the diagram, the other ends of the strings are attached to the two fixed points A and B on a horizontal table. The distance between A and B is $2l$. There is a thin hole at C , the midpoint of AB. The particle is hanging through C, below the table. Show that the particle hangs in equilibrium at D a distance $\frac{l}{3}$ below C and the strings are taut.

Now the particle P is placed at C and projected vertically downwards with speed $\sqrt{\frac{8gl}{3}}$.



In the subsequent motion, show that, while the string is not slack, the distance x from C to the particle P satisfies the equation $\ddot{x}+\frac{3g}{l}\left(x-\frac{l}{3}\right)=0$.

Again write the above equation in the form $\ddot{X}-\omega^2 X=0$ where $X==x-\frac{l}{3}$ and $\omega^2=\frac{3g}{l}$.

Find the amplitude of motion and the velocity of P at D.

As the particle passes the point D, the right side of the string is cut. In the subsequent motion, show that y satisfies the equation $\ddot{y}+\frac{3g}{l}\left(y-\frac{2l}{3}\right)=0$ where $y=CP$.

Let the solution of the above equation is $y=\frac{2l}{3}+A\cos\omega t+B\sin\omega t$. Evaluate the values of A, B and ω.

Hence or otherwise, show that the time taken by the particle to reach the lowest point is $\sqrt{\frac{2l}{3g}}\left\{\frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(-2\sqrt{3}\right)\right\}$

14.a. Using the **vector method** in the usual notation of a triangle ABC, show that $a^2=b^2+c^2-2bc\cos A$

Let the position vectors of A and B are $\mathbf{i}+\alpha\mathbf{j}$ and $\beta\mathbf{i}+\mathbf{j}$ where α and β are the real constants. If the point C lies on AB such that $AC:CB=2:1$, show that position vector of C is $\overrightarrow{OC}=\frac{(2\beta+1)\mathbf{i}+(\alpha+2)\mathbf{j}}{3}$.

Let $OADC$ is a parallelogram such that $\overrightarrow{AD}=3\mathbf{i}+\mathbf{j}$, show that $\alpha=1$ and $\beta=4$. If $O\hat{A}B=A$, obtain $\cos A=-\frac{1}{\sqrt{2}}$.

b. The point A, B, C and D are the vertices of a square of side a meter. E lies on the extended CD such that $CD = DE$. Forces of magnitude $P, 2P, 3P, lP, mP$ and nP

Newtons act along AB, AD, CD, AC, EA and BC respectively, in the sense indicated by the order of the letters.

If the system is in equilibrium, evaluate l, m and n .

Now the force acting along EA is replaced by a force with the same magnitude acting along DB .

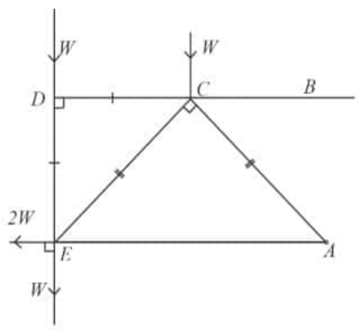
If it is needed to keep the new system in equilibrium, find the magnitude and the sense of the couple to be added to the system

15.a. Three uniform heavy rods each of length $2a$ and weight W are smoothly jointed at the ends B and C. The system is in equilibrium with AB and CD on two smooth pegs at a distance $3a$ apart on the same level. Ends A and D are below BC rod which is horizontal in a vertical plane.

If AB and CD rods inclined an angle θ with the horizontal, show that $\theta = \cos^{-1}\left[\left(\frac{3}{4}\right)^{\frac{1}{3}}\right]$

Find the reaction at B then show that it makes an angle $\tan^{-1}\left(\frac{1}{3}\cot\theta\right)$ with the **horizontal.**

b. The frame work shown in the diagram consists of six light rods AC, AE, CE, BC, CD and DE are freely jointed at the ends. The system kept in equilibrium in a vertical plane by loading with weight w at C, D and E and by a horizontal force $2w$ at E. It is given that $AC = CE, DC = DE$ and BCD is horizontal

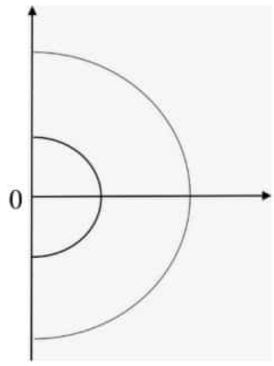


i. Draw a stress diagram, using **Bow's notation,** for the joints C, D and E . **Hence,** find the stresses in the rods, stating whether they are tensions or thrusts

ii. Also find the reactions at the hinge A and B.

16. Show that the centre of mass of a uniform hollow hemispherical shell of radius a is at a distance $\frac{a}{2}$ from its centre.

Also show that the centre of mass of a uniform solid right circular cone of height h is at a distance $\frac{h}{4}$ from the centre of the base.

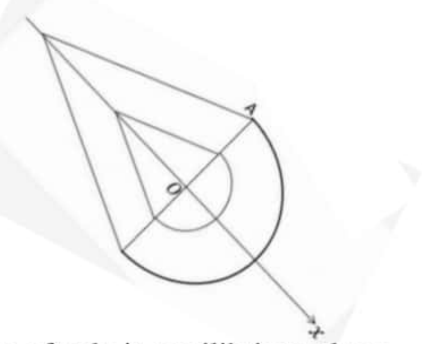


The object P is made by removing a solid hemisphere of radius a from a solid hemisphere of radius $2a$. Both have the same centre O.

Using the integration, show that the centre of mass of the object P from O on the axis is $\frac{45}{56}a$.

Another object Q is made by removing a solid cone with height $\frac{h}{2}$ and radius a from a solid cone with height h and radius $2a$.

A composite body is made by rigidly fixing the objects P and Q so that their bases coincide as shown in the adjoining figure. Show that the centre of mass of the composite body thus formed lies on its axis of symmetry at a distance $\frac{15(12a^2-h^2)}{56(4a+h)}$ from O.



The composite body is suspended from the point A and hangs freely in equilibrium, show that the inclination of the axis of symmetry with the downward vertical is $\tan^{-1}\left[\frac{112a(h+4a)}{15(h^2-12a^2)}\right]$ If the composite body can rest in equilibrium with any point of the spherical surface on a smooth horizontal plane, show that $h=2\sqrt{3}a.$

17.a. Let the set of data $\{x_1, x_2, ..., x_n\}$ has the corresponding frequencies $\{f_1, f_2, ..., f_n\}$ in a grouped Frequency distribution, Define the mean and the variance of the distribution.

Hence, deduce the mean and the variance of the set of data $\{x_1, x_2, ..., x_n\}$ in the ungrouped distribution.

Let $x=\{x_1, x_2, x_3, ...x_n\}; \bar{x}, s_x$ where $\bar{x}$ and s_x be the mean and the standard deviation of the data set. Given that the set is transformed to a set $y_i=\{y_1, y_2, ...y_n\}; \bar{y}, s_y$ by means of the linear transformation $y_i=\alpha x_i+\beta$ where $\alpha, \beta \in \mathbb{R}$. Establish the following results.

a) ( ) $\bar{y}=\alpha\bar{x}+\beta$ **b)** $s_y^2=\alpha^2 s_x^2$ **c)** $s_y=|\alpha| s_x$

Calculate the mean and the variance of all the positive integers **less than** 11 .

Hence, deduce the mean, variance and the standard deviation of the set $\{4.1, 5.2, 6.3, ..., 14\}$

b. Let P is an unbiased cubical die, $1,2,3,4,5,6$ on its six separate faces and Q is another unbiased die with 4 sides $1,2,3,4$ are on its faces.

A player start a game using two dice, P,Q and an unbiased coin.

First toss the coin, if the **Head** is appeared then the die P is tossed twice accordingly.

If the **Tail** is appeared the die Q is tosses twice accordingly.

Let

$H = \{\text{getting Head of the coin}\}$ $T = \{\text{getting Tail of the coin}\}$

$A = \{\text{getting 1 in } P\}$ $B = \{\text{getting 1 in } Q\}$

Draw a tree diagram, to represent the above information.

The winning and losing of the game as follows.

In die P

1 is appeared twice can win Rs.100

1 is appeared only once win Rs.50

and if 1 is **not appeared** alleast once, the player has to pay a fine of Rs.10.

In die Q, 1 appeared once only, can win Rs.60

1 is appeared twice, can win Rs.90

1 is **not appeared** at least once, the player has to play a certain amount.

Find the probability of

i. A player wining a certain amount of money.
ii. A player has to pay a fine.

At the end, if a player wins Rs 2022, find the fine he has to pay for the die. Q to the nearest rupee.

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