## Part A

01. Using the principle of mathematical induction, prove that $\sum_{r=1}^{n}(3r-2)=\frac{n(3n-1)}{2}$ for all $n \in Z^{+}$



02. Sketch the graph of $y=|x-3|$ and $y=5-|x|$ in the same diagram. Hence or otherwise, find all real values of $x$ satisfying the inequality $2|x|+|2x-3|>5$ .

03. Dark the region given by $|z| \leq 5$ and $Arg(z-i-1) > \frac{\pi}{3}$, Hence find $z$ such that $|z-1|$ is maximum.

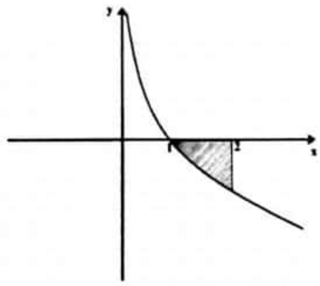


04 . If the coefficient $x^7$ in $\left(ax^2 + \frac{1}{bx}\right)^{11}$ and $x^{-7}$ in $\left(ax - \frac{1}{bx^2}\right)^{11}$ are equal. Then show that $ab = 1$

05. Find the value of $\lim_{x \to \frac{\pi}{3}} \dfrac{\sin\left(\frac{\pi}{3} - x\right)}{2\cos x - 1}$.



06. Find area bounded by $y = \log_{\frac{1}{2}} x$ and x-axis between $x = 1$ and $x = 2$.



07. If $lx + my = 1$ is normal to the parabola $y^2 = 4ax$ then show that $al^3 + 2alm^2 = m^2$.



08. A variable line, drawn through the point of intersection of the straight lines $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{b} + \frac{y}{a} = 1$, meets the coordinate axes in A & B . Show that the locus of the mid-point of AB is the curve $2xy(a+b) = ab(x+y)$

09. The straight line $2y+x-1=0$ passes through the x-axis and y-axis at the points A and B respectively. Find equation of the circle which touches the line AB at A and passes through the origin. Find the equation of the other tangent drawn to the circle from B and show that it is perpendicular to AB.



10. If $\tan^{-1}\dfrac{1}{a-1}=\tan^{-1}\dfrac{1}{x}+\tan^{-1}\dfrac{1}{a^2-x+1}$ , find the values of $x$ in terms of $a$ .

Part B

❖ *Answer only five questions.*

11. *a.* The equation $x^2 + mx + 15 = 0$ has roots $\alpha$ and $\beta$ and the equation $x^2 + hx + k = 0$ has roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

*i.* Write down the value of $k$.

*ii.* Find an expression for $h$ in terms of $m$.

*iii.* Find the two possible values of $\alpha$ such that $\beta - 2\alpha = 1$

*iv.* Hence, find the two possible values of $m$ and $h$

*b.* A curve has equation $y = x^3 + 2x^2 - 11x - m$, where $m$ is a positive integer. The curve crosses the $x-$ axis at the point $(-4,0)$

*i.* Find the value of $m$

*ii.* For that value of $m$, factorise $x^3 + 2x^2 - 11x - m$ completely. The curve also crosses the $x-$ axis at two other points.

*iii.* Find the coordinates of those points.

*c.* Let $f(x) = ax^4 + bx^3 + x^2 + cx - 14$, where $a, b, c \in R$. If $(x-1)$ and $(x-2)$ are factors of $f(x)$ and when $f(x)$ is divided by $(x+1)$, the remainder is $6$. Find the values of $a, b, c$. Write $f(x)$ as a product of linear factors and find the remainder when $f(x)$ is divided by $(3x-1)$.



12. *a.* Find the number of arrangements of the letters of the word "INDEPENDENCE". In how many of these arrangements,

(i) do the words start with P

(ii) do all the vowels always occur together

(iii) do the vowels never occur together

(iv) do the words begin with I and end in P?

*b.* If $f(r)=\{r(r+1)\}^2$, prove that $f(r)-f(r-1)=4r^3$

Hence, deduce that $\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$.

Write down the $r^{th}$ term $Ur$ of the series $\frac{3}{1^3}+\frac{5}{1^3+2^3}+\frac{7}{1^3+2^3+3^3}+\ldots$

Find $v(r)$ such that $U(r)=v(r)-v(r+1)$.

Show that $\sum_{r=1}^{n} U_r = 4-\frac{4}{(n+1)^2}$.

Show that the infinite series $\sum_{r=1}^{\infty} U_r$ is convergent.



13. *a.* Given $A=\begin{pmatrix} 5 & 2 \\ -7 & -3 \end{pmatrix}$

i. Find $A^2$ and find $B$ such that $B=3I+A-A^2$.

ii. Calculate $AB$.

iii. Deduce $A^{-1}$.

*b.* The complex number $w$ is defined by $w=\frac{22+4i}{(2-i)^2}$. Find $w$ in the form $w=x+iy$

It is given that $p$ is a real number such that $\frac{1}{4}\pi \le \arg(w+p) \le \frac{3}{4}\pi$. Find the set of possible values of $p$.

The complex conjugate of $w$ is denoted by $\bar{w}$. The complex numbers $w$ and $\bar{w}$ are represented in an Argand diagram by the points S and T respectively. Find, in the form $|z-a|=k$, the equation of the circle passing through S,T and the origin.

*c.* State the De Movier's Theorem for positive integer index

By writing $\frac{\pi}{12}=\frac{\pi}{3}-\frac{\pi}{4}$, show that

i. $\sin\left(\frac{\pi}{12}\right)=\frac{1}{4}\left(\sqrt{6}-\sqrt{2}\right)$ ii. $\cos\left(\frac{\pi}{12}\right)=\frac{1}{4}\left(\sqrt{6}+\sqrt{2}\right)$

Hence find the exact values of $z$ for which

$z^4=4\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$ Give your answers in the form $Z=a+ib$ where $a,b\in\Re^+$

14. *a.* Let $f(x)=\dfrac{x^2+1}{(x-1)^2}$ for $x\neq 1$.

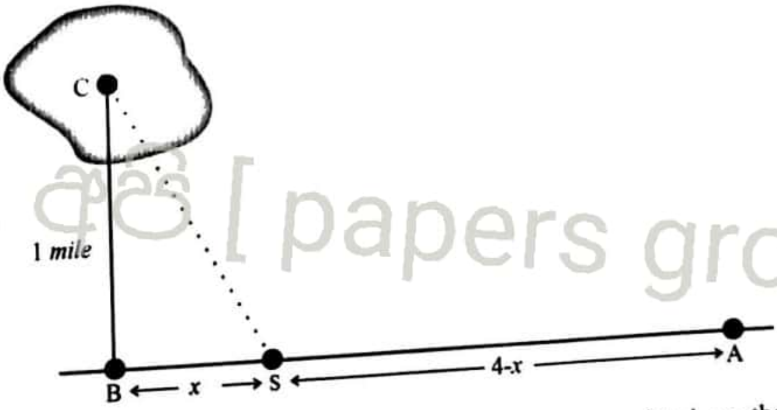
Show that $f'(x)$, the derivative of $f(x)$, is given by $f'(x)=\dfrac{-2(x+1)}{(x-1)^3}$ for $x\neq 1$.

It is given that $f''(x)=\dfrac{4(x+2)}{(x-1)^4}$ for $x\neq 1$.

Find the $x$-coordinates of the points of inflection of the graph of $y=f(x)$.

Sketch the graph of for $y=f(x)$ indicating the asymptotes, $y$-intercept and the turning points.

*b.*






A power line is to be constructed from a power station at point A on the shore to an island at point C. B is the nearest point on the shore from C. Also B is 4 miles away from A. It costs \$5000 per mile to lay the power line under water and \$3000 per mile to lay the line underground.

If a point S is located on BA, $x$ miles away from B, find $C(x)$ as the total cost for the power line from A to C across S.

Examine the sign of $\dfrac{d}{dx}C(x)$ as $x$ increases from 0 to 4 miles.

Find the most suitable position of S on BA in order to minimize cost.

15. *a.* Find the value of the constants $A, B, C$ and $D$ such that $\dfrac{2x^2+3}{(x^2-1)(x^2+4)}=\dfrac{A}{x-1}+\dfrac{B}{x+1}+\dfrac{Cx+D}{x^2+4}$. Hence integrate $\dfrac{2x^2+3}{(x^2-1)(x^2+4)}$ with respect to $x$.

*b.* Using integration by parts, Find $\int_1^3 \sqrt{x}\tan^{-1}\sqrt{x}\,dx$

c. Show that $\int_0^{\frac{\pi}{2}} f(x)\,dx = \int_0^{\frac{\pi}{2}} f\left(\frac{\pi}{2}-x\right)dx$.

Let $I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{1+\cos x+\sin x}dx$ and $J = \int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos x+\sin x}dx$ . Show that $I = J$.

Hence show that $I = \frac{\pi}{4} - \frac{1}{4}\int_0^{\frac{\pi}{2}} \frac{\sec^2\frac{x}{2}}{1+\tan\frac{x}{2}}dx$.

Using the substitution $\tan\frac{x}{2} = t$ or otherwise, show that $I = \frac{\pi}{4} - \frac{1}{2}\ln 2$.

d. Using the substitution $x = u^6$, Find $\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}dx$

16. a. Obtain the equation of the straight line that makes intercepts $a$ and $b$ on the $x$ and $y$ axes respectively.

The straight line $u \equiv 4x+3y-12=0$ meets the $x$ and $y$ axes at the points $L$ and $M$. Find the coordinates of $L$ and $M$.

A straight line perpendicular to $u=0$ meets $x$ and $y$ axes at the points $P$ and $Q$ respectively. If $MP$ and $LQ$ lines intersect at $T$, show that the locus of point $T$ is given by, $s \equiv x^2+y^2-3x-4y=0$.

Show that $LM$ is a diameter of the circle $s=0$. $O$ is the origin. If the point $P$ lies on the bisector of $L\hat{M}O$, find the coordinates of $P$ and $Q$.

Find also the circle that passes through $L, M$ and $Q$.



17. a. If $\operatorname{cosec}\theta + \cot\theta = \frac{11}{2}$, then find the value of $\tan\theta$.

b. State and prove the **sine rule** for any triangle $ABC$ in the usual notation.

Show that

$(b^2-c^2)\cot A + (c^2-a^2)\cot B + (a^2-b^2)\cot C = 0$ and

If $a^2+c^2=2b^2$ , then $\cot A + \cot C = 2\cot B$ .

c. Find the solution of the equation $\sin x + \sin\frac{\pi}{8}\sqrt{(1-\cos x)^2+\sin^2 x} = 0$.