DEVI BALIKA VIDYALAYA – COLOMBO
First Term Test – May 2023
Combined Mathematics I
Grade 13

* Answer all questions.

Part B

which have α_1 & α_2.

(11) a) Find two values of α ~~interms~~ of a and b such that the roots of the quadratic equation $\frac{\alpha}{2x} = \frac{a}{x+1} + \frac{b}{x-1}$ are equal where $a \neq b$ and $a > 0,\ b > 0$. show that $\alpha_1 \alpha_2 = (a-b)^2$.

b) If the roots of the quadratic equation $(x-p)^2 + (x-q)^2 = 2$ are α and β then
Here, p & q are real no, where $q^2 \neq \frac{2}{5}$

i) Show that $(p\alpha + q\beta) + (q\alpha + p\beta) = (p+q)^2$

ii) If $p = 2q$ in above quadratic equation then,

a) Show that zero is not a root of the quadratic equation.

b) Show that there are two values of q if the roots are real and equal in the given quadratic equation. Here $p, q \in \mathbb{R}$

iii) Find the quadratic equation in terms of p and q which the roots are $\frac{\alpha+\beta}{\alpha}$ and $\frac{\beta+\alpha}{\beta}$

c) $f(x) = x^3 + px + qr$ and $g(x) = x^3 + qx^2 + pr$. If (x-2) is a factor of both f(x) and g(x) then show that **(p/q) = (4-r) / (2-r)**

$h(x) = f(x) - g(x)$

If $h(x) = f(x) + g(x)$ then show that the remainder of h(x) when it devided by (x+1) is **~~((2-r)q / (3r-1))~~.** $\frac{6q}{r-2}$.

Using a parameter

(12) a) Express the coordinates of a point on a straight line, where the straight line is perpendicular to the line $px + qy + r = 0$ and it is passing through the point $A(\alpha, \beta)$

$px + qy + r = 0$ and $ax + by + c = 0$ are two parallel straight lines. The point $A(\alpha, \beta)$ is in between the two parallel lines. Show that $\frac{a}{b}\left(\frac{a\alpha + b\beta + c}{p\alpha + q\beta + r}\right)\left(\frac{p^2+q^2}{\underset{a^2+b^2}{\cancel{ap+bq}}}\right)^2 < 0$

b) The coordinates of the points E and G are (– 2, 5) and (–6, 3) respectively. Show that the coordinates of any point on ~~the straight line which is perpendicular~~ to EG line as bisector of the line EG as, $(t-4,\ 4-2t)$ where t is a parameter.
F and H are points where EFGH is a rhombus which named in anticlockwise directions. If the length of the side FH is $2\sqrt{5}$ then find the equations of all the other sides and diagonals of the rhombus.

(13) a) Let $f(x) = \dfrac{x^2}{(2-x)(x+1)}$ where $x \neq 2$ and $x \neq -1$

Show that the first derivative of $f(x)$ as $f'(x) = \dfrac{x(x+4)}{(2-x)^2(x+1)^2}$.

Sketch the curve of $f(x)$ by indicating the turning points and the asymptotes.

b) Two poles, one 6 meters tall and one 15 meters tall are 20 meters apart. A length of wire is attached to the top of each pole and it is staked to the ground somewhere between the two poles. Where should the wire be staked so that the minimum amount of wire is used?

(14) a) Separate into partial fractions $\dfrac{1-3x^2}{(x-2)^2(x^2+1)}$

Hence evaluate $\displaystyle\int \frac{1-3x^2}{(x-2)^2(x^2+1)}\,dx$

b) Integrates by using integration parts $\int x^2 \cos^{-1} x\,dx$.

c) If $I = \displaystyle\int_{\pi/4}^{\pi/2} \sin^2 x\,dx$ and $J = \displaystyle\int_{\pi/2}^{\pi/4} \cos^2 x\,dx$ then find $I - J$ and $I + J$ and hence find I and J separately.

d) Evaluate $\displaystyle\int_0^{\pi/3} x\,\text{cosec}^2 x\,dx$, by using the formula $\displaystyle\int_0^a f(x)dx = \int_0^a f(a-x)dx$

(15) i) If $\tan\alpha + \tan\beta = a$ and $\cot\alpha + \cot\beta = b$ and $\tan(\alpha+\beta) = c$, then find a relation in a, b and c. (B) prove that $\sqrt{3}\,\text{cosec}\,\frac{\pi}{9} - \sec\frac{\pi}{9} = 4$

ii) Rewrite the function $f(x) = 3\sin^2 x + 13\cos^2 x + 10\sin x\cos x$ in the form.
$f(x) = A\sin(2x+\alpha) + B$ where A, B and α are constants to be determined.
Hence find minimum and maximum of $f(x)$ and sketch the curve $f(x)$ in $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

iii) In a triangle ABC, $B\hat{A}C = \frac{\pi}{4}$ and $A\hat{B}C = \frac{5\pi}{12}$ then find $a + \sqrt{2}c = 2b$ interms of b where a, b and c are lengths of the triangle ABC in the usual notations.

DEVI BALIKA VIDYALAYA – COLOMBO

First Term Test – April 2023

Combined Mathematics II

Grade 13

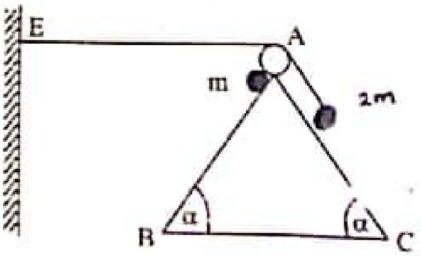
★ Answer all the questions

Part B

(11) a) Two motor cars M and N travel in the same direction. At t = 0 the car M is behind the car N with 110 meter distance. Initially the velocities of M and N are v and v + 10 respectively. The accelerations of M and N are $\left(a + \frac{1}{5}\right)$ and a respectively. After a some time the two cars M and N start to move in decelerations with magnitude a and $\left(a + \frac{1}{5}\right)$ respectively.
Draw the velocity time graph for the motion of M relative to N.
Hence, find the time spent to overtakes N by M if M can overtakes N before N stops the motion.

b) A battle ship 'A' sailing with uniform velocity u kmh^{-1} towards the south. An enemy ship B is sailing with uniform velocity $4\sqrt{2}\ \underset{a}{*}$ kmh^{-1} towards the south west.
Find the velocity of B relative to A.
At some instant B is on l km distance east of A. The enemy ship cannot escape if the distance between A and B is less than $\frac{3l}{5}$ km.
Show that the enemy ship can escape safely.
Find the time spent by B to reach the minimum risk point, from the initial point.

(12) a) The triangle ABC in the given figure represents a vertical cross-section through the centre of gravity of a uniform smooth wedge of mass M. The lines AC, AB are lines of greatest slope of the face containing α angle with BC. BC side is on a smooth horizontal table. An inelastic light string passes over a smooth small pulley at A. One end of that string connected to a particle of mass 2m which is on the AC and the other end connected to a fixed point E on a vertical plane as shown in the figure.



Another particle of mass m has kept on AB. Initially the particle m is at A and the system released from rest. Find until the particle m leaves the wedge, the acceleration of the wedge and find the distance travelled by the wedge, until the particle m leaves the wedge.

The length AE and AC are $4l$ and l meters respectively.

b) i) A simple pendulum is started so as to make complete revolutions in a vertical plane, what must be the least velocity of projection?

ii) In the subsequent motion, w_1 and w_2 are the greatest and least angular velocities and T_1 and T_2 are the greatest and least tensions. Prove that when the pendulum makes an angle θ with the vertical, the angular velocity is, $\left(w_1^2\cos^2\frac{\theta}{2}+w_2^2\sin^2\frac{\theta}{2}\right)^{1/2}$ and that the tension is, $T_1\cos^2\frac{\theta}{2}+T_2\sin^2\frac{0}{2}$. Initially the simple pendulum is at the lowest point of the circular motion.

(13) a) Two equal spheres B and C, each of mass 4m, lie at rest on a smooth horizontal table. A third sphere A of the same radius as B and C but of mass m, moves with velocity V along the line of centers of B and C. The sphere A collides with B which then collides with C. If A is brought to rest by the first collision find the coefficient of restitution between A and B. If the coefficient of restitution between B and C is e' find the velocities of B and C after the second collision interms of v and e'. If $e' = \frac{1}{2}$ show that the total loss of kinetic energy due to the two collision is $\frac{27mv^2}{64}$.

b) A particle is projected with a velocity u from the foot of an inclined plane whose inclination to the horizen is β. It strikes the plane at right angles. Show that the range of the projectile on the inclined plane is $\frac{2u^2\sin\beta}{g(1+3\sin^2\beta)}$ and the vertical height of point struck is $\frac{2u^2\sin^2\beta}{g(1+3\sin^2\beta)}$ and the time of flight is $\frac{2u}{g\sqrt{1+3\sin^2\beta}}$

BD : DC

(14) a) ABC is a triangle, where point D is on BC such that BC : CD is 2 : 1. E is on CA such that CE : EA is 3 : 1. The line DE and BA can intersect at F. The position vectors of A and B relative to C are a

any point on $\overline{BA}$ and $\overline{DE}$ relative to the point C as

and b . Express the position vectors of any point on $\overrightarrow{BA}$ and $\overrightarrow{DE}$ relative to the point C as, $\underline{r} = ta - (1-t)\mathrm{b}$ and $\underline{s} = \frac{3\lambda}{4}a - \frac{(1-\lambda)}{3}\mathrm{b}$

Where t and λ are parameters. Find t and λ and show that $\overline{CF} = \frac{(6a - \mathrm{b})}{5}$. Hence find the ratio of the line AB can be divided at F.

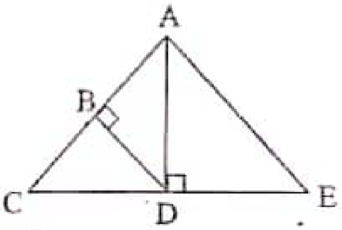
b) PQRSTU is a regular hexagon of side 'a' metres. Forces 2p, p, 3p, 4p, 5p and 2p newtons act along the sides PQ, RQ, RS, TS, TU and UP respectively. Show that the system can be reduces to a couple and a single force of magnitude P along TQ. Find the magnitude of the couple and the direction. If the system reduces to a single force then find the point that intersect PQ by the line of action of the single force.

(15) a) Five uniform heavy rods AB, BC, CD, DE and EA are smoothly jointed at their ends to forms a frame work in the shape of a pentagon ABCDE. The system hangs at the mid point of AB rod. The shape of the frame maintained by using a light rod joining C and E. The vertex D is the lowest joint of the system.

Each rod has weight w and equal length

Find the vertical and horizontal components of the reactions at the joints D and A on the rods DE and EA respectively.

b) Seven light rods AB, BC, CD, DE, AE, BD and AD rods smoothly jointed as in the bellow figure. The weights 200N, 150 N and 50 N are connected at the joints A, B and E respectively. The system is in equilibrium in the vertical plane.



CD and DA are in equal length and length of DE is half of CD.

Two verticle forces P and R are at on the joints C and D respectively. Draw the stres diagram for all the joints and find tensions and thrusts of any five rods.